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LETTERS TO THE EDITOR

COMMENTS ON "DYNAMICS OF A TIRE–WHEEL—SUSPENSION ASSEMBLY"

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I have been studying Dr Dohrmann's excellent article [1] and have come across several inconsistencies that may need to be addressed. Respectfully, I submit that by applying Hamilton's principle to the kinetic energy, potential energy, and applied work formulations, equations (9)–(11), I cannot reproduce the author's results, specifically, the equations of motions, equations (13) and (16).

The Ω^2 terms in equations (9) and (10) do not cancel out, and should have produced a term along the lines of $\Omega^2(2w_\theta'' + w_\theta^{\mathrm{iv}} - w_\theta^{\mathrm{vi}})$ in either equation (13) or (16). In addition, the p_0 terms in equations (10) and (11) should have combined to produce a term along the lines of $p_0(4w_\theta + 2w_\theta'' - 2w_\theta^{\mathrm{vi}})$ in either equations (13) or (16). Where did these terms go?

This leads to the next question, regarding equation (10): where did the $(\rho ah\Omega^2 + p_0)(w_\theta + w_\theta''')^2$ term come from? Searching reference [2], as referenced in Dr Dohrmann's article proved fruitless. Several authors [3, 4] of similar, though non-pressurized, models include a hoop stress term due to rotation of the form, $\rho ah\Omega^2(w_\theta + w_\theta'')^2$. The difference in the number of derivatives could be due to either Dr Dohrmann's insight or to typography. The inclusion of the pressure term appears to be redundant when considering the external work term, due to pressure loading, included in equation (1). Clarification is requested.

Regarding equation (11), it should be noted that the pressure term had been included without first applying the virtual operator, or at least indicating that the variational derivative was still required of that term.

In addition, there appears to be several typographical errors. For example, in equation (13) the d_θ term should probably be f_θ and in equation (16), the $c_r\Omega_\theta'''$ term should be $c_r\Omega w_\theta^{\mathrm{iv}}$.

Note that equations (23) and beyond are consistent with equations (13) and (16) as printed in the article, with the exception of a sign in equation (23): The $-k_x m_r$ and the $-k_y m_r$ terms should be $+k_x m_r$ and $+k_y m_r$ to be consistent with the earlier equations.

I remain impressed with the depth of Dr Dohrmann's analysis, though I truly wish the fundamental equations were more clearly derived.

REFERENCES

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AUTHOR'S REPLY

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The author wishes to thank C. D. Morgan [1] for his interest in the material presented in reference [2]. It appears that three typographical errors were the source of some confusion. First, the coefficient of $(\rho a h \Omega^2 + p_0)$ in equation (10) should be $(w_\theta + w''_\theta)$ instead of $(w_\theta + w'''_\theta)$. Second, the term $(\dot{w}'_\theta + \Omega w'''_\theta)$ in equation (11) should read $(\dot{w}'_\theta + \Omega w''_\theta)$. Finally, a variational symbol δ should be inserted right after p_0 in equation (11). Once these corrections are made, equations (13–18) follow from equations (9–11) as shown below. An explanation is also provided for the internal pressure terms appearing in equations (10) and (11).

Taking the variations of equations (9–11) in reference [2] and integrating by parts yields

$$\begin{aligned}\delta T = \rho a b h \int_0^{2\pi} &\{\ddot{w}''_\theta + 2\Omega(\dot{w}'_\theta + \dot{w}'''_\theta) + 2\ddot{x}\sin\psi - 2\ddot{y}\cos\psi - \ddot{w}_\theta \\ &+ \Omega^2(w_\theta + 2w''_\theta + w^{\mathrm{IV}}_\theta)\}\,\delta w_\theta\,\mathrm{d}\psi \\ &+ \left[\rho a b h \int_0^{2\pi} \cos\psi\{\ddot{w}'_\theta + \Omega(\dot{w}_\theta + \dot{w}''_\theta) - \ddot{x} + \ddot{w}_\theta \sin\psi\}\,\mathrm{d}\psi - m_w \ddot{x}\right]\delta x \\ &+ \left[\rho a b h \int_0^{2\pi} \sin\psi[\ddot{w}'_\theta + \Omega(\dot{w}_\theta + \dot{w}''_\theta) - \ddot{y} - \ddot{w}_\theta \cos\psi\}\,\mathrm{d}\psi - m_w \ddot{y}\right]\delta y, \end{aligned} \tag{1}$$

$$\begin{aligned}\delta U = b \int_0^{2\pi} &\{-D(w''_\theta + 2w^{\mathrm{IV}}_\theta + w^{\mathrm{IV}}_\theta)/a^3 + (\rho a h \Omega^2 + p_0)(w_\theta + 2w''_\theta + w^{\mathrm{IV}}_\theta) \\ &+ a(k_\theta w_\theta - k_r w''_\theta)\}\,\delta w_\theta\,\mathrm{d}\psi + k_x x\,\delta x + k_y y\,\delta y, \end{aligned} \tag{2}$$

$$\begin{aligned}\delta W = ab \int_0^{2\pi} &\{f'_r + f_\theta + c_r(\dot{w}''_\theta + \Omega w'''_\theta) - c_\theta(\dot{w}_\theta + \Omega w'_\theta) + p_0(w_\theta + w''_\theta)/a\}\,\delta w_\theta\,\mathrm{d}\psi \\ &+ \left[ab \int_0^{2\pi} (f_r \cos\psi - f_\theta \sin\psi)\,\mathrm{d}\psi + f_x - c_x \dot{x}\right]\delta x \\ &+ \left[ab \int_0^{2\pi} (f_r \sin\psi + f_\theta \cos\psi)\,\mathrm{d}\psi + f_y - c_y \dot{y}\right]\delta y. \end{aligned} \tag{3}$$