Journal of Sound and Vibration (2000) **231**(3), 519–536
doi:10.1006/jsvi.1999.2542, available online at http://www.idealibrary.com on IDEAL®

A REVIEW OF THE MODELLING OF WHEEL/RAIL NOISE GENERATION

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(*Received in final form* 23 *September* 1999)

Mechanisms associated with the interaction of the wheel and the rail dominate the noise production of railway operations at conventional speeds and remain significant even for high-speed trains. This wheel/rail noise may be divided into three main categories. Rolling noise occurs on straight track and is predominantly caused by undulations of the wheel and rail surfaces which induce a vertical relative vibration. Impact noise can be considered as an extreme form of rolling noise occurring at discontinuities of the wheel or rail surface. The excitation is again vertical, but non-linearities play a greater role. Squeal noise, occurring on sharp radius curves, is usually induced by a lateral excitation mechanism. A review of theoretical models that have been developed to predict these phenomena is given.

1. INTRODUCTION

Railways are usually seen as an environmentally friendly option for transport. This has recently led to considerable interest in the expansion of their role in the movement of freight, in long-distance high-speed passenger travel, and also to solve congestion in densely populated areas, in the form of light rail and tramway systems. Railways are therefore entering a new era of higher speeds and higher capacities both for intercity and urban systems and are set to play their part in reducing the environmental burden caused by the steady growth in road transport. Unfortunately, the environmental effects of noise and vibration can work against this promotion of rail. The prospect of new railway construction has led to resistance from residents, partly based on noise and vibration issues, which promoters are required to take into account. There are many railway projects in Europe affected in this way. Moreover, in recent years, in response to growing public concerns, a number of countries have added railway noise to the issues covered by noise regulations. Without reducing the noise from individual trains, a reduction in the permissible level can imply a significant restriction to the train service or speed. Noise barriers, seen by many as a routine solution to excess noise from roads, are already widely used in some countries along the railway lines. However, these have the disadvantage that, to be effective, they also have to be visually intrusive for the lineside residents as well as the passengers, and they are

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also expensive. Moreover, the acoustic effect of such barriers is limited and may be insufficient in some European countries to achieve compliance with new national noise legislation.

There is therefore a growing awareness in the railway community that methods of reducing noise at source are needed. Since the rolling stock and track have an expected life in excess of 30 years, solutions are required which can be applied to existing vehicles and infrastructure as well as to new systems. For effective solutions to be developed, there is a requirement for increased fundamental understanding of how the noise is generated. In fact, since the early 1970s, work has been underway to develop theoretical models for railway noise, to validate these models against full-scale running tests and to use the models to aid the design of quieter trains. Even before the first railway noise workshop in 1976 [1], Remington *et al.* published their first models of wheel/rail noise [2–6].

The main source of noise from railway operations on open line is rolling noise, generated by unevennesses of the wheel/rail running surfaces. However, impact noise generated by the wheel running over discontinuities at rail joints, dipped welds or points and crossings is also an important source of noise, particularly in built-up areas close to stations and yards. A third type of wheel/rail noise is squealing noise generated in sharp curves. This paper reviews recent developments in the theoretical modelling of each of these forms wheel/rail noise. The reader is also referred to review papers from previous railway noise workshops [7–11].

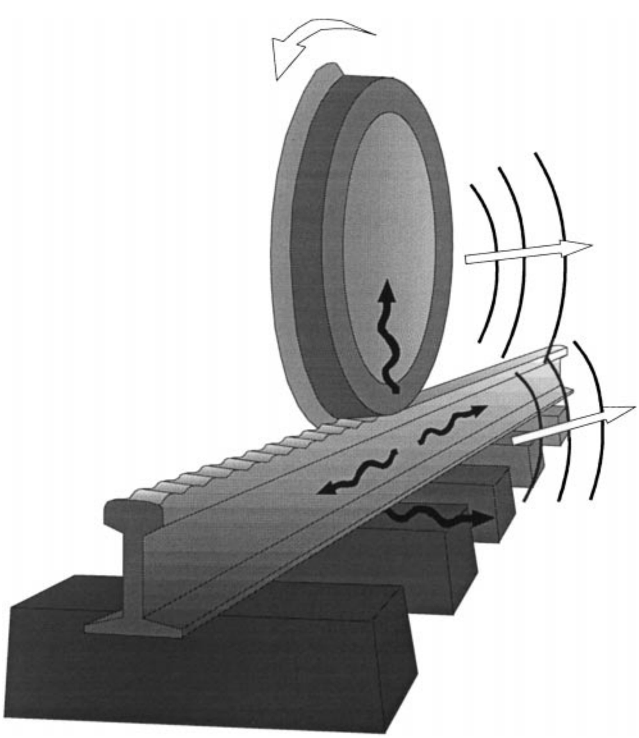

Figure 1. Schematic diagram of how rolling noise is generated by the wheel/rail interaction and radiated by the wheel, the rail and the sleepers.

2. ROLLING NOISE

2.1. THE TWINS MODEL

The first theoretical model of rolling noise, produced by Remington [2, 5, 12, 13], was based on the premise that irregularities of the wheel and rail running surfaces ("roughness") cause the wheel and rail to vibrate relative to one another; these vibrations are transmitted through the structures and thereby radiate noise. This model was developed further and extended by Thompson [14–19]. Subsequent research funded by the European Rail Research Institute (ERRI) resulted in the implementation of the prediction model in a computer program, TWINS (Track-Wheel Interaction Noise Software) [20]. Extensive full-scale validation experiments [21] have shown that this model is capable of predicting the noise, from a range of typical wheel and track designs, to within about 2 dB. The variations within individual 1/3 octave bands is greater. Figure 1 depicts the basis of these theoretical models and Figure 2 shows the TWINS prediction model schematically. The various parts of this modelling scheme are discussed further in the following sections. A similar, although more simplified model, known as RIM, is presented in reference [22].

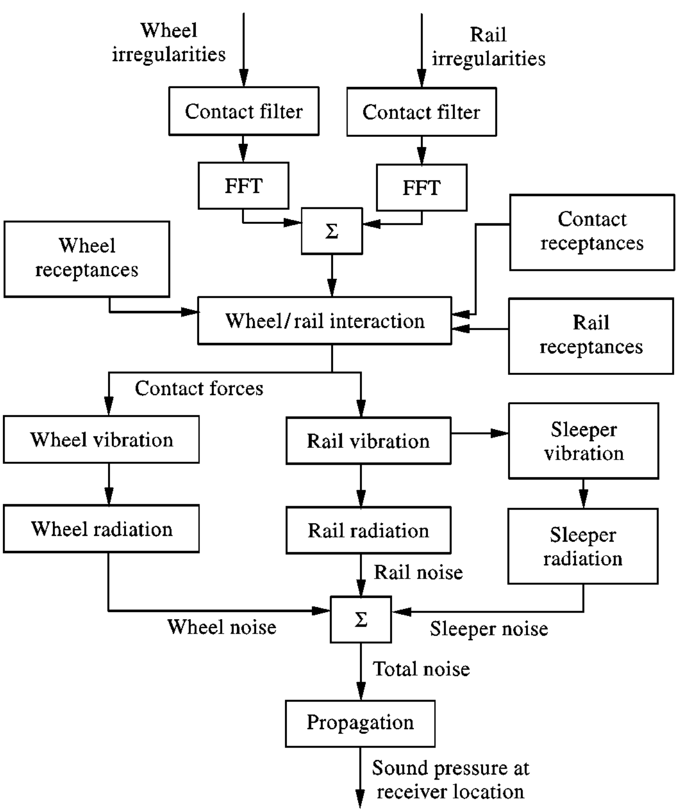

Figure 2. Flow diagram of the TWINS calculation model.

2.2. EXCITATION

The dominant excitation of rolling noise is now widely agreed to be due to surface unevennesses and irregularities, usually referred to as "roughness". The wavelengths responsible are typically 5–200 mm with amplitudes from below 1 up to 50 μm. These induce a relative vertical motion between the wheel and rail [5, 15]. This is demonstrated qualitatively by the differences between the noise produced by wheels equipped with cast-iron block brakes, which have been found to exhibit corrugation, and those with disc brakes, which are smooth, and also between smooth and corrugated track [7]. Moreover, it is demonstrated quantitatively by the good agreement found in validation experiments of models based on roughness input [13, 21]. Figure 3 compares measured and predicted overall noise levels for 25 different cases (wheel, track and speed), from reference [21], and Figure 4 shows the average spectral differences between predicted and measured results.

The results in Figure 4 are slightly different to those in reference [21] due to a modification in the rail response calculation implemented since the publication of reference [21]. The consistent over-prediction by about 2 dB in Figure 3 is principally due to inadequacies in the sound propagation model which lead to an over-emphasis of some frequency bands, e.g., around 500 Hz, and under-emphasis of others, e.g., below 250 Hz. In reference [23], various published roughness and noise data are compared and found to be broadly consistent with a linear relationship. Nevertheless, the correct quantification of the roughness that acts at the wheel/rail interface presents problems. Uncertainties in the roughness input were identified as the most likely cause of the large standard deviations in the comparisons shown in Figure 4; if there are other causes these are masked by the large uncertainties in the input.

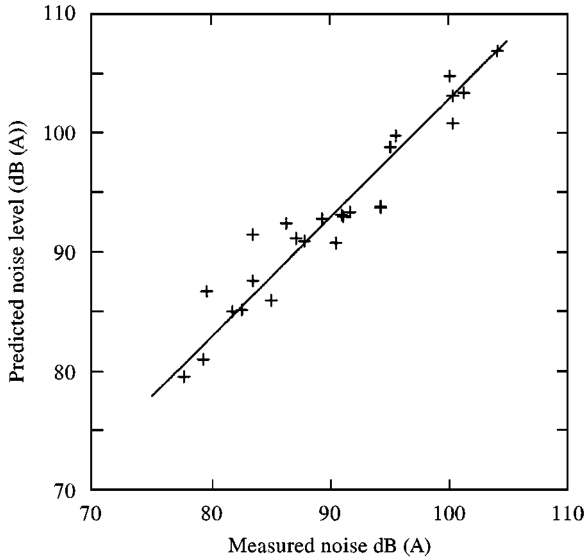

Figure 3. Overall noise predicted using TWINS versus measured noise for 25 combinations of wheel, track and train speed (from reference [21]).

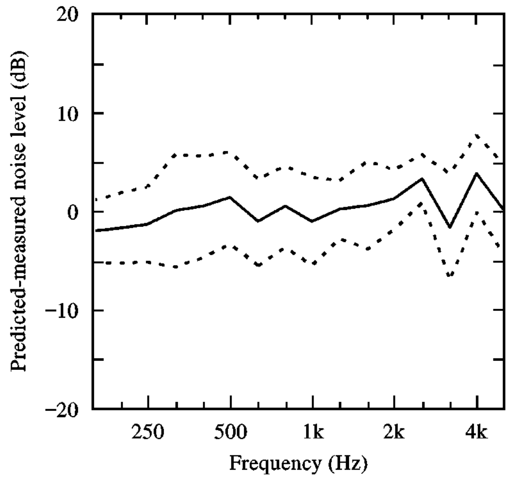

Figure 4. Mean and standard deviation of difference between spectra of noise predicted using TWINS and measured noise for 25 combinations of wheel, track and train speed (modified from reference [21]).

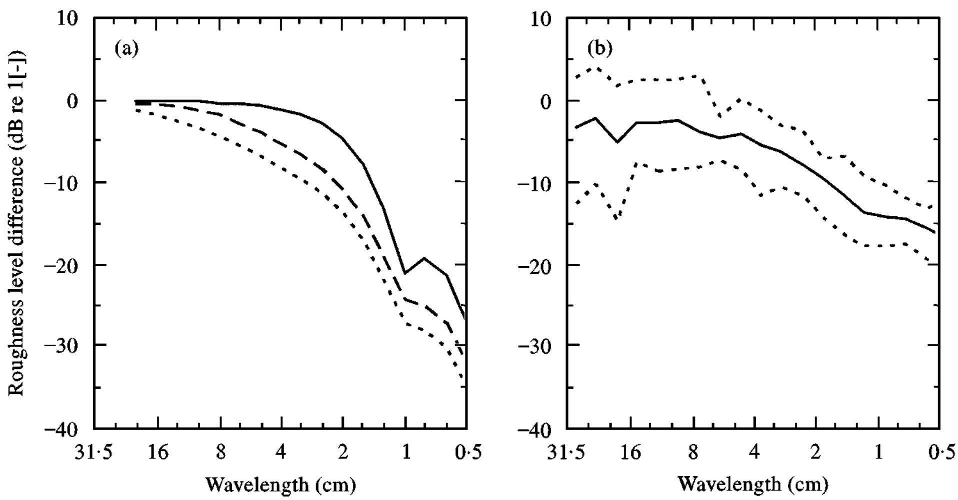

Figure 5. Contact filter effect for a contact patch length of 11 mm. (a) From theory for different values of correlation parameter α [12]: ——, high correlation $\alpha = 1$; – – –, medium correlation $\alpha = 5$; - - -, low correlation $\alpha = 10$. (b) Derived from time-domain analysis of roughness data: mean, maximum and minimum per one-third octave band of six sets of data (from reference [23]).

As the contact exists over an area, typically 10–15 mm long, a filtering effect is found to occur through which short wavelength irregularities have a reduced influence on the system. Such "contact filtering" was originally formulated in the inverse wavelength domain [5, 12]. In that work, the extent of the correlation of the roughness across the width of the contact had to be assumed. Figure 5(a) shows results from this model for a contact patch length of 11 mm. More recently, Remington has developed a discrete point reacting spring (DPRS) model and also compared its results with a more exact Boussinesq integral model [24, 25]. These models are intended to be used with roughness measurements performed on multiple parallel lines a few millimetres apart. In reference [23] it is shown, using a series of such measurements in combination with the DPRS model, that the filtering effect is not so severe at high frequencies as the analytical model of references [5, 12] indicated. These results are reproduced in Figure 5(b). This model has also been used in reference [26] to show the effect of conforming transverse profiles on noise.

In reference [14], it was shown that small variations in transverse profile, due to the addition of roughness to the quasi-constant profile, could lead to "wandering" of the contact across the wheel and rail surfaces even if the transverse position of the wheelset on the track is fixed. This could lead to an additional excitation mechanism alongside the vertical roughness excitation. Work is still underway with the DPRS model to quantify this potential moment excitation [26].

Although roughness has been shown to be the dominant mechanism of excitation of the wheel/rail system, other mechanisms can also play a role [27–32]. Particularly at low frequencies, parametic excitation due to the sleeper passing frequency can be significant [28, 32]. However, this is more relevant to ground-borne vibration and noise than to (air-borne) rolling noise.

2.3. WHEEL VIBRATION

Although simple analytical models were used in the earlier work [2, 12], and even a simple mass will suffice for lower frequency applications [33–37], modelling of the vibrations of wheels for noise prediction is now generally based on the finite element method [16, 38, 39]. Heiss [38] used a finite element mesh of three-dimensional solid elements containing over 8000 degrees of freedom to model a wheelset, but for axisymmetric wheels this can be refined considerably by use of a formulation in which only the cross-section is meshed in two dimensions and a separate solution is obtained for modes with each given number of nodal diameters. In many cases, it is not necessary to model the axle, but the wheel can be constrained at the inner edge of the hub. Although the 0- and 1-nodal-diameter modes, which are strongly coupled with axle vibration, are incorrectly predicted by such a procedure, these modes usually have a higher modal damping and are less significant in the response and hence noise [16]. Nevertheless, it is important that the rigid-body modes of the whole wheelset are included in the modal basis.

Figure 6 shows the modes of vibration of a standard 920 mm freight wheel [40]. The modes of most importance in rolling noise are those of the axial one-nodal-circle set and those of the predominantly radial set, in each case with two or more nodal diameters. The axial modes with zero nodal circles have a predominantly lateral motion at the wheel/rail contact point. As a result, they are not excited in rolling noise, where the principal excitation is in the vertical direction, but are often excited in curve squeal.

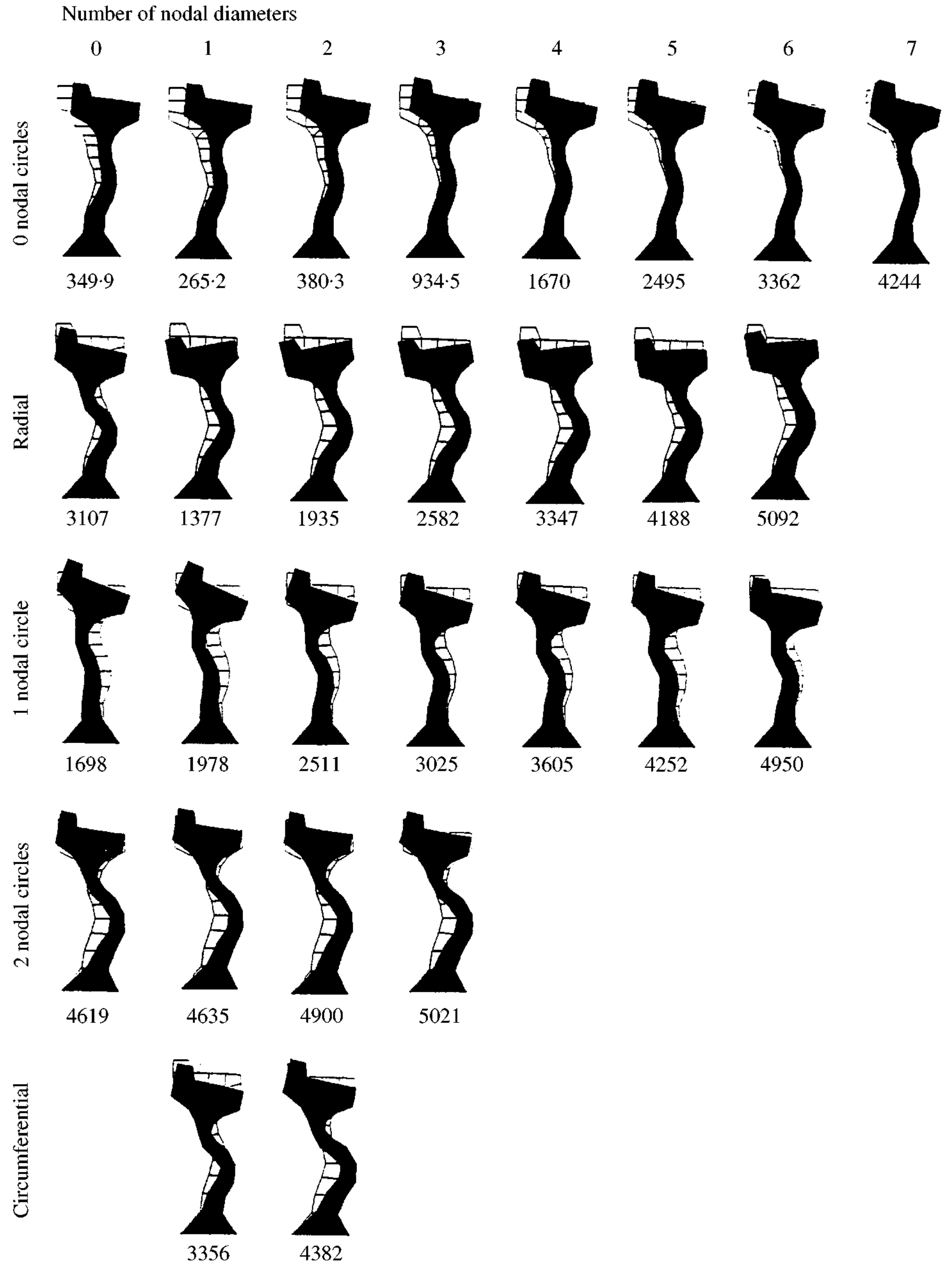

Figure 6. Modes of vibration and natural frequencies (in Hz) of a standard 920 mm freight wheel (from reference [40]).

In all-steel wheels, the modal damping ratios of modes with 2 or more nodal diameters are typically 10^{-4}, which is considerably smaller than the damping induced by the coupling with the track. Exact values are therefore not required for rolling noise prediction (although they are for curve squeal). Where damping treatments are added to wheels, the modal damping has to be known more precisely. In reference [41], the modelling philosophy for wheels is described in more detail, including the aspect of predicting the damping for such cases.

In order to construct the frequency response functions at the interaction, a modal basis from a finite element model is used in a modal superposition [16]. The rotation of the wheel leads to a splitting of each resonance peak associated with a mode having nodal diameters ($n \geqslant 1$) into two peaks, one corresponding to a wave rotating in the same direction as rolling, the other rotating in the opposite direction [19]. These two resonant waves are thus excited by different frequency components at the wheel/rail interface, but in a frame of reference rotating with the wheel they both occur at the original resonance frequency. Since they are excited by different frequency components of the interaction force, no fixed interference can occur between them in the wheel, either in the frame of reference that is rotating with the wheel or that fixed relative to the contact point.

The wheel/rail interaction calculation described in references [14–19], and subsequently in reference [20], is based on calculations in the frequency domain in which the frequency resolution is increased greatly in the region of wheel resonances. It is shown in reference [42] that an analysis based only on one-third octave band receptances of wheel and track cannot give results to the same accuracy. These results are reproduced in Figure 7. In particular, although the rail response can be predicted fairly well by using a logarithmic average of the wheel receptance within each band, the wheel response in the resonant region, i.e., above 1 kHz, is under-predicted by about 10 dB by using this approach. Moreover, it is

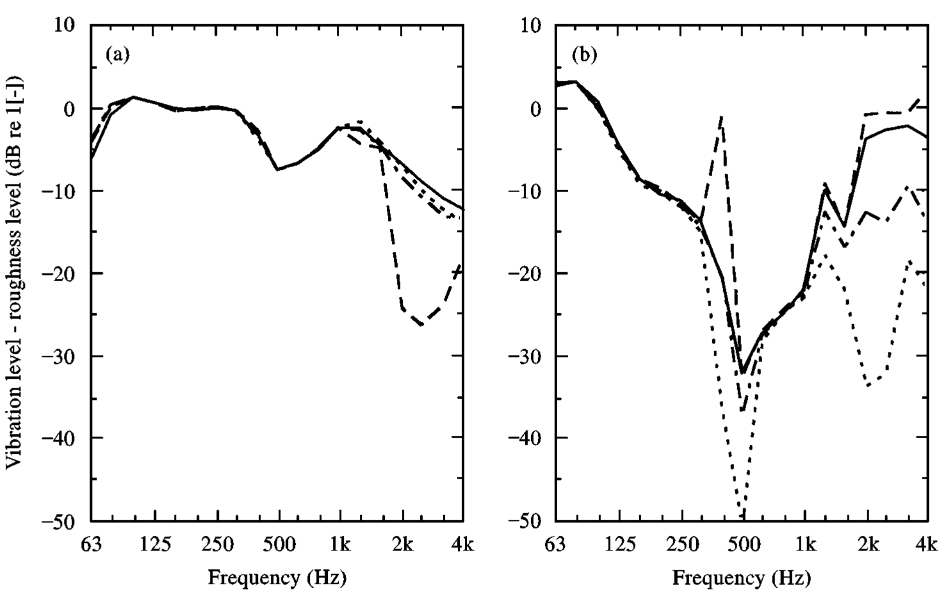

Figure 7. Predictions of (a) rail vibration and (b) wheel vibration, normalized to roughness input, from full narrow-band model [15–19] (——) and from one-third octave model with different wheel receptances: – – – from r.m.s. response for assumed flat force spectrum; - - -, from r.m.s. force for an assumed flat response spectrum; –·–·–, average of logarithms of the receptance [42].

not possible to investigate correctly the effect of added damping treatments on the wheel by such a one-third octave approximation. This is a weakness of the simplified approach adopted, for example, in reference [22].

2.4. TRACK VIBRATION

Compared to the modelling of wheels, the track still poses researchers with many challenges. A comprehensive review of track and interaction models has been presented by Knothe and Grassie [33]. For the current paper, emphasis is given to developments of the track models within TWINS, which have been described in reference [20], and to models published more recently. Specific to noise generation is the need to include not just vertical but also lateral vibration models of the track, and the need to model correctly the damping of wave propagation along the track.

Three alternative track models are included in TWINS [20] (see also references [43, 44] for a fuller description and comparison with field measurements): a Timoshenko beam on a continuous two-layer foundation; the same beam on a periodic foundation; and the model from reference [17] which includes cross-section deformation of the rail in combination with a continuous track foundation. These models each have advantages and disadvantages; the first is simple to use but lacks detail, the second allows the effects of periodicity to be added, which are important for the vertical vibration, and the third gives more reliable results for the lateral direction where torsion and web bending are important as well as lateral bending. A recent development has been to include in these models a flexible sleeper, using an approach similar to that developed by Grassie [45].

A number of more detailed rail models have been developed recently, based on a modified finite element or finite strip approach [46–49]. Of these, the two papers by Gry [48, 49] present the more useful approaches in as much as the supports are also included.

Nordborg's track models [30–32] allow the effect of a moving load on a periodically supported beam to be studied using an elegant analytical approach. However, since the rail is modelled as an Euler–Bernoulli beam, the models are limited in validity to low frequencies where shear deformation and rotational inertia effects can be ignored. Rather than use a Timoshenko beam formulation, this is partially overcome by using a correction to the bending stiffness to ensure that the correct pinned–pinned frequency of just over 1 kHz is obtained. This correction is only valid at that frequency and can lead to incorrect wavenumbers at other frequencies. The model only considers the vertical direction.

In another development, Heckl has studied random sleeper spacing [50]. This approach is useful, since sleepers are never exactly equally spaced, although the analysis stops short of a full prediction of the effect on noise. This model is also considered in detail in a review by Kurze [51]. Experimental validation of the effects of random sleeper spacing as well as randomness in the ballast stiffness under each sleeper have been considered in a recent study [52].

Wu and Thompson [53] have developed a methodology for studying the effect of the local preload close to the wheel. This preload leads to a local increase in the ballast and pad stiffnesses, which reach their unloaded values by about the fifth or sixth sleeper from the excitation. The static load distribution is modelled with a non-linear model and the appropriate stiffnesses are then used in a dynamic model similar to that developed in reference [50]. The point receptances of the track are found to be dominated by the local loaded stiffnesses, whereas the wave propagation and decay rates are dominated by the unloaded region.

2.5. NOISE RADIATION

Models for predicting the noise radiation from a given velocity distribution on the wheel and rail were included in Remington's early work [2, 12] on the basis of analytical models. In reference [54], Thompson developed more advanced radiation models and used them in conjunction with measured wheel and rail vibration data to show that the rail radiation dominates at frequencies up to 1 kHz and the wheel radiation at higher frequencies. The precise frequency at which this change takes place corresponds to the first wheel mode with two nodal diameters from either the radial or one-nodal-circle axial set of modes. This frequency is therefore dependent on the wheel type.

Such a division between wheel and rail radiation is now widely acknowledged and has encouraged parallel development of noise control measures for wheel and track. Omitted from reference [54] was a prediction of radiation from the sleepers, now realized to contribute the main part of the noise radiated at frequencies below about 500 Hz. For track with stiff rail pads they may dominate the noise up to 1 kHz and can contribute a greater component than the rail [55].

The wheel radiation model in reference [54] was based on the Rayleigh integral in which a plane baffled source is assumed. In order to avoid having to consider modes with different numbers, n, of nodal diameters separately, the result for $n = 2$ was used for frequencies up to 1·6 kHz, and that for $n = 4$ for higher frequencies. The radiation efficiency is in any case close to 1 for frequencies above approximately 500 Hz. Below this frequency, modes with $n = 0$, 1 and 2 are present. The use of $n = 2$ for the calculation at low frequencies was chosen in order to compensate partially for the use of a baffled model, as the radiation efficiency for $n = 2$ in a baffled model is similar to that for $n = 1$ in an unbaffled model. Later work, based on the boundary element method [56], has shown that the radiation predicted by the Rayleigh integral method is much too directional at high frequencies, due mainly to suppression of the effects of the wheel cross-sectional shape. However, when considering an average over a train pass-by this did not lead to large errors. It has also been shown that the radial motion of the wheel, omitted in reference [54], contributes significantly to the radiation, especially for straight-webbed wheels and for resilient wheels [41]. These features have now been included in the TWINS prediction model, although not directly as boundary element calculations, but in the form of semi-analytical formulae derived by comparison with the results of boundary element calculations for a range of cases

[56]. Fingberg [57] also predicted the radiation from a wheel using boundary element calculations. In his model, only the wheel cross-section was defined and the number of nodal diameters were included explicitly in the boundary element formulation.

To predict the radiation from the rail, the boundary element method can be used in two dimensions [54], as long as the wavelength in the rail is long compared to that in air and the decay of vibration along the rail is not too great [58]. Petit *et al.* [59] used an alternative two-dimensional approach based on replacing the vibrating rail by a series of equivalent line monopoles and dipoles located within the surface. This model was later implemented in the TWINS program [20].

In reference [58], the validity of such a two-dimensional approach (boundary element or multipole) has been assessed using a model based on a line of simple sources. Figure 8 shows an example result from reference [58] in which the sound power from a rail is plotted as a surface plot on the complex domain of rail wavenumber normalized to the wavenumber in air. This is predicted from a line of dipole sources, the amplitude and phase of which correspond to a wave in the rail emanating from a single excitation point with a given wavenumber and decay rate. The results are normalized to the power that would be produced by the same mean-square amplitude in a two-dimensional model. The results are therefore seen to be similar to those of the two-dimensional model in the rear part of the graph and only to deviate at the front and the left, where the waves have a short wavelength or a high decay rate along the rail respectively.

The waves in actual rails are located mostly in the plateau region for frequencies above about 250 Hz. This justifies the use of two-dimensional models in most cases. For certain wavenumbers where such a two-dimensional model is not strictly valid,

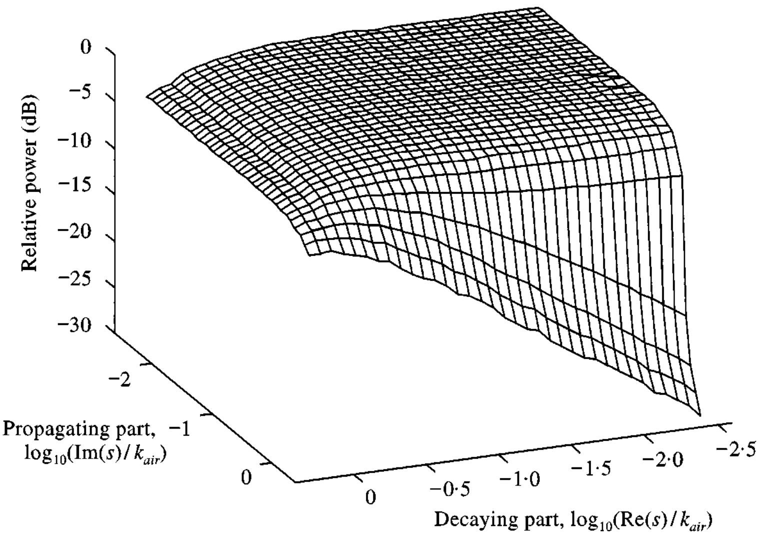

Figure 8. Surface plot of the sound power radiated by a line of dipole sources representing the rail against $\log_{10}$ of the real and imaginary parts of the propagation constant, s of the rail normalized to the wavenumber in air. The sound power is normalized to the power radiated by the same mean square velocity in a two-dimensional model.

it can nevertheless be used with some simple corrections to the radiation efficiency and directivity.

2.6. VALIDATION

A number of full-scale validation tests have been performed over the years. One notable example occurred in 1984 when ORE, now ERRI, commissioned a series of running tests, performed by British Rail and London Underground at Coppull on the British West Coast Main Line [60, 61]. Over a period of two weeks, 30 passages of a test train over a test site were achieved between the normal timetabled service. These included measurements of wheel and rail vibration and wayside noise. Associated measurements of wheel and rail roughness were also performed. The purpose of these tests was to investigate the assumption that a linear relationship exists between the roughness, the vibration and the noise without using a specific model. The relationship between vibration and noise was indeed shown to be linear, and moreover, these test results were also used in conjunction with wheel and rail radiation models to demonstrate the respective contributions to radiated noise [54]. However, the tests indicated that the relationship between roughness and vibration was not completely linear. Nevertheless, with the benefit of hindsight, it can be observed that there were short-comings in the tests which may have led to a false indication of non-linearities. In particular, no account was taken of the contact patch location across the rail head at the two different sites (smooth and corrugated). This may have led to differences between the two sites other than their roughness: for example, greater excitation of lateral vibration where the contact point is further from the centre of the rail. The effects of wheels in the adjacent bogie on the rail vibration were also not accounted for. These had a different roughness to that of the test wheels, so that on smooth rails they would excite the track much less than the rougher of the test wheels but on corrugated track would excite it just as much as the test wheels. Also omitted were measurements of the vibration of the sleepers and estimates of their sound radiation, measurements of the track mobility and the decay rate in the rail.

Many lessons were learnt from these early tests in planning validation tests for the TWINS models [20, 21]. These included three different types of wheel and track to avoid limiting validation to a single wheel/track combination. Extensive static measurements were performed to ensure that the wheel and track were correctly modelled, for example the resonance frequencies and damping of the wheel, and the stiffness and damping of the rail pads and ballast. Very detailed roughness measurements were performed with many parallel measurement lines across the running surface for input to the DPRS model discussed above. Nevertheless, omissions were made, notably the location of the contact zone on the wheel and rail running surfaces was not measured. In this respect, transverse profiles of the wheel and rail would have been useful. This meant that uncertainties of up to ± 5 dB in a given 1/3 octave band existed in the roughness applying during the tests and this limited the accuracy of the predictions (see Figure 4). The results for the radiation part of the predictions from vibration to noise were much more reliable [21].

In another set of tests, some of the problems with defining the excitation were overcome by using wheels machined with a special sinusoidal profile [62]. By gradually increasing the train speed, the single-frequency excitation was allowed to sweep through the frequency range. Six wheels with profiles of different wavelengths were used to cover a wide range of excitation frequencies. These tests allowed some specific aspects of the theoretical models to be verified. It was confirmed that peaks in the wheel vibration occur at frequencies up to 20 Hz above the resonance frequencies of the free wheel due to coupling with the rail and that the damping experienced by a rolling wheel is significantly greater than that of a free wheel, as predicted in reference [19].

Additional validation tests of the TWINS model have been performed in France on high-speed trains [63, 64]. Also tests of optimized freight wheels and track (the "OFWHAT" project) allow some extension of validation to less conventional wheels and track [65, 66]. The ongoing European projects "Silent Freight" and "Silent Track" should allow this to be extended further.

2.7. NON-LINEAR EFFECTS

The TWINS model, like its predecessors, is linear. This means that roughness of a particular wavelength is assumed to generate noise at the corresponding frequency, allowing for the train speed. An increase in roughness amplitude at a particular wavelength is assumed to lead to a corresponding increase in noise at this frequency. Although the linear theory has been demonstrated to be effective over a wide range of roughnesses [23], it is not valid at high roughness levels such as those occurring when rails are corrugated [67] or at discontinuities such as rail joints or wheel flats [68]. This is because of non-linearities in the vertical contact spring between the wheel and rail, and the lateral creep terms. The wheel and track can be considered as linear systems, coupled by a non-linear contact element. At large enough amplitudes, loss of contact can occur, and the resulting impact of the wheel on the rail is a source of significant additional noise.

Although such non-linear models have been developed in the past in relation to track dynamics and corrugation growth [33–35, 67–70] they must be extended to considerably higher frequencies to allow them to predict noise radiation (at least 5 kHz instead of, typically, 1500 Hz). Compared to rolling noise, very little theoretical work has been performed on wheel/rail impact noise [3, 9].

3. CURVE SQUEAL

Curve squeal, although involving interaction of the wheel and rail, is a very different type of noise to rolling or impact noise. The excitation mechanism essentially generates lateral forces due to frictional instability. The review paper by Remington [9] summarizes the work up to the mid-1980s of which that by Rudd [4] is seminal. Van Ruiten [71] also used Rudd's model as a basis for comparisons with measurement results. In this model, wheel yaw angles are assumed to lead to large constant creepages. In the saturated regime, a creep force versus velocity curve with an assumed negative slope is equated to a damper with negative damping coefficient applied between the wheel and rail. Where this negative damping exceeds the positive damping of the wheel modes, squeal can occur. Heckl has recently extended this simple model with a more rigorous mathematical analysis [72].

Recent modelling work on curve squeal has been limited to a few researchers. Fingberg [57, 73] extended earlier models by Schneider *et al.* [74], his main contribution being to develop more accurate predictions of wheel radiation using the boundary element approach (see section 2.5). Their models solved non-linear equations in the time domain starting from an assumed yaw angle of the wheelset relative to the track. Track dynamics were also included.

Périard [75] has developed a time-domain model for the curve squeal from trams. This incorporates non-steady vehicle curving behaviour in the same calculation as wheel/rail interaction, wheel and rail vibration and wheel noise radiation. Some compromises had to be made in the various sub-models, for example transient contact is not included, although used by Fingberg [73], the wheel/rail transverse profiles are very simplified and the wheel radiation is based on a Rayleigh integral rather than boundary element prediction. Nevertheless, this is the first complete model of the generation of curve squeal.

Work on curve squeal lacks the extensive validation performed for rolling noise. This is partly due to the large fluctuations in the occurrence of squeal, the noise levels obtained and the wheel mode which squeals that are found in practice even for apparently identical conditions. Nevertheless, such validation is urgently required. The use of statistical methods will need to be considered rather than a purely deterministic approach.

4. DISCUSSION

In the past 10 years, since the review of rolling noise published in reference [10], many of the remaining questions have been answered through the development and validation of advanced theoretical models. It is now well established that both wheel and rail are major radiators of sound, their exact balance depending on wheel and track design parameters. The sleepers are also seen to contribute significantly at low frequencies, and for very stiff rail pads may dominate the noise from the track. The theoretical models, although still requiring refinement, have now reached maturity and are being widely used in the railway community to assess wheel and track designs and to aid in the development of intrinsically quieter components. Meanwhile many details of the models remain to be investigated further, for example aspects of the contact zone behaviour, the radiation models, and the relation between rail fastener properties and the damping experienced by the rail. However, the major outstanding question in the field of rolling noise is the cause and control of wheel and rail surface irregularity. If this can be understood, significant further gains in the noise level can be achieved.

Much less research has been carried out into curve squeal or impact noise in the period since the review of reference [9]. This is probably because these are more localized phenomena than rolling noise. Nevertheless, they are generally most common in built-up areas due to the nature of the track layout, and their high levels can still cause considerable annoyance. As with rolling noise, the development of thorough theoretical models and their systematic validation against experiments will provide the understanding necessary to reduce these sources of noise.

REFERENCES

1. 1977 *Journal of Sound and Vibration* **51**, *Proceedings of the 1st Workshop on Railway and Tracked Transit System Noise*, Derby, UK, 1976.
2. P. J. Remington 1976 *Journal of Sound and Vibration* **46**, 359–379. Wheel/rail noise, Part I: characterization of the wheel/rail dynamic system.
3. I.L. Vér, C. S. Ventres and M. M. Myles 1976 *Journal of Sound and Vibration* **46**, 395–417. Wheel/rail noise, Part II: impact noise generation by wheel and rail discontinuities.
4. M. J. Rudd 1976 *Journal of Sound and Vibration* **46**, 381–394. Wheel/rail noise, Part III: wheel squeal.
5. P. J. Remington 1976 *Journal of Sound and Vibration* **46**, 419–436. Wheel/rail noise, Part IV: rolling noise.
6. A. G. Galaitsis and E. K. Bender 1976 *Journal of Sound and Vibration* **46**, 437–451. Wheel/rail noise, Part V: measurement of wheel and rail roughness.
7. B. Hemsworth 1979 *Journal of Sound and Vibration* **66**, 297–310. Recent developments in wheel/rail noise research.
8. T. ten Wolde and C. J. M. van Ruiten 1983 *Journal of Sound and Vibration* **87**, 147–160. Sources and mechanisms of wheel/rail noise: state-of-the-art and recent research.
9. P. J. Remington 1987 *Journal of Sound and Vibration* **116**, 339–353. Wheel/rail squeal and impact noise: what do we know? What don't we know? Where do we go from here?
10. P. J. Remington 1988 *Journal of Sound and Vibration* **120**, 203–226. Wheel/rail rolling noise: what do we know? What don't we know? Where do we go from here?
11. P. H. de Vos 1996 *Journal of Sound and Vibration* **193**, 77–82. Railway noise: the physics of airborne sound generation and propagation.
12. P. J. Remington 1987 *Journal of the Acoustical Society of America* **81**, 1805–1823. Wheel/rail rolling noise, I: theoretical analysis.
13. P. J. Remington 1987 *Journal of the Acoustical Society of America* **81**, 1824–1832. Wheel/rail rolling noise, II: validation of the theory.
14. D. J. Thompson 1990 *Ph.D. thesis, University of Southampton.* Wheel/rail noise—theoretical modelling of the generation of vibrations.
15. D. J. Thompson 1993 *Journal of Sound and Vibration* **161**, 387–400. Wheel-rail noise generation, Part I: introduction and interaction model.
16. D. J. Thompson 1993 *Journal of Sound and Vibration* **161**, 401–419. Wheel-rail noise generation, Part II: wheel vibration.
17. D. J. Thompson 1993 *Journal of Sound and Vibration* **161**, 421–446. Wheel-rail noise generation, Part III: rail vibration.
18. D. J. Thompson 1993 *Journal of Sound and Vibration* **161**, 447–466. Wheel-rail noise generation, Part IV: contact zone and results.
19. D. J. Thompson 1993 *Journal of Sound and Vibration* **161**, 467–482, Wheel-rail noise generation, Part V: inclusion of wheel rotation.
20. D. J. Thompson, B. Hemsworth and N. Vincent 1996 *Journal of Sound and Vibration* **193**, 123–135. Experimental validation of the TWINS prediction program for rolling noise, Part 1: description of the model and method.

21. D. J. Thompson, P. Fodiman and H. Mahé 1996 *Journal of Sound and Vibration* **193**, 137–147. Experimental validation of the TWINS prediction program for rolling noise, Part 2: results.
22. R. Diehl, R. Görlich and G. Hölzl 1997 *Proceedings of World Congress on Railway Research, Firenze, Italy, Vol. E*, 421–427. Acoustic optimization of railroad track using computer aided methods.
23. D. J. Thompson 1996 *Journal of Sound and Vibration* **193**, 149–160. On the relationship between wheel and rail surface roughness and rolling noise.
24. P. J. Remington and J. Webb 1996 *Journal of Sound and Vibration* **193**, 83–102. Estimation of wheel/rail interaction forces in the contact area due to roughness.
25. P. J. Remington and J. Webb 1996 *Journal of Sound and Vibration* **193**, 335–348. Wheel/rail noise reduction through profile modification.
26. D. J. Thompson and P. J. Remington 2000 *Journal of Sound and Vibration* **231**, 537–548. The effects of transverse profile on the excitation of wheel/rail noise.
27. M. Heckl 1990 *Proceedings of DAGA* 90, 135–140. Körperschallentstehung bei Schienenfahrzeugen (Structure-borne noise excitation by railway vehicles).
28. M. Heckl 1994 *Proceedings of Transport Noise* 94, *St. Petersburg*, 109–116. The role of the sleeper passing frequency in rail-wheel noise.
29. M. A. Heckl 1989 *Workshop on Rolling Noise Generation, Technische Universität Berlin, 10–11 October*, 112–117. Flutter-like instabilities caused by feed back in the rail-wheel interaction.
30. A Nordborg 1995 *Doctoral thesis, TRITA-FKT Report* 9535, *Royal Institute of Technology, Stockholm.* Vertical rail vibrations: noise and structure-borne sound generation.
31. A. Nordborg 1998 *Acustica* **84**, 280–288. Vertical rail vibrations: pointforce excitation.
32. A. Nordborg 1998 *Acustica* **84**, 289–300. Vertical rail vibrations: parametric excitation.
33. K. Knothe and S. L. Grassie 1993 *Vehicle System Dynamics* **22**, 209–262. Modeling of railway track and vehicle/track interaction at high frequencies.
34. J. C. O. Nielsen 1993 *Doctoral thesis, Division of Solid Mechanics, Chalmers University of Technology, Göteborg, Sweden.* Train/track interaction. Coupling of moving and stationary dynamic systems—theoretical and experimental analysis of railway structures considering wheel and track imperfections.
35. A. Igeland 1997 *Doctoral thesis, Division of Solid Mechanics, Chalmers University of Technology, Göteborg, Sweden.* Dynamic train/track interaction: simulation of railhead corrugation growth under a moving bogie using mathematical models combined with full-scale measurements.
36. S. L. Grassie 1995 *Vehicle System Dynamics Supplement* **24**, 355–362. Benchmark test for models of railway track and of vehicle/track interaction at relatively high frequencies.
37. S. L. Grassie 1996 *Vehicle System Dynamics Supplement* **25**, 243–262. Models of railway track and vehicle/track interaction at high frequencies: results of a benchmark test.
38. P. Heiss 1986 *Doctoral thesis, Technical University of Berlin.* Untersuchungen über das Körperschall- und Abstrahlverhalten eines Reisezugwagenrades (Investigations into the vibration and noise behaviour of a passenger coach wheel).
39. U. J. Kurze and H. Horn 1990 *Acustica* **70**, 166–178. Schwingungen von Eisenbahnrädern (Vibrations of railway wheels).
40. D. J. Thompson and M. G. Dittrich 1991 *ORE Technical Document DT*248 (*C*163), *Utrecht*. Wheel response and radiation—laboratory measurements of five types of wheel and comparisons with theory.
41. C. J. C. Jones and D. J. Thompson 2000 *Journal of Sound and Vibration* **231**, 779–790. Rolling noise generated by wheels with visco-elastic layers.
42. D. J. Thompson 1994 *Journal of Sound and Vibration* **171**, 427–432. Author's reply.

43. D. J. Thompson and N. Vincent 1995 *Vehicle System Dynamics Supplement* **24**, 86–99. Track dynamic behaviour at high frequencies. Part 1: theoretical models and laboratory measurements.
44. N. Vincent and D. J. Thompson 1995 *Vehicle System Dynamics Supplement* **24**, 100–114. Track dynamic behaviour at high frequencies. Part 2: experimental results and comparisons with theory.
45. S. L. Grassie 1995 *Journal of Sound and Vibration* **187**, 799–813. Dynamic modelling of concrete railway sleepers.
46. K. Knothe, Z. Strzyzakowski and K. Willner 1994 *Journal of Sound and Vibration* **169**, 111–123. Rail vibrations in the high frequency range.
47. L. Gavric 1995 *Journal of Sound and Vibration* **185**, 531–543. Computation of propagative waves in free rail using a finite element technique.
48. L. Gry 1996 *Journal of Sound and Vibration* **195**, 477–505. Dynamic modelling of railway track based on wave propagation.
49. L. Gry and C. Gontier 1997 *Journal of Sound and Vibration* **199**, 531–558. Dynamic modelling of railway track: a periodic model based on a generalized beam formulation.
50. M. A. Heckl 1995 *Acustica* **81**, 559–564. Railway noise—can random sleeper spacings help?
51. U. J. Kurze 1997 *Acustica* **83**, 506–515. Refined calculations or improved understanding of rail vibrations?
52. G. de France 1998 *Master of Science Dissertation, ISVR, University of Southampton.* Railway track: effect of rail support stiffness on vibration and noise.
53. T. X. Wu and D. J. Thompson 1999 *Journal of Sound and Vibration* **219**, 881–904. Effects of local preload on the foundation stiffness and vertical vibration of railway track.
54. D. J. Thompson 1988 *Journal of Sound and Vibration* **120**, 275–280. Predictions of acoustic radiation from vibrating wheels and rails.
55. N. Vincent, P. Bouvet, D. J. Thompson, P. E. Gautier 1996 *Journal of Sound and Vibration* **193**, 161–171. Theoretical optimization of track components to reduce rolling noise.
56. D. J. Thompson, N. S. Ferguson and C. J. C. Jones Modelling the sound power radiated by a railway wheel, to be published.
57. U. Fingberg 1990 *VDI Fortschritt-berichte, Reihe* 11, *No.* 140. Ein Modell für das Kurvenquietschen von Schienenfahrzeugen (A model for the curve squeal of railway vehicles).
58. D. J. Thompson, C. J. C. Jones and N. Turner Investigation of the validity of two-dimensional models for rail radiation, to be published.
59. M.-F. Petit, M. Heckl, J. Bergemann and J. Baae 1992 *Proceedings of DAGA* 92, 997–1000. Berechnung des Abstrahlmaßes von Eisenbahnschienen anhand der Multipolsynthese (Calculation of the radiation efficiency of railway rails using multipole synthesis).
60. O.R.E. 1986 *Question C*163, *Railway Noise, Report RP*7, *Utrecht.* Wheel/rail contact noise—comparative study between theoretical model and measured results.
61. O. R. E. 1987 *Question C*163, *Railway Noise, Report RP*8, *Utrecht.* Wheel/rail contact noise—further comparative analysis of noise, vibration and roughness data.
62. D. J. Thompson, N. Vincent and P. E. Gautier 1999 *Journal of Sound and Vibration* **223**, 587–609. Validation of a model for railway rolling noise using field measurements with sinusoidally profiled wheels.
63. L. Castel, P. E. Gautier, N. Vincent and J. P. Goudard 1993 *Proceedings of Inter Noise* 93, *Leuven*, 1467–1470. 350 kph running tests to assess a new railway noise model.
64. P. Fodiman, L. Castel and G. Gaborit 1995 *Proceedings of* 11*th International Wheelset Congress, Paris.* Validation of a TGV-A trailing-car wheel with an acoustically optimised profile.
65. P. Fodiman 1996 *Proceedings of World Congress on Railway Research, CO, USA.* Line test validation of low noise railway components.

66. C. J. C. Jones and J. W. Edwards 1996 *Proceedings of Inter Noise* 96, *Liverpool*, 403–408. Development of wheels and track components for reduced rolling noise from freight trains.
67. B. Ripke 1995 *VDI Fortschrittberichte*, *Reihe* 12, *No*. 249. Hochfrequente Gleismodellierung und Simulation der Fahrzeug-Gleis-Dynamik unter Verwendung einer nichtlinearen Kontaktmechnik (High frequency track modelling and simulation of the vehicle-track dynamics taking into account the influence of non-linear contact mechanics).
68. S. G. Newton and R. A. Clark 1979 *Journal of Mechanical Engineering Science* **21**, 287–297. An investigation into the dynamic effects on the track of wheelflats on railway vehicles.
69. R. A. Clark, P. A. Dean, J. A. Elkins and S. G. Newton 1982 *Journal of Mechanical Engineering Science* **24**, 65–76. An investigation into the dynamic effects of railway vehicles running on corrugated rails.
70. H. Ilias 1996 *VDI Fortschrittberichte*, *Reihe* 12, *No*. 297. Nichtlineare Wechselwirkungen von Radsatz und Gleis beim Überrollen von Profilstörungen (Non-linear interaction of wheel and track during rolling over profile variations).
71. C. J. M. van Ruiten 1988 *Journal of Sound and Vibration* **120**, 245–253. Mechanism of squeal noise generated by trams.
72. M. A. Heckl 1998 *Proceedings of Euronoise* 98, *München*, 85–90. Curve squeal of trains—which wheel modes are prone to squeal?
73. U. Fingberg 1990 *Journal of Sound and Vibration* **143**, 365–377. A model of wheel-rail squealing noise.
74. E. Schneider, K. Popp and H. Irretier 1988 *Journal of Sound and Vibration* **120**, 227–244. Noise generation in railway wheels due to rail-wheel forces.
75. F. Périard 1998 *Doctoral thesis*, *Technical University of Delft*. Wheel-rail noise generation: curve squealing by trams.