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SYNCHRONOUS ELIMINATION OF VIBRATION IN THE PLANE, PART 1: ANALYSIS OF OCCURRENCE OF SYNCHRONOUS MOVEMENTS*

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A synchronous vibration eliminator applied to an object moving in a plane is investigated. The equation of motion of a system, which consists of a rigid object and several free rotating vibrators placed on this object, are introduced. Necessary conditions are found under which vibrators, moving synchronously with a force acting on an object, can eliminate object vibration. Computer simulation of an object and vibrators behaviour is done. Some time-runs of object vibrations and co-ordinates of vibrators positioning are shown. It was proven that in reality, elimination of vibration occurrence and vibrator movements stabilize in such a way that they move with constant velocities and phases. Simplified solutions of equation describing object movements were found, and then vibrator forces that induced vibrator movements were defined. It was found on which parameters these forces depend and how they change in relation to vibrator positions. The zero points of these forces were obtained and these points are the vibrators equilibrium positions. It was proven that by fulfilling certain conditions such positions of vibrators give full reduction of an object's vibration.

1. INTRODUCTION

Vibrations are dangerous for machines and their work. They decrease the working time, lower the quality of executed operations and at the same time, are harmful for service personnel or for people in the vicinity.

To decrease harmful vibrations we try to eliminate their causes first, for example, by balancing rotating elements. If it is not possible to eliminate these forces, then we try to decrease their effects, for example, by placing periodical forces on a larger surface of an elastic system or by increasing their acting time. It is possible to decrease the effects of dynamic loads by offsetting resonance frequencies of a system according to the frequency of the force applied [1].

Many methods of elimination or damping of vibrations in mechanical systems are known. The most popular are passive methods which are simple and in many cases give positive results. Introduction of elastic or elastic-dissipative supports is a well-known method to protect objects from vibrations [2–5]. The ratio of frequency of the excitation to the natural frequency determines the degree of vibrato-isolation. To make it as large as possible, we use extremely elastic elements in certain deformation ranges. Using a vibrato-isolator of a constant reaction force [6, 7] gave an interesting solution to the problem of reducing noise and vibration in pneumatic chisels to meet the strict standards for stroke tools.

*In memory of my wife Elzbieta

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A classical method to decrease machine vibrations is to attach an additional system. Examples of such systems are dynamic Frahm eliminators for discrete systems [2–4] and Stockbridge eliminators for continuous systems [8, 9]. For elimination of torsion vibrations we use a Taylor pendulum and Salomon external and internal shafts [2, 3, 10]. Other kinds of eliminators are striking eliminators in which free elements (metal spheres, shot, and sand) move in a closed space under the influence of vibrations [3].

To eliminate vibrations of vehicles or ships, eliminators based on the gyroscope effect are used [3, 4].

Advantages of passive elements are their simplicity, low production prices and high infallibility, but they cannot fulfill all requirements of modern construction.

Active methods in which automatic systems are applied are used more often. By measuring vibrations or forces acting on an object and having an appropriate exciter with an automatic steering system, it is possible to operate upon an object in a controlled way to minimize vibrations. They are complicated and expensive; in addition they can be unstable for certain parameters [3, 10].

The main source of vibrations in mechanical systems are unbalanced rotors. Methods of automatic balancing of rotors are described in references [11–14].

In reference [15] the author proposed a new method of vibration elimination which is based on a synchronized effect [16, 17]. An eliminator discussed in this paper reduces vibrations of an object that is moving in one direction. Additional information can be found in reference [18].

This paper has the aim of checking the possibility of application of such an eliminator for vibration damping in a single plane. By knowing the properties of such an eliminator, it is possible to eliminate vibrations of any configuration.

2. DESCRIPTION OF A SYSTEM

A rigid object is suspended on many elastic and damping elements placed in a single plane. Object vibrations are caused by replacement of the base on which the object is placed and by forces that are acting directly upon it. Such forces reduced to a point give a main vector and a main moment which acts in the plane being considered. It is necessary to generate equal but opposite forces for elimination of object vibrations.

Rotating cylinders in which there are free elements such as spheres, rollers or shots are placed on an object. They can be also inertia vibrators, this means, unbalanced rotors placed freely on a rotating axis. Their aim is to generate appropriate forces and moments, which are needed for the elimination of vibrations, and therefore, they have to be placed at different points on this object. Figure 1 shows a scheme of this system.

It was assumed that each axis of the cylinder rotates with constant velocity. Balls or rollers move relative to the cylinders without slipping. They do not hit each other and they do not detach from the race. Free element masses that will be called later as inertia vibrators are much smaller in mass than the object. It is also assumed that base vibrations are independent from object vibrations and that elastic and damping elements characteristics are linear.

3. EQUATION OF MOVEMENT

The movement of an object is examined according to stationary reference system XAY. A system of xOy co-ordinates is related to an object. A position of the O_i-axis is described by

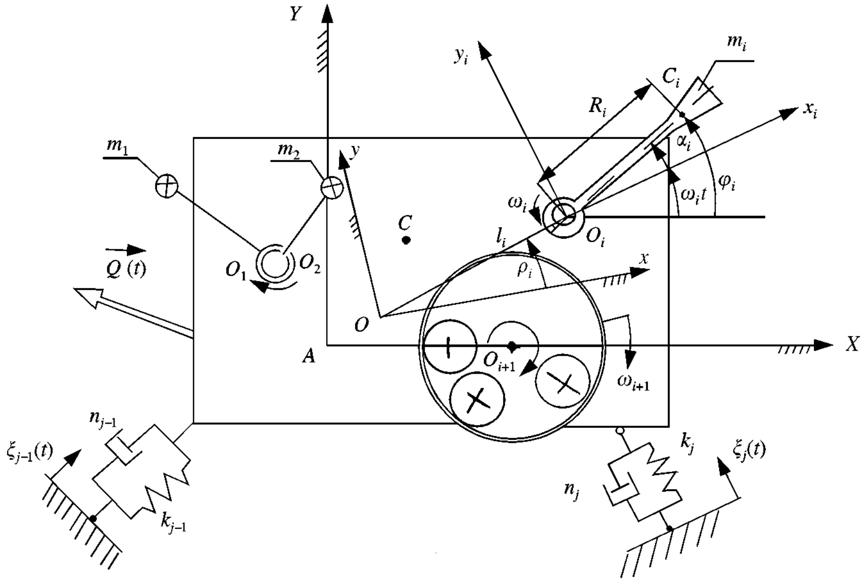

Figure 1. Object scheme with synchronous eliminators of vibrations.

co-ordinates x_i, y_i, which are calculated according to a system related to an object. Co-ordinate system $x_iO_iy_i$ is related to a rotating vibrator axis. The position of the vibrator according to such a system is described by the angle co-ordinate α_i. It is assumed that initially each vibrator has the same angular velocity as the axis on which it is placed.

The direction of axis rotation and also vibrator rotation is described by the coefficient s_i. It takes the value $+1$ or -1. If a vector of angular velocity $\boldsymbol{\omega}_i$ is in agreement with the axis Z of a dextrorotatory system ZYZ then $s_i = +1$, otherwise $s_i = -1$.

General co-ordinates for an object consist of co-ordinates x, y of a replacement of a point O and the rotation angle γ of an object around the axis Z and also the angle α_i that describes the position of a certain vibrator according to co-ordinates $x_iO_iy_i$. Object vibrations are assumed to be small.

The kinetic energy of a system is given by

$$T = T_0 + \sum_{i=1}^{N} T_i, \tag{1}$$

where T_0 is the kinetic energy of the object and T_i is the energy of the ith vibrator.

$$T_0 = 0{\cdot}5M^*(v_{xc}^2 + v_{yc}^2) + 0{\cdot}5J_{zc}\dot{\gamma}^2 \tag{2}$$

$$T_i = 0{\cdot}5m_i(v_{xi}^2 + v_{yi}^2) + 0{\cdot}5I_{ci}(\omega_i + \dot{\alpha}_i)^2 \tag{3}$$

where

$$v_{xc} = \dot{x} - \dot{\gamma}(y_c \cos\gamma + x_c \sin\gamma),$$

$$v_{yc} = \dot{y} + \dot{\gamma}(x_c \cos\gamma - y_c \sin\gamma),$$

$$v_{xi} = \dot{x} - \dot{\gamma}(y_i \cos\gamma + x_i \sin\gamma) - \dot{\varphi}_i R_i \sin\varphi_i,$$

$$v_{yi} = \dot{y} + \dot{\gamma}(x_i \cos\gamma - y_i \sin\gamma) + \dot{\varphi}_i R_i s_i \cos\varphi_i,$$

$$\varphi_i = \omega_i t + \alpha_i, \quad \dot{\varphi}_i = \omega_i + \dot{\alpha}_i.$$

If we do not take into consideration the force of gravity [15, 18], then the potential energy stored in elastic elements can be described by the relation

$$\begin{aligned} V &= 0{\cdot}5 \sum_{j=1}^{K} [(k_{xj}(x_j - \xi_{xj})^2 + k_{yj}(y_j - \xi_{yj})^2 + k_{\gamma j}(\gamma - \xi_{\gamma j})^2] \\ &= 0{\cdot}5[k_x(x - \xi_x)^2 + k_y(y - \xi_y)^2 + k_\gamma(\gamma - \xi_\gamma)^2 + 2k_{xy}(x - \xi_x)(y - \xi_y) \\ &\quad + 2k_{x\gamma}(x - \xi_x)(\gamma - \xi_\gamma) + 2k_{y\gamma}(y - \xi_y)(\gamma - \xi_\gamma)] \end{aligned} \tag{4}$$

where $k_{xj}, k_{yj}, k_{\gamma j}$ indicate stiffnesses of the jth spring in the direction of the axis of a stationary co-ordinate system, x_j, y_j are component replacements of the end of the elastic element at the point of contact with the object, K is the number of elastic elements that support the object, and ξ_x, ξ_y, ξ_γ are the component replacements of a base.

The components of the stiffness of object suspension are described by relations

$$k_x = \sum_{j=1}^{K} k_{xj}, \quad k_{x\gamma} = -\sum_{j=1}^{K} k_{xj} y_j, \quad k_{y\gamma} = \sum_{j=1}^{K} k_{yj} x_j, \quad k_\gamma = \sum_{j=1}^{K} (k_{xj} y_j^2 + k_{yj} x_j^2). \tag{5}$$

The axis position of the ith vibrator in a system xOy can be described with the help of a radius l_i and angle ρ_i that are related to axis co-ordinates x_i, y_i as

$$l_i = \sqrt{x_i^2 + y_i^2}, \quad \cos\rho_i = x_i/l_i, \quad \sin\rho_i = s_i y_i/l_i \tag{6}$$

A relation similar to equation (4) describes the energy dissipative function, but we replace $k_x, \ldots, k_\gamma$ by $n_x, \ldots, n_\gamma$, the coefficients of viscous damping, and replace the general co-ordinates of the object by their first derivatives with respect to time.

Equations of motion are obtained from Lagrange's equations and for small object vibrations they have the forms

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{n}\dot{\mathbf{q}} + \mathbf{k}\mathbf{q} = \mathbf{k}\xi + \mathbf{n}\dot{\xi} + \mathbf{Q}(t) + \sum_{i=1}^{N} \mathbf{Q}_i(t), \tag{7}$$

$$\mathbf{I}\ddot{\mathbf{q}}_w = \mathbf{B}\ddot{\mathbf{q}} - \mathbf{F}, \tag{8}$$

where $\mathbf{q}$ is the general co-ordinates matrix for the object, $\mathbf{q}_w$ the general co-ordinates matrix for vibrators, $\mathbf{M}$ the inertia matrix of the object, $\mathbf{n}$ the damping matrix for the object, $\mathbf{k}$ the suspension stiffness matrix for the object, ξ the base replacement matrix, $\mathbf{Q}$ the matrix of force acting directly on the object, $\mathbf{Q}_i$ the matrix of action on the object for the ith vibrator, $\mathbf{I}$ the inertia matrix of vibrators, $\mathbf{B}$ the matrix of position coefficients for vibrators, and $\mathbf{F}$ the matrix of general frictional forces for vibrators.

The matrices have the forms

$$\mathbf{q}=\begin{bmatrix} x \\ y \\ \gamma \end{bmatrix}, \qquad q_w=\begin{bmatrix} \alpha_1 \\ \dots \\ \alpha_N \end{bmatrix}, \qquad \mathbf{M}=\begin{bmatrix} M & 0 & -S_x \\ 0 & M & S_y \\ -S_x & S_y & J_z \end{bmatrix}, \qquad \mathbf{Q}=\begin{bmatrix} Q_x(t) \\ Q_y(t) \\ Q_\gamma(t) \end{bmatrix}, \tag{9}$$

$$\mathbf{Q}_i=\begin{bmatrix} m_i R_i(\dot{\varphi}_i^2 \cos\varphi_i + \ddot{\varphi}_i \sin\varphi_i) \\ m_i R_i s_i(\dot{\varphi}_i^2 \sin\varphi_i - \ddot{\varphi}_i \cos\varphi_i) \\ m_i R_i l_i[\dot{\varphi}_i^2 \sin(\varphi_i-\rho_i) - \ddot{\varphi}_i \cos(\varphi_i-\rho_i)] \end{bmatrix}, \tag{10}$$

where $M = M^* + \sum_{i=1}^{N} m_i$, $S_x = M^* y_c + \sum_{i=1}^{N} m_i y_i$, $S_y = M^* x_c + \sum_{i=1}^{N} m_i x_i$, and J_z is mass moment of inertia of a system with respect to the z-axis. Also,

$$\mathbf{B}=\begin{bmatrix} \mathbf{B}_1 \\ \dots \\ \mathbf{B}_N \end{bmatrix}=\begin{bmatrix} m_1R_1\sin\varphi_1 & -m_1R_1s_1\cos\varphi_1 & -m_1R_1l_1\cos(\varphi_1-\rho_1) \\ \dots & \dots & \dots \\ m_NR_N\sin\varphi_N & -m_NR_Ns_N\cos_N & -m_NR_Nl_N\cos(\varphi_N-\rho_N) \end{bmatrix}. \tag{11}$$

The resistance moment which acts against the movement of the ith vibrator can be shown to be the sum of three components,

$$F_i = F_{iw} + F_{it} + F_{is}, \tag{12}$$

where F_{iw} is the viscous resistance proportional to instantaneous vibrator velocity in relation to a cylinder or an axis on which the vibrator is placed, F_{it} the frictional moment proportional to the reactive force of the race or axis on the vibrator, $F_{iw} = n_i R^2 \dot{\alpha}_i$, $F_{it} = N_i R_i f_i / r_i \operatorname{sgn}(\dot{\alpha}_i)$, F_{is} the motor torque in the case of directional positioning of vibrator on its axis, $F_{is} \cong n_{is}(\dot{\varphi}_{is} - \dot{\varphi}_i)$ for an asynchronous motor, $N_i \cong m_i R_i^2 \dot{\varphi}_i^2$ for the normal reaction of the ith race, n_i, f_i the viscous resistance and vibrator rolling resistance coefficients, $\dot{\varphi}_{is}$ the angular velocity of a motor magnetic field, and $\varphi_i = \omega_i t + \alpha_i$.

The origin of the co-ordinate system xOy relative to the object can be chosen in such a way that static moments S_x, S_y become zero. The origin of the co-ordinate systems becomes the center of mass of a system consisting of an object and N vibrators. Then the inertia matrix of an object $\mathbf{M}$ becomes diagonal. Additionally, the co-ordinate system xOy can be rotated by an angle so that $k_{xy} = 0$.

By taking into account the fact that general co-ordinates $\alpha_i(t)$ change slowly in time, we can simplify the form of the matrix of a vibrator acting on an object [15, 18]

$$\mathbf{Q}_i \cong \begin{bmatrix} m_i R_i(\omega_i + \dot{\alpha}_i)^2 \cos(\omega_i t + \alpha_i) \\ m_i R_i s_i(\omega_i + \dot{\alpha}_i)^2 \sin(\omega_i t + \alpha_i) \\ m_i R_i l_i(\omega_i + \alpha_i)^2 \sin(\omega_i t + \alpha_i - \rho_i) \end{bmatrix}. \tag{13}$$

Equations (7) describe small vibrations of an object in relation to a static balancing position. This is a linear system of differential equations of second degree. Equations (8) describe movements of certain vibrators in relation to an axis or a cylinder on which they are placed. This is a strongly non-linear system of differential equations.

4. CONDITIONS NECESSARY TO ELIMINATE VIBRATIONS

Before starting to analyze system movement equations (7) and (8), it is necessary to know the conditions which should be fulfilled in the case where vibrators really could eliminate object vibrations. Subsequent solutions of movement equations can be compared with those conditions. It is obvious that solutions of equations (7) tend to zero only when the right-hand sides of those equations become zero. Such a possibility takes place when vibrators rotate with an absolute velocity appropriate to the frequency of the force. It is assumed that the dynamic load is mono-harmonic. For poly-harmonic forces it would be necessary to devote some vibrators for certain frequencies of force.

Therefore, the vibrator movements should be described by the relations

$$\alpha_i(t) = (\Omega - \omega_i)\,t + \delta_i, \tag{14}$$

where Ω is the circular frequency of dynamic load, ω_i the angular velocity of an axis or a cylinder and δ_i the movement phase of a vibrator.

All forces have the same frequency, so

$$\xi(t) = \xi_0 \cos(\Omega t + \beta_\xi), \quad \mathbf{Q}(t) = \mathbf{Q}_0 \cos(\Omega t + \beta_Q).$$

The kinematic mono-harmonic excitation, which exists on the right-hand side of equation (7), can be presented in the form

$$\mathbf{k}'\xi + \mathbf{n}\dot{\xi} = \mathbf{k}\xi_0 \cos(\Omega t + \beta), \tag{15}$$

where, for example,

$$k_x = \sqrt{(k'_x)^2 + (n_x\Omega)^2}, \quad \beta_x = \beta'_x + \arctan(n_x\Omega/k_x).$$

Combining equation (15) with the force applied directly to the object, we get

$$\mathbf{Q} + \mathbf{k}\xi + \mathbf{n}\dot{\xi} = \mathbf{Q}_c^Q \cos\Omega t - \mathbf{Q}_s^Q \sin\Omega t. \tag{16}$$

For movement of the vibrators as described by equation (14), the action on the object also can be presented as

$$\sum_{i=1}^{N} \mathbf{Q}_i = \mathbf{Q}_c^w \cos\Omega t - \mathbf{Q}_s^w \sin\Omega t. \tag{17}$$

The right-hand side of equation (7) acquires the form

$$\mathbf{k}\xi + \mathbf{n}\dot{\xi} + \mathbf{Q} + \sum_{i=1}^{\mathrm{N}} \mathbf{Q}_i = (\mathbf{Q}_c^Q + \mathbf{Q}_c^w)\cos\Omega t - (\mathbf{Q}_s^Q + \mathbf{Q}_s^w)\sin\Omega t$$

$$= \mathbf{Q}_c \cos\Omega t - \mathbf{Q}_s \sin\Omega t. \tag{18}$$

$$\mathbf{Q}_c^Q = \begin{bmatrix} Q_x \cos\beta_{qx} + k_x \cos\beta_x + k_{x\gamma}\cos\beta_\gamma \\ Q_y \cos\beta_{qy} + k_y \cos\beta_y + k_{y\gamma}\cos\beta_\gamma \\ Q_\gamma \cos\beta_{q\gamma} + k_\gamma \cos\beta_\gamma + k_{x\gamma}\cos\beta_x + k_{y\gamma}\cos\beta_y \end{bmatrix},$$

$$\mathbf{Q}_c^w = \begin{bmatrix} \sum_{i=1}^{N} m_i R_i \Omega^2 \cos\delta_i \\ -\sum_{i=1}^{N} m_i s_i R_i \Omega^2 \sin\delta_i \\ -\sum_{i=1}^{N} m_i l_i R_i \sin(\delta_i - \rho_i) \end{bmatrix}, \qquad \mathbf{Q}_s^w = \begin{bmatrix} \sum_{i=1}^{N} m_i R_i \Omega^2 \sin\delta_i \\ -\sum_{i=1}^{N} m_i s_i R_i \Omega^2 \cos\delta_i \\ -\sum_{i=1}^{N} m_i l_i R_i \cos(\delta_i - \rho_i) \end{bmatrix}. \tag{19}$$

The equation (7) becomes autonomous for the phases of movement of the vibrators $\delta_{1kt}, \ldots, \delta_{Nkt}$ and then

$$\mathbf{Q}_c(\delta_{1kt}, \ldots, \delta_{Nkt}) = 0, \qquad \mathbf{Q}_s(\delta_{1kt}, \ldots, \delta_{Nkt}) = 0. \tag{20}$$

From this we can conclude that for full reduction of object vibrations it is necessary to fulfill the six conditions (20).

Therefore, both things are important, not only the final arrangements described by phases δ_{ikt} but also their positioning on the object and their directions of rotation.

For example, for the force acting only in the X-axis direction and for suspension of an object in such a way that $k_{x\gamma} = k_{y\gamma} = 0$, it is possible to apply at least two vibrators (Figure 2(a)) rotating in opposite directions. Their static moments have to fulfill the condition

$$m_1 R_1 = m_2 R_2 = 0{\cdot}5 k'_x \xi_{0x} / \Omega^2.$$

The final arrangements of vibrators are described by angles

$$\delta_{1kt} = \delta_{2kt} = \pi.$$

The vibrator axes have to be positioned symmetrically with respect to the x-axis.

For kinematic excitation $\xi_\gamma(t) = \xi_{0\gamma} \cos \Omega t$, we need at least two vibrators (Figure 2(b)) rotating in the same direction. Their static moments have to be the same,

$$m_1 R_1 = m_2 R_2 = k_\gamma \xi_{0\gamma} / [(l_1 + l_2)\, \Omega^2],$$

and the final phases must be

$$\delta_{1kt} = \pi/2, \qquad \delta_{2ky} = -\pi/2.$$

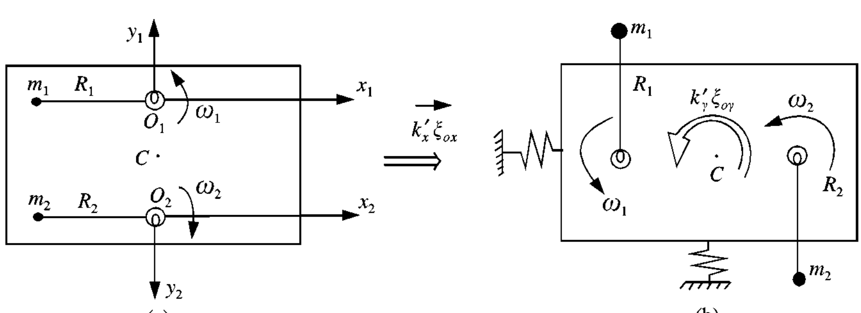

Figure 2. Required setting of vibrators according to (a) excitation in X-axis direction, (b) moment excitation.

If two kinematic excitations act in principal directions and they are matched in such a way, that $k_x\xi_{0x} = k_y\xi_{0y}$ with mutual phase replacements $\beta_x - \beta_y = \pi/2$, then the resultant force acting on the object has a constant value and rotates with a constant velocity according to the z-axis. In this case a single vibrator rotating in the same direction can compensate for vibrations of the object. Its static moment has to have the value $mR = k_x\xi_{0x}/\Omega^2$ and its final arrangement is $\delta_{1kt} = \pi$ (Figure 3(a)).

For the same forces but without mutual phase replacement $\beta_x - \beta_y = 0$, a resultant force has a constant direction and a changing value. It is necessary to use at least two vibrators (Figure 3(b)) rotating in opposite directions. Their static moment should satisfy the condition $m_1R_1 = m_2R_2 = 0{\cdot}5k_x\xi_{0x}/\Omega^2$, and the final phases $\delta_{1kt} = 5\pi/4$, $\delta_{2kt} = 3\pi/4$.

Figure 4(a) presents the positioning of vibrators that is needed in the case of vibration coupling $k_{xy} \neq 0$. Object vibrations can be compensated using four vibrators. Two of them compensate for a force acting in the X direction and the other two generate a moment that is in anti-phase to the moment $k_{xy}\xi_{0x}$. The final phases are $\delta_{1kt} = \delta_{2kt} = \pi$, $\delta_{3kt} = \pi/2$, $\delta_{4kt} = -\pi/2$.

Unfortunately, the examples of using one or two vibrators presented here are successful only for certain parameters of excitation. For example, a change of base vibration amplitude can unbalance the system. By using a larger number of vibrators it is possible to avoid this inconvenience.

The case of a single force in the X direction that is compensated for by four vibrators of the same static moment was shown in Figure 4(b). Vibrators are able to balance a force in the range $k'_x\xi_{0x} \in \langle 0, 4mR\Omega^2\rangle$.

The vibrators that are moving synchronously with force are able to eliminate object vibrations if their static moment relating to the rotating axis, final phases δ_{ikt}, rotating directions and vibrators positioning on an object, fulfill conditions (20).

The relation, which describes their movements, has the form

$$\alpha_i(t) = (\Omega - \omega)t + \delta_{ikt}. \tag{21}$$

5. NUMERICAL SOLUTIONS

Deliberations mentioned in the previous section had as their aim to check conditions that should be fulfilled in order to completely eliminate the vibrations of an object under forces $\mathbf{Q}(t)$ and kinematic excitation $\xi(t)$. It is necessary to show that this happens in reality.

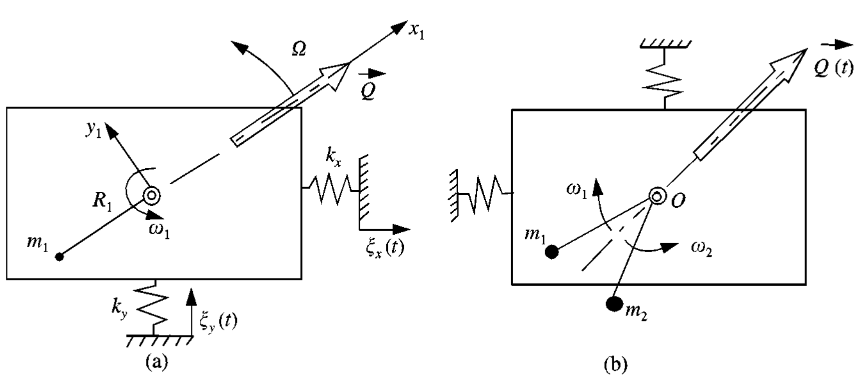

Figure 3. Required setting of vibrators according to (a) rotating constant excitation, (b) excitation with constant direction.

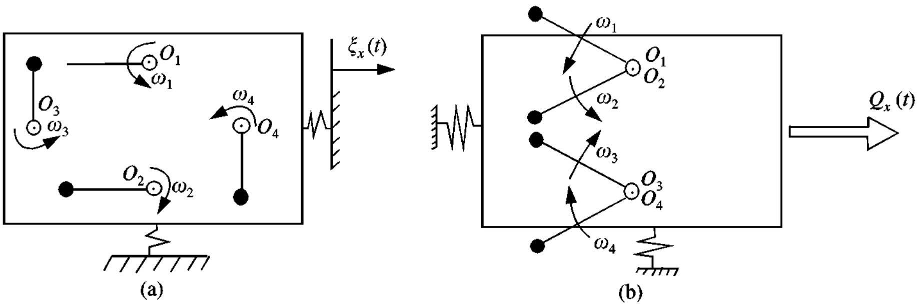

Figure 4. Required setting of vibrators according to (a) vibrators positioning in the case of vibrations coupling, (b) single excitation compensated by four vibrators.

Therefore, it is important to solve equations (7, 8) and show that their solutions satisfy

$$\lim_{t\to 0}\mathbf{q}_w(t) = (\Omega - \omega_i)\,t + \delta_{kt},\ \lim_{t\to 0}\mathbf{q}(t) = \mathbf{0}. \tag{22}$$

The equations of motion are very complicated and it is impossible to find their solutions analytically. Therefore, it is necessary to use numerical methods. We introduce a parameter $\tau = \Omega t$ and solve the equation system (7, 8). Many data sets were tested for different forces, different number of vibrators and their positioning on an object. Many excitation frequencies were also tested. Examples of the results of these calculations are presented in Figures 5–8.

First, Figures 5 and 6 show the kinematic excitation case matched in such a way that amplitudes of both forces have the same value in both directions, $k_x\xi_{ox} = k_y\xi_{oy}$, and these are the main directions of suspension, this means that $k_{x\gamma} = k_{y\gamma} = 0$. Force phases are matched in such a way ($\beta_x - \beta_y = \pi/2$) that the resultant force rotates in the same direction as the Z-axis.

Figure 5 presents the behaviour of vibrators and the object when the force frequency is the same as the initial vibrator velocity and is smaller than the natural frequency of the object. In this case, vibrators, instead of eliminating vibration, cause their amplification when they are placed almost in phase with the excitation. On the other hand, for $\Omega, \omega_i > \omega_{ox}, \omega_{oy}$, vibrators set in positions δ_{1kt}, δ_{2kt} cause vibrations of the object to disappear.

Figure 6 presents the same excitation case but here its frequency is not in agreement with the vibrator initial velocity and is larger than the object's natural frequency. In this case, the vibrator tries to keep up with the force and becomes anti-phase to it and causes the object vibration to disappear.

For excitation in the form of a pair of forces, it is necessary to apply at least two vibrators rotating in the same direction (Figure 7). If their static moment fulfills conditions (20) and the force frequency is larger than the system's natural frequencies, the vibrators are able to compensate vibrations. Their behaviour from the initial appearance of the force is presented in Figure 7. Vibrators tend to final positions that are described by phases δ_{1kt}, δ_{2kt} according to relation (20).

The graphs in Figure 8 refer to the case where excitation acts in a constant direction — $\xi_y(t) = \xi_{oy}\cos\Omega t$. Two similar vibrators placed symmetrically around the Y-axis were used. Only rolling resistance exists. We have vibration elimination. Vibrators placed randomly at the beginning tend to acquire positions $\delta_{1kt} = -\pi/2$, $\delta_{2kt} = \pi/2$.

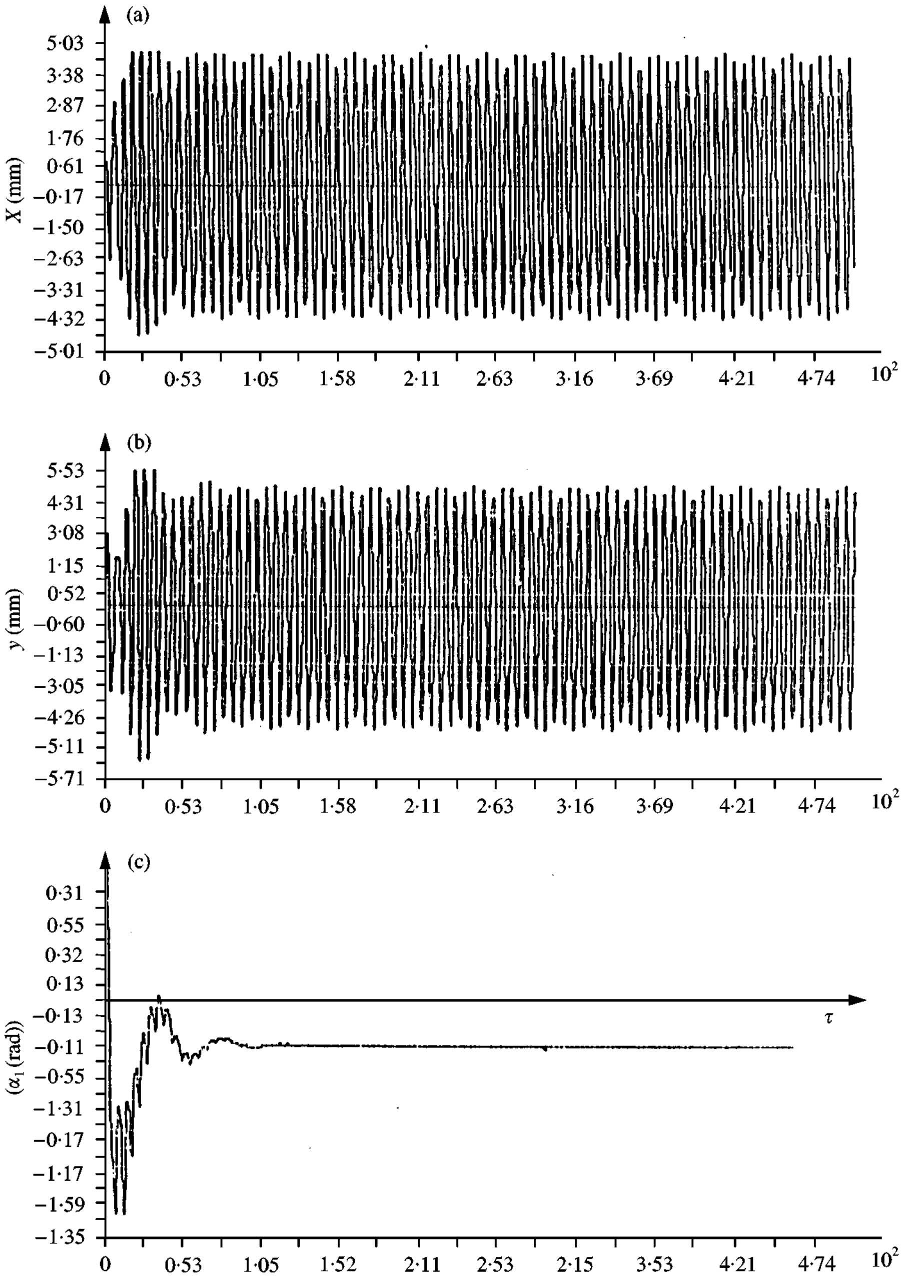

Figure 5. Behaviour of an object and vibrator for $\omega = \Omega < \omega_{ox}, \omega_{oy}$, $\xi_{ox} = \xi_{oy} = 1$ mm, $\beta_x = \beta_y = \pi/2$, $\Omega = 30$ rad/s, $m = 0{\cdot}22$ kg, $R = 0{\cdot}03$ m; (a, b) component vibrations of object $x(\tau)$ and $y(\tau)$; (c) position of vibrator $\alpha_1(\tau)$.

If coupling between certain directions exists, then during appearance of kinematic excitation, vibrations in another direction, Y for example, also appear. If it is possible to fulfill the conditions (20) and the force frequency and velocity of the vibrators is greater than the natural frequencies of the object, then the vibrators eliminate vibrations.

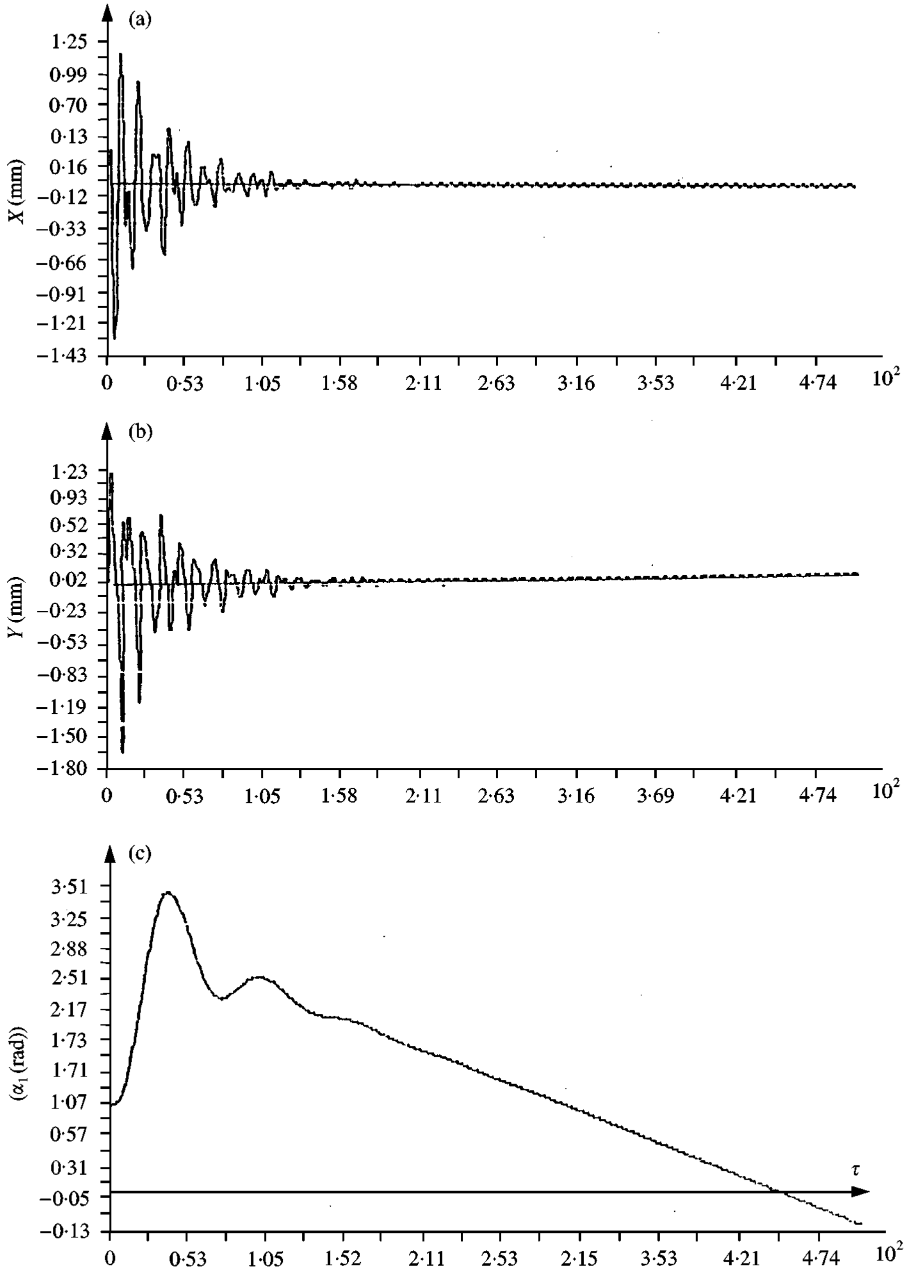

Figure 6. Behaviour of an object and vibrator for $\omega \neq \Omega > \omega_{ox}, \omega_{oy}$ $\xi_{ox} = \xi_{oy} = 1$ mm, $\omega - \Omega = 0{\cdot}5$ rad/s, $\Omega/\omega_{ox} = 1{\cdot}54$, $n_1 = 0{\cdot}2$ kg/s (a, b) component vibrations of object $x(\tau)$ and $y(\tau)$, (c) position of vibrator α_1.

From computer simulations that were carried out, we can state that for different forces the vibrators are able to organize themselves in such a way that they can eliminate vibrations when the forcing frequency is greater than the natural frequency of the object and the initial difference between excitation frequency and initial vibrator's velocity is not too large. In addition, if the motion resistance of vibrators is not too great, they move according to relation (14) and the final phases of the positions are close to values that result from relation (20).

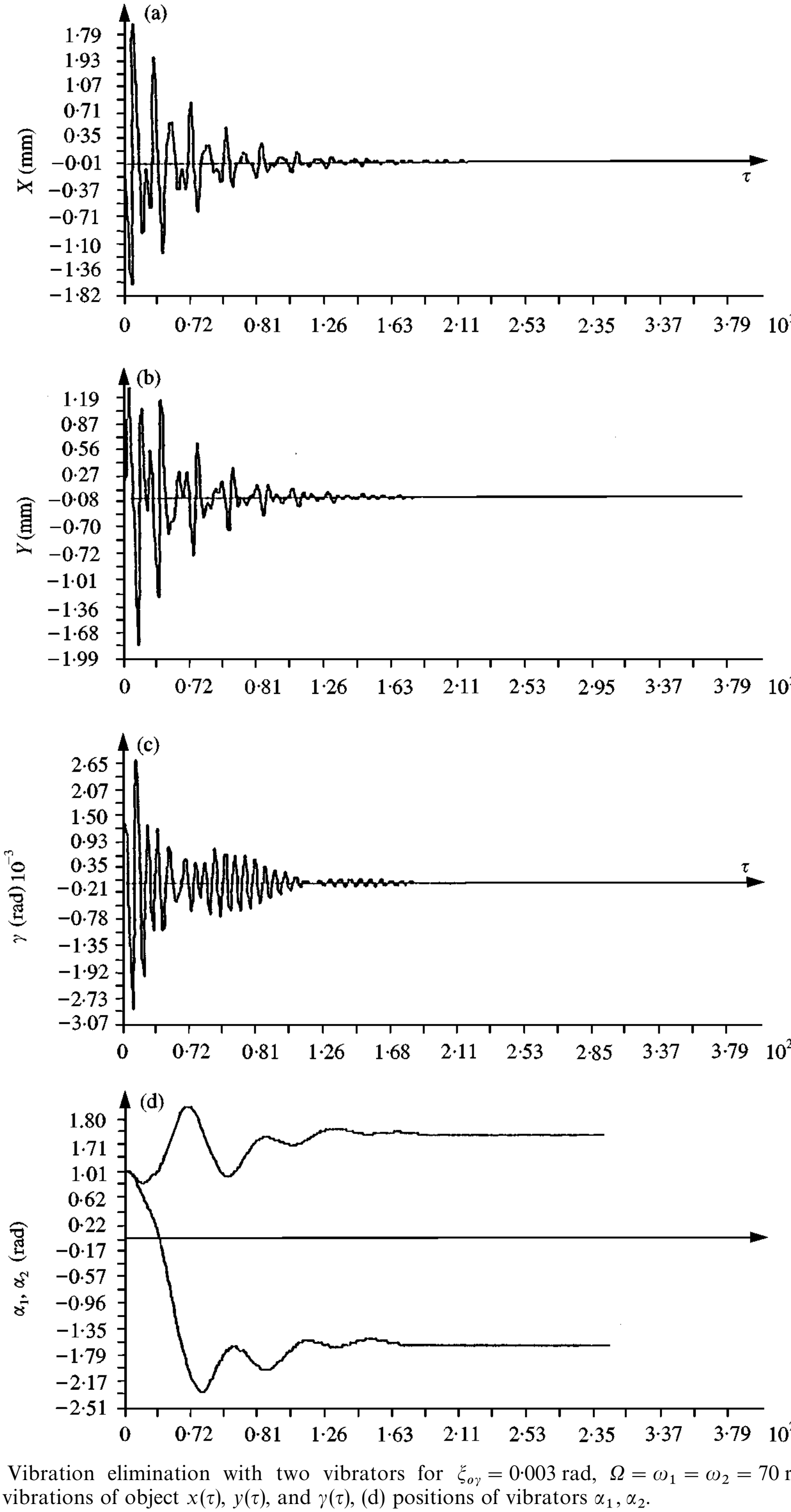

Figure 7. Vibration elimination with two vibrators for $\xi_{o\gamma} = 0{\cdot}003$ rad, $\Omega = \omega_1 = \omega_2 = 70$ rad/s; (a, b, c) component vibrations of object $x(\tau)$, $y(\tau)$, and $\gamma(\tau)$, (d) positions of vibrators α_1, α_2.

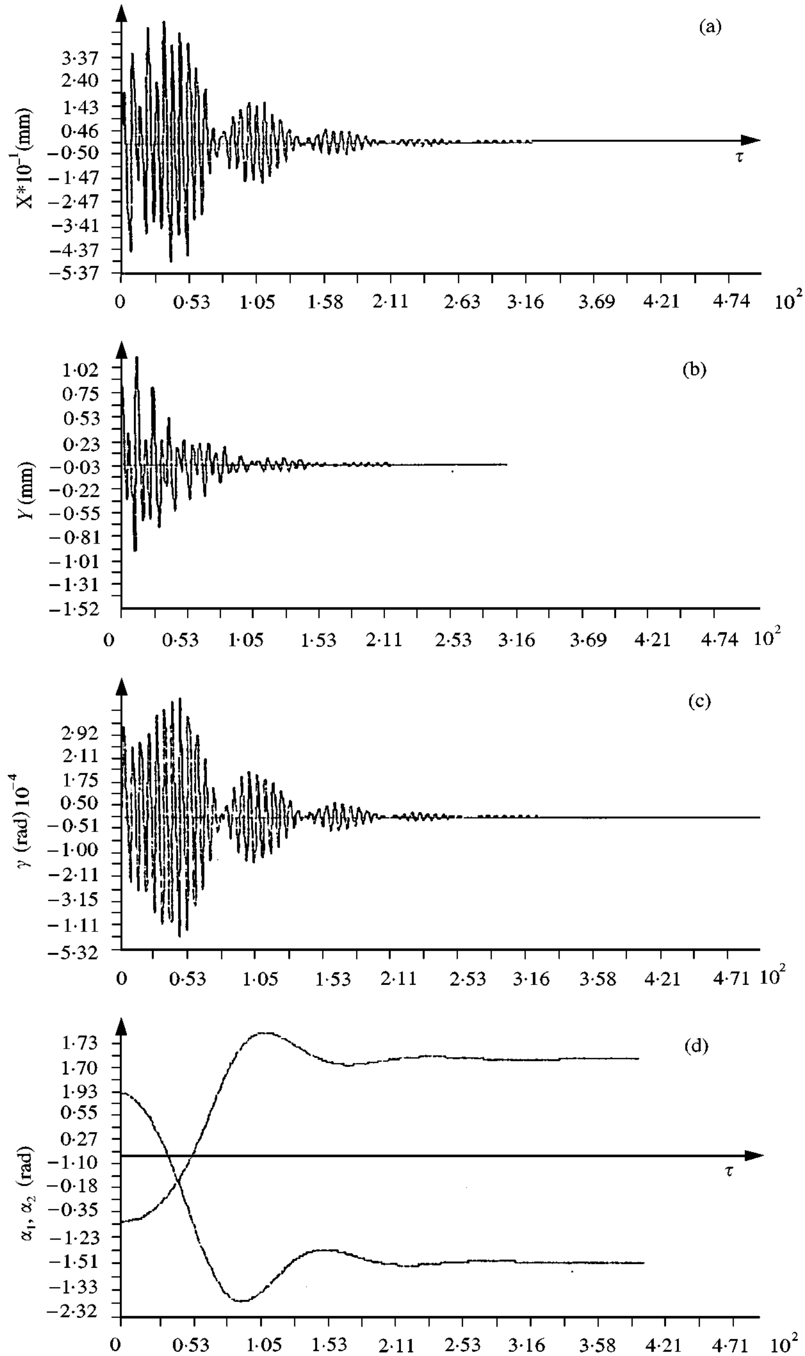

Figure 8. Vibration elimination with two vibrators if $\xi_{oy} = 1$ mm, $\Omega = \omega_1 = \omega_2 = 70$ rad/s (a, b, c) component vibrations of object $x(t\tau)$, $y(\tau)$, and $\gamma(\tau)$, (d) positions of vibrators α_1, α_2.

6. ANALYSIS OF SYNCHRONOUS MOVEMENTS

Vibrations depend on stabilized movements of vibrators. The frequency of vibration of the vibrator is much smaller than the frequency of vibration of the object. The movements of the vibrators are similar to the movements of a fixed inertial system subjected to a force that acts like a step function. By taking into consideration the fact that accelerations of vibrators are very small, their action on an object can be put in the form (13). With this simplification and the fact that equations (7) are linear with respect to the general co-ordinates of the object, we can anticipate their solutions as a sum of fixed vibrations from the forces acting on the object and the centrifugal forces of the vibrators:

$$\mathbf{q}(t) = \mathbf{a}_0 \cos(\Omega t + \beta - \varphi) + \sum_{i=1}^{N} \mathbf{a}_i \cos(\omega_i t + \alpha_i - \varphi_i). \tag{23}$$

When the behaviour of the movements of the vibrators that are synchronous with the vibrations of the base satisfies equations (14), then

$$\mathbf{q}(t) = \mathbf{a}_0 \cos(\Omega t + \beta - \varphi) + \sum_{i=1}^{N} \mathbf{a}_i \cos(\Omega t + \delta_i - \varphi_i), \tag{24}$$

where $\mathbf{a}_0^{\mathrm{T}} = [a_{ox}, a_{oy}, a_{o\gamma}]$ are the amplitudes of object vibrations from the forces acting on it, $\mathbf{a}_i^{\mathrm{T}} = [a_{ix}, a_{iy}, a_{i\gamma}]$ the amplitudes of object vibrations caused by the ith vibrator, $\varphi_i^{\mathrm{T}} = [\varphi_x, \varphi_y, \varphi_\gamma - \rho_i]$ the $\varphi_x, \varphi_y, \varphi_\gamma$ phase replacements of component vibrations according to the excitation.

Vibration amplitudes depend on the values of the acting forces, on the ratio of frequency of acting forces to the natural frequencies of the object and on its damping. Phases φ depend on the last two conditions. If directions x, y, and γ are the main directions of elastic suspension, then the relation between amplitude and phase replacements have a simple form.

For example, vibrations in the direction of the y-axis and rotating vibrations have the forms

$$a_{oy} = \frac{k'_x \xi_{ox}}{M\sqrt{(\omega_{oy}^2 - \Omega^2)^2 + (n_y \Omega/M)^2}}, \quad \varphi_y = \arctan(n_y \Omega/(M(\omega_{oy}^2 - \Omega^2))),$$

$$a_{i\gamma} = \frac{m_i R_i l_i \Omega^2}{I_z\sqrt{(\omega_{o\gamma}^2 - \Omega^2)^2 + (n_\gamma \Omega/I_z)^2}}, \quad \varphi_\gamma = \arctan(n_\gamma \Omega/(I_z(\omega_{o\gamma}^2 - \Omega^2))).$$

The natural frequencies $\omega_{ox}, \omega_{oy}, \omega_{o\gamma}$ in this case describe the relation

$$\omega_{ox} = \sqrt{k_x/M}, \quad \omega_{oy} = \sqrt{k_y/M}, \quad \omega_{o\gamma} = \sqrt{k_\gamma/I_z}.$$

If the object vibrations are known, then it is possible to describe the force P'_i acting on the ith vibrator

$$P'_i = m_i R_i [\ddot{x} \cos(\Omega t + \delta_i) - s_i \ddot{y} \sin(\Omega t + \delta_i) - l_i \ddot{\gamma} \sin(\Omega t + \delta_i - \rho_i)]. \tag{25}$$

After substituting relation (24) into equation (25), we can get a relation of generalized force $\mathbf{P}'$ depending on the external force, and all vibrators' positions and time. The behaviour of the system depends mainly on the average value of this force [11–15]:

$$\mathbf{P} = \frac{1}{T} \int_0^{\mathrm{T}} \mathbf{P}' \, \mathrm{d}t. \tag{26}$$

After making the above calculation, we get

$$P_i = -0{\cdot}5 m_i R_i \Omega^2 \left\{ \left[a_{ox} \sin(\delta_i - \beta_x + \varphi_x) + \sum_{j=1}^{N} a_{jx} \sin(\delta_i - \delta_j + \varphi_x) \right] \right.$$
$$+ s_i \left[-a_{oy} \cos(\delta_i - \delta_j + \varphi_y) + \sum_{j=1}^{N} a_{jy} s_j \sin(\delta_i - \delta_j + \varphi_y) \right]$$
$$\left. + l_i \left[-a_{o\gamma} \cos(\delta_i - \rho_i - \beta_\gamma + \varphi_\gamma) + \sum_{j=1}^{N} a_{j\gamma} \sin(\delta_i - \delta_j - \rho_i + \rho_j + \varphi_\gamma) \right] \right\} \quad (27)$$

The relation mentioned above can be written in the form

$$P_i = \left(P_{iox} + \sum_{j=1}^{N} P_{ijx} \right) + \left(P_{ioy} + \sum_{j=1}^{N} P_{ijy} \right) + \left(P_{io\gamma} + \sum_{j=1}^{N} P_{ij\gamma} \right), \quad (28)$$

where

$$P_{iox} = -0{\cdot}5 m_i R_i \Omega^2 a_{ox} \sin(\delta_i - \beta_x + \varphi_x),$$

$$P_{ijx} = -0{\cdot}5 m_i R_i \Omega^2 a_{jx} \sin(\delta_i - \delta_j + \varphi_x),$$

$$\ldots$$

$$P_{ij\gamma} = -0{\cdot}5 m_i R_i \Omega^2 l_i a_{j\gamma} \sin(\delta_i - \delta_j - \rho_i + \rho_j + \varphi_\gamma).$$

According to relation (28), the vibration force acting on the ith vibrator is a sum of forces coming from component vibrations $x(t)$, $y(t)$, $\gamma(t)$ of the object. It is much greater than the vibrating force of an object with one degree of freedom [15]. The vibrations of the object result from the forces acting directly on it and from the actions of all vibrators. Every vibrator acts on the ith vibrator, which is represented by a member P_{ij}, and the value of this force depends on mutual phase positions of these vibrators, $\delta_i - \delta_j$. The vibration force component $P_{i\gamma}$ coming from rotational vibrations depends also on the distance from an axis of this vibrator to the centre of mass of the system l_i and the phase angle ρ_i according to the system xOy associated with an object. The ith vibrator acts on itself with a force

$$P_{ii} = -0{\cdot}5 m_i R_i [a_{ix} \sin \varphi_x + a_{iy} \sin \varphi_y + l_i a_{i\gamma} \sin \varphi_\gamma]. \quad (29)$$

This moment is always opposite to the motion of the vibrator.

Figure 9(a) presents the change in the generalized vibration force as a function of the position of a single vibrator and angle velocity from Figure 3(a). If the excitation frequency is larger than both natural vibration frequencies ω_{ox}, ω_{oy}, the vibration force components P_x, P_y are additive. At the point where $\delta_{kt} = \pm\pi$, the force P has value zero and the derivative of this force with respect to phase δ is negative. This means that the vibrator position is stable. For frequency $\Omega \in (\omega_{ox}, \omega_{oy})$, the force components P_{ix}, P_{iy} have opposite signs and they are subtractive (assuming $\omega_{ox} < \omega_{oy}$). In this case the slope of the graph of $P(\delta)$ depends on whether Ω is closer to ω_{ox} or to ω_{oy}. Depending on which of those components dominate we can state whether the vibrator compensates vibrations of the object or increases them. Charts in Figure 9(b) represent an object with a large damping effect, $\varepsilon_x = \varepsilon_y = 0{\cdot}5$. Increases in damping of the object result in a shift of the vibration force

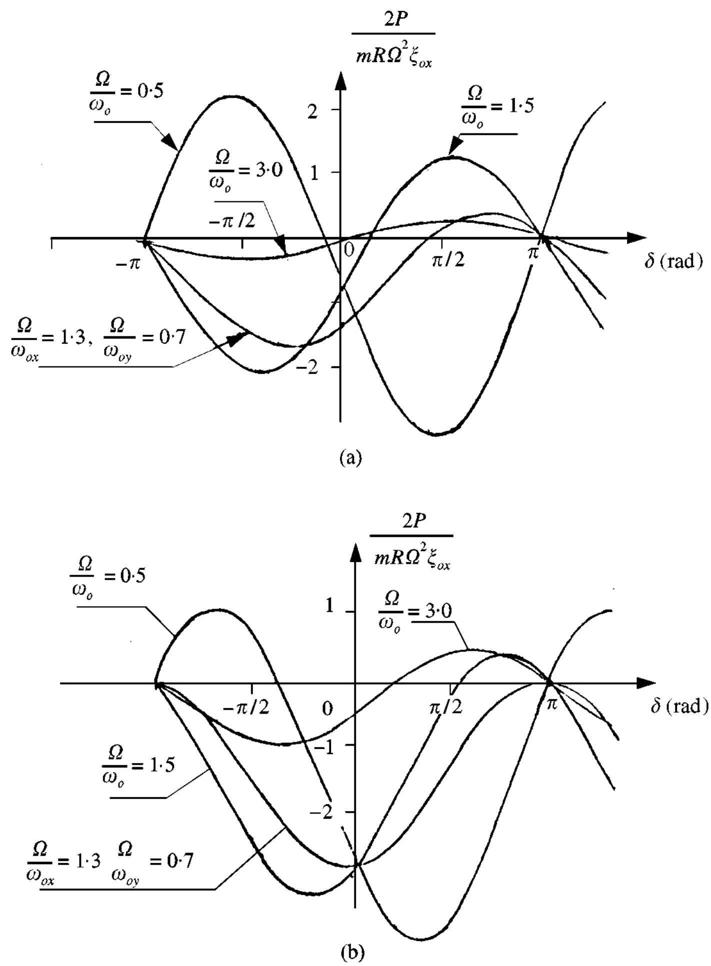

Figure 9. Change of vibration moment in function of vibrator positioning for (a) object damping $\varepsilon_x = \varepsilon_y = 0{\cdot}1$, (b) object damping $\varepsilon_x = \varepsilon_y = 0{\cdot}5$.

in the direction of negative values of the P force and, additionally, it is shifted in the horizontal direction of positive values δ. For frequencies $\Omega < \omega_{ox}, \omega_{oy}$, the characteristics of P are qualitatively different than for $\Omega > \omega_{ox}, \omega_{oy}$.

If vibration force components P_x, P_y have opposite signs, the resultant force is small, the time required for the vibrator to arrive in final position is greater, and the significance of the friction of the vibrator is greater. If movement friction is greater than the vibration forces, then vibration will not be eliminated, as discussed earlier.

For an excitation with frequency $\Omega = 0{\cdot}5\omega_{ox} = 0{\cdot}5\omega_{oy}$, the vibrator equilibrium position is described by the angle $-\pi/12$ for object damping $\varepsilon_x = \varepsilon_y = 0{\cdot}1$ and by the angle $-\pi/3$ for $\varepsilon_x = \varepsilon_y = 0{\cdot}5$ (Figure 9):

$$P_i(\delta_{1kt}, \ldots, \delta_{Nkt}) = 0, \quad i = 1, \ldots, N. \tag{30}$$

The system of equations describes the position of the vibrators δ_{ikt} for which all forces P_i become zero.

After transformations the relation (27) of a vibration force can be written in the form

$$\begin{aligned}
&\sin(\delta_{ikt} + \varphi_x)\left[a_{ox}\cos\beta'_x + \sum_{j=1}^{N} a_{jx}\cos\delta_{jkt}\right] \\
&\quad - \cos(\delta_{ikt} + \varphi_x)\left[a_{ox}\sin\beta'_x + \sum_{j=1}^{N} a_{jx}\sin\delta_{jkt}\right] \\
&\quad - s_i\left\{\cos(\delta_{ikt} + n_y)\left[a_{oy}\cos\beta'_y + \sum_{j=1}^{N} s_j a_{jy}\sin\delta_{jkt}\right]\right. \\
&\quad \left. + \sin(\delta_{ikt} + \varphi_y)\left[a_{oy}\sin\beta'_y - \sum_{j=1}^{N} s_j a_{jy}\cos\delta_{jkt}\right]\right\} \\
&\quad + l_i\left\{\cos(\delta_{ikt} - \rho_i + \varphi_\gamma)\left[a_{o\gamma}\cos\beta'_\gamma + \sum_{j=1}^{N} a_{j\gamma}\sin(\delta_{jkt} - \rho_j)\right]\right. \\
&\quad \left. + \sin(\delta_{ikt} - \rho_i + \varphi_y)\left[a_{o\gamma}\sin\beta_\gamma - \sum_{j=1}^{N} a_{j\gamma}\cos(\delta_{jkt} - \rho_j)\right]\right\} = 0. \qquad (31)
\end{aligned}$$

Expressions in square brackets are of value zero because of equation (20) and, therefore, all vibration forces are of value zero at the same time for vibrators placed in positions $\delta_{1kt}, \ldots, \delta_{Nkt}$ described in Section 4. These are positions of equilibrium.

Therefore, for vibrators synchronously moving with excitation and phases $\delta_{1kt}, \ldots, \delta_{Nkt}$ we have

$$\mathbf{P}(\delta_{1kt}, \ldots, \delta_{Nkt}) = \mathbf{0}, \quad \mathbf{q}(t, \delta_{1kt}, \ldots, \delta_{Nkt}) = \mathbf{0}. \qquad (32)$$

It has been proved that vibrators can organize themselves in such a way with respect to the object that they can move synchronously with the object vibrations and in addition, they can generate forces that are opposed to the excitation. If the parameters of the vibrators satisfy conditions (20) and the frequency of excitation is higher than the natural frequencies of the object, then the vibrations could be completely eliminated. In practice, the complete elimination of vibrations is limited by other factors that will be considered in Part 2 of this paper.

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