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SYNCHRONOUS ELIMINATION OF VIBRATION IN THE PLANE, PART 2: METHOD EFFICIENCY AND ITS STABILITY

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This paper is a continuation of Part 1 in which the idea of a synchronous vibration eliminator was investigated. The vibrator movement friction causes phase shifts of vibrators. Appearance of the vibrators' phase displacements causes that their effect on an object does not compensate fully the excitation and, therefore, residual vibrations remain. A parameter describing the efficiency of this method of vibration elimination was introduced. The influence of parameters of a system was investigated based on values of phase displacements. A detailed analysis was carried out for one or two vibrators and results are presented in a chart form. The efficiency of the method was tested. The stability of synchronous movement of the eliminators with excitation was verified. A criterion of this stability was given and conditions were set in which vibrator motions assuring vibration elimination may occur. The maximum difference between excitation frequency and initial velocity of vibrators was investigated for which synchronous interruptions may evolve. It was shown that synchronous vibrator motions and the elimination of object vibrations can occur only in supercritical ranges of excitation frequencies. On the contrary, in undercritical ranges vibrators increase object vibrations.

1. DEVIATION OF VIBRATOR POSITIONS

Vibration forces acting on vibrators attempt to replace them in positions in which they have zero values. These positions must be stable so that they can be physically realized. In every movement there are frictions which change this motion in correlation to these friction values. Therefore, movement is going to be slower and the positions of equilibrium are not going to overlap with those described before. The vibrator friction, in general, can consist of viscous resistance, rolling resistance, bed friction, and motor driving torque. The last of these is of equal importance as the others; however, it may have positive and negative values.

New equilibrium positions are described by phases δ_{ik} which differ by Δ_i from position δ_{ikt} for which object vibrations fully disappeared:

$$\delta_{ik} = \delta_{ikt} + \Delta_i \tag{1}$$

The vibration forces balance vibrator movement friction in the new equilibrium positions. The equations from which position deviations of vibrators $\Delta_1, \ldots, \Delta_N$ according to "ideal" positions that can be calculated have the form

$$\mathbf{P}(\delta_{1kt} + \Delta_1, \ldots, \delta_{Nkt} + \Delta_N) - \mathbf{F} = \mathbf{0}, \tag{2}$$

where **P** is described by equation (27) of Part 1 and the movement friction **F** by equation (12) of Part 1. A relation of general vibration force can be expanded in power series according to deviation Δ_i and if these deviations are small we need only take into consideration the first two terms:

$$P_i(\delta_{1kt} + \Delta_1, \ldots, \delta_{Nkt} + \Delta_N) \cong P_i(\delta_{1kt}, \ldots, \delta_{Nkt}) + \sum_{j=1}^{N} \frac{\partial P_i}{\partial \Delta_j} \Delta_j. \tag{3}$$

After using equation (17) of Part 1 we arrive at

$$\begin{aligned} P_i \cong -0{\cdot}5 m_i R_i \Omega^2 \{ & [a_{ox} \cos(\delta_{ikt} - \beta_x + \varphi_x) \Delta_i + \sum_{j=1}^{N} a_{jx} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_x)(\Delta_i - \Delta_j)] \\ & + s_i [a_{oy} \sin(\delta_{ikt} - \beta_y + \varphi_y) \Delta_i + \sum_{j=1}^{N} s_j a_{jy} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_y)(\Delta_i - \Delta_j)] \\ & + l_i [a_{o\gamma} \cos(\delta_{ikt} - \rho_i - \beta_\gamma + \varphi_\gamma) \Delta_i + \sum_{j=1}^{N} a_{j\gamma} \cos(\delta_{ikt} - \delta_{jkt} - \rho_i + \rho_j + \varphi_\gamma)(\Delta_i - \Delta_j)]. \end{aligned} \tag{4}$$

If we replace equation (2) with equation (4) we obtain

$$\begin{aligned} & \sum_{j=1}^{N} a_{jx} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_x) \Delta_j + s_i \sum_{j=1}^{N} s_j a_{jy} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_y) \Delta_i \\ & \quad + l_i \sum_{j=1}^{N} a_{j\gamma} \cos(\delta_{ikt} - \delta_{jkt} - \rho_i + \rho_j + \varphi\gamma) \Delta_j - \frac{2F_i}{m_i R_i \Omega^2} = 0, \\ & i = 1, \ldots, N. \end{aligned} \tag{5}$$

This is a system of N equations with unknowns $\Delta_1, \ldots, \Delta_N$, realized by movement friction F and the system's parameters.

If object vibrations can be compensated by one vibrator then from the above relation we get

$$\Delta \cong \frac{2F}{mR\Omega^2(a_x \cos\varphi_x + a_y \cos\varphi_y)}. \tag{6}$$

For an object suspended in such a way that $k_x = k_y$, $\varepsilon_x = \varepsilon_y$ the following occurs: $a_x = a_y$, $\varphi_x = \varphi_y$, and deviation of vibrator positioning placed on an object moving with plane movement is two times smaller than for a vibrator on the object with one degree of freedom [1–3].

According to previous assumptions, this method is efficient in a supercritical range of frequencies for which $\varphi_x, \varphi_y \in (\pi/2, \pi)$ and the denominator in relation (6) becomes negative. For positive resistance force $F = n_1 R^2 \Delta\omega$, position deviation is negative, this means that the vibrator movement is delayed according to excitation. The vibrator position deviation (shown in Figure 1(a)) is presented according to forcing frequency. The viscous friction coefficient of the vibrator is constant and its mass becomes smaller with forcing frequency. The frequency difference is either $\Delta\omega = \Omega - \omega = \pm 5$ rad/s. For small damping of an object, vibrator positioning deviation becomes bigger according to exciting frequency increase. For large object damping ($\varepsilon = 0{\cdot}5$), the vibrator positioning deviation is smaller and almost constant in the analyzed frequency range.

The charts from Figure 1(b) deal with a case where the vibrator viscous coefficient n_1 is proportional to vibrator mass changing with excitation frequency according to the

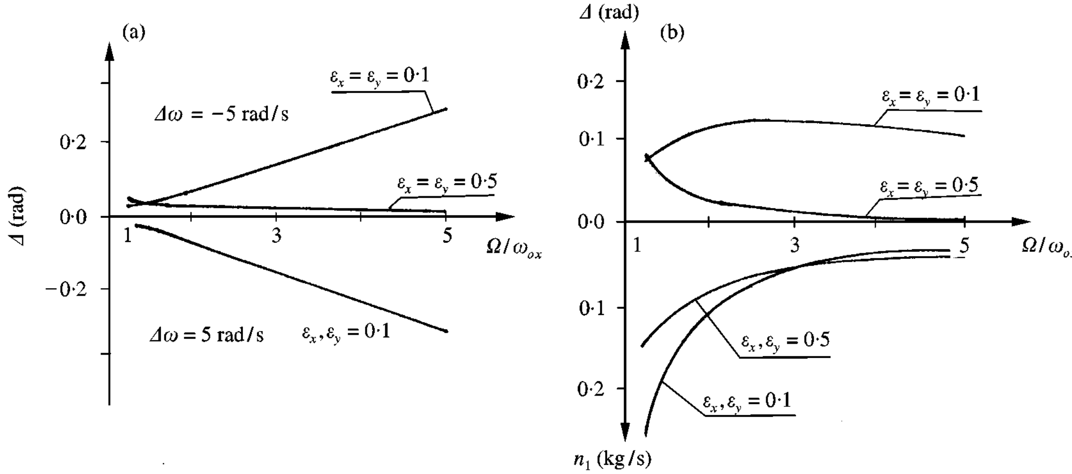

Figure 1. Vibrator positioning deviation in relation to forcing frequency for (a) constant coefficient of resistance $n_1 = 0{\cdot}05$ kg/s = const., (b) resistance coefficient proportional to vibrator mass.

following relation:

$$m = k_x \xi_{ox} \sqrt{1 + (2\varepsilon_x \Omega/\omega_{ox})^2}/(R\Omega^2). \tag{7}$$

In this case, vibrator positioning deviation for small damping ($\varepsilon = 0{\cdot}1$) first becomes larger as frequency Ω increases and then it becomes smaller. For large damping ($\varepsilon = 0{\cdot}5$), the deviation Δ decreases as Ω increases.

Far from resonance, when $\Omega \gg \omega_{ox}, \omega_{oy}$ we can assume that $\varphi_x, \varphi_y \cong \pi$ and then

$$\Delta \cong -\frac{F}{mR\Omega^2 a_x}. \tag{8}$$

For undeveloped friction, which can happen when rolling friction or sliding friction exists and at the same time $\omega = \Omega$, the vibrator positioning deviation is not exactly defined and it can have different values in a certain range. The condition of balance, in this case, has the form

$$|P_1| - F_i \leqslant 0. \tag{9}$$

We get from this

$$-\frac{2F_i}{mR\Omega^2 |a_{ox}\cos\varphi_x + a_{oy}\cos\varphi_y|} < \Delta < \frac{2F_i}{mR\Omega^2 |a_{ox}\cos\varphi_x + a_{oy}\cos\varphi_y|}. \tag{10}$$

For a larger number of vibrators their shifts $\Delta_1, \ldots, \Delta_N$ as function of excitation have to be obtained from equation system (5). In matrix notation they have the form

$$\mathbf{A}\Delta - \mathbf{F} \leqslant \mathbf{0}, \tag{11}$$

and from this we get

$$\Delta = \mathbf{A}^{-1}\mathbf{F}, \tag{12}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{11}, \ldots, a_{1N} \\ \cdots\cdots\cdots\cdots \\ a_{N1}, \ldots, a_{NN} \end{bmatrix}, \qquad \mathbf{F} = \begin{bmatrix} F_1 \\ \cdots \\ F_N \end{bmatrix},$$

$$a_{ij} = 0{\cdot}5 m_i R_i \Omega^2 \Bigg[\sum_{j=1}^{N} a_{jx} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_x) \Delta_j + s_i \sum_{j=1}^{N} a_{jy} s_j \cos(\delta_{ikt} - \delta_{jkt} + \varphi_y) \Delta_i$$
$$+ l_i \sum_{j=1}^{N} a_{j\gamma} \cos(\delta_{ikt} - \delta_{jkt} + \varphi_\gamma) \Delta_j \Bigg].$$

For two vibrators meeting only viscous resistance, phase shifts Δ_1, Δ_2 were calculated from the above relation and are shown in Figure 2(a, b). A case of kinematic excitation in Y direction was tested. It introduces changing forces in time in this direction through elastic and damping elements on an object. Also, a case of constant coefficients, $n_1 = n_2 = const$, was analyzed as well as coefficients proportional to changing vibrator mass. One vibrator exhibits a similar characteristic of change.

The relation given makes sense only when real solutions of equations exist and this takes place for $|P_i|_{max} > F_i$.

For large deviations $\Delta_1, \ldots, \Delta_N$ it is necessary to take into consideration a large number of terms of force P_i explicit in series expansion. Then we get a non-linear algebraic equation system from which it is possible to calculate values of these deviations.

A vibrator's arrangement in replacement positions of about $\Delta_1, \ldots, \Delta_N$ according to positions $\delta_{1kt}, \ldots, \delta_{Nkt}$ causes that conditions (20) of Part 1 cannot be satisfied and, therefore, object vibrations are not fully compensated through vibrator actions. A certain excitation residual $W(t)$ is left and it causes existence of object residual vibrations $\mathbf{q}_r(t)$. This results from forces acting directly on an object, kinematic excitation and vibrator actions, can be written as

$$\mathbf{W}(t) = \begin{bmatrix} W_x(t) \\ W_y(t) \\ W_\gamma(t) \end{bmatrix}$$

$$= \begin{bmatrix} Q_x(t) + k_x \xi_x(t) + k_{x\gamma} \xi_\gamma(t) + \sum_{i=1}^{N} m_i R_i \Omega^2 \cos(\Omega t + \delta_{ikt} + \Delta_i) \\ Q_y(t) + k_y \xi_y(t) + k_{y\gamma} \xi_\gamma(t) + \sum_{i=1}^{N} m_i s_i R_i \Omega^2 \sin(\Omega t + \delta_{ikt} + \Delta_i) \\ Q_\gamma(t) + k_\gamma \xi_\gamma(t) + k_{x\gamma} \xi_x(t) + k_{y\gamma} \xi_y(t) + \sum_{i=1}^{N} m_i l_i R_i \Omega^2 \sin(\Omega t + \delta_{ikt} - \rho_i + \Delta_i) \end{bmatrix}. \tag{13}$$

For small $\Delta_1, \ldots, \Delta_N$ we can continue as with the vibration force $\mathbf{P}$, and then $\mathbf{W}$ has the form

$$\mathbf{W}(t) = \begin{bmatrix} W_x \\ W_y \\ W_\gamma \end{bmatrix} \cong \begin{bmatrix} -\sum_{i=1}^{N} m_i R_i \Omega^2 \sin(\Omega t + \delta_{ikt}) \Delta_i \\ \sum_{i=1}^{N} m_i s_i R_i \Omega^2 \cos(\Omega t + \delta_{ikt}) \Delta_i \\ \sum_{i=1}^{N} m_i l_i R_i \Omega^2 \cos(\Omega t + \delta_{ikt} - \rho_i) \Delta_i \end{bmatrix}. \tag{14}$$

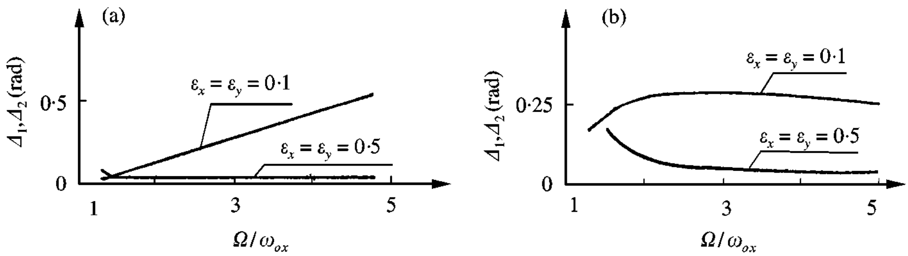

Figure 2. Vibrator positioning deviation for $N = 2$, $\Delta\omega = \Omega - \omega = -5$ rad/s, and $n_1 = n_2 = 2{\cdot}5 \times 10^{-2}$ kg/s: (a) constant coefficient of resistance $n_1 = const.$, (b) resistance coefficient proportional to vibrator mass.

This can be presented in another form as

$$\mathbf{W}(t) = \mathbf{W}_c \cos \Omega t + \mathbf{W}_s \sin \Omega t = \mathbf{W}_o \cos(\Omega t + \beta_w), \tag{15}$$

where

$$\mathbf{W}_c = \begin{bmatrix} -\sum_{i=1}^{N} m_i R_i \Omega^2 \sin \delta_{ikt} \Delta_i \\ \sum_{i=1}^{N} m_i s_i R_i \Omega^2 \cos \delta_{ikt} \Delta_i \\ \sum_{i=1}^{N} m_i l_i R_i \Omega^2 \cos(\delta_{ikt} - \rho_i) \Delta_i \end{bmatrix}, \qquad \mathbf{W}_s = \begin{bmatrix} \sum_{i=1}^{N} m_i R_i \Omega^2 \cos \delta_{ikt} \Delta_i \\ -\sum_{i=1}^{N} m_i s_i R_i \Omega^2 \sin \delta_{ikt} \Delta_i \\ -\sum_{i=1}^{N} m_i l_i R_i \Delta^2 \sin(\delta_{ikt} - \rho_i) \Delta_i \end{bmatrix},$$

$$W_{oj} = \sqrt{W_{cj}^2 + W_{sj}^2}, \qquad tg\beta_{wj} = W_{sj}/W_{cj}, \qquad j = 1, 2, 3.$$

The object motion equation

$$\mathbf{M}\ddot{\mathbf{q}}_r + \mathbf{n}\dot{\mathbf{q}}_r + \mathbf{k}\mathbf{q}_r = \mathbf{W}(t) \tag{16}$$

can be treated as an imaginary part of the equation

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{n}\dot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{G}e^{i\Omega t}, \tag{17a}$$

where

$$G_j = W_{0j} e^{i\beta_{wj}}, \quad j = 1, 2, 3. \tag{17b}$$

Equation solution (16) has the form

$$\mathbf{q}_r = \mathrm{Im}(\mathbf{Z}^{-1}\mathbf{G}e^{i\Omega t}), \tag{18}$$

where

$$\mathbf{Z} = \mathbf{k} - \Omega^2\mathbf{M}.$$

Therefore, the co-ordinates change according to

$$x_r(t) = a_{xr} \sin(\Omega t + \beta_{w1} - \varphi_x),$$

$$y_r(t) = a_{yr} \sin(\Omega t + \beta_{w2} - \varphi_y),$$

$$\gamma_r(t) = a_{\gamma r} \sin(\Omega t + \beta_{w3} - \varphi_\gamma). \tag{19}$$

Residual vibration amplitudes are related to the amplitude force $\mathbf{W}_0$, the closeness of the exciting frequency to an object's natural frequency, and its damping. If the directions assumed are principal directions of a suspended object, then relations for vibration amplitudes are rather simple. For example,

$$a_{xr} = \frac{\Omega^2\sqrt{(\sum_{i=1}^{N} m_i R_i \sin(\delta_{ikt})\Delta_i)^2 + (\sum_{i=1}^{N} m_i R_i \cos(\delta_{ikt})\Delta_i)^2}}{M\sqrt{(\omega_{0x}^2 - \Omega^2)^2 + (n_x\Omega/M)^2}}.$$

As a measurement of vibration reduction v, the following ratio (of residual vibration amplitude to object vibration amplitude, which does not have eliminators) was defined:

$$\mathbf{v} = \frac{\mathbf{a}_r}{\mathbf{a}_0} 100\%. \tag{20}$$

For full elimination of vibrations, v becomes zero.

For example, for the excitation presented in Figure 3(a) of Part 1, which is compensated by one vibrator, we get

$$\mathbf{W}_{ox} \cong m_1 R_1 \Omega^2 \Delta \cong \frac{2F}{|a_x \cos\varphi_x + a_y \cos\varphi_y|}, \tag{21}$$

$$v_x = \frac{a_{xr}}{a_{ox}} \cong \frac{2F}{k_x \xi_{ox} |a_x \cos\varphi_x + a_y \cos\varphi_y| \sqrt{1 + (2\varepsilon_x \Omega/\omega)^2}}. \tag{22}$$

For this case of excitation, the vibration reduction as a function of frequency Ω was presented in Figure 3.

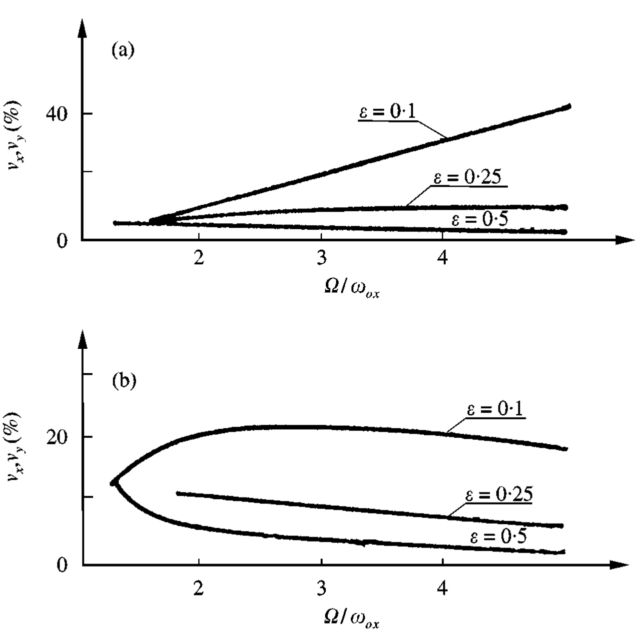

Figure 3. Vibration reduction degree for (a) constant coefficient of resistance $n_1 = 0{\cdot}05$ kg/s $= const.$, $\Delta\omega = 5$ rad/s, $\varepsilon_x = \varepsilon_y = \varepsilon$; (b) viscous coefficient proportional to vibrator mass.

For small object damping and for $\Omega \gg \omega_{ox}, \omega_{oy}$, where $\varphi_x, \varphi_y \cong \pi$, the above relationship can be simplified to

$$v \approx \frac{F}{k_x \xi_{ox}^2} |1 - (\Omega/\omega)^2|. \tag{23}$$

The efficiency of this method is greater for excitation with a frequency which is somewhat greater than an object's natural frequency. This means that in such a range, where object vibrations are most dangerous, the method's efficiency increases. Also, when vibrator movement friction decreases, the efficiency increases.

It was shown that vibrators can move synchronously with excitation, but because of the friction which they encounter, their movement is described as

$$\alpha_i(t) = (\Omega - \omega_i)t + \delta_{ikt} + \Delta_i, \tag{24}$$

and because of deviations Δ_i, the efficiency of the method is reduced.

Also, such a situation can occur when vibrators decrease vibrations in one direction and increase them in another direction.

2. STABILITY RESEARCHERS

Under the action of general vibration forces **P**, vibrators place themselves in positions of equilibrium. From the run of forces, we can state that vibrator movements with phases close to δ_{ikt} can be stable after fulfilling certain requirements. In order for the physical possibility of movements to exist in equation (24), the motion with final phases must be stable. It is necessary to check if for the vibrator with movement phases δ_{ikt}, giving full elimination of vibrations, or if, for δ_{ik}, for which certain residual vibrations remain, stable conditions are fulfilled.

Asymptotically stability movements (24) related to phases $\delta_{1k}, \ldots, \delta_{Nk}$ exist, if for this solution all algebraic roots of equation of N degree,

$$\left| \frac{\partial P_i}{\partial \delta_j} - \lambda_{ij}\chi \right| = 0, \quad i, j = 1, \ldots, N, \qquad \lambda_{ij} = \begin{cases} 1 & \text{iba } i = j, \\ 0 & \text{iba } i \neq j \end{cases} \tag{25}$$

have negative real parts:

$$\mathrm{Re}(\chi) < 0.$$

If a real part of only one of the roots is positive then movements having form (24) are unstable and cannot be realized physically [1, 2, 4].

Individual derivatives are described by

$$\begin{aligned} \frac{\partial P_i}{\partial \delta_i} = & -0{\cdot}5 m_i R_i \Omega^2 \Bigg\{ [a_{ox} \cos(\delta_{ik} - \beta_x + \varphi_x) + \sum_{j=1}^{N} a_{jx} \cos(\delta_{ik} - \delta_{jk} + \varphi_x)] \\ & + s_i [a_{oy} \cos(\delta_{ik} - \beta_y + \varphi_y) + \sum_{i=1}^{N} s_j a_{jy} \cos(\delta_{ik} - \delta_{jk} + \varphi_y)] \\ & + l_i [a_{oy} \cos(\delta_{ik} - \beta_\gamma + \varphi_\gamma) + \sum_{j=1}^{N} s_j a_{j\gamma} \cos(\delta_{ik} - \delta_{jk} - \rho_i + \rho_j + \varphi_\gamma)] \Bigg\}, \end{aligned} \tag{26}$$

but in the above relation the term $j = i$ does not exist in the summation.

For $j \neq i$

$$\frac{\partial P_i}{\partial \delta_j} = -0{\cdot}5 m_i R_i \Omega^2 \{ -a_{jx} \cos(\delta_{ik} - \delta_{jk} + \varphi_x) - s_i s_j a_{jy} \cos(\delta_{ik} - \delta_{jk} + \varphi_y)$$
$$- l_i a_{j\gamma} \cos(\delta_{ik} - \delta_{jk} - \rho_i + \rho_j + \varphi_\gamma)\},$$
$$\delta_{ik} = \delta_{ikt} + \Delta_i. \tag{27}$$

Derivatives are calculated in positions $\delta_{1k}, \ldots, \delta_{Nk}$. Phases δ_{ikt} satisfy equation (20) of Part 1 and derivations of vibrator positioning are derived from relations (11).

To check the stability of one vibrator is the easiest task because the relations are not complicated and it is possible to find if there is stability or not by using any number. One vibrator can compensate only for the excitation shown in Figure 3(a) of Part 1. The equilibrium position of the vibrator can be described as

$$P(\delta_{1kt} + \Delta) - F = 0. \tag{28}$$

By substituting suitable relations we can get

$$-0{\cdot}5 m R \Omega^2 \{ a_{ox} [\sin(\delta_{ikt} + \Delta - \beta_x + \varphi_x) + \sin\varphi_x] - a_{oy} [\cos(\delta_{1kt} + \Delta - \beta_y + \varphi_y)$$
$$- \sin\varphi_y]\} - F = 0, \tag{29}$$

where

$$\delta_{1kt} = \pi, \quad \beta_x = 0, \quad \beta_v = -\pi/2.$$

The resistance F can be described by equation (12) of Part 1. For extreme resistance, a solution of equation (29) exists when

$$|\mathbf{P}| - |\mathbf{F}_{eks}| > 0. \tag{30}$$

If the parameters of object suspension are identical in both directions then $a_{ox} = a_{oy}$, $\varphi_v = \varphi_v$, and from the above relation we can get

$$F_{eks} < 0{\cdot}5 m R \Omega^2 |\sin(\Delta_{eks} + \varphi_x) + \sin\varphi_x|. \tag{31}$$

Values of vibrator positioning deviation can be in the range

$$\Delta \in (\pi/2 - \varphi_x, 3\pi/2 - \varphi_x), \tag{32}$$

and accordingly,

$$F_{cks} = m R \Omega^2 |\pm 1 + \sin\varphi_x|. \tag{33}$$

If vibrator resistance combines with viscous friction in the form $F = n_1 R^2 \Delta\omega$, then from equation (30) we can calculate the maximal frequency difference. Real solutions of equation (28) exist when the difference $\Delta\omega = \Omega - \omega$ between forcing frequency and initial velocity of the vibrator is within the range

$$\Delta\omega \in \left(\frac{m\Omega^2 a_x (\sin\varphi_x - 1)}{n_1 R}, \frac{m\Omega^2 a_x (\sin\varphi_x + 1)}{n_1 R} \right). \tag{34}$$

The admissible frequency difference is not only a function of vibrator friction but also depends on exciting frequency and object damping. It is much bigger than for an object of one degree of freedom [1, 2]. This results from the fact that general vibration forces are much larger because they come from a few component vibrations $x(t)$, $y(t)$ and $\gamma(t)$. In

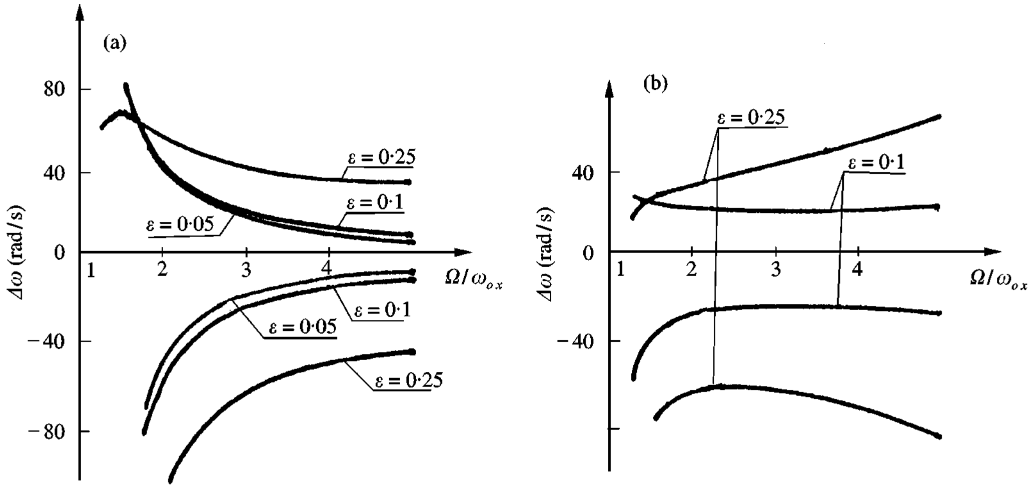

Figure 4. Admitting range of frequency difference $\Delta\omega$ for (a) constant coefficient of resistance $n_1 = 0{\cdot}05$ kg/s $=$ *const.*, (b) resistance coefficient proportional to vibrator mass.

Figure 4, an extreme frequency difference $\Delta\omega$ was shown in the range of the above critical frequency of excitation as a function of a ratio of forcing frequency to an object's natural frequency, Ω/ω_{ox}, and the object damping ε_x. Figure 4(a) shows a case where the resistance coefficient n_1 is constant. Figure 4(b) relates to coefficient n_1, which is proportionate to the object mass. For constant resistance, the admissible frequency difference gets smaller with an increase of exciting frequency. This change is strong for small object damping ε_x, ε_y.

In agreement with condition (25), the vibrator position $\delta_{kt} = \pi$ is stable only when

$$\chi = -0{\cdot}5mR\Omega^2[a_{ox}\cos(\delta_{kt} + \Delta - \beta_x + \varphi_x) + a_{oy}\sin(\delta_{kt} + \Delta - \beta_y + \varphi_y)] < 0. \quad (35)$$

The above-mentioned condition can be written as

$$a_{ox}\cos(\varphi_x + \Delta) + a_{oy}\cos(\varphi_y + \Delta) < 0. \quad (36)$$

If a derivation of vibrator positioning is in range (32) then condition (36) for a vibrator in position $\delta_k = \pi + \Delta$ is fulfilled for φ_x, $\varphi_y > \pi/2$. If placed in the "ideal" position $\delta_{1kt} = \pi$ then the stability condition is of form

$$a_x\cos\varphi_x + a_y\cos\varphi_y < 0. \quad (37)$$

It can be satisfied only in the supercritical range of exciting frequencies $\Omega > \omega_{ox}, \omega_{oy}$ for which $\varphi_x, \varphi_y \in (\pi/2, \pi)$. For frequencies $\Omega < \omega_{ox}, \omega_{oy}$, $\varphi_x, \varphi_y \in (0, \pi/2)$ occur and the left side of equation (37) gets positive values. This leads to conclude that under critical exciting frequencies the vibrator position $\delta_{1kt} = \pi$ is unstable.

For deviations of vibrator positioning $\Delta \neq 0$, the equilibrium position is moved according to $\delta_{1kt} = \pi$ and stability condition (25) is also satisfied for $\Omega > \omega_{ox}, \omega_{oy}$ provided the shift angle is not too large. The vibrator position with phase $\delta_{1kt} = \pi$ can be also stable in a certain frequency range $(\omega_{ox}, \omega_{oy})$. The resultant vibration moment $P = P_x + P_y$ determines the vibrator's stability in this range. One component is positive and the other is negative. If moment P_x which comes from object vibrations in the X direction outweighs P_y (assuming that $\omega_{ox} < \Omega < \omega_{oy}$) then position δ_{1k} is stable. But the deviation of position Δ becomes large even with small vibrator movement resistance. Method effectiveness, in this

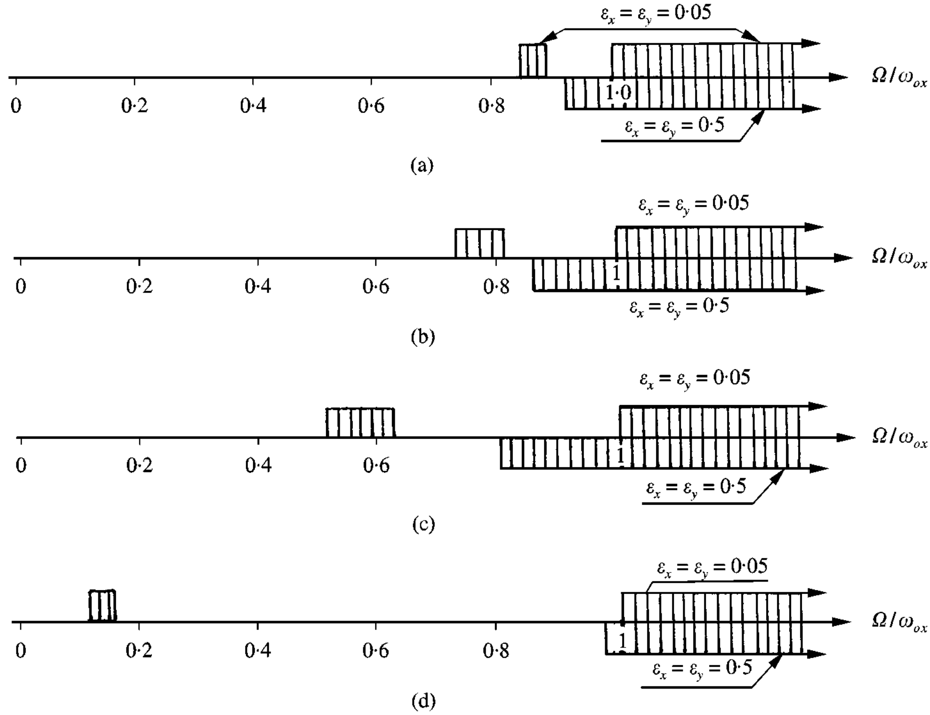

Figure 5. Stability ranges for vibrators moving with viscous resistance: (a) $\omega_{0x}/\omega_{0y} = 1{\cdot}2$, (b) $\omega_{0x}/\omega_{0y} = 1{\cdot}4$, (c) $\omega_{0x}/\omega_{0y} = 2$, (d) $\omega_{0x}/\omega_{0y} = 10$.

range of frequencies, is very small. In Figure 5 the range of stability for different ratios ω_{oy}/ω_{ox} and object damping was indicated.

For two similar vibrators, which compensate kinematic excitation in the direction of the Y-axis, condition (25) has the form

$$\chi^2 - B\chi + C = 0. \tag{38}$$

where

$$B = \frac{\partial P_1}{\partial \delta_1} + \frac{\partial P_2}{\partial \delta_2}, \quad C = \frac{\partial P_1}{\partial \delta_1}\frac{\partial P_2}{\partial \delta_2} - \frac{\partial P_1}{\partial \delta_2}\frac{\partial P_2}{\partial \delta_1}.$$

Derivatives are calculated for the phases $\delta_1 = \delta_{1k}, \delta_2 = \delta_{2k}$.

For an excitation assumed to be $\delta_{1kt} = -\pi/2$, $\delta_{2kt} = \pi/2$, $\beta_y = 0$, $\rho_1 = \rho_2 = 0$, $l_1 = l_2$ and zero derivations of the vibrator's positioning we get

$$\frac{\partial P_1}{\partial \delta_1} = \frac{\partial P_2}{\partial \delta_2} = 0{\cdot}5mR\Omega^2(a_x \cos\varphi_x + a_y \cos\varphi_y + l_1 a_\gamma \cos\varphi_\gamma), \tag{39}$$

$$\frac{\partial P_1}{\partial \delta_2} = \frac{\partial P_2}{\partial \delta_1} = -\,0{\cdot}5mR\Omega^2(a_x \cos\varphi_x - a_y \cos\varphi_y + l_1 a_\gamma \cos\varphi_\gamma), \tag{40}$$

Roots of equation (38) are

$$\chi_{1,2} = \frac{\partial P_1}{\partial \delta_1} \mp \frac{\partial P_1}{\partial \delta_2}$$

therefore

$$\chi_1 = mR\Omega^2(a_x \cos\varphi_x + l_1 a_\gamma \cos\varphi_\gamma), \qquad \chi_2 = mR\Omega^2 a_y \cos\varphi_y. \tag{41}$$

Roots of equation (38) are negative ($\chi_1, \chi_2 < 0$) for exciting frequencies $\Omega > \omega_{ox}, \omega_{oy}$. The vibrator's positions $\delta_{1kt} = 3\pi/2, \delta_{2kt} = \pi/2$ are stable for forcing frequencies which are larger than the object's natural frequencies. For a larger number of vibrations, we reach the same conclusions.

If vibrators move only with rolling friction then stability conditions can be fulfilled only in a certain narrow range of forcing frequencies Ω and in an overcritical area which is near the resonance. This range increases with the decrease of the rolling resistance coefficient.

The stability of synchronous movement, in form (24) was also tested in relation to the variational equation which was obtained from the full equation system (7, 8) of Part 1. Small disturbances $\mathbf{q}_1(t)$ of object position and $\mathbf{q}_w^*(t)$ for vibrators were introduced. General co-ordinates of an object and vibrators have the form

$$\mathbf{q}(t) = \mathbf{q}_r + \mathbf{q}_1, \qquad \mathbf{q}_w(t) = \mathbf{q}_{w0} + \mathbf{q}_w^*, \tag{42}$$

where $\mathbf{q}_r$-co-ordinates of fixed object movement (residual vibrations), and $\mathbf{q}_{w0}$-co-ordinates of vibrators in fixed movement:

$$\mathbf{q}_r = \begin{bmatrix} x_r \\ y_r \\ \gamma_r \end{bmatrix}, \qquad \mathbf{q}_1 = \begin{bmatrix} x_1 \\ y_1 \\ \gamma_1 \end{bmatrix}, \qquad \mathbf{q}_{w0} = \begin{bmatrix} (\Omega - \omega)t + \delta_{1k} \\ \cdots\cdots\cdots\cdots\cdots \\ (\Omega - \omega)t + \delta_{Nk} \end{bmatrix}, \qquad \mathbf{q}_w^* = \begin{bmatrix} \eta_1 \\ \cdots \\ \eta_N \end{bmatrix}. \tag{43}$$

By substituting the above relations into equations (7, 8) of Part 1 by assuming that $\mathbf{q}_i$ and $\mathbf{q}_w^*$ are small quantities and that vibrator vibration frequencies around the final position are, at least, 10 times smaller than forcing frequencies, we obtained the linear differential equation system

$$\mathbf{M}\ddot{\mathbf{q}}_i + \mathbf{n}\dot{\mathbf{q}}_i + \mathbf{k}\mathbf{q} = -\begin{bmatrix} \sum_{i=1}^{N} m_i R_i[-\eta_i \sin(\tau + \delta_{ik}) + 2\dot{\eta}_1 \cos(\tau + \delta_{ik})] \\ \sum_{i=1}^{N} m_i R_i s_i[\eta_i \cos(\tau + \delta_{ik}) + 2\dot{\eta}_1 \sin(\tau + \delta_{ik})] \\ \sum_{i=1}^{N} m_i R_i l_i[\eta_i \cos(\tau + \delta_{ik}) + 2\dot{\eta}_1 \sin(\tau + \delta_{ik} - \rho_i)] \end{bmatrix}, \tag{44}$$

$$I_i\ddot{\eta}_i = m_i R_i \begin{bmatrix} \sin(\tau + \delta_{ik}) \\ -s_i \cos(\tau + \delta_{ik}) \\ -l_i \cos(\tau + \delta_{ik} - \rho_i) \end{bmatrix}^{\mathrm{T}} \ddot{\mathbf{q}}_1 - \frac{\partial F_i}{\partial \dot{\alpha}_i}\dot{\eta}_i, \quad i = 1, \ldots, N \tag{45}$$

The above equations are for time $\tau = \Omega t$.

The solutions of these equations confirmed stability of vibrators in positions $\delta_{1k}, \ldots, \delta_{Nk}$ in an overcritical range of forcing frequencies and a lack of stability for $\Omega < \omega_{ox}, \omega_{oy}, \omega_{o\gamma}$. Exemplary results are shown in Figures 6 and 7.

Free vibration frequencies of a vibrator around the equilibrium position for $\Omega > \omega_{ox}, \omega_{oy}$ and lack of damping are described by the relation

$$\omega_{oi} = \sqrt{\frac{\partial P_i}{\partial \delta_i}/I_i}, \tag{46}$$

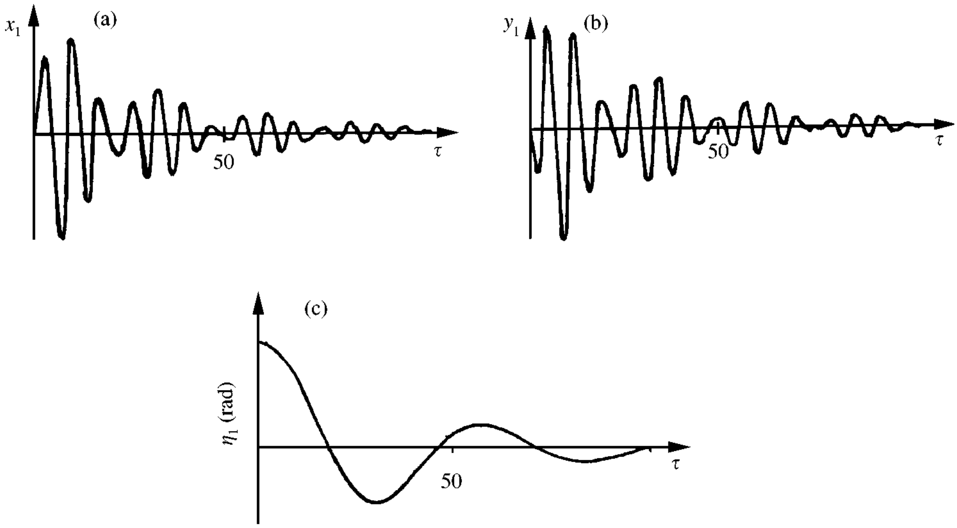

Figure 6. Behaviour of an object and vibrator during disturbance of the initial position of vibrator in an overcritical range ($N = 1$): (a, b) component disturbances of object vibration $x_1(\tau)$, $y_1(\tau)$: (c) disturbance of vibrator position $\eta_1(\tau)$.

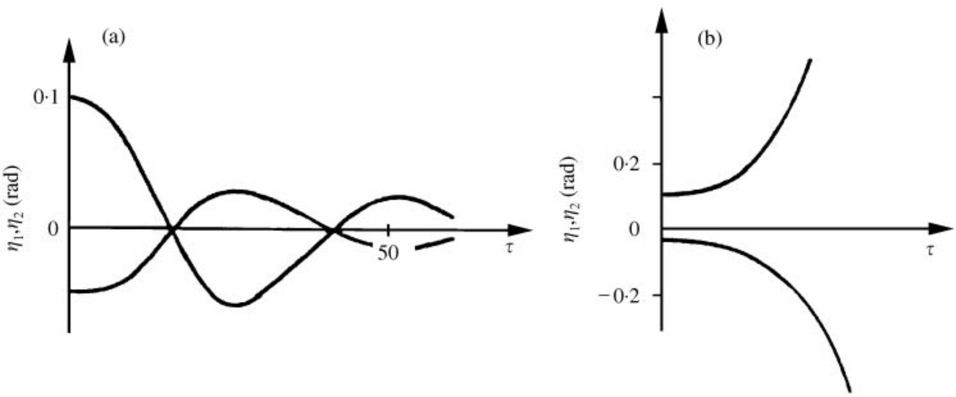

Figure 7. Behaviour of two vibrators during disturbance of its initial position: (a) in the supercritical range $\Omega > \omega_{0x}, \omega_{0y}$, (b) in the undercritical range $\Omega < \omega_{0x}, \omega_{0y}$.

where P_i is described by relation (27) and derivatives are calculated at points $\delta_{1k}, \ldots, \delta_{Nk}$. For one excitation from Figure 3(a) of Part 1 which is compensated by one vibrator we obtained

$$\omega_{ot} = \Omega\sqrt{(-\,mRa_x\cos\varphi_x)/I}. \tag{47}$$

By knowing the free vibrations of a vibrator around the equilibrium position we can also describe the vibrator's critical damping and non-dimensional damping coefficient ε_i for certain viscous resistance coefficient, of vibration n_i:

$$\frac{n_{1kr}R^2}{2I} = \omega_{o1}. \tag{48}$$

From this critical damping coefficient we find

$$n_{1kr} = \frac{2\Omega}{R^2}\sqrt{(-\,ma_x RI\cos\varphi_x)}. \tag{49}$$

The ratio of critical damping for the vibrator is

$$\varepsilon_1 = \frac{n_1}{n_{1kr}} = \frac{n_1 R^2}{2\Omega\sqrt{(-\,mRa_x I\cos\varphi_x)}}. \tag{50}$$

Coefficient ε_i determines the character of vibrator movement to the final position and the ratio of disappearance of its vibrations after arriving at this position.

3. CONCLUSIONS

Vibration eliminators such as spheres or rolls placed in a rotating cylinder or physical pendulum can compensate for changing loads acting on an object when certain conditions are satisfied. Vibrators rotating with individual initial velocities and in different directions organize themselves in such a way that their resultant force is an anti-phase to the excitation if only the frequency of this excitation is larger than the natural frequency of an object.

Such types of vibrator movements, which are synchronous with excitation, can exist only if the vibrator movement resistance does not exceed a certain level. If the maximal value of vibration force is smaller than the movement resistance then these types of vibrator movements do not appear. In this case, vibrators organize themselves in such a way as to compensate each other.

The largest vibration elimination takes place when initial velocities of vibrators are equal to forcing frequency and viscous resistance is against vibrator movements. In this situation object vibrations will be fully eliminated. But when these frequencies are disimilar or rolling resistance exists or if there is bed friction then vibrators move with phase delay according to “ideal” positions. Therefore, vibrators do not compensate entirely for excitation of an object and residual vibrations remain. The obtained runs give a view of the behaviour of vibrators and how this process is developed in time.

Vibrators move according to the assumed reference system under action of vibration forces coming from the vibrations of the object. Certain relations were introduced, from which we can find what vibration forces depend on and where they have their zero places. From a run of these forces we can determine which positions are stable. If an object cannot vibrate then these forces do not exist.

For the forcing frequencies that are between the smallest and the largest natural frequencies of the object, there is a range in which vibrators can compensate object vibrations. For these small values of vibration forces, vibrators coming to final positions take longer and position deviations become larger which causes the method efficiency in this range to be small.

Vibrators can fully eliminate object vibrations when their number, static momentum, rotating directions, axis co-ordinates and final phases are able to justify conditions (20) of Part 1. For certain particular cases of excitation, we can find a minimal number of vibrators which is needed to eliminate vibrations. To use the basic attribute of this method, that is, the self-organizing of a system without human participation and for unknown excitation, the number of vibrators placed on an object has to be large enough to satisfy these conditions for an individual excitation. If the conditions mentioned in this paper are satisfied then vibrators can recognize the system's excitation and organize themselves in such a way as to eliminate it.

The analysis give in this paper deals with a case of mono-harmonic forces. If the excitation is poly-harmonic then we have to use a vibrator set tuned to individual frequencies appearing in an excitation. This is a multiplication of the problem presented. For excitation with a continuous frequency spectrum, inertia vibrators can eliminate only excitation components that have frequencies in agreement with the velocity of vibrators.

For the excitation with frequencies that are smaller than the system's natural frequencies, vibrators organize themselves in such a way that they move almost in phase with the excitation. This causes an increase of object vibrations.

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