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LETTERS TO THE EDITOR

A METHOD FOR PARAMETER IDENTIFICATION OF STRONGLY NON-LINEAR SYSTEMS

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1. INTRODUCTION

On the basis of the approximate solution obtained by the MLP method [1] and the new stroboscopic method [2], the mathematical model for parameter identification of strongly non-linear systems is set up in this paper. Using measured data of vibration response and optimal technique, we present a method for estimating system parameters. The examples of the autonomous system and the non-autonomous system show that the results of optimal calculation are satisfactory. This method is suitable for any parameter identification problem in which the approximate solution of strongly non-linear system can be determined.

2. MATHEMATICAL MODEL OF PARAMETER IDENTIFICATION

The differential equation of motion of the strongly non-linear system is

$$\ddot{x} + f(x, \dot{x}, t, \beta, \gamma, \dots) = 0, \tag{1}$$

where $\beta, \gamma, \dots$ are the parameters of the vibration system. They are the arbitrary constant. We assume that the approximate solution of equation (1) can be written as

$$x = x(t, \beta, \gamma, \dots). \tag{2}$$

The response data $\bar{x}(t_i)$ for $i = 1, 2, \dots, m$ can be obtained by the vibration test of the strongly non-linear system. Introducing the concepts of optimization [4], we may identify the system parameters $\beta, \gamma, \dots$ based on the least-square method.

The mathematical model of parameter identification subject to constraints can be stated as follows:

$$\min \quad S(\beta, \gamma, \dots) = \sum_{i=1}^{m} (x(t_i, \beta, \gamma, \dots) - \bar{x}(t_i))^2 \tag{3}$$

$$\text{s.t.} \quad g_j(\beta, \gamma, \dots) = 0 \quad \text{for } j = 1, 2, \dots, l_1, \tag{4}$$

$$h_k(\beta, \gamma, \dots) \leqslant 0 \quad \text{for } k = 1, 2, \dots, l_2. \tag{5}$$

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Applying the optimization method for minimizing the object function (3), we can seek the optimal value of system parameters $\beta, \gamma, \ldots$.

3. AN EXAMPLE OF AUTONOMOUS SYSTEM

Consider the free vibration of a typical Duffing system

$$\ddot{x} + \omega_0^2 x + \varepsilon(x^3 + \beta x^5) = 0 \tag{6}$$

with the initial conditions

$$x(0) = a, \quad \dot{x}(0) = 0, \tag{7}$$

where ε and β are not the small parameters. Using MLP method we obtain the approximate solution of equation (6) as follows:

$$x = a\cos\omega t + \alpha(C_1\cos\omega t + C_2\cos 3\omega t + C_3\cos 5\omega t), \tag{8}$$

where

$$\alpha = \frac{\varepsilon\omega_1}{\omega_0^2 + \varepsilon\omega_1}, \qquad \omega_1 = \frac{3}{4}a^2 + \frac{5}{8}\beta a^4,$$

$$C_1 = -\frac{1}{32\omega_1}a^3 - \frac{1}{24\omega_1}\beta a^5, \qquad C_2 = \frac{1}{32\omega_1}a^3\left(1 + \frac{5}{4}\beta a^2\right),$$

$$C_3 = \frac{1}{384\omega_1}\beta a^5, \qquad \omega = \omega_0\left[1 + \frac{1}{2}\alpha + \left(\frac{3}{8} + \frac{\delta_2}{2}\right)\alpha^2\right],$$

$$\delta_2 = \frac{1}{32\omega_1}a^2 + \frac{1}{24\omega_1}\beta a^4 + \frac{1}{8\omega_1}C_1 a(18 + 25\beta a^2) + \frac{1}{16\omega_1}C_2 a(12 + 25\beta a^2)$$
$$+ \frac{5}{16\omega_1}\beta C_3 a^3. \tag{9}$$

We measure the data $\bar{x}(t_i)$ for $i = 1, 2, \ldots, m$ from the free-vibration test and estimate the system parameters ω_0, ε and β by minimizing $\sum_{i=1}^{m}[x(t_i) - \bar{x}(t_i)]^2$. The optimal problem in which there is no constraint may be written as

$$\min S(\omega_0, \varepsilon, \beta) = \sum_{i=1}^{m}(a\cos\omega t_i + \alpha(C_1\cos\omega t_i + C_2\cos 3\omega t_i$$
$$+ C_3\cos 5\omega t_i) - \bar{x}(t_i))^2, \tag{10}$$

where $\alpha, \omega_1, C_1, C_2, C_3, \omega$ and δ_2 are given by equation (9). In order to inspect the accuracy of identification, we make an analogue test by using computer.

The simulated values of response $\bar{x}(t_i)$ of equations (6) and (7) for $\omega_0 = 3$, $\varepsilon = 2$, $\beta = 1{\cdot}5$ and $a = 1$ are obtained by the Runge–Kutta method as shown in Table 1.

The parameter values sought by using the optimizaton method are

TABLE 1

The numerical solution of equation $\ddot{x} + 9x + 2(x^3 + 1{\cdot}5x^5) = 0$

t	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9
$\ddot{x}$	1	0·932	0·744	0·474	0·158	−0·172	−0·487	−0·754	−0·937	−1·000
t	1·0	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	
$\bar{x}$	−0·926	−0·734	−0·461	−0·144	0·186	0·499	0·764	0·942	0·999	

$$\omega_0 = 2{\cdot}938, \qquad \varepsilon = 2{\cdot}068, \qquad \beta = 1{\cdot}495.$$

They are in agreement with the system parameters.

4. AN EXAMPLE OF NON-AUTONOMOUS SYSTEM

Here we will deal with the non-autonomous system in the form

$$\ddot{x} + \omega_0^2 x + \beta x^3 = \gamma(1 - x^2)\dot{x} + \delta \cos \Omega t, \tag{11}$$

where γ and δ are small parameters, and β may not be small. Using the stroboscopic method, we can obtain the approximate forced vibration solution of equation (11) as follows:

$$x = \frac{\gamma\Omega a_0^4}{\delta}\cos \Omega t + \frac{\gamma\Omega a_0^2}{\delta}\left(\frac{a_0^2}{4} - 1\right)\sin \Omega t, \tag{12}$$

where

$$a_0 = 2\sqrt{\frac{\Omega^2 \omega_0^2}{3\beta}}. \tag{13}$$

The parameters of equation (11) satisfy the relation

$$a_0^2\Omega^2\gamma^2\left(\left(1 - \frac{a_0^2}{4}\right)^2 + a_0^4\right) - \delta^2 = 0. \tag{14}$$

Measuring the data $\tilde{x}(t_i)$ for $i = 1, 2, \ldots, m$ from the test in the case of known excitation, i.e., δ and Ω are given, we estimate the system parameters. Substituting equation (13) into equations (12) and (14), we get the optimal problem subject to the constraint (14) as follows:

$$\min \quad S(\omega_0, \beta, \gamma) = \sum_{i=1}^{m}\left(\frac{16}{9\delta\beta^2}(\Omega^2 - \omega_0^2)^2\,\gamma\,\Omega\cos\Omega t_i + \frac{4}{3\delta\beta}(\Omega^2 - \omega_0^2)\right.$$
$$\left.\times\left(\frac{\Omega^2 - \omega_0^2}{3\beta} - 1\right)\gamma\Omega\sin\Omega t_i - \bar{x}(t_i)\right)^2 \tag{15}$$

$$\text{s.t.} \quad \frac{4}{3\beta}(\Omega^2 - \omega_0^2)\Omega^2\gamma^2\left(\left(1 - \frac{\Omega^2 - \omega_0^2}{3\beta}\right)^2 + \frac{16}{9\beta^2}(\Omega^2 - \omega_0^2)^2\right) - \delta^2 = 0. \tag{16}$$

TABLE 2

The numerical solution of equation $\ddot{x} + 3x + 5x = 0{\cdot}2(1 - x^2)\dot{x} + 0{\cdot}2\cos \Omega t$

t	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0	1·1	1·2
$\ddot{x}$	0·142	0·041	−0·011	−0·162	−0·258	−0·345	−0·420	−0·480	−0·520	−0·538	−0·533	−0·505	−0·456
t	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2·0	2·1	2·2	2·3	2·4	2·5
$\bar{x}$	−0·389	−0·308	−0·217	−0·119	−0·017	0·086	0·186	0·280	0·365	0·438	0·493	0·528	0·541

For example, we consider the equation

$$\ddot{x} + \omega_0^2 x + \beta x^3 = \gamma(1 - x^2)\dot{x} + 0{\cdot}2\cos \Omega t \tag{17}$$

while $\omega_0 = 1{\cdot}732$, $\beta = 5$ and $\gamma = 0{\cdot}2$, values of response $\bar{x}(t_i)$ of equation (17) are obtained by using numerical simulations as shown in Table 2.

The results of the optimal calculation are listed below:

$$\omega_0 = 1{\cdot}709, \qquad \beta = 5{\cdot}098, \qquad \gamma = 0{\cdot}194.$$

We find satisfaction in these identification data.

5. CONCLUSION

In this paper we have presented a method for parameter identification of strongly non-linear systems. This method is suitable for any parameter identification problem of strongly non-linear systems in which the approximate solution can be determined.

It is noted that the accuracy of parameter identification depends on the degree of accuracy of approximate solutions.

We also note that if there is no expression of an approximate solution, system parameters will not be identified. Therefore, it is important that the approximate solution of non-linear systems is investigated.

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