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MEASUREMENT OF THE ISOTROPIC DYNAMIC YOUNG'S MODULUS IN A SEISMICALLY EXCITED CANTILEVER BEAM USING A LASER SENSOR

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An approach to the direct measurement of the dynamic Young's modulus for a viscoelastic material using a contactless sensor, based on a laser emitter–receiver, is presented in this paper. The proposed method consists in exciting a cantilever beam specimen by means of a seismic acceleration. The acceleration of the base is recorded by means of a piezoelectric accelerometer, and the vertical displacement of a suitable point of the specimen is recorded by means of an accurate laser sensor. Using a contactless sensor avoids introducing any perturbation due to contact that could locally change the mechanical properties of the material. This enables one to accurately determine Young's modulus as a function of frequency. A mathematically accurate treatment of the experimental measurement of Young's modulus is also presented, yielding a final expression for $\mathbf{E}(i\omega)$ as a function of the two measured entities, namely the acceleration of the base and the vertical displacement of an adequate point of the specimen. Experimental curves of Young's modulus at different temperatures are reported.

1. INTRODUCTION

According to the theory of viscoelasticity [1–3] the mechanical behavior of an isotropic viscoelastic material can be completely defined by means of two characteristic parameters. Such parameters are dependent on the environmental and operating conditions, and particularly on frequency and temperature.

Usually, the two characteristic parameters that are measured at different temperatures and frequencies are two moduli (e.g., Young's and shear modulus, etc.), or one modulus and the Poisson ratio. Then, the elastic–viscoelastic correspondence principle allows one to compute all the other characteristic parameters by means of relations that are formally identical to those of the theory of elasticity.

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Much relevant work was done in the past decades regarding the direct measurement of Young's modulus and of other characteristic moduli of a material.

Gottenberg and Christensen [4] described an experimental technique to determine the complex shear modulus of a linear, isotropic, viscoelastic solid and its dependence on frequency and temperature.

Pritz [5–8] did relevant work on the measurement of Young's modulus. In reference [5] he described an investigation of the complex modulus of acoustic materials by using a transfer function method. In this method, a cylindrical or prismatic specimen is excited into longitudinal vibration at one end, the other end being loaded by a mass, so as to realize a spring-like specimen. Then, the specimen is modelled by lumped parameters mechanical elements and the transfer function of the specimen from the excited end to the loaded one is theoretically investigated and experimentally measured. In references [6, 7], Pritz theoretically investigated and experimentally measured the complex modulus of an acoustic material by considering a rod-like specimen being excited by a shaker at one end. Again, the transfer function measured is used, which involves the measurement of the vibration amplitudes at the specimen ends and the phase angle between them. More recently, the same technique has been used in reference [8] to measure the dynamic Young's modulus and the loss factor of plastic foams for impact sound isolation.

Holownia [9] presented a technique, based on holographic interferometry, to measure the dynamic Young's modulus and the dynamic bulk modulus for rubbers, so as to obtain a thorough knowledge of such materials, and particularly of the Poisson ratio, which is difficult to be measured directly. A later work by Holownia and Rowland [10] described the measurement of the dynamic bulk modulus for rubbers by using a technique called Electronic Speckle Pattern Interferometry (ESPI). The measurement of bulk modulus was achieved by direct measurements of the volume contractions of submerged specimens subjected to sinusoidal pressure changes.

Sim and Kim [11] presented a method to estimate the properties of viscoelastic materials for finite element method application. A first estimate of Young's modulus and of the loss factor of a viscoelastic material were first derived from the transmissibility measurements made on a specimen. Then, a first estimate of the Poisson ratio (assumed constant with frequency) was obtained on the basis of a theoretical development. Then, Young's modulus and the Poisson ratio were evaluated after an iterative process.

Ödeen and Lundberg [12] presented a method for determination of the complex modulus of a linearly viscoelastic material from measured endpoint accelerations of an impact-loaded rod specimen. Young's modulus was obtained by an iterative numerical scheme. Trendafilova *et al.* [13] used the same technique, but measured the displacements, instead of the accelerations, of the specimen ends by means of electro-optical transducers. Ostiguy and Ewan-Iwanowski [14] used a laser vibrometer for measurement of the complex Young's modulus of linear viscoelastic solids.

This paper presents an approach to the direct measurement of the dynamic Young's modulus of a viscoelastic material by using a contactless sensor, based on a laser emitter–receiver. A small specimen is mounted on to an electrodynamic shaker and seismically excited by a sine-sweep input signal so as to bend under the boundary conditions of a cantilever beam. The acceleration of the base is recorded by means of a piezoelectric accelerometer, and the vertical displacement of a suitable point of the specimen is recorded by means of an accurate laser sensor. In this way, no load effect perturbation is introduced in the measurement chain by any sort of contact sensor (such as a strain gauge) that could locally change the mechanical properties of the material. This enables one to determine accurately. Young's modulus as a continuous function of frequency. Moreover, the measurements are carried out in a temperature controlled chamber, so as to obtain several curves of Young's modulus at different temperatures.

2. THEORY OF YOUNG'S MODULUS FROM MEASUREMENTS IN A BEAM-LIKE SPECIMEN USING A CONTACTLESS SENSOR (LASER)

In this section a theoretical discussion about how Young's modulus can be obtained, by experimental measurements in a beam-like specimen seismically excited by a sinusoidal force input, is carried out. It is shown that Young's modulus can be determined by simultaneously measuring the vertical displacement of a suitable point of the bending beam and the acceleration of the supporting basement.

First, it should be recalled that Young's modulus $E(t)$ for an isotropic material is defined as

$$\sigma(t) = \int_{-\infty}^{t} E(t-\tau)\frac{\mathrm{d}\varepsilon(\tau)}{\mathrm{d}\tau}\,\mathrm{d}\tau, \tag{1}$$

where $\sigma(t)$ is the stress and $\varepsilon(t)$ the strain in condition of uniaxial stress relaxation conditions. The strain history can be specified as a harmonic function of time, according to

$$\varepsilon(t) = \varepsilon_0 \mathrm{e}^{\mathrm{i}\omega t}, \tag{2}$$

where ε_0 is the amplitude and ω is the oscillation frequency. Then, $E(t)$ must be decomposed into the sum of an asymptotic constant term E_∞ and a time-variable one $E'(t)$,

$$E(t) = E_\infty + E'(t), \tag{3}$$

where $E'(t) \to 0$ as $t \to \infty$. Such a decomposition is necessary to obtain an expression for the complex Young's modulus whenever the viscoelastic body is subjected to steady state oscillatory conditions. As a matter of fact, it can be proved [1] that the expression for Young's modulus in the frequency domain can be obtained by substituting equations (2) and (3) into equation (1):

$$\sigma(t) = E_\infty \varepsilon_0 \mathrm{e}^{\mathrm{i}\omega t} + \mathrm{i}\omega\varepsilon_0 \int_{-\infty}^{t} E'(t-\tau)\mathrm{e}^{\mathrm{i}\omega t}\,\mathrm{d}\tau. \tag{4}$$

With the change of variable $t - \tau = \eta$, equation (4) can be rewritten as

$$\sigma(t) = \left[E_\infty + \mathrm{i}\omega \int_{-\infty}^{\infty} E'(\eta) \mathrm{e}^{-\mathrm{i}\omega\eta} \, \mathrm{d}\eta \right] \varepsilon_0 \mathrm{e}^{\mathrm{i}\omega t}. \tag{5}$$

To be consistent with the steady state conditions assumed for strain history, the stress will be taken to have the same steady state form,

$$\sigma(t) = \mathbf{E}(\mathrm{i}\omega) \varepsilon_0 \mathrm{e}^{\mathrm{i}\omega t}, \tag{6}$$

and finally, upon introducing complex numbers,

$$\mathbf{E}(\mathrm{i}\omega) = \sigma(\mathrm{i}\omega)/\varepsilon(\mathrm{i}\omega). \tag{7}$$

Thus, from equations (5), (6) and (7), the complex Young's modulus, $E(\mathrm{i}\omega)$ results

$$\mathbf{E}(\mathrm{i}\omega) = E_\infty + \mathrm{i}\omega \mathbf{E}'(\mathrm{i}\omega). \tag{8}$$

However, equation (8) is not of practical use, since stress measurements cannot be easily and accurately performed. It is then necessary to obtain a practical expression for Young's modulus, which can be computed from experimental measurements carried out in a laboratory.

The equation of a seismically excited beam is

$$m \frac{\partial^2 u(x,t)}{\partial t^2} + J \int_{-\infty}^{t} E(t-\tau) \frac{\partial}{\partial t} \frac{\partial^4 u(x,t)}{\partial x^4} \, \mathrm{d}\tau = -m a_b(t), \tag{9}$$

where m is the mass per unit length of the beam, $u = u(x,t)$ is the relative vertical displacement from the non-deformed configuration, J is the moment of inertia, $E(t)$ is the relaxation function for Young's modulus of the specimen and $a_b(t)$ is the absolute vertical acceleration of the supporting basement.

Modal decoupling of equation (9) can be done by using a complete eigenfunction expansion,

$$u(x,t) = \sum_{i=1}^{\infty} \phi_i(x) q_i(t), \tag{10}$$

and by integrating over the free length

$$\int_0^L m\phi_i\phi_i \, \mathrm{d}x \ddot{q}_i + J \int_0^L \phi_i \frac{\mathrm{d}^4\phi_i}{\mathrm{d}x^4} \mathrm{d}x \left(\int_{-\infty}^{t} E(t-\tau) \frac{\mathrm{d}q_i}{\mathrm{d}\tau} \mathrm{d}\tau \right) = -m a_b \int_0^L \phi \, \mathrm{d}x. \tag{11}$$

Now, by setting

$$\int_0^L \phi_i \frac{\mathrm{d}^4\phi_i}{\mathrm{d}x^4} \mathrm{d}x = \frac{\gamma_i}{m} = \frac{b_i^4}{mL^4}, \qquad \int_0^L m\phi_i \, \mathrm{d}x = B_i, \tag{12}$$

where the b_i represent the solutions of the frequencies equation and the B_i are the model loads for a unit acceleration of the base, the modal decoupled equations can be obtained in the form

$$\ddot{q}_i + J\frac{b_i^4}{mL^4}\int_{-\infty}^{t} E(t-\tau)\frac{\mathrm{d}q_i}{\mathrm{d}\tau}\,\mathrm{d}\tau = -B_i a_b(t). \tag{13}$$

Expression (13) in the frequency domain can be obtained in a similar manner as equations (7) and (8), namely,

$$\left(-\omega^2 + J\frac{b_i^4}{mL^4}E(\mathrm{i}\omega)\right)\mathbf{q}_i(\mathrm{i}\omega) = -B_i\mathbf{a}_b(\mathrm{i}\omega). \tag{14}$$

Equation (14) can be rearranged so as to get an expression for Young's modulus as a complex number:

$$\mathbf{E}(\mathrm{i}\omega) = -B_i\frac{mL^4}{Jb_i^4}\frac{\mathbf{a}_b(\mathrm{i}\omega)}{\mathbf{q}_i(\mathrm{i}\omega)} + \frac{mL^4}{Jb_i^4}\omega^2. \tag{15}$$

Now, the model co-ordinate $\mathbf{q}_i$ must be expressed in terms of a measurable entity. If a strain gauge is employed in the experimental set-up, the strain at a suitable point of the specimen can be measured. In our first experimental tests we used a stain gauge, but the results were not satisfactory because the glue fastening the gauge to the specimen caused a local stiffening of the material, which heavily affected the measurements (see section 3 for further details). Therefore, a contactless sensor had to be used. A laser sensor was chosen for this purpose. The entity the laser measures is the absolute local vertical displacement of the specimen, hence, the modal co-ordinate $\mathbf{q}_i$ in equation (15) must now the expressed in terms of the absolute displacement $\mathbf{u}$ measured by the laser sensor (see Figures 1 and 2 for reference).

Upon recalling equation (10) and assuming that the second eigenmode is not excited due to the particular positioning of the laser at a distance d from the clamped edge of the beam, and that the higher order modes can be neglected (because their amplitude greatly decreases by increasing the order, as shown in Appendix A), it can be written as

$$u(d,t) = \sum_{i=1}^{\infty}\phi_i(d)q_i(t) = \sum_{i=1}^{\infty}D_i q_i \cong D_1 q_1(t) \quad \text{with } D_1 = \phi_1(d), \tag{16}$$

where d, i.e., the distance of the point where the laser is positioned from the clamped edge of the beam, is defined as meeting the requirement: $\phi_2(d) = 0$.

In the frequency domain, equation (16) is written as

$$u(\mathrm{i}\omega) = D_1\mathbf{q}_1(\mathrm{i}\omega). \tag{17}$$

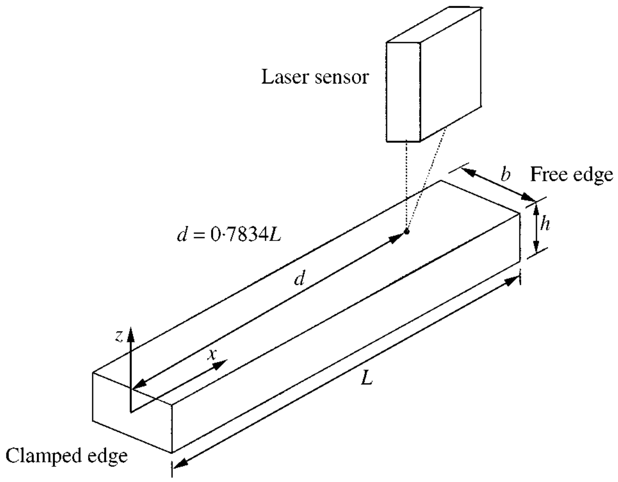

Figure 1. Specimen characteristics.

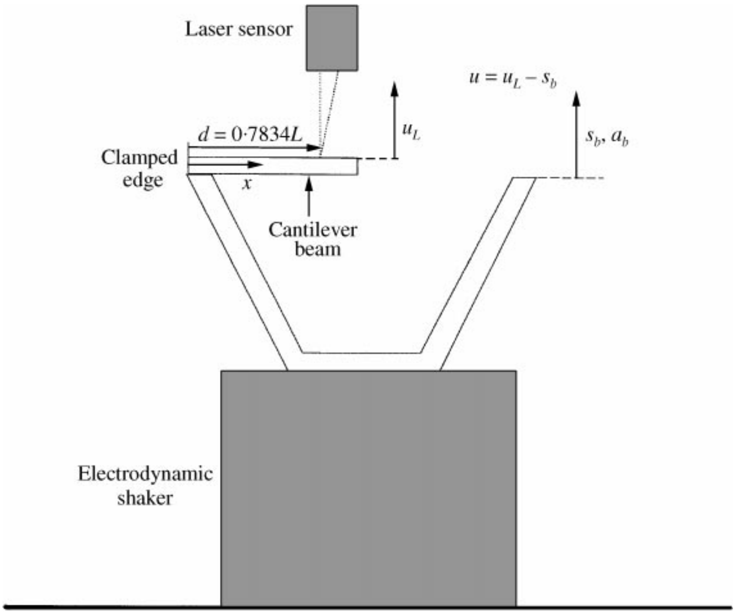

Figure 2. Set-up of the cantilever beam, the support and the shaker.

Thus,

$$\frac{\mathbf{a}_b(\mathrm{i}\omega)}{\mathbf{q}_1(\mathrm{i}\omega)} = D_1 \frac{\mathbf{a}_b(\mathrm{i}\omega)}{\mathbf{u}(\mathrm{i}\omega)}. \tag{18}$$

Then, by substituting equation (18) into equation (15), one obtains

$$\mathbf{E}(\mathrm{i}\omega) = -B_1 D_1 \frac{mL^4}{Jb_1^4} \frac{\mathbf{a}_b(\mathrm{i}\omega)}{\mathbf{u}(\mathrm{i}\omega)} + \frac{mL^4}{Jb_1^4}\omega^2. \tag{19}$$

Now, $\mathbf{u}(\mathrm{i}\omega)$ is the relative displacement of the specimen. Upon recalling that the laser measures an absolute displacement $\mathbf{u}_L(\mathrm{i}\omega)$ and letting $\mathbf{s}_b(\mathrm{i}\omega)$ be the displacement of the base (see Figure 2), it can be written as

$$\mathbf{u}_L(\mathrm{i}\omega) = \mathbf{u}(\mathrm{i}\omega) + \mathbf{s}_b(\mathrm{i}\omega) = \mathbf{u}(\mathrm{i}\omega) + \mathbf{a}_b(\mathrm{i}\omega)/(\mathrm{i}\omega)^2, \tag{20}$$

and thus

$$\frac{\mathbf{u}(\mathrm{i}\omega)}{\mathbf{a}_b(\mathrm{i}\omega)} = \frac{\mathbf{u}_L(\mathrm{i}\omega)}{\mathbf{a}_b(\mathrm{i}\omega)} + \frac{1}{\omega^2}. \tag{21}$$

By substituting equation (21) into equation (19), one obtains

$$\mathbf{E}(\mathrm{i}\omega) = -B_1 D_1 \frac{mL^4}{Jb_1^4}\left(\frac{\mathbf{u}_L(\mathrm{i}\omega)}{\mathbf{a}_b(\mathrm{i}\omega)} + \frac{1}{\omega^2}\right)^{-1} + \frac{mL^4}{Jb_1^4}\,\omega^2. \tag{22}$$

Equation (22) can now be rearranged, so as to make it independent of the dimensions and the inertial characteristics of the beam. This can be done as shown in the following, by introducing as many dimensionless parameters as possible. A set of normalization parameters w_i can be introduced, so that a set of normalized eigenfunctions $\phi_i(x)$ can be defined

$$\phi_i(x) = \frac{1}{\sqrt{mL}}\, w_i \hat{\phi}_i(x). \tag{23}$$

Here $\hat{\phi}_i(x)$ are the non-normalized eigenfunctions and $w_i = \sqrt{1/\int_0^1 \hat{\phi}_i^2(y)\,\mathrm{d}y}$. A dimensionless co-ordinate $y = x/L$ can also be introduced.

Now, the B_i in equation (12) and D_i in equation (16) can be rewritten as

$$B_i = \int_0^L m\phi\,\mathrm{d}x = m\mathrm{L}\int_0^1 \phi_i(y)\,\mathrm{d}y = mL\frac{1}{\sqrt{mL}}\,w_i\int_0^1 \hat{\phi}_i(y)\,\mathrm{d}y = \sqrt{mL}w_i t_i \tag{24}$$

$$D_i = \phi_i(d) = \frac{1}{\sqrt{mL}} w_i \hat{\phi}_i(d) = \frac{1}{\sqrt{mL}} w_i r, \tag{25}$$

by setting $t_i = \int_0^1 \hat{\phi}_i(y)\,\mathrm{d}y$ and $r_i = \hat{\phi}_i(d/L)$.

Finally, upon recalling that for a squared-section beam $m/J = 12\rho hb/bh^3 = 12\rho/h^2$, the final expression for $E(\mathrm{i}\omega)$ as a function of the vertical absolute displacement (measured by the laser sensor) and the acceleration of the basement (measured by the accelerometer) can be written as

$$\mathbf{E}(\mathrm{i}\omega) = \frac{\rho L^4}{h^2}\left[F_1\left(\frac{\mathbf{u}_L(\mathrm{i}\omega)}{\mathbf{a}_b(\mathrm{i}\omega)} + \frac{1}{\omega^2}\right)^{-1} + F_2\omega^2\right], \tag{26}$$

where the dimensionless parameters F_1 and F_2 are given by $F_1 = -12w_1^2 t_1 r_1 / b_1^4$, $F_2 = 12/b_1^4$.

By applying equation (26) to the case of a cantilever beam (see Appendix A), the expression to obtain Young's modulus of the tested specimen can be obtained

$$\mathbf{E}(\mathrm{i}\omega) = \frac{\rho L^4}{h^2}\left[-1{\cdot}0686\left(\frac{\mathbf{u}_L(\mathrm{i}\omega)}{\mathbf{a}_b(\mathrm{i}\omega)} + \frac{1}{\omega^2}\right)^{-1} + 0{\cdot}9707\omega^2\right]. \tag{27}$$

Some care should be taken when applying the method described above, since the useful frequency range is a frequency interval centered at the resonance frequency of the specimen. The resonance frequency of the specimen used in the experimental tests varies with the test temperature, ranging from 224 (at a temperature of 46°C) to 307 Hz (at a temperature of 8°C). The frequency interval from 150 to 400 Hz turned out to be the useful frequency range of the method. Outside this interval some errors could affect the results. For instance, at higher frequencies the so-called "plate effect" of the specimen could introduce an error in the measurement. The plate effect for a specimen can be qualitatively explained as follows: if a beam is excited in bending at increasing frequency, the "wavelength" of the displacement configuration becomes shorter and the width-to-wavelength ratio increases. Hence, the more the excitation frequency increases, the more the behavior of the specimen is different from that of an ideal beam, to which the theory in this paper refers. For a complete proof of this fact, the interested reader can refer to a previous work of one of the authors [15].

3. EXPERIMENTAL RESULTS

Measurements of the complex dynamic Young's modulus have been carried out for a viscoelastic material, namely a mixture of polypropylene and calcium carbonate. A beam-like specimen (see Figure 1) has been mounted onto an *ad hoc* support and fastened at one extremity, so as to realize a cantilever beam (see Figure 2). The support with the specimen has been mounted onto an electrodynamic shaker set inside a temperature-controlled chamber and excited by means of a sine-sweep acceleration. In this way, several curves of Young's modulus at different temperatures can be obtained.

A scheme of the whole measurement system is depicted in Figure 3.

The temperature-controlled chamber is implemented by means of a closed ring insulated with polyurethane in order to minimize the thermal dispersion. A refrigerator and a resistor are located inside the ring; the driving voltage of the resistor is modulated so as to realize a temperature control by means of a microprocessor and an I/O board. The temperature transducer is a thermometric probe HD8605 by DeltaOhm, which outputs a voltage signal proportional to the air temperature. This signal is sent through a National Instrument AT-MIO-16 I/O board to a PC, where a dedicated program in the Labview environment implements the temperature control. The temperature range for the measurements goes from 8°C (lower bound due to the thermal dispersion of the chamber) to 46°C

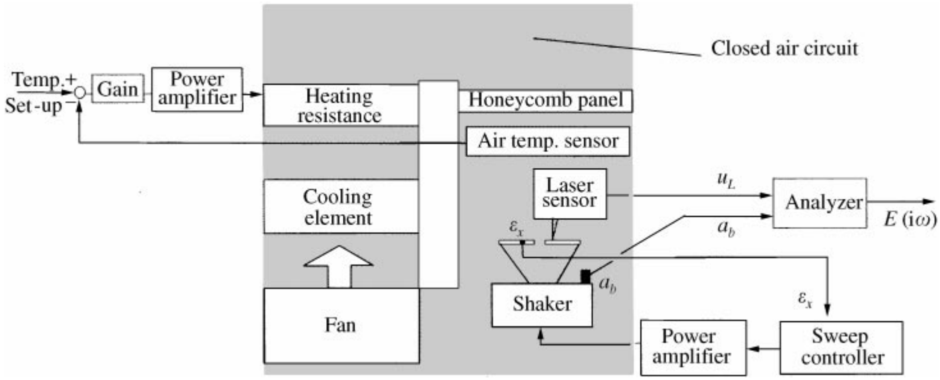

Figure 3. A scheme of the whole measurement system.

(higher temperatures could damage the specimen and the laser sensor, which is to be located right above the specimen, inside the chamber).

The measurement chain is composed of an electrodynamic shaker with a sweep controller, a specimen of the viscoelastic material mounted onto a suitable support, an accelerometer, an optical sensor (laser) for contactless measurement of displacement and a dynamic analyzer software.

The electrodynamic shaker used in a commercial MB Dynamics PM25. The sweep controller is a commercial Brüel & Kjaer 1047A which drives the electrodynamic shaker. The range of frequency on which the sweep is carried out goes from 150 to 400 Hz. For each test at any temperature, a sweep up and a sweep down are made and averaged, so as to reduce the measurement errors.

The specimen support is a cup in aluminum, adequately designed and realized, to which a cantilever beam (the specimen) is fastened by means of glue. The choice of a cantilever beam, i.e., a beam with clamped-free boundary conditions, fastened to the support, is the result of many years of tests. As a matter of fact, at the beginning the tests were carried out with the specimen set as a beam with pinned–pinned boundary conditions, exactly as one of the authors did in a previous work [15] in order to perform a direct measurement of the Poisson ratio of a viscoelastic material. However, such boundary conditions turned out to be inadequate to perform a direct measurement of the dynamic Young's modulus by using the method described in this paper, because the counteracting screws used to realize the pinned–pinned conditions introduced some frictions, which remarkably affected the experimental measures. Thus, the boundary conditions were changed and a clamped–clamped beam mounted on the cup support by means of clamps was realized. Another problem then arose, since the difference between the thermal expansion coefficient of the specimen and the one of the support caused the beam to bend at the highest test temperatures (greater than 35°C), thus basically invalidating the measurement. Hence, another configuration was tried, namely a cantilever beam fixed to the support at one end by means of a clamp, but this did not work, either, because the boundary conditions were not repeatable, depending on the torque the clamp applied to the beam end in any single test. Finally, the most suitable configuration turned out to be a cantilever beam with the fixed end glued to the support. This kind of fastening realized the clamped boundary condition with good accuracy and repeatability.

The specimen used in the tests is made of a viscoelastic material (a mixture of polypropylene and calcium carbonate), and its physical features are: 3·2 mm height, 7·15 mm, width 61·5 mm free length; 1235 kg/m^3 density (see Figure 1). The specimen is excited by the shaker, and it is subject to a bending inertia load. A piezoelectric accelerometer set on the edge of the support measures the acceleration of the base, which is required to determine Young's modulus according to equation (27). A laser sensor measured the absolute displacement of the specimen at a certain point, namely at a distance for which the second eigenmode yields a null relative displacement (see Figure 2). In this way the error made by approximating the whole eigenfunction expansion with the first eigenmode only is reduced. For a cantilever beam, the second eigenmode is zero at a distance from the fastened extremity equal to 0·7834 times the free length of the beam (see Appendix A).

The laser sensor is employed in our experimental apparatus in order to measure small displacements without contact. As a matter of fact, a major problem encountered in the past years while trying to perform accurate measurement of the complex dynamic Young's modulus was due to the fact that a strain gauge was used to measure the deformation of the specimen. However, the glue used to fasten the stain gauge to the beam caused a local stiffening of the specimen, thus making the measurement of Young's modulus inaccurate. The existence of this problem was highlighted by some bending tests, which showed that a double grid stain gauge glued to the specimen did locally increase the stiffness of the viscoelastic material by more than 8%.

Hence, a contactless sensor was used, namely a low-cost emitter–receiver laser sensor by MicroEpsilon Type OptoNCDT series 1605-2 (output power: 1 mW). Its (static) resolution is 0·5 μm on a measurement range of ± 1 mm. The measurement of this sensor is based on triangulation; the output is an analog voltage signal linearly proportional to the measured displacement in the range ± 10 V.

An accurate calibration of the laser sensor was done before the tests. The calibration procedure consisted in setting the accelerometer on the electrodynamic shaker, and the laser sensor right above it, in such a way that they measured the same dynamic entity, namely the vertical displacement of the shaker top. The output of the laser sensor was then compared with the output of the accelerometer, integrated twice, in order to determine the characteristics of the laser sensor. It was verified that the laser sensor used did not introduce any signal distortion, but just a linear phase delay ($\delta_{lin} = 6{\cdot}385 \times 10^4$ rad/Hz) and a slight constant reduction of the absolute value of the signal ($K_{red} = 0{\cdot}984$). These errors were then easily corrected by the analyser software.

A second cantilever beam of the same viscoelastic material, identical to the one tested, but instrumented with a strain gauge, was mounted onto the support. Its purpose is just to provide a feedback signal, namely the longitudinal strain of the beam, to the sweep controller that drives the shaker. The control value of the longitudinal strain was set at 20 μm/m. Other tests, carried out with a control

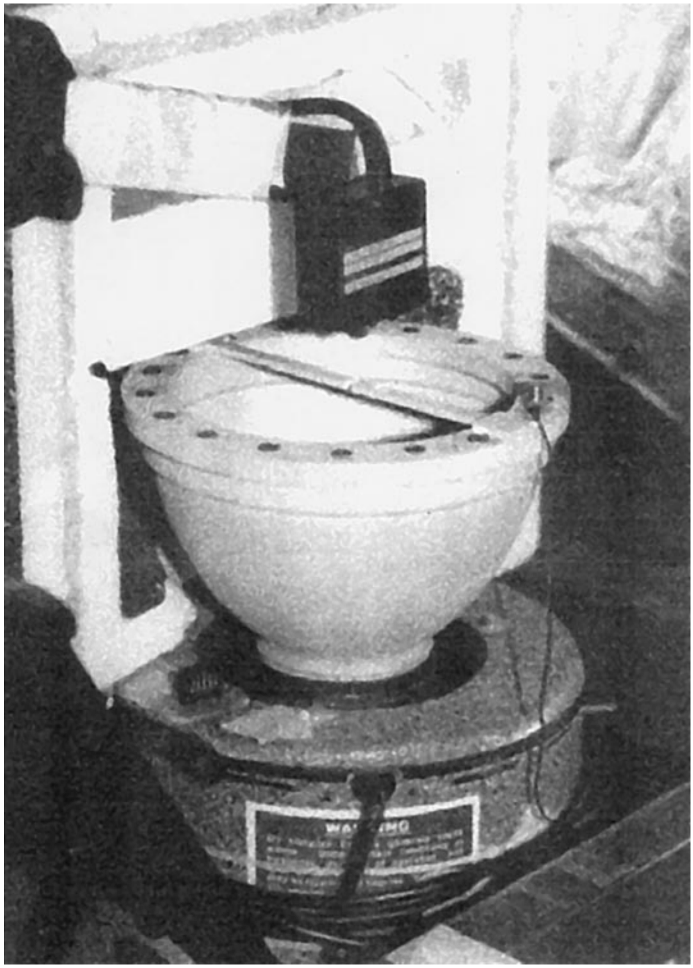

Figure 4. The experimental set-up.

values of 10 and 30 μm/m, yielded identical results, thus ensuring the linearity of the tested material.

Finally, the dynamic analyzer is a dedicated software, realized by the authors, running on a PC in a Labview environment. The measurement signals are input to the PC through a National Instrument AT-A2150 I/O board, allowing four simultaneous signals without multiplexing. Such integrals are analyzed and elaborated by the software, so as to get curves of Young's modulus at different temperatures.

Figure 4 is a picture of the experimental set-up, with the laser sensor above the specimen and the accelerometer set on the specimen support. The support is located on the shaker top, inside the temperature controlled chamber. The presence of the second identical cantilever beam fastened to the support can also be noticed.

Figure 5 and 6 show the experimental results of the measurement of Young's modulus for the tested specimen. The curves in Figure 5 represent the absolute

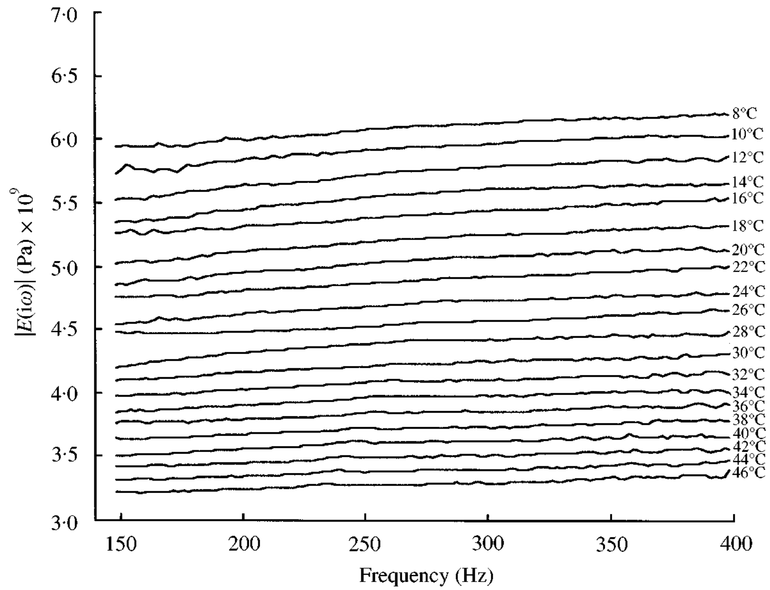

Figure 5. Absolute values of Young's modulus of a viscoelastic material at different temperatures.

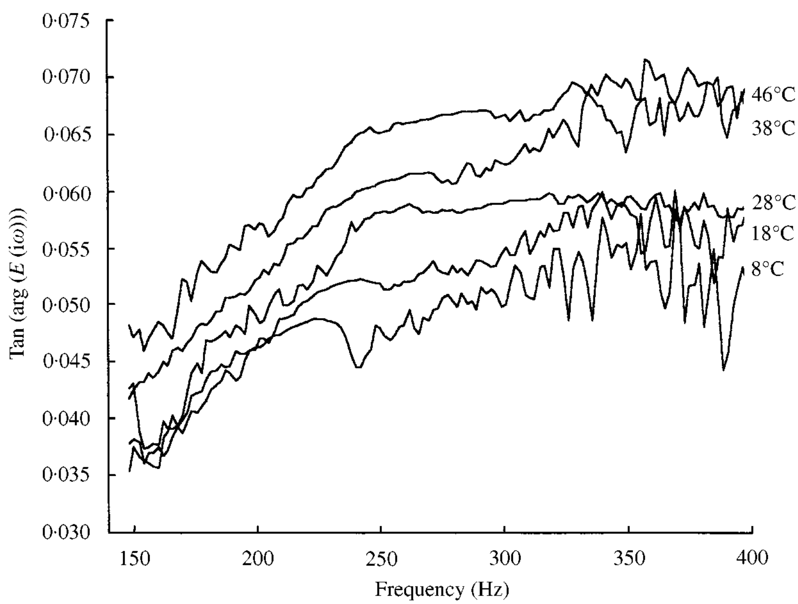

Figure 6. Loss factors of Young's modulus of a viscoelastic material at different temperatures.

values of Young's modulus for the tested material at different temperatures as a function of frequency. The test temperatures are: 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44 and 46°C. It results that the experimental data of Young's modulus are not in contrast with the theoretical behavior described for instance in reference [2]. In fact, the curves of the absolute values have higher values for lower temperatures; conversely, the curves at higher temperatures have lower values. Moreover, the absolute values of Young's modulus at all temperatures increase with frequency. The curves in Figure 6 represent the loss factors of Young's modulus for the tested material as a function of frequency. For sake of clarity, only a few curves have been reported, using a step of 10°C. It can be noticed that, for the viscoelastic material considered, the loss factors are very small, ranging from 0·035 to 0·070. The fact that the loss factor is very low also explains why some "fluctuation" is observed in the experimental curves: namely, the unavoidable measurement errors affect more the loss factors than the absolute values, the former being very small in value.

4. CONCLUSIONS

This paper has presented an approach to the measurement of the dynamic Young's modulus for a viscoelastic material by using a contactless sensor, based on a laser emitter–receiver. The proposed method consists in exciting a cantilever beam specimen by means of a seismic acceleration. The acceleration of the base is recorded by means of a piezoelectric accelerometer, and the vertical displacement of a suitable point of the specimen, in order to isolate the contribution of the fundamental mode, is recorded by means of an accurate laser sensor. Using a contactless sensor avoids introducing any perturbation due to contact that could locally change the mechanical properties of the material. This enables one to accurately determine Young's modulus as a function of frequency. Moreover, the measurements are carried out in a temperature controlled chamber, so as to obtain several curves of Young's modulus at different temperatures.

A mathematically accurate treatment of the experimental measurement of Young's modulus has also been presented, yielding a final expression for $\mathbf{E}(\mathrm{i}\omega)$ as a function of the two measured entities, namely the acceleration of the base and the vertical displacement of an adequate point of the specimen.

The experimental set-up has been described in detail, and the experimental curves of Young's modulus at different temperatures have been reported.

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APPENDIX A

Given the general expression for the eigenfunction for a beam,

$$\hat{\phi}_i(x) = \hat{C}_{i1}\sinh\left(b_i\frac{x}{L}\right) + \hat{C}_{i2}\cosh\left(b_i\frac{x}{L}\right) + \hat{C}_{i3}\sin\left(b_i\frac{x}{L}\right) + \hat{C}_{i4}\cos\left(b_i\frac{x}{L}\right), \quad \text{(A1)}$$

the first eigenfrequencies can be obtained by considering the four boundary conditions and by solving the characteristic equation associated with them. In the case of a cantilever beam, the characteristic equation is

$$\cosh(b)\cos(b) + 1 = 0, \quad \text{(A2)}$$

and its solutions are

$$b_i = 1{\cdot}8751,\ b_2 = 4{\cdot}6941,\ b_3 = 7{\cdot}8548,\ b_4 = 14{\cdot}1372. \quad \text{(A3)}$$

By arbitrarily setting: $\hat{C}_{i1} = 1$, the values of the $\hat{C}_{ij}$ coefficients can be computed, by solving the linear system

$$\begin{vmatrix} \cosh(b_i) & \sin(b_i) & \cos(b_i) \\ 0 & 1 & 0 \\ \sinh(b_i) & \cos(b_i) & -\sin(b_i) \end{vmatrix} \begin{vmatrix} \hat{C}_{i2} \\ \hat{C}_{i3} \\ \hat{C}_{i4} \end{vmatrix} = -\begin{vmatrix} \sinh(b_i) \\ 1 \\ \cosh(b_i) \end{vmatrix}. \quad \text{(A4)}$$

For the fundamental mode of vibrations, the values are

$$\hat{C}_{11} = 1,\ \hat{C}_{12} = -1{\cdot}3622,\ \hat{C}_{13} = -1,\ \hat{C}_{14} = 1{\cdot}3622.$$

Then the entities defined in section 2 can be computed, namely

$$w_1 = \sqrt{1/\int_0^1 \hat{\phi}_1^2(y)\,\mathrm{d}y} = 0{\cdot}7341, \qquad t_1 = \int_0^1 \hat{\phi}_i(y)\,\mathrm{d}y = -1{\cdot}0666. \qquad \text{(A5, A6)}$$

According to equation (A1), the distance d from the fastened end of the beam, where the laser has to be positioned, so that the second mode of vibration is null, can be obtained by the equation

$$\hat{C}_{21}\sinh\left(b_2\frac{d}{L}\right) + \hat{C}_{22}\cosh\left(b_2\frac{d}{L}\right) + \hat{C}_{23}\sin\left(b_2\frac{d}{L}\right) + \hat{C}_{24}\cos\left(b_2\frac{d}{L}\right) = 0. \quad \text{(A7)}$$

that yields: $d = 0{\cdot}7834L$, L being the free length of the beam.

Finally $r_1 = \hat{\phi}_1(d/L) = -1{\cdot}9152$, $F_1 = -12w_1^2 t_1 r_1/b_1^4 = -1{\cdot}0686$ and $F_2 = 12/b_1^4 = 0{\cdot}9707$.

Knowing the b_i coefficients also enables one to evaluate the influence of higher modes on the vibration of a cantilever beam, with respect to the fundamental mode. It is known that the amplitudes of the higher modes are inversely proportional to the squared natural frequency of the mode, or to the fourth power of the b_i coefficient of that mode.

For the second mode: $\omega_1^2/\omega_2^2 = b_1^4/b_2^4 = 2{\cdot}55\%$. However, the second vibratory mode is not present in our measurement of the beam displacement because the laser is positioned at a distance d from the clamped edge of the beam where such a mode is null, so that $\phi_2(d) = 0$.

The influence of higher modes on the beam vibration decreases very rapidly. In fact, for the third mode $\omega_1^2/\omega_3^2 = b_1^4/b_3^4 = 0{\cdot}32\%$ and for the fourth mode: $\omega_1^2/\omega_4^2 = b_1^4/b_4^4 = 0{\cdot}03\%$.