



LETTERS TO THE EDITOR



MODAL SYNTHESIS METHOD FOR VIBRATION ISOLATION DESIGN OF MASSIVE ROTATING MACHINES RESILIENTLY SUPPORTED BY AN ELASTIC STRUCTURE

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(Received 7 July 1998, and in final form 21 August 1999)

1. INTRODUCTION

In industrial applications machines are usually mounted on springs so as to isolate them from the foundation. Generally, it is assumed that the vibration isolation system consists of a rigid body representing the equipment connected to an absolutely rigid foundation through an isolator which possesses resilient and energy dissipation elements, and the rigid body is constrained to move along vertical direction only [1].

For more general cases, six degrees of freedom are considered, and the six simultaneous equations of motion with numerous terms are presented by Himelblau and Rubin [2]. A more general case is studied by Yam *et al.* [3]. The vibration of a rigid body supported by springs at multiple points and along arbitrary directions can be analyzed using a three-step simple matrix calculation.

In this paper, a component synthesis method for vibration isolation design of a massive rotating machine resiliently supported by an elastic structure is presented.

This system can be considered as being composed of two parts, i.e., the machine and the supporting structure connected through resilient elements.

The model of vibration calculation for this kind of coupled system is established in this paper. Using this model, the stiffness matrix for the connecting resilient elements of any configuration and location can be obtained by a simple and clear procedure. Then, the equations of motion and the vibration characteristics of the synthetical system can be obtained.

In general, in order to reduce the vibration transmissibility, springs with lower stiffness are used as the connecting elements between the machine and the elastic supporting structure. For such a case, the natural frequencies of the coupled system

will be distinguished into low band and high band. For machines of small size, the supporting frame is comparatively stiff, and the calculation model presented in this paper becomes the equations of motion for a resiliently supported rigid body with six degrees of freedom. As the machine size becomes more and more larger, the stiffness of the frame will become relatively more and more weaker and the natural frequencies in the high band will be more and more lower. When the lowest frequency in the high band approaches the operating frequency of the rotating machine, resonance of the supporting structure will be induced. As an example, this component synthesis method is applied to the vibration isolation design of a rolling-drum-type washing machine. In this example, the natural frequencies in the low band are important to the operating behaviour under washing condition; while the operation behaviour under the dehydrating condition is affected by both the highest natural frequency in the low band and the lowest natural frequency in the high band. In other words, in the design the dehydrating frequency ought to be somewhat higher than the highest natural frequency in the low band and somewhat lower than the lowest one in the high band to avoid resonance.

2. EQUATIONS OF MOTION OF THE COUPLED SYSTEM

The coupled system is considered as being composed of two parts, i.e., the part of the massive machine which can be treated as a rigid body, and the elastic supporting structure. These two parts are connected through resilient elements.

The motion of the rigid body is referred to as an inertial co-ordinate system (O, X, Y, Z) , and another co-ordinate system $(c, \bar{X}, \bar{Y}, \bar{Z})$ is fixed on the machine body with the origin at the mass center c (see Figure 1). When the machine body is in equilibrium under the action of gravity alone, the two sets of co-ordinates are coincident. The motion of the machine body is described by the displacements of the body axes relative to the inertial axes. The translational displacements of the mass centre of the machine body are x_c, y_c and z_c in the X -, Y - and Z -directions, respectively, and the rotational displacements are characterized by the rotational angles of the body axes \bar{X}, \bar{Y} and \bar{Z} around the X -, Y - and Z -axis, respectively, i.e., α, β , and γ . These displacements are shown graphically in Figure 1. Then the small motion of an arbitrary point $b(\bar{x}_b, \bar{y}_b, \bar{z}_b)$ on the machine body will be

$$\mathbf{X}_b = \mathbf{L}_b \mathbf{X}_c, \quad (1)$$

where

$$\mathbf{X}_b = [x_b \quad y_b \quad z_b]^T, \quad \mathbf{X}_c = [x_c \quad y_c \quad z_c \quad \alpha \quad \beta \quad \gamma]^T, \quad (2)$$

$$\mathbf{L}_b = \begin{bmatrix} 1 & 0 & 0 & 0 & \bar{z}_b & -\bar{y}_b \\ 0 & 1 & 0 & -\bar{z}_b & 0 & \bar{x}_b \\ 0 & 0 & 1 & \bar{y}_b & -\bar{x}_b & 0 \end{bmatrix}. \quad (3)$$

Assume that three springs k_{bx}, k_{by} and k_{bz} with their principal elastic axes parallel to axes X, Y and Z , respectively, are connected between point b on the machine body and the elastic supporting structure, i.e., the frame. Let \mathbf{X}_{Ib} represent the

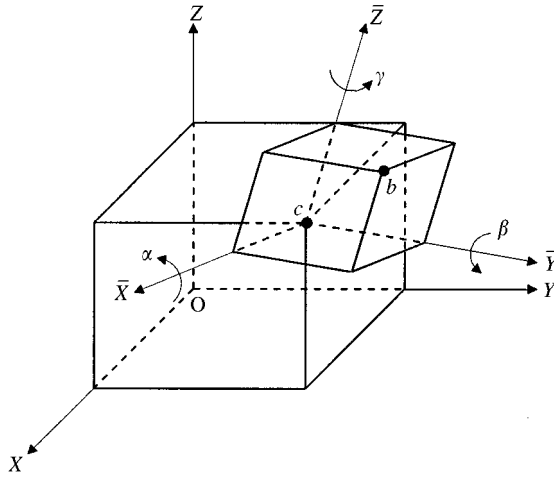


Figure 1. Inertial co-ordinate system and body fixed co-ordinate system.

displacement vector of those points on the frame connected to k_{bx} , k_{by} and k_{bz} , then

$$\mathbf{X}_{Ib} = [x_{Ib} \quad y_{Ib} \quad z_{Ib}]^T \quad (4)$$

and the force vector applied to the machine body produced by the springs connected to point b is

$$\mathbf{F}_b = -\mathbf{K}_b(\mathbf{X}_b - \mathbf{X}_{Ib}), \quad (5)$$

where

$$\mathbf{K}_b = \text{diag}[k_{bx} \quad k_{by} \quad k_{bz}]. \quad (6)$$

When \mathbf{F}_b is transformed to the mass centre of the machine body, it becomes

$$\begin{aligned} \mathbf{F}_{cb} &= \mathbf{L}_b^T \mathbf{F}_b = -\mathbf{L}_b^T \mathbf{K}_b (\mathbf{X}_b - \mathbf{X}_{Ib}) \\ &= -\mathbf{L}_b^T \mathbf{K}_b (\mathbf{L}_b \mathbf{X}_c - \mathbf{X}_{Ib}) \\ &= [f_{cx} \quad f_{cy} \quad f_{cz} \quad m_\alpha \quad m_\beta \quad m_\gamma]^T, \end{aligned} \quad (7)$$

where \mathbf{F}_{cb} is the equivalent force vector applied to the mass centre of the machine body, and it has six components, f_{cx} represents the force along X -direction, m_α is the moment about X -axis, and so forth.

The equation of motion of the machine body is

$$\mathbf{M}_c \ddot{\mathbf{X}}_c = \mathbf{F}_{cb}$$

or

$$\mathbf{M}_c \ddot{\mathbf{X}}_c + \mathbf{L}_b^T \mathbf{K}_b (\mathbf{L}_b \mathbf{X}_c - \mathbf{X}_{Ib}) = \mathbf{0}, \quad (8)$$

where \mathbf{M}_c is the inertia matrix of the machine body. Using a selection matrix \mathbf{S}_{Ib} (see Appendix A) which selects vector \mathbf{X}_{Ib} from the displacement vector \mathbf{X}_I of the frame structure, we have

$$\mathbf{X}_{Ib} = \mathbf{S}_{Ib}\mathbf{X}_I. \quad (9)$$

Then, equation (8) can be rewritten as

$$\mathbf{M}_c\ddot{\mathbf{X}}_c + \mathbf{L}_b^T\mathbf{K}_b(\mathbf{L}_b\mathbf{X}_c - \mathbf{S}_{Ib}\mathbf{X}_I) = \mathbf{0}. \quad (10)$$

The equation of motion of the frame is

$$\mathbf{M}_I\ddot{\mathbf{X}}_I + \mathbf{K}_I\mathbf{X}_I = \mathbf{F}_{Ib}, \quad (11)$$

where \mathbf{M}_I and \mathbf{K}_I are the inertia matrix and stiffness matrix of the frame respectively. \mathbf{F}_{Ib} is the force vector developed by the springs connected to the frame, and

$$\mathbf{F}_{Ib} = \mathbf{S}_{Ib}^T(-\mathbf{F}_b) = \mathbf{S}_{Ib}^T\mathbf{K}_b(\mathbf{L}_b\mathbf{X}_c - \mathbf{S}_{Ib}\mathbf{X}_I).$$

Thus, equation (11) can be rewritten as follows:

$$\mathbf{M}_I\ddot{\mathbf{X}}_I + \mathbf{K}_I\mathbf{X}_I - \mathbf{S}_{Ib}^T\mathbf{K}_b(\mathbf{L}_b\mathbf{X}_c - \mathbf{S}_{Ib}\mathbf{X}_I) = \mathbf{0},$$

i.e.,

$$\mathbf{M}_I\ddot{\mathbf{X}}_I + (\mathbf{K}_I + \mathbf{S}_{Ib}^T\mathbf{K}_b\mathbf{S}_{Ib})\mathbf{X}_I - \mathbf{S}_{Ib}^T\mathbf{K}_b\mathbf{L}_b\mathbf{X}_c = \mathbf{0}. \quad (12)$$

A combination of equations (10) and (12) yields the coupled equation of motion (13) of the integrate system, in which only one point b on the machine body is connected to the frame by springs k_{bx} , k_{by} and k_{bz} :

$$\begin{bmatrix} \mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_I \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_c \\ \ddot{\mathbf{X}}_I \end{Bmatrix} + \begin{bmatrix} \mathbf{L}_b^T\mathbf{K}_b\mathbf{L}_b & -\mathbf{L}_b^T\mathbf{K}_b\mathbf{S}_{Ib} \\ -\mathbf{S}_{Ib}^T\mathbf{K}_b\mathbf{L}_b & \mathbf{K}_I + \mathbf{S}_{Ib}^T\mathbf{K}_b\mathbf{S}_{Ib} \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{X}_I \end{Bmatrix} = \mathbf{0}. \quad (13)$$

For a multiple-connected system

$$\begin{aligned} \mathbf{F}_c &= \sum_{b=1}^n \mathbf{F}_{cb} = - \sum_{b=1}^n \mathbf{L}_b^T\mathbf{K}_b(\mathbf{L}_b\mathbf{X}_c - \mathbf{S}_{Ib}\mathbf{X}_I), \\ \mathbf{F}_I &= \sum_{b=1}^n \mathbf{F}_{Ib} = \sum_{b=1}^n \mathbf{S}_{Ib}^T(-\mathbf{F}_b) = \sum_{b=1}^n \mathbf{S}_{Ib}^T\mathbf{K}_b(\mathbf{L}_b\mathbf{X}_c - \mathbf{S}_{Ib}\mathbf{X}_I), \end{aligned} \quad (14)$$

where n is the number of connected points on the machine body.

Let

$$\mathbf{K} = \text{diag}[\mathbf{K}_1 \ \mathbf{K}_2 \ \cdots \ \mathbf{K}_n],$$

$$\mathbf{L}_p = [\mathbf{L}_1 \ \mathbf{L}_2 \ \cdots \ \mathbf{L}_n]^T,$$

$$\mathbf{S}_I = [\mathbf{S}_{I1} \ \mathbf{S}_{I2} \ \cdots \ \mathbf{S}_{In}]^T,$$

where suffixes 1, 2, ..., n mean that $b = 1, 2, \dots, n$. Then equation (14) is reduced to

$$\begin{aligned} \mathbf{F}_c &= -\mathbf{L}_p^T \mathbf{K} (\mathbf{L}_p \mathbf{X}_c - \mathbf{S}_I \mathbf{X}_I), \\ \mathbf{F}_I &= \mathbf{S}_I^T \mathbf{K} (\mathbf{L}_p \mathbf{X}_c - \mathbf{S}_I \mathbf{X}_I). \end{aligned} \quad (15)$$

Thus the equation of motion of a coupled system connected at multiple points on the machine body can be represented as follows:

$$\begin{bmatrix} \mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_I \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_c \\ \ddot{\mathbf{X}}_I \end{Bmatrix} + \begin{bmatrix} \mathbf{L}_p^T \mathbf{K} \mathbf{L}_p & -\mathbf{L}_p^T \mathbf{K} \mathbf{S}_I \\ -\mathbf{S}_I^T \mathbf{K} \mathbf{L}_p & \mathbf{K}_I + \mathbf{S}_I^T \mathbf{K} \mathbf{S}_I \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{X}_I \end{Bmatrix} = \mathbf{0}. \quad (16)$$

Using the modal co-ordinates of the frame,

$$\mathbf{X}_I = \Phi \mathbf{Q},$$

where

$\Phi = [\{\varphi_1\} \{\varphi_2\} \dots \{\varphi_m\}]$ is the mode shape matrix of the frame, and

$\mathbf{Q} = [q_1 \ q_2 \ \dots \ q_m]^T$ is the modal co-ordinate vector, then

$$\begin{Bmatrix} \mathbf{X}_c \\ \mathbf{X}_I \end{Bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Phi \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{Q} \end{Bmatrix} = [\mathbf{T}] \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{Q} \end{Bmatrix},$$

where

$$[\mathbf{T}] = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \Phi \end{bmatrix}.$$

Substituting these equations into equation (16) and premultiplying the results by $[\mathbf{T}]^T$, Equation (16) becomes

$$\begin{bmatrix} \mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_r \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_c \\ \ddot{\mathbf{Q}} \end{Bmatrix} + \begin{bmatrix} \mathbf{L}_p^T \mathbf{K} \mathbf{L}_p & -\mathbf{L}_p^T \mathbf{K} \mathbf{S}_I \Phi \\ -\Phi^T \mathbf{S}_I^T \mathbf{K} \mathbf{L}_p & \mathbf{K}_r + \Phi^T \mathbf{S}_I^T \mathbf{K} \mathbf{S}_I \Phi \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{Q} \end{Bmatrix} = \mathbf{0}, \quad (17)$$

where

$$\mathbf{M}_r = \Phi^T \mathbf{M}_I \Phi, \quad \mathbf{K}_r = \Phi^T \mathbf{K}_I \Phi.$$

3. MORE GENERAL CASES

In general, the principal elastic axes of the connecting springs may not coincide with the co-ordinate axes. In this case, \mathbf{K}_b is replaced by $\tilde{\mathbf{K}}_b$, and

$$\tilde{\mathbf{K}}_b = A_b^T \mathbf{K}_{\lambda b} A_b, \quad (18)$$

where

$$\mathbf{K}_{\lambda b} = \text{diag} [k_{bp} \quad k_{bq} \quad k_{br}], \quad (19)$$

$$A_b = \begin{bmatrix} \lambda_{xp} & \lambda_{yp} & \lambda_{zp} \\ \lambda_{xq} & \lambda_{yq} & \lambda_{zq} \\ \lambda_{xr} & \lambda_{yr} & \lambda_{zr} \end{bmatrix}, \quad (20)$$

where k_{bp} is the stiffness of a spring with principal elastic axis along the p -direction and connected with the machine body at point b , and so forth; λ_{ij} is the cosine of the angle between the elastic principal axis j of the spring connected at b and the co-ordinate axis i . For example, λ_{xp} is the cosine of the angle between the X -axis and the elastic principal axis of k_{bp} (see Appendix B). Using equation (18), $\mathbf{K}_{\lambda b}$ can be transformed into the inertial co-ordinate system (O, X, Y, Z) and becomes $\tilde{\mathbf{K}}_b$ [3]. The details are shown in the Appendix. The equation of motion of the coupled system for the above-mentioned more general case can be obtained through a similar procedure of derivation as

$$\begin{bmatrix} \mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_I \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_c \\ \ddot{\mathbf{X}}_I \end{Bmatrix} + \begin{bmatrix} \mathbf{L}_p^T \tilde{\mathbf{K}} \mathbf{L}_p & -\mathbf{L}_p^T \tilde{\mathbf{K}} \mathbf{S}_I \\ -\mathbf{S}_I^T \tilde{\mathbf{K}} \mathbf{L}_p & \mathbf{K}_I + \mathbf{S}_I^T \tilde{\mathbf{K}} \mathbf{S}_I \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{X}_I \end{Bmatrix} = \mathbf{0}, \quad (21)$$

where

$$\tilde{\mathbf{K}} = \text{diag} [\tilde{\mathbf{K}}_1 \quad \tilde{\mathbf{K}}_2 \quad \cdots \quad \tilde{\mathbf{K}}_n] \quad (22)$$

and the suffixes 1, 2, ..., n indicate that $b = 1, 2, \dots, n$.

Using modal co-ordinates for the frame, equation (21) is transformed into

$$\begin{bmatrix} \mathbf{M}_c & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_r \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{X}}_c \\ \ddot{\mathbf{Q}} \end{Bmatrix} + \begin{bmatrix} \mathbf{L}_p^T \tilde{\mathbf{K}} \mathbf{L}_p & -\mathbf{L}_p^T \tilde{\mathbf{K}} \mathbf{S}_I \Phi \\ -\Phi^T \mathbf{S}_I^T \tilde{\mathbf{K}} \mathbf{L}_p & \mathbf{K}_r + \Phi^T \mathbf{S}_I^T \tilde{\mathbf{K}} \mathbf{S}_I \Phi \end{bmatrix} \begin{Bmatrix} \mathbf{X}_c \\ \mathbf{Q} \end{Bmatrix} = \mathbf{0}. \quad (23)$$

In equations (19) and (20), only three springs connected to point b on the machine body are considered. The case for more than three springs can be treated in a similar way. In these cases, there will be more rows in $\mathbf{K}_{\lambda b}$ and A_b , and the calculation of the stiffness matrix in equations (16) and (17) or equations (21) and (23) will be slightly more complex, but the number of rows in \mathbf{K} or $\tilde{\mathbf{K}}$ will remain unchanged.

The natural frequency matrix and mode shape matrix of the coupled system obtained from equations (16) or (21) are notated as follows:

$$\Omega = \text{diag} [\Omega_1 \quad \Omega_2 \quad \cdots \quad \Omega_N], \quad (24)$$

$$\Psi = \begin{bmatrix} \Psi_c \\ \Psi_I \end{bmatrix} = [\{\psi_1\} \quad \{\psi_2\} \quad \cdots \quad \{\psi_N\}], \quad (25)$$

where Ψ_c is the mode shape matrix of the machine body, and Ψ_I is that of the elastic supporting structure. If equation (25) is the solution of equations (17) or (23), it is

necessary to carry out the following transformation to convert the frame mode \mathbf{U}_I back to the inertial co-ordinate system, and

$$\mathbf{U}_I = \Phi \Psi_I. \quad (26)$$

Thus, the mode shape matrix of the integrate system is

$$\mathbf{U} = \begin{bmatrix} \Psi_c \\ \mathbf{U}_I \end{bmatrix} = \begin{bmatrix} \Psi_c \\ \Phi \Psi_I \end{bmatrix}. \quad (27)$$

4. FREQUENCY CALCULATION OF A WASHING MACHINE

Due to the special functional requirement of a washing machine, it is desired to make the natural frequencies of the system fall into two frequency bands, in order to realize vibration isolation in both the washing and dehydrate stages.

The dynamic characteristics of a rolling-drum-type washing machine with 20 kg capacity are calculated using the method introduced in this paper. The drum body was treated as a rigid body, which is suspended in a frame by four springs (see Figure 2). In order to reduce the vibration level, soft springs are used as the suspension elements while the frame is made relatively stiff. In this case, the distribution of nature frequencies of the integrated machine and suspension system can be divided into low band and high band. In general, the washing frequency is designed to be lower than the lowest frequency among the low-band natural frequencies of the integrated machine system. The designed dehydrate frequency will be between the low band and high band. It will be higher than the highest one in the low band and will be lower than the lowest one in the high band. Modal analysis of the frame is carried out using FEM software developed by ourselves. The reliability of this software has been checked using the general-purpose software. The discretization of the frame in the FEM calculation is shown in

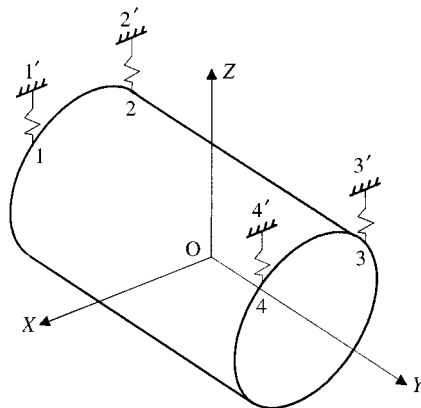


Figure 2. Machine body of the washing machine.

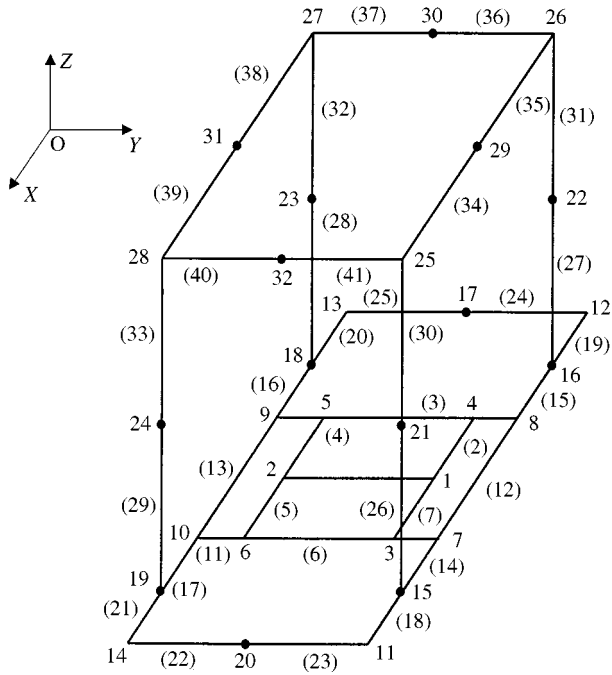


Figure 3. Frame structure of the washing machine.

TABLE 1

Natural frequencies of the frame

No.	1	2	3	4	5	6	7
Freq. (Hz)	40.50	53.54	54.29	109.50	118.62	118.62	132.78

TABLE 2

Natural frequencies of the coupled system

No.	1	2	3	4	5	6	7	8	9	10	11
Freq. (Hz)	2.86	3.74	4.17	5.24	5.61	9.80	44.68	53.55	54.30	110.45	118.65

Figure 3. The frame is made of structural steel with Young's modulus $E = 200$ GPa, the Poisson ratio $\nu = 0.3$ and density $\rho = 7.8 \times 10^3$ kg/m³.

The natural frequencies and mode shapes of the whole system were obtained by solving the eigenvalue problem of equation (17). As mentioned above, the frequencies of the integrated system are distributed over the low band and high band. Tables 1 and 2 show the natural frequencies of the frame and the coupled

TABLE 3

Natural frequencies obtained from calculations when the frame is treated as an absolutely rigid foundation and the experimental results

No.		1	2	3	4	5	6
Cal.	Freq. (Hz)	2.86	3.75	4.27	5.51	5.79	9.88
Exp.	Freq. (Hz)	2.46	3.89	5.02	5.85	6.38	8.91

system, respectively. It is found that the frequencies in the high band of Table 2 are very close to those listed in Table 1.

Experiment measurements were carried out using B & K 4321 three-dimensional and B & K 4370 one-dimensional accelerometers, a B & K 2635 charge amplifier and a HP5423 signal analysis system. Table 3 shows the calculated frequencies of the coupled system when the frame is treated as an absolutely rigid foundation and the experimental results of the frequencies in the low band. It can be seen that they are very close to each other. In general, if the lowest natural frequency in the high band and the highest one in the low band are in the ratio of 5 to 1 or even higher, the modal frequencies in the low band and the high band can be calculated separately. For the low band, the frame can be treated as a rigid foundation and equations (17) and (23) are reduced to

$$\mathbf{M}_c \ddot{\mathbf{X}}_c + \mathbf{L}_p^T \mathbf{K} \mathbf{L}_p \mathbf{X}_c = \mathbf{0} \quad (28)$$

and

$$\mathbf{M}_c \ddot{\mathbf{X}}_c + \mathbf{L}_p^T \tilde{\mathbf{K}} \mathbf{L}_p \mathbf{X}_c = \mathbf{0} \quad (29)$$

respectively. For the high band, only the frame is taken into consideration in the calculation.

The vibrations of the whole system were measured for both washing and dehydrating operations. Good vibration isolation effects were achieved. The maximum vibration amplitude can meet the requirement of quality control of the washing machine.

5. CONCLUSIONS

The modal synthesis method for vibration isolation design of a rolling-drum-type washing machine with two critical frequencies, i.e., the washing frequency and the dehydrating frequency, is discussed in this paper. The machine is treated as a rigid body, which is suspended in an elastic frame by springs. The motion of the machine is described by three translational displacements of the mass centre of the machine, and three rotational displacements about the inertial axes. Modal characteristics of the elastic frame are calculated using the finite element method. The modal synthesis method is adopted to derive the general equations of motion

of the coupled system. These equations are general, because they can deal with any configuration and location of the suspended connecting springs.

Moreover, a relatively simple procedure for obtaining the stiffness matrix of the coupled system is given; the support of the system can be either solid or elastic. The number of equations is much lower than that of existing methods. This proposed approach has a clear concept and is easy to use by engineers. The readers may feel somewhat confused about this method at first; however, they will find its merit through a little more practice. The numerical calculation for the isolation design of a rolling-drum-type washing machine is conducted; the results show good agreement with those from experiments. The method proposed in this paper is also suitable for vibration isolation design of various rotating machinery.

ACKNOWLEDGMENTS

This study is funded by both The Research Committee of The Hong Kong Polytechnic University and The Beijing Service Machinery Research Institute. The authors greatly appreciate the support of these two institutions. Thanks are also due to Mr. D. P. Yan and Mr. Y. Y. Zhang for their assistance in the calculation and experiments.

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APPENDIX A: SELECTION MATRIX S_{Ib}

There are two co-ordinate vectors:

$$\mathbf{X}_b = [x_b, y_b, z_b]^T \text{ related to point b on the rigid body,}$$

$$\mathbf{X}_I = [x_{I1}, x_{I2}, \dots, x_{IN}]^T \text{ related to system } I, \text{ i.e., the frame.}$$

Assume that x_b , y_b and z_b are connected to x_{I1} , x_{I3} and x_{I5} through springs k_{bx} , k_{by} and k_{bz} , respectively, then the force vector acting on system I will be

$$\mathbf{F}_b = \begin{Bmatrix} F_{bx} \\ F_{by} \\ F_{bz} \end{Bmatrix} = - \begin{Bmatrix} k_{bx} & & \\ & k_{by} & \\ & & k_{bz} \end{Bmatrix} \left(\begin{Bmatrix} x_b \\ y_b \\ z_b \end{Bmatrix} - \begin{Bmatrix} x_{I1} \\ x_{I3} \\ x_{I5} \end{Bmatrix} \right). \quad (\text{A1})$$

Let

$$\mathbf{S}_{Ib} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \end{bmatrix}_{3 \times N}. \quad (\text{A2})$$

Then

$$[x_{I1}, x_{I3}, x_{I5}]^T = \mathbf{S}_{Ib} [x_{I1}, x_{I2}, \dots, x_{IN}]^T$$

or

$$\mathbf{X}_{Ib} = \mathbf{S}_{Ib} \mathbf{X}_I,$$

where \mathbf{S}_{Ib} is called the selection matrix. Then we have

$$\mathbf{F}_b = -\mathbf{K}_b (\mathbf{X}_b - \mathbf{X}_{Ib}) = -\mathbf{K}_b (\mathbf{X}_b - \mathbf{S}_{Ib} \mathbf{X}_I)$$

and

$$\mathbf{F}_{Ib} = \begin{Bmatrix} 0 \\ -F_{bx} \\ 0 \\ -F_{by} \\ 0 \\ -F_{bz} \end{Bmatrix} = \mathbf{S}_{Ib}^T \mathbf{F}_b.$$

APPENDIX B: STIFFNESS MATRIX FOR GENERAL CASE

In a general case, the connecting springs can be along any direction. Assume that there are three springs connected at point b on the machine body; they are along bp , bq , and br directions, respectively, and their principal stiffness are k_{bp} , k_{bq} and k_{br} , respectively. Take k_{bp} as an example. Assume that the angles between its principal stiffness direction and the three reference axes X , Y and Z are θ_{xp} , θ_{yp} and θ_{zp} , respectively, as shown in Figure 4(a). If point b has a negative displacement, say, $-\Delta y$ along the y -direction, it is equivalent to a positive displacement Δy at point p along the y -direction (see Figure 4(b)). For small deformation, the elongation of the spring due to this displacement is $\Delta p = \Delta y \cos \theta_{yp}$. When point p has displacement components Δx , Δy and Δz along the X -, Y - and Z -direction, respectively, the elongation of spring bp should be (see Figure 4(b)),

$$\Delta p = \Delta x \cos \theta_{xp} + \Delta y \cos \theta_{yp} + \Delta z \cos \theta_{zp} = \Delta x \lambda_{xp} + \Delta y \lambda_{yp} + \Delta z \lambda_{zp}, \quad (\text{B1})$$

where $\lambda_{xp} = \cos \theta_{xp}$, $\lambda_{yp} = \cos \theta_{yp}$, $\lambda_{zp} = \cos \theta_{zp}$.

The expressions for Δq and Δr can be obtained similarly; then, using equation (20), we have

$$\begin{Bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{Bmatrix} = \begin{Bmatrix} \lambda_{xp} & \lambda_{yp} & \lambda_{zp} \\ \lambda_{xq} & \lambda_{yq} & \lambda_{zq} \\ \lambda_{xr} & \lambda_{yr} & \lambda_{zr} \end{Bmatrix} \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} = [\lambda_b] \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix}. \quad (\text{B2})$$

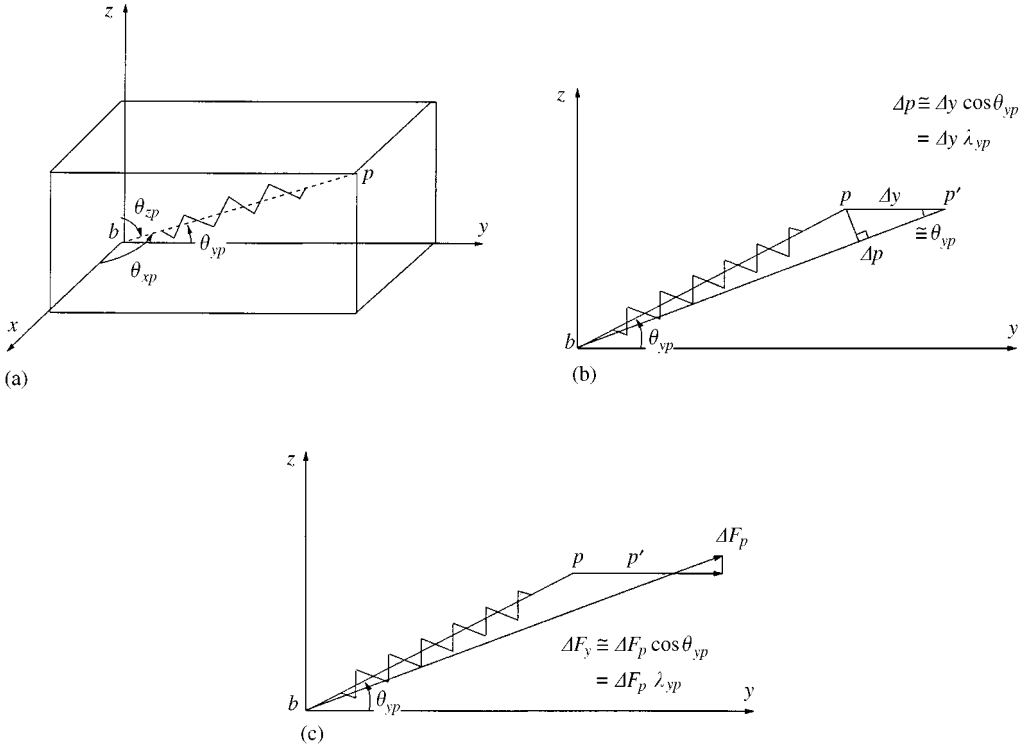


Figure 4. Connecting spring along bp direction at point b .

The tensile force due to the elongation of the springs is (see Figure 4(c))

$$\begin{Bmatrix} \Delta F_p \\ \Delta F_q \\ \Delta F_r \end{Bmatrix} = [\mathbf{K}_{\lambda b}] \begin{Bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{Bmatrix}, \tag{B3}$$

where

$$[\mathbf{K}_{\lambda b}] = \text{diag} [k_{bp} \quad k_{bq} \quad k_{br}]. \tag{19}$$

By resolving these force into the X -, Y - and Z -directions (see Figure 4(c)), the following can be obtained:

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_z \end{Bmatrix} = [\lambda_b]^T \begin{Bmatrix} \Delta F_p \\ \Delta F_q \\ \Delta F_r \end{Bmatrix}. \tag{B4}$$

Substituting equation (B2) into equation (B3) and then into equation (B5), we have

$$\begin{Bmatrix} \Delta F_x \\ \Delta F_y \\ \Delta F_z \end{Bmatrix} = [\lambda_b]^T [\mathbf{K}_{\lambda b}] [\lambda_b] \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix} = [\tilde{\mathbf{K}}_b] \begin{Bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{Bmatrix}, \quad (\text{B5})$$

where

$$[\tilde{\mathbf{K}}_b] = [\lambda_b]^T [\mathbf{K}_{\lambda b}] [\lambda_b]. \quad (\text{18})$$

APPENDIX C: NOMENCLATURE

(O, X, Y, Z)	inertial reference co-ordinate system
$(c, \bar{X}, \bar{Y}, \bar{Z})$	co-ordinate system fixed on the massive machine body
$b(\bar{x}_b, \bar{y}_b, \bar{z}_b)$	an arbitrary point b on the machine body with position co-ordinates $(\bar{x}_b, \bar{y}_b, \bar{z}_b)$
\mathbf{F}_b	force vector applied at point b
\mathbf{F}_{cb}	force vector applied at the mass centre of the machine body
\mathbf{K}_b	stiffness matrix of the resilient elements connected to point b
\mathbf{K}_I	stiffness matrix of the supporting elastic structure
\mathbf{L}_b	matrix of co-ordinate transformation
\mathbf{S}_{Ib}	selection matrix which selects vector \mathbf{X}_{Ib} from the displacement vector \mathbf{X}_I
\mathbf{M}_c	inertia matrix of the machine body
\mathbf{M}_I	inertia matrix of the supporting elastic structure
\mathbf{X}_b	displacement vector of point b
\mathbf{X}_c	displacement vector of the mass centre of the machine body
\mathbf{X}_I	displacement vector of the supporting elastic structure
\mathbf{X}_{Ib}	displacement vector of the points on the elastic structure connected to point b through resilient elements