for each ω (circular eigenfrequency of the undamped system). Equation (3) simply indicates that none of the eigenvalues of the undamped system is at the same time also an eigenvalue of the damped system.

Of course, other criteria for controllability can also be used to check the pervasiveness of damping. For example, a criterion for pervasive damping, not involving the circular eigenfrequencies of the undamped system results from Kalman's criterion

$$\mathrm{rank}(\mathbf{D}(\mathbf{M}^{-1}\mathbf{K})(\mathbf{M}^{-1}\mathbf{D})(\mathbf{M}^{-1}\mathbf{K})^2(\mathbf{M}^{-1}\mathbf{D})\ldots(\mathbf{M}^{-1}\mathbf{K})^{n-1}(\mathbf{M}^{-1}\mathbf{D})) = n \tag{4}$$

(see e.g. [reference 4, Theorem 6.9, p. 165]; the German expression "*durchdringende Dämpfung*" is used for *pervasive damping*).

The author feels that the concept of pervasive damping is extremely relevant to engineering vibrations, since engineers may wish to damp all the free vibrations of a system, introducing a few dampers or dashpots only. In this case, the damping matrix **D** will not be positive definite, but damping should still be pervasive. The concept is therefore used exhaustively in all vibration courses taught by the author. It can and should of course also be generalized to continuous and also to non-linear systems and this is regularly done. For a discussion of the history of the concept of pervasive damping see [reference 5, p. 156].

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AUTHORS' REPLY

E. V. Wilms

4301-65 Swindon Way, Winnipeg, Manitoba, Canada R3P0T8

AND

H. Cohen

Department of Mathematics, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

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Professor Hagedorn points out that engineers may wish to damp out free vibrations with as few dashpots as possible.

It appears that all motion will be damped out with just one dashpot, if it is located between two points which always have a non-zero relative displacement for all the undamped mode shapes. Stephen [1] pointed this out for the special case of two degrees of freedom. (Note that it resulted in a highly unsymmetrical system.)

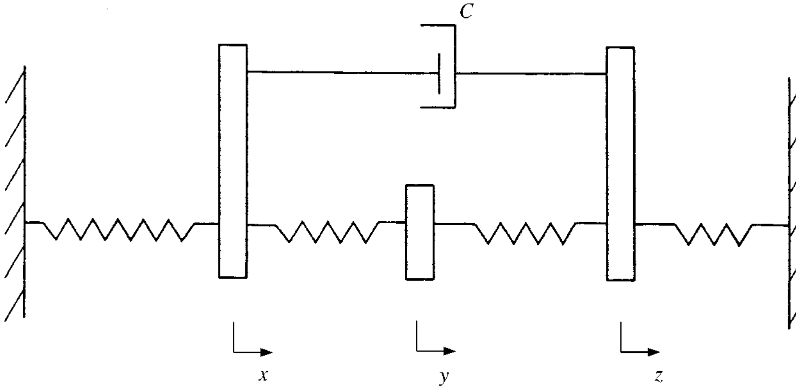

Figure 1. System A.

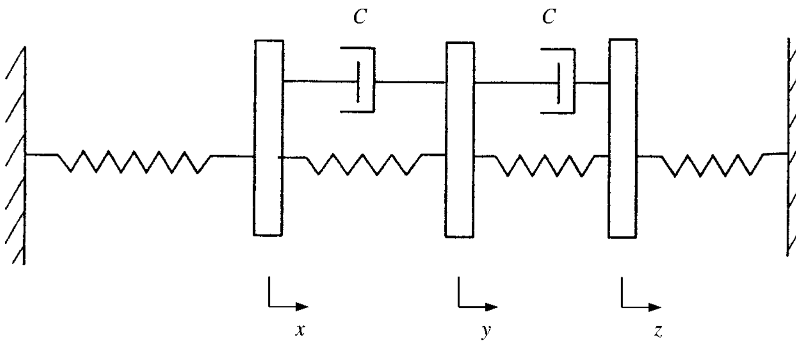

Figure 2. System B.

It appears that finite residual motion always remains, for any number of dashpots, provided they are all located between points which have zero relative displacement for at least one undamped mode shape.

In the three-degree-of-freedom system there was a non-zero relative displacement between x and y and between y and z for all three undamped mode shapes (all motion was damped out for System B). One can show that all motion is still damped out if there is only one dashpot located either between x and y or between y and z.

Again in the three-degree-of-freedom case, there was zero relative displacement between x and z for two mode shapes, namely (b) and (c); (finite residual motion for System A). See Figures 1 –3.

Professor Hagedorn discusses a procedure (involving vibration and control theory), which is similar to the procedure used by Gürgöze [2], This approach is useful for determining whether or not pervasive damping occurs.

We are currently extending our work to four- and perhaps five-degree-of-freedom systems.

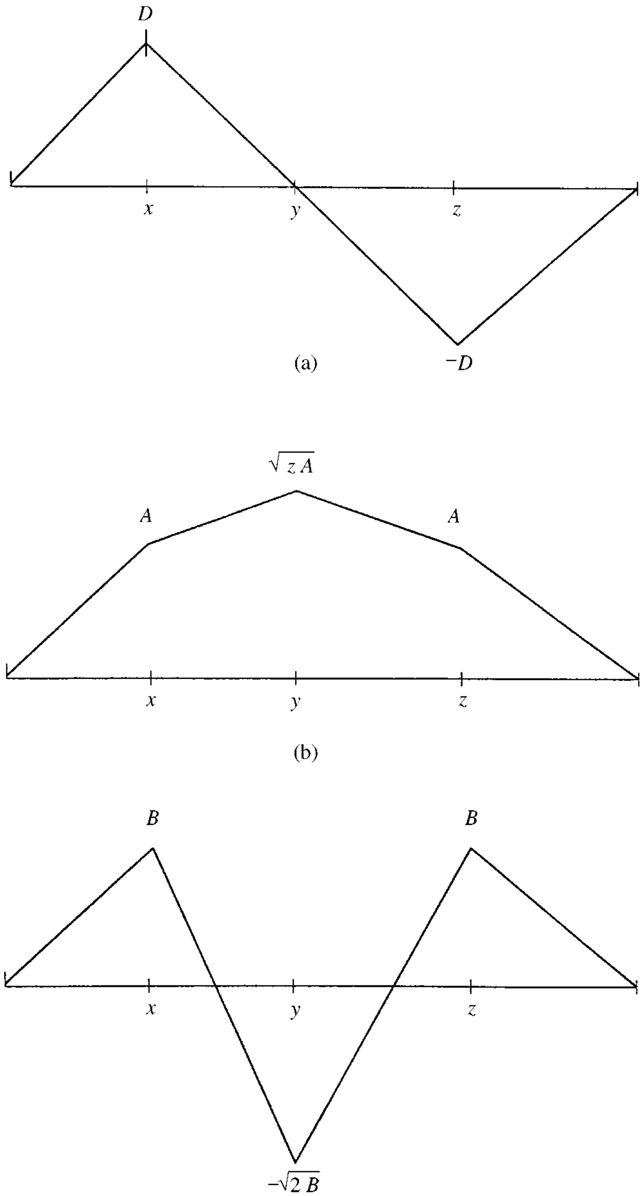

Figure 3. The undamped mode shapes: (a) $\lambda_1 = \sqrt{2}$, (b) $\lambda_2 = (2 + \sqrt{2})^{1,2}$, (c) $\lambda_3 = (2 - \sqrt{2})^{t/2}$.

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