Journal of Sound and Vibration (2000) **233**(4), 617–641
doi:10.1006/jsvi.1999.2841, available online at http://www.idealibrary.com on IDEAL®

A STUDY OF THE ACOUSTICAL TERMINATION ON PRACTICAL GAS PULSATION MEASUREMENT

P. C.-C. LAI†

Sound and Vibration Laboratory, Copeland Corporation of Emerson Electric Co., Sidney OH 45365, *U.S.A*

(*Received* 31 *August* 1998, *and in final form* 3 *December* 1999)

It has been well recognized in the past that an anechoic termination, which can effectively eliminate the reflective acoustic wave, is required for measurement of exhaust gas pulsation from engines or machinery. In academic environment, the acoustic termination on the exhaust line can be well controlled by appropriate treatment. However, it is not unusual in practical industrial applications that the anechoic termination is not available. Therefore, a theoretical investigation was performed in order to understand the impact on the gas pulsation measurement without an anechoic termination. A simplified model of an exhaust line with different acoustic terminations was analyzed by both analytical and experimental approaches. Both one-microphone and two-microphone measurement methods, which are commonly used, were evaluated. The results clearly demonstrate that without an anechoic termination, the variations of the measurements will be substantial due to the reflective acoustic wave, as has been argued for years in the industry.

1. INTRODUCTION

Similar to the exhaust gas noise from the auto engine, the gas pulsation originating from the machinery has been an important quality index for the product because it may create noise or interact with the whole piping system. More and more customers prefer to buy products with low exhaust gas pulsation. In order to provide customers accurate gas pulsation information of a certain product, an industrial measurement specification has to be identified. However, the appropriate measurement methods to obtain consistent gas pulsation measurement data have been disputed in the positive displacement machinery industry for years [1].

The anechoic termination has been recognized as a standardized termination for gas pulsation measurement for a long time [2, 3]. It can be generally achieved by careful treatments such as long exhaust tubing, absorption materials or horn shaped pipes in academic- or research-type experiments [4–10]. But the situation is often encountered in the industrial applications that an anechoic termination may not be available. For example, in the refrigeration compressor industry, it is difficult to control the acoustical impedance in the suction or discharge lines of the regular testing facility in the factory environment, especially for a mass production quality audit program. It has been noticed that the exhaust gas pulsation data measured in practical applications without qualified anechoic termination could vary dramatically due to the acoustic wave reflected by the non-anechoic termination in the piping system. Because this is a specific concern to industry only,

†Currently affiliated with Structural Dynamics, Hughes Space and Communications, El Segundo, CA, U.S.A.

academic researchers often neglected the importance of this topic in the past as a result of which very limited literature is available. Nowadays, this problem becomes more serious when the International Standard Organization (ISO) is trying to standardize the industrial measurement technique. Therefore, a study was conducted and supported by the related industrial community to understand the discrepancies of the non-anechoic termination measurement methods. Some theoretical discussion of both anechoic and non-anechoic terminations on mathematical modelling of gas pulsation has been reported in references [11, 12]. But unfortunately, the influence of the non-anechoic termination on the practical measurement has not been thoroughly discussed as yet, nor has any experiment been implemented to support the conclusion in references [11, 12]. In this investigation, an analysis approach similar to that reported in references [11, 12] will then be followed.

The gas pulsation exhausted from the engine or machinery is often affected by the exhaust piping line length and the termination impedance of the adjacent system or the testing stand. The abrupt termination or discontinuation induces the reflection wave travelling back to the source. Unlike the automobile, whose exhaust line terminates in the atmosphere, the termination acoustic impedance for machinery is particularly difficult to control because it varies with regard to not only the systems connected to the machine but also the machine operating conditions. With an anechoic termination, the gas pulsation measurements by single microphone or pressure transducer will be inherently very consistent at a certain machine running condition because there is no reflection wave superposed on the outgoing incident wave. The single pressure measurement is primarily used in the machinery industry [2, 3]. Another popular measurement technique is the so-called two-microphone method. This two-microphone method has been used for a long time to measure the acoustical intensity [13, 14] and the muffler transmission loss [15]. It can also be used to decompose the incident and reflected waves [16]. Intuitively, this method may obtain a consistent result of the incident gas pulsation wave even without an anechoic termination. Unfortunately, both single-microphone and two-microphone methods will not have consistent measurement results without a proper anechoic termination, as will be demonstrated later in this paper.

The manifold system from the valve or port of the machinery cylinder to the entrance of the exhaust piping line usually consists of tubing, cavity and/or muffler(s) and can be viewed as a kind of "muffler" as a whole (see Figure 1). Therefore, without losing generality, a muffler reported in reference [8] can represent a gas management system inside any machine for modelling purpose. It is usually assumed that for the positive displacement machine the exhausted flow rate from the valve or port at a certain operating condition remains constant whatever the output condition in the exhaust piping line is [17–19]. In general, this is true because flow back to the cylinder(s) through the valve or port is inhibited due to the large pressure difference across the valve or port.

A model as shown in Figure 1 was used to simulate the gas management system from machine valve or port to the testing stand or adjacent system. All the components were picked such that a simple one-dimensional (1-D) acoustic wave theory in terms of four pole parameters [15, 17, 18] could be used for analysis. The first part of the investigation was to demonstrate the basic characteristics of the gas pulsation measurement by either single- or two-microphone method from both theory and experiment. The second part was then to simulate the measurement results of this gas management system by this simple model while the exhaust line lengths, damping treatments in the piping line, and output impedances at the testing stand or attaching system vary. Finally, the peak-to-peak pulsating pressure from the machine measured by the single pressure transducer, which is widely used in the refrigeration compressor industry [2], with a finite length exhaust line and a non-anechoic termination was calculated and compared to the case with an anechoic termination.

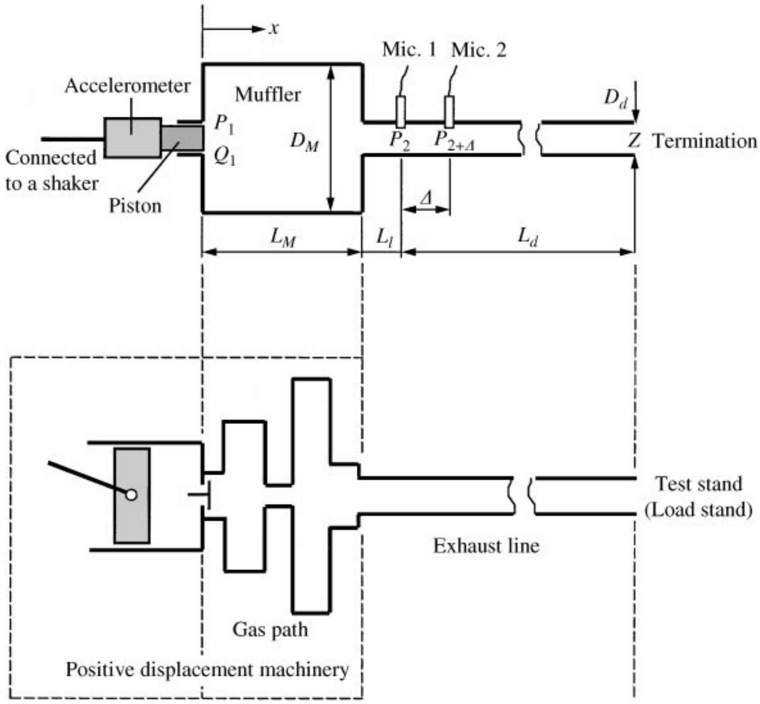

Figure 1. Schematic illustration of muffler bench test set-up and its corresponding gas path in a typical machine.

2. THEORETICAL BASIS

First of all, the theory used in this study will be reviewed briefly following references [11, 12].

2.1. FOUR POLE PARAMETERS

A gas cavity can be represented by the following format in the frequency domain [18]:

$$\begin{bmatrix} Q_1 \\ P_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} Q_2 \\ P_2 \end{bmatrix}, \tag{1}$$

where Q_1, P_1 are the input volume velocity and acoustic pressure, Q_2, P_2 are the output volume velocity and acoustic pressure. A, B, C, D are the so-called four pole parameters for a particular gas cavity. The 1-D model is usually used for modelling a long pipe or muffler element, whose cross-sectional dimension is small compared to the shortest wavelength of interest. The four pole parameters of the 1-D model with constant cross-section are [17, 18]

$$A = \cosh(\gamma L), \quad B = \frac{\mathrm{j}\omega A_0}{\rho c^2 \gamma} \sinh(\gamma L), \tag{2,3}$$

$$C = \frac{\rho c^2 \gamma}{\mathrm{j}\omega A_0} \sinh(\gamma L), \quad D = \cosh(\gamma L). \tag{4,5}$$

A_0 is the cross-sectional area of the cavity, $\gamma = \xi\sqrt{\omega}/(2cd) + \mathrm{j}(\omega/c)$, ξ the damping ratio, d the diameter of the cavity, and L the length of this cavity. ρ and c are the gas density and speed of sound, ω represents the frequency.

2.2. NORMALIZED OUTPUT ACOUSTIC PRESSURE

2.2.1. *Anechoic termination*

If the termination in the exhaust piping line is anechoic with output impedance $Z = \rho c/S_d$ [17, 18], the transfer function between the input volume velocity and output acoustic pressure of a gas cavity is obtained

$$\frac{P_2}{Q_1} = \frac{1}{A(S_d/\rho c) + B}, \tag{6}$$

where S_d is the cross-sectional area of the exhaust pipe. However, the output acoustic or pulsating pressure can be further normalized as

$$P_2^* = \frac{P_2}{Q_1}\cdot\frac{S_d}{\rho c} = \frac{(S_d/\rho c)}{A\,(S_d/\rho c) + B}, \tag{7}$$

based on the assumption that the volume flow rate (velocity) from the valve or port of a particular positive displacement machine at a certain operating condition is constant. This normalized variable is very convenient for the analysis and will be frequently used in this study.

There is a short distance between the output of the muffler and the location of the first microphone as shown in Figure 1. The four poles of this section can be obtained from equations (2)–(5) as well. The corrected four pole parameters between the muffler input and the first microphone location can then be calculated by simple matrix manipulation:

$$\begin{bmatrix} A_C & B_C \\ C_C & D_C \end{bmatrix} = \begin{bmatrix} A_M & B_M \\ C_M & D_M \end{bmatrix}\begin{bmatrix} A_l & B_l \\ C_l & D_l \end{bmatrix}, \tag{8}$$

where the subscripts M and l represent the muffler itself and the tube section between the output of the muffler and the first microphone respectively.

2.2.2. *Single microphone method*

If the muffler or say the machine is attached to a finite length exhaust line with an output impedance Z in the testing stand or adjacent system as shown in Figure 1, the four poles of the exhaust line itself can be still modelled by the 1-D linear acoustic theory (equations (2)–(5)). Therefore, the normalized acoustic pressure can be derived as [11, 12]

$$P_2^* = \frac{P_2}{Q_1}\frac{S_d}{\rho c} = \frac{\mathrm{S}_d}{\rho c}\frac{C_d + D_d Z}{A_C A_d + A_C B_d Z + B_C C_d + B_C \mathrm{D}_d Z}, \tag{9}$$

where the subscript d represents the finite length exhaust line. A_d, B_d, C_d, and D_d are the four pole parameters of the exhaust line L_d. This equation represents the gas pulsation measured by the single-microphone or pressure transducer method without an anechoic termination.

2.2.3. *Theoretical incident wave*

It was suspected that the incident wave in the exhaust line subjected to the same input flow rate might always remain the same regardless of the pipe length and output impedance. It is potentially a candidate of acoustical property to be measured in mass production audit programs. The incident wave can be calculated directly from the theory. Again, the relation of the muffler output and input is as illustrated in equation (1). The acoustical pressure in the exhaust line consists of the incident and reflective waves,

$$P_2 = P_{2+} + P_{2-}, \tag{10}$$

where the subscripts " + ' and " − " stand for the incident and reflective directions respectively. Therefore, the output volume velocity can be rewritten as [20]

$$Q_2 = -\frac{S_d}{\mathrm{j}\omega\rho}\frac{\mathrm{d}P_2}{\mathrm{d}x} = \frac{S_d}{\rho c}P_{2+} - \frac{S_d}{\rho c}P_{2-}. \tag{11}$$

Since the representation of normalized output volume velocity can be obtained by the same way as equation (9),

$$Q_2^* = \frac{Q_2}{Q_1} = \frac{A_d + B_d Z}{A_C A_d + A_C B_d Z + B_C C_d + B_C D_d Z}, \tag{12}$$

the normalized incident acoustic pressure is then derived from equations (9) through (12)

$$P_{2+}^* = \frac{P_{2+}}{Q_1}\frac{S_d}{\rho c} = \frac{1}{2}\frac{A_d + B_d Z + (S_d/\rho c)(C_d + D_d Z)}{A_C A_d + A_C B_d Z + B_C C_d + B_C D_d Z}. \tag{13}$$

2.2.4. *Two-microphone method*

If the normalized acoustic pressures measured by both microphones 1 and 2 in Figure 1 in terms of incident and reflective waves are

$$P_2^* = P_{2+}^* + P_{2-}^*, \quad P_{2+\Delta}^* = P_{2+}^* \mathrm{e}^{-\mathrm{j}k\Delta} + P_{2-}^* \mathrm{e}^{\mathrm{j}k\Delta}, \tag{14,15}$$

where $k = \omega/c$ is the wave number, P_2^* and $P_{2+\Delta}^*$ can be obtained by either equation (9) or experiment. The incident normalized acoustic wave becomes

$$P_{2+}^* = \frac{\mathrm{e}^{\mathrm{j}k\Delta}P_2^* - P_{2+\Delta}^*}{2j\sin k\Delta}. \tag{16}$$

3. EXPERIMENTAL APPROACH

The experimental approach was used to verify the theory. The experimental set-up simulating a typical positive displacement machinery without losing generality is shown in Figure 1, basically following the transfer function measurement technique reported in reference [4]. A piston at the cavity entrance was attached to a dynamic shaker to generate sound, which simulates the pulse input from the machinery cylinder head. The acceleration of this oscillating piston was measured and then the input volume velocity was obtained by integration. The muffler was used to simulate the gas path between the gas pulsation source from the machinery cylinder head and the exit of the machinery. Two microphones were installed at the exit of the cavity to measure the output acoustic pressure. Therefore, the

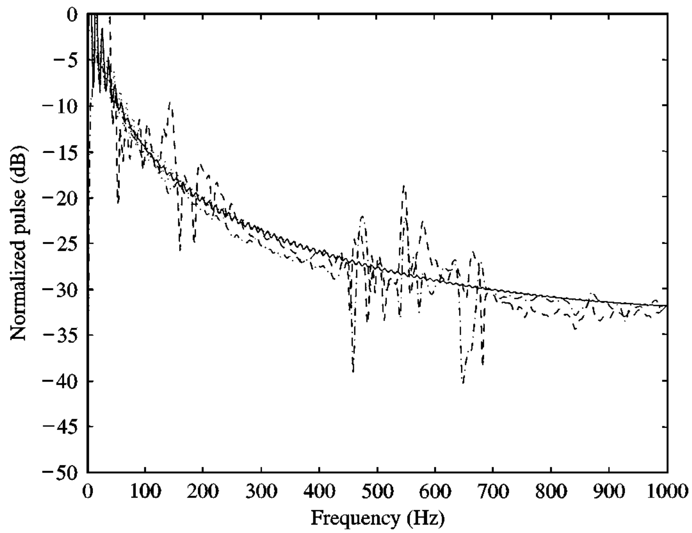

Figure 2. Comparison of normalized pulsating pressure from both theory and experiment for a discharge pipe length $L_d = 15{\cdot}24$ m. (—) One microphone method (theory); (···) Two microphone method (theory); (–·–·) One microphone method (experiment); (– – –) Two microphone method (experiment).

transfer function between the output acoustic pressure and input volume velocity or the normalized pulsating pressure was measured by either the single- or two-microphone method. Either a very long exhaust pipe, which simulated the anechoic termination, or a finite length tube was used. The medium used here was air at room temperature ($\rho = 1{\cdot}21$ kg/m^3, $c = 343$ m/s). The dimensions used were $L_M = 0{\cdot}072$ m, $D_M = 0{\cdot}070$ m, $D_d = 0{\cdot}011$ m, and $L_l = 0{\cdot}013$ m.

4. CASE STUDY AND DISCUSSION

4.1. ANECHOIC TERMINATION VERSUS NON-ANECHOIC TERMINATION

In the first case studied, a very long exhaust tube ($L_d = 15.24$ m) was used to simulate the anechoic termination. The damping factors chosen for the muffler and the exhaust line were 0·001 and 0·02, respectively, based on the experimental data. The normalized pulsating pressure by both single-microphone (equations (7) or (9)) and two-microphone (equation (16)) methods predicted from the theory were compared to the experimental data as shown in Figure 2. The simulation agreed with the measurement satisfactorily except that some measurement noise existed in the frequency ranges of 0–250 and 450–700 Hz. Both the single-microphone and two-microphone methods generated almost the same result while an anechoic termination was used. Note that the frequency range was picked on purpose such that no muffler resonance showed up in this frequency range, which may cause some distortion in the comparison. Also, the spacing between the two microphones, $\Delta = 0{\cdot}0254$ m, was chosen such that no singularity would be encountered except 0 Hz [16].

In contrast, two cases with non-anechoic terminations were analyzed in the same way. If a finite length tube ($L_d = 1.09$ m) with an open end (approximate output impedance $Z = 0$ [11, 12, 18]) was utilized, a comparison of the predictions and measurements is illustrated in Figure 3. The theory matched the experiment very satisfactorily. For both single- and two-microphone measurement methods, the exhaust tube resonances came into play and

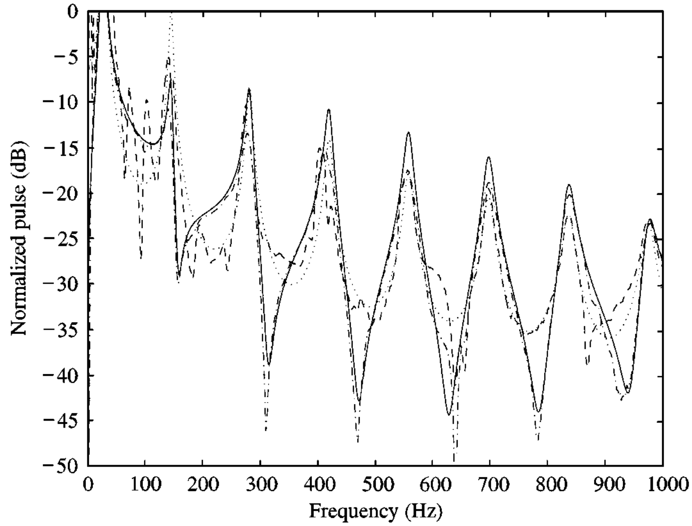

Figure 3. Comparison of normalized pulsating pressure from both theory and experiment for a discharge pipe length $L_d = 1{\cdot}09$ m with an open end. (—) One microphone method (theory); (···) Two microphone method (theory); (–··–) One microphone method (experiment); (– – –) Two microphone method (experiment).

seriously affected the measurement result, as reported in references [11, 12]. If the output impedance was modified by using a metal cap to close the end of the tube (rigid end boundary condition), which demonstrated the extreme acoustic impedance change at the testing stand or system due to different operating conditions, the simulations and experimental results were compared in Figure 4. The randomly picked output impedance in the simulation was $Z = 10^{16}$ kg/m^4 s, which was extremely large compared to the impedance for anechoic termination, $Z = \rho c/S_d = 1.34 \times 10^8$ kg/m^4 s [17, 18]. Similarly, the gas pulsation measurement for both methods would be seriously distorted, since the termination was not anechoic. In addition, by comparing Figures 3 and 4, it was found that the resonance frequencies induced from the exhaust tube would shift due to the change of output impedance (boundary condition) in the testing stand.

In sum, both the one- and two-microphone measurements give the same results as long as an appropriate anechoic termination is applied.

4.2. DIFFERENT EXHAUST PIPE CONDITIONS

The previous section has demonstrated that without proper anechoic termination (boundary condition) in the exhaust pipe, different results will be obtained by various measurement schemes. In this section, the one-microphone method (equation (9)), theoretical incident wave (equation (13)), and two-microphone method (equation (16)) measurements with a non-anechoic termination were compared to the case with an anechoic termination (equation (7)) on a theoretical basis in order to demonstrate what would be seen in the actual measurement where no anechoic termination is available.

4.2.1. *Output impedances*

Figure 5 shows a comparison of the above methods while the machinery was connected to an open end ($Z = 0$) finite length exhaust tube to the case with an anechoic termination. As expected, the incident wave predicted from theory (equation (13)) was exactly the same as

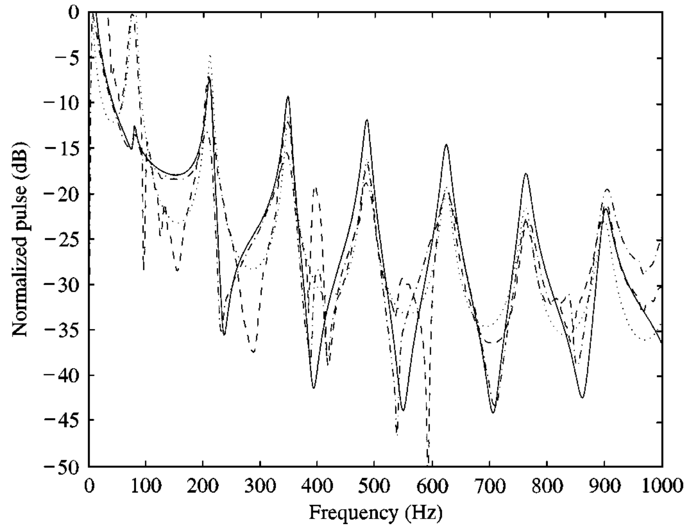

Figure 4. Comparison of normalized pulsating pressure from both theory and experiment for a discharge pipe length $L_d = 1{\cdot}09$ m with a closed end. (—) One microphone method (theory); (···) Two microphone method (theory); (–·–·) One microphone method (experiment); (– – –) Two microphone method (experiment).

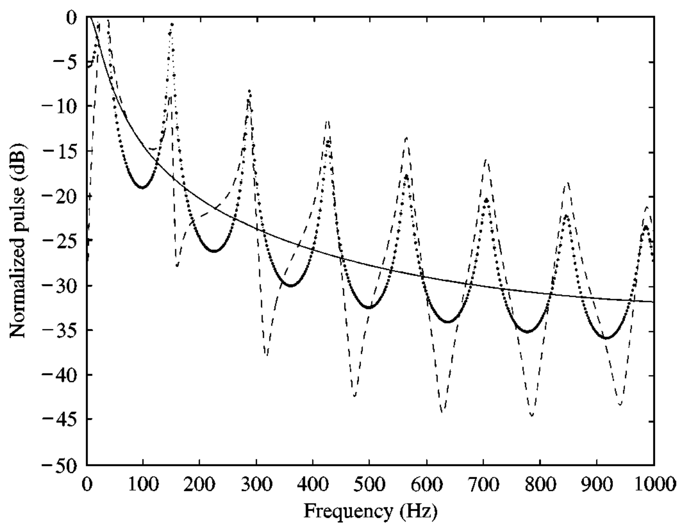

Figure 5. Normalized pulsating pressure when the discharge line length is finite ($L_d = 1{\cdot}09$ m) with an output impedance $Z = 0$. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

the result of the two-microphone method (equation (16)) because the two-microphone method was actually used to measure the incident wave. However, the one-microphone method and the two-microphone method yielded different results because one gave the combination of the incident and reflective waves while the other gave only the incident wave. Both the single pressure measurement and the incident wave pressure results without

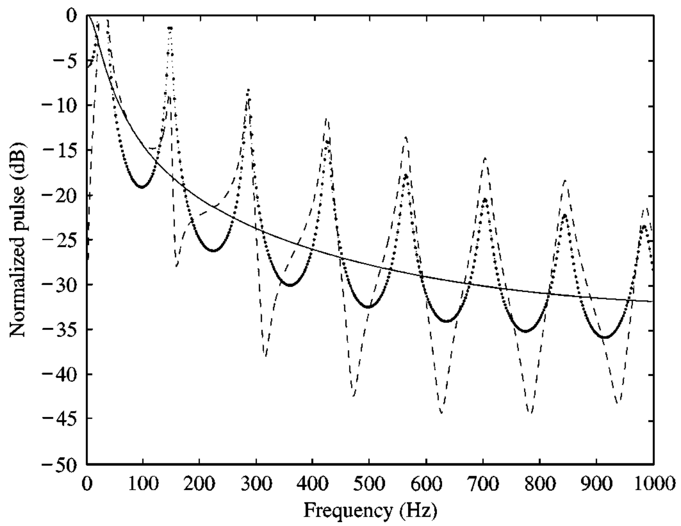

Figure 6. Normalized pulsating pressure when the discharge line length is finite (L_d = 1·09 m) with an output impedance, which includes the end correction. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

an anechoic termination were all different from the pulsating pressure with an anechoic termination.

Note that the simulation with a finite length open end tube could be further improved by using the end corrected output impedance (radiation impedance) reported in reference [20]

$$Z = \frac{\rho c}{S_d}\,[0{\cdot}0625\cdot(kd)^2 + \mathrm{j}0{\cdot}3kd]. \tag{17}$$

The results, using this end correction, are shown in Figure 6. Since there was no large difference between Figures 6 and 5, $Z = 0$ was used for the open-end boundary condition throughout this study.

The case with a closed-end finite length tube was calculated and compared in Figure 7. Similarly, both the single pressure measurement and incident wave pressure data were quite different from the result with an anechoic termination. If the output impedance of the finite-length tube was artificially assigned to be $Z = \rho c/S_d$, a comparison is illustrated in Figure 8. Both the single- and two-microphone methods gave identical answers to the case with an anechoic termination condition. In fact, $Z = \rho c/S_d$ is the boundary condition of the so-called anechoic termination. It was also interesting to see what would happen if the output boundary condition is out of phase with the acoustic impedance for an anechoic termination. Figure 9 shows the results of the computer simulation. The tube resonances still came into play for this case. Note that the resonance frequencies were different from those cases with either open-end or closed-end exhaust tubes, i.e., different output impedances.

From the above illustration, it is very clear that the inconsistent measurements result from the reflection wave in the exhaust pipe, which is controlled by the boundary condition, i.e., acoustic impedance of the piping outlet. The resonances of the exhaust pipe are excited by the incident and reflective waves bouncing back and forth inside the pipe and appear in the measurement. Furthermore, the reflective wave controlled by the boundary condition

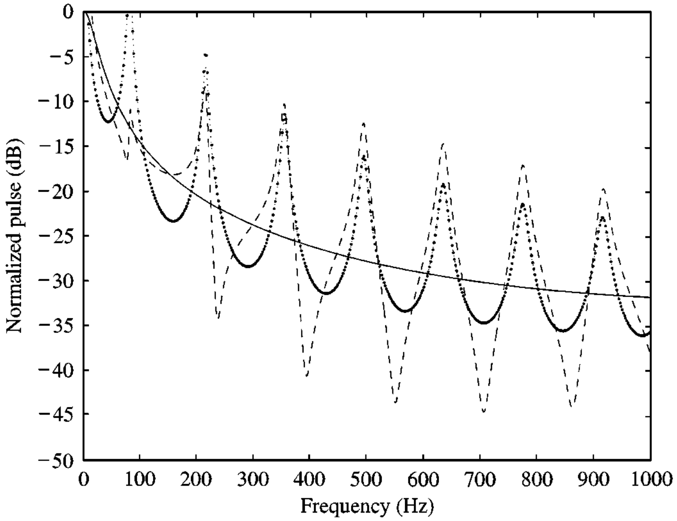

Figure 7. Normalized pulsating pressure when the discharge line length is finite ($L_d = 1{\cdot}09$ m) with an infinite output impedance. (—) Anechoic termination (baseline); (– – –) One microphone method; (· · ·) Incident wave from theory; (●) Two microphone method.

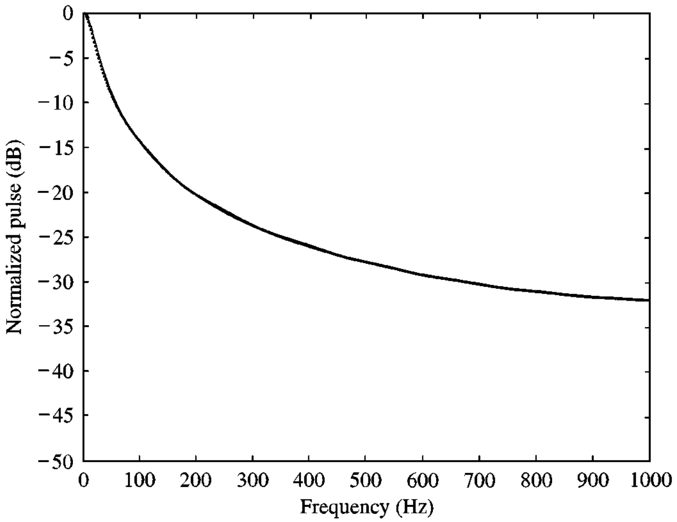

Figure 8. Normalized pulsating pressure when the discharge line length is finite ($L_d = 1{\cdot}09$ m) with an output impedance $Z = \rho c/S_d$. (—) Anechoic termination (baseline); (– – –) One microphone method; (· · ·) Incident wave from theory; (●) Two microphone method.

not only induces the resonances in the exhaust pipe but also changes the source impedance and therefore the incident wave. Thus, even the incident wave measurement (two-microphone method) cannot produce a consistent result when different acoustic terminations (boundary condition) are applied.

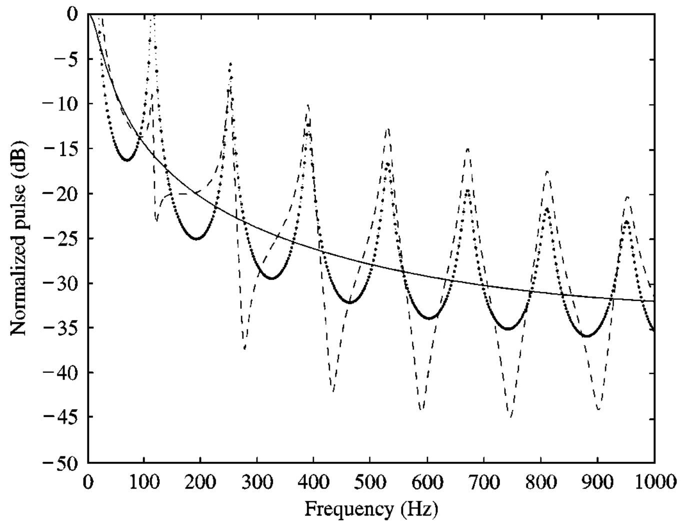

Figure 9. Normalized pulsating pressure when the discharge line length is finite (L_d = 1·09 m) with an output impedance $Z = j\rho c/S_d$. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

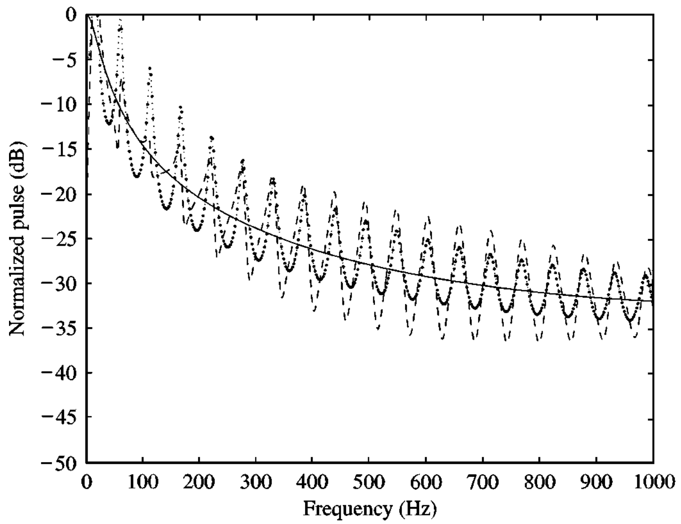

Figure 10. Normalized pulsating pressure when the discharge line length is L_d = 3m with an output impedance $Z = 0$. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

4.2.2. *Exhaust pipe length effect*

It has been shown that different output impedances (boundary conditions) would not improve the measurement except in the artificial anechoic termination case, and the inconsistent measurement came from the reflection wave. Therefore, the only way to obtain consistent measurement results was to reduce or eliminate the reflection wave by increasing either the exhaust tube length or damping. Figure 10 shows the results of simulation by

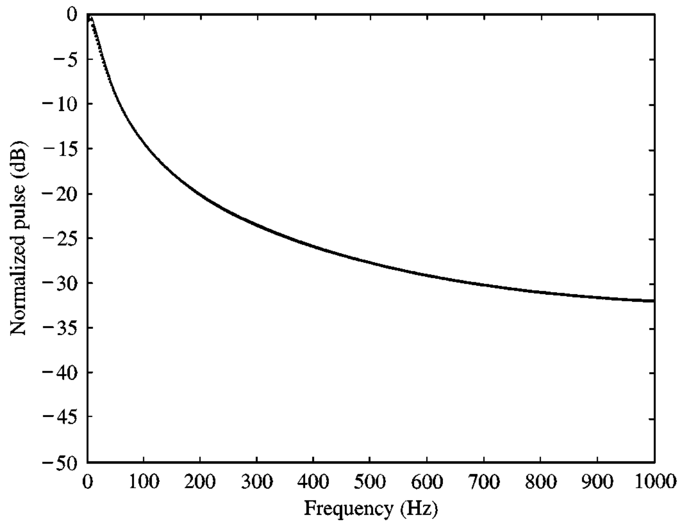

Figure 11. Normalized pulsating pressure when the discharge line length is $L_d = 100$ m with an output impedance $Z = 0$. (—) Anechoic termination (baseline); (– – –) One microphone method; (· · ·) Incident wave from theory; (●) Two microphone method.

increasing the tube length to 3 m. The tube resonances are still found to exist in the measurement while the magnitudes were attenuated by the dissipation. Note that the tube resonance frequencies changed due to different tube lengths. If the tube length was further increased to 100 m, which was much closer to an anechoic termination condition, both single microphone and incident wave measurements were the same as in the case with an anechoic termination as illustrated in Figure 11. Therefore, a long exhaust tube can achieve near anechoic termination condition and give consistent gas pulsation measurement irrespective of using the one- or two-microphone method. Unfortunately, a very long pipe is not very practical in most industrial applications since the length is limited by the laboratory environment.

4.2.3. *Damping factor effect*

Damping materials have long been used to dissipate the acoustic wave in noise control applications. By adding the absorption materials to the exhaust line, the gas pulsation from the machinery can be effectively reduced. However, the application is limited in the machinery industry because the absorption materials may soak up liquid, lubrication oil, or debris, which occasionally travel along with the operating medium, and cause a clog in the piping line. If the above-mentioned problems can be overcome, this may be a natural solution for standardizing the measurement method. Therefore, the impact of damping in the exhaust line is discussed in this section for completeness. Again, Figure 12 shows a comparison of various measurement methods while the damping factor in the exhaust tube was increased from 0·02 to 0·05. The magnitudes of the pipe resonances occurred in the measurements were reduced due to the larger dissipation while the resonance frequencies remained the same. It is obvious that the variation between different methods was minimized. If the damping factor in the exhaust pipeline was further increased to 0·2 as illustrated in Figure 13, both one-microphone and two-microphone methods yielded almost the same results as the case with an anechoic termination.

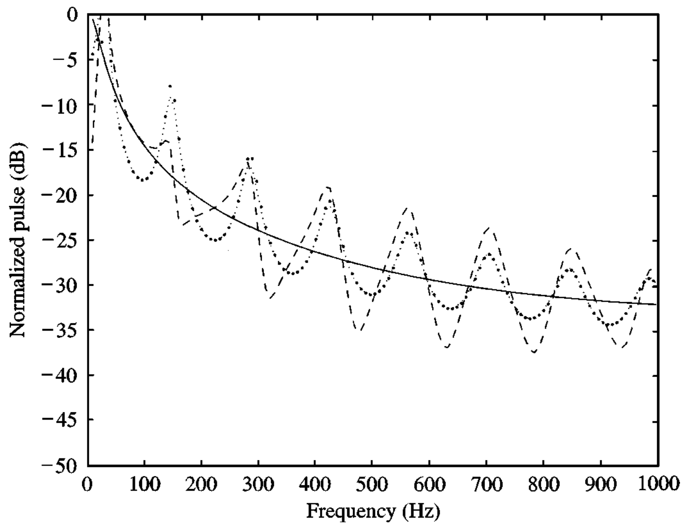

Figure 12. Normalized pulsating pressure when the discharge line damping is $\xi_d = 0{\cdot}05$ with a discharge line length $L_d = 1{\cdot}09$ m and an output impedance $Z = 0$. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

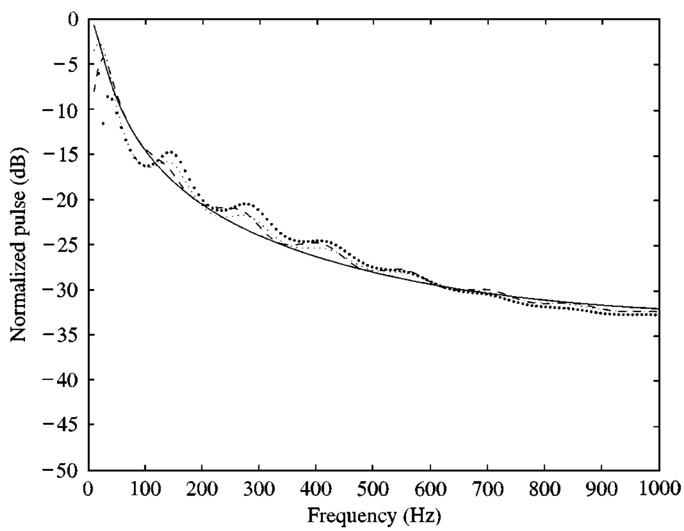

Figure 13. Normalized pulsating pressure when the discharge line damping is $\xi_d = 0{\cdot}2$ with a discharge line length $L_d = 1{\cdot}09$ m and an output impedance $Z = 0$. (—) Anechoic termination (baseline); (– – –) One microphone method; (···) Incident wave from theory; (●) Two microphone method.

With sufficient damping, consistent measurement in the exhaust line can be achieved provided that the practical application can be implemented.

4.3. PEAK-TO-PEAK PRESSURE

In the refrigeration compressor industry, peak-to-peak magnitude is the standard measure for the gas pulsation [2]. It was then interesting to see what pulsating pressure in the practical applications would be measured based on the single pressure method if the anechoic termination is not available. The procedure reported in references [17, 19] for the analysis of compressor discharge gas pulsation was followed in this investigation. Assume that the time series of the input volume flow rate from the valve or port of the machine is known. Since the volume flow rate is periodic for cyclic machine, it can be represented as

$$q_1(t) = \sum_{n=0}^{N-1} c_n \mathrm{e}^{\mathrm{j}n\omega t}, \tag{18}$$

where N is the total number of data sampling points and ω is the machine running frequency. The Fourier coefficients are

$$c_n = Q_1(n\omega) = \sum_{k=0}^{N-1} q_1(k\Delta t)\,\mathrm{e}^{-\mathrm{j}2\pi kn/N}, \tag{19}$$

where Δt is the sampling time and $n = 0, 1, \ldots, N-1$. The pressure response at the exit of the muffler subjected to the input volume velocity can be calculated by multiplying the volume flow rate by the transfer function in the frequency domain:

$$P_2(n\omega) = \frac{P_2(n\omega)}{Q_1(n\omega)} \cdot Q_1(n\omega). \tag{20}$$

Finally, the pulsating pressure in the time domain can be obtained by an inverse Fourier transformation:

$$p_2(k\Delta t) = \sum_{n=0}^{N-1} P_2(n\omega)\mathrm{e}^{\mathrm{j}2\pi kn/\mathrm{N}}, \tag{21}$$

where $k = 0, 1, 2, \ldots, N-1$.

In the following simulation, the medium used was refrigerant with gas properties $\rho = 71{\cdot}25$ kg/m^3 and $c = 181{\cdot}26$ m/s, which are typical for the discharge gas pulsation from a refrigeration compressor. Two artificial input volume velocities, case (a): squared wave and case (b): saw-toothed wave, as shown in Figure 14, were used to represent two different machines. These two volume flow rate assumptions are commonly used for refrigeration compressor simulation [18]. The transfer function of the muffler, which stands for the whole machine internal exhaust system in reality, can be calculated from equations (6) and (9) for both anechoic and non-anechoic termination cases respectively. The transfer function of this specific muffler example in the frequency domain with a discharge pipe length of 1 m was compared to that of an anechoic termination case as shown in Figure 15. Note that the damping factor in the discharge line here was 0·002 instead of 0·02 in order to exaggerate the effect of the non-anechoic termination. In the real application, the damping ratio has to be determined from experiment. An open-end assumption was used for this 1 m pipe. The pulsating pressures in the time domain as illustrated in Figure 16 were then obtained from equations (20) and (21) for both squared wave and saw-toothed wave inputs. Comparing to the baseline cases with an anechoic termination, the shapes of the output acoustic waves for both input waveforms in the time domain were distorted when the non-anechoic termination was applied. The peak-to-peak magnitudes of the baselined output gas pulsation for both cases (a) and (b) with the anechoic termination were $1{\cdot}4 \times 10^4$ and $1{\cdot}9 \times 10^4$ Pa respectively. However, the pulsating pressures with the 1 m long open-end exhaust tube were only $1{\cdot}3 \times 10^4$ and $1{\cdot}5 \times 10^4$ Pa, which were both smaller than the baselines due to the effect of the reflective waves.

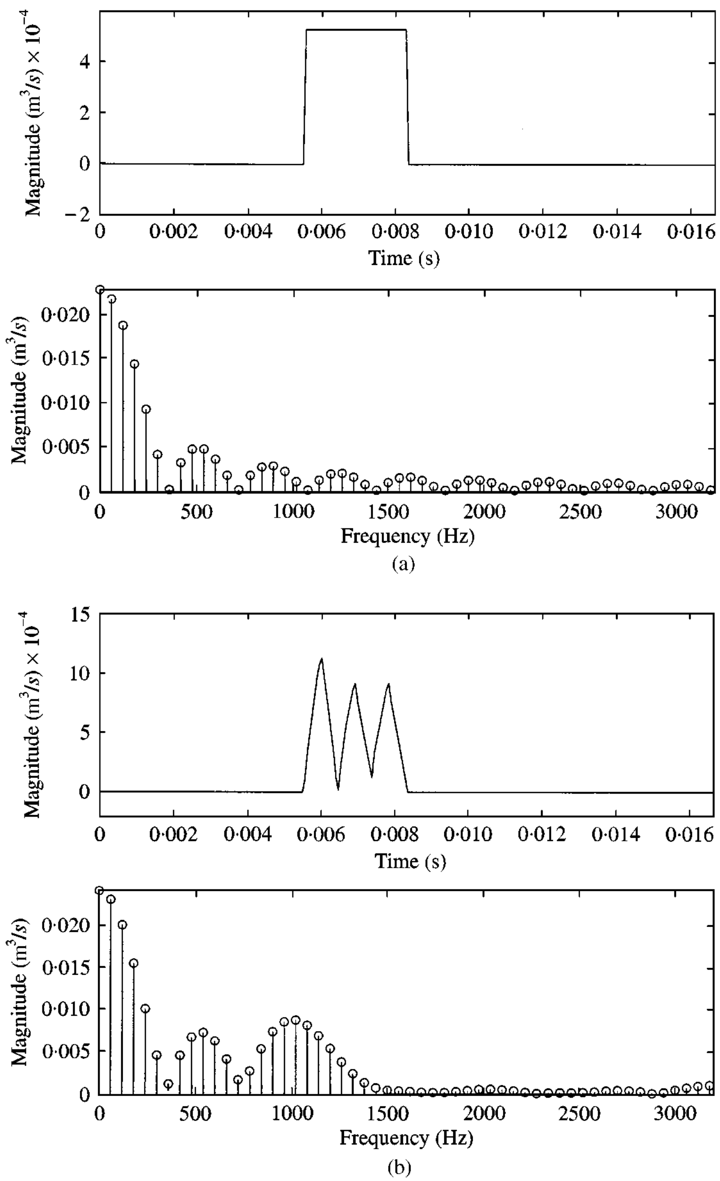

Figure 14. Two different input flow rates used for compressor discharge pulsating pressure parametric study. (a) Squared wave; (b) Saw-toothed wave.

If the tube length was increased to 1·8 m, the pulsating pressure for the squared wave input was $1{\cdot}4 \times 10^4$ Pa, which was very close to the baseline with an anechoic termination. However, the waveform was different from the baseline as shown in Figure 17 because the transfer functions in the frequency domain for anechoic termination and non-anechoic termination cases were different. In addition, the pulsating pressure for the saw-toothed wave input was $1{\cdot}7 \times 10^4$ Pa, which was still different from the baseline (anechoic termination). This comparison demonstrates that even if the tube length may be tuned in order to obtain an approximate peak-to-peak value, which is close to that of the anechoic termination case, for a certain machine, it may not work the same way for another machine.

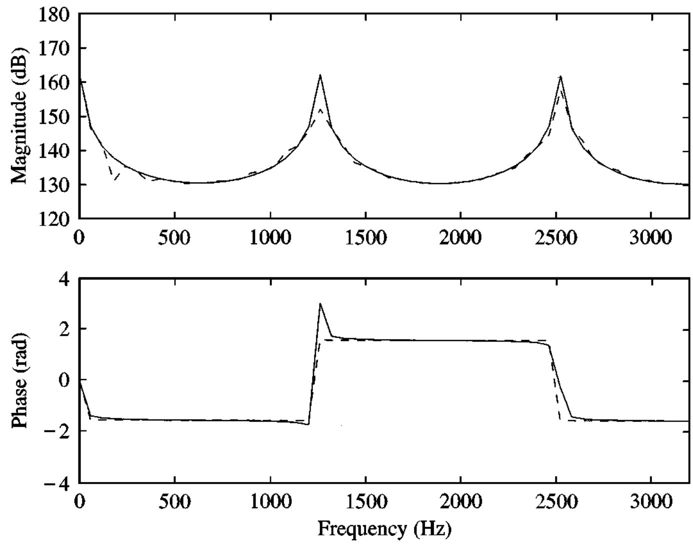

Figure 15. Transfer function of the muffler connected to a finite length pipe ($L_d = 1$ m) compared to that of having an anechoic termination. (—) Anechoic termination; (– – –) Non-anechoic termination.

Figure 18 shows the simulation results while the tube length was 2 m long. The peak-to-peak output pressure for the squared wave input was $2{\cdot}6 \times 10^4$ Pa, while the output pressure magnitude for the saw-toothed wave input volume velocity was $1{\cdot}8 \times 10^4$ Pa. It demonstrates that the peak-to-peak value of the output pulsating wave from machinery may not be always bigger or smaller than the baseline value (with an anechoic termination). It is essentially system dependent and could move either way depending on the combination of incident and reflective waves. The computer simulation results of cases with various discharge tube lengths are listed in Table 1 for reference.

Similarly, the cases of a 1 m discharge tube with closed end were simulated and the peak-to-peak gas pulsation magnitudes, as shown in Figure 19 for both volume velocity inputs, were $2{\cdot}0 \times 10^4$ and $2{\cdot}5 \times 10^4$ Pa, which were both larger than the baseline values and those of the open-end cases with the same length pipe. This means that while the output impedance varies with the operating conditions in the testing stand, the magnitude of the output gas pulsation also changes even though the input volume velocity directly from the machine still remains the same. If an artificial anechoic termination impedance can be applied to the testing stand or adjacent system, the output gas pulsation from the machinery will be the same as the baselines as illustrated in Figure 20. Different output impedances were used for simulation and the peak-to-peak gas pulsation results are summarized in Table 2 for reference.

Finally, the peak-to-peak pressure values versus the lengths of the discharge tubing were investigated, even though the tuning of tube length has been discussed earlier. The simulated results for different tube lengths with an open-end termination were compared in Figure 21. It shows that the peak-to-peak magnitude of the gas pulsation is very sensitive to the tube length such that an optimization is essentially impractical. Various output impedances were used for simulation in the same way. In this comparison, two different pipe lengths were used and the results are illustrated in Figures 22 and 23. It would appear that the peak-to-peak gas pulsation measurement is not so sensitive to the output impedance as to the tube length. However, it is still machine dependent.

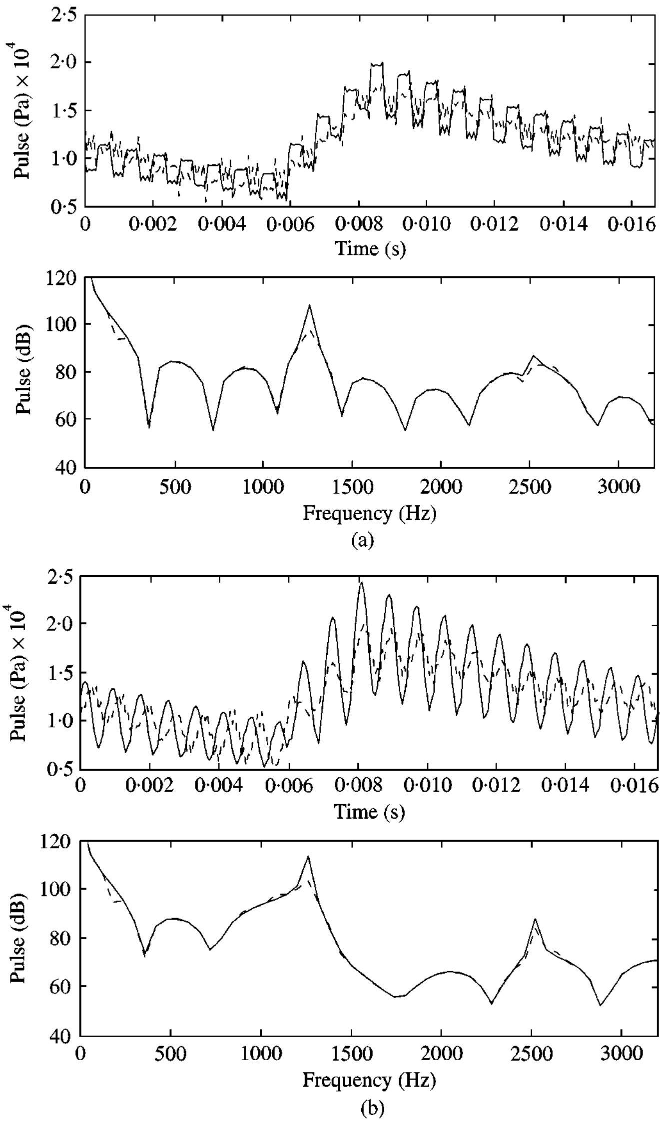

Figure 16. Compressor discharge pulsating pressure excited by (a) Squared wave input and (b) Saw-toothed wave input with $L_d = 1$ m and $Z = 0$. (—) Anechoic termination; (– – –) Non-Anechoic termination.

5. SUMMARY AND CONCLUSION

The gas pulsation measurement in practical applications with and without an anechoic termination was thoroughly studied through a simplified model. This model comprised a circular expansion muffler, which represented the internal gas management system inside a real machine from the valve or port to the exit of the machine gas path, and a tube, which represented the exhaust line and termination acoustic impedance (or boundary condition) in the adjacent system or testing facility. The oscillating piston at the entrance of the muffler simulated the volume flow rate from the valve or port of a positive displacement machine. The normalized acoustic pressure was then predicted or measured by either the single-microphone or the so-called two-microphone method.

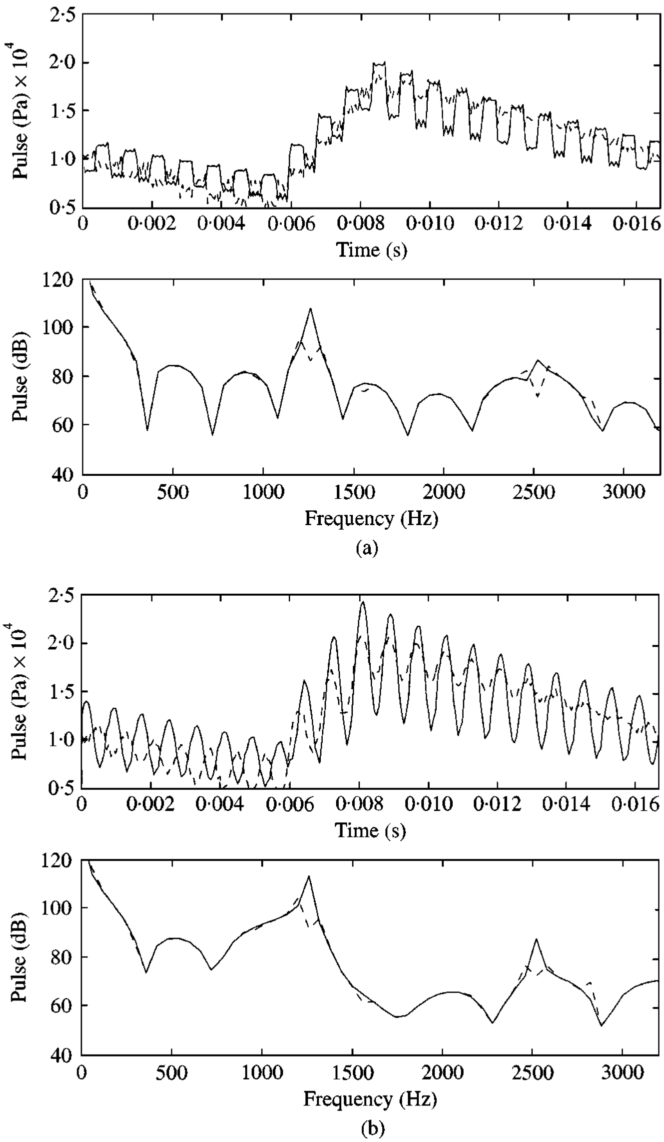

Figure 17. Compressor discharge pulsating pressure excited by (a) squared wave input and (b) saw-toothed wave input with $L_d = 1{\cdot}8$ m and $Z = 0$. (—) Anechoic termination; (– – –) Non-anechoic termination.

The results of this investigation are summarized as follows:

- The conclusions reported in references [11, 12] regarding the anechoic termination affecting the acoustic manifold modelling were proven by both theory and experiment. Without an anechoic termination, the resonances of the exhaust tube show up in the pulsating pressure spectrum. Besides, the acoustic pressure spectrum would also vary with the acoustic impedance in the connecting system or testing stand. Similar to the structural vibrations, these acoustic resonances are due to reflections at boundaries. In the absence of such boundaries, there are no resonances (infinite media have no modes) [21].

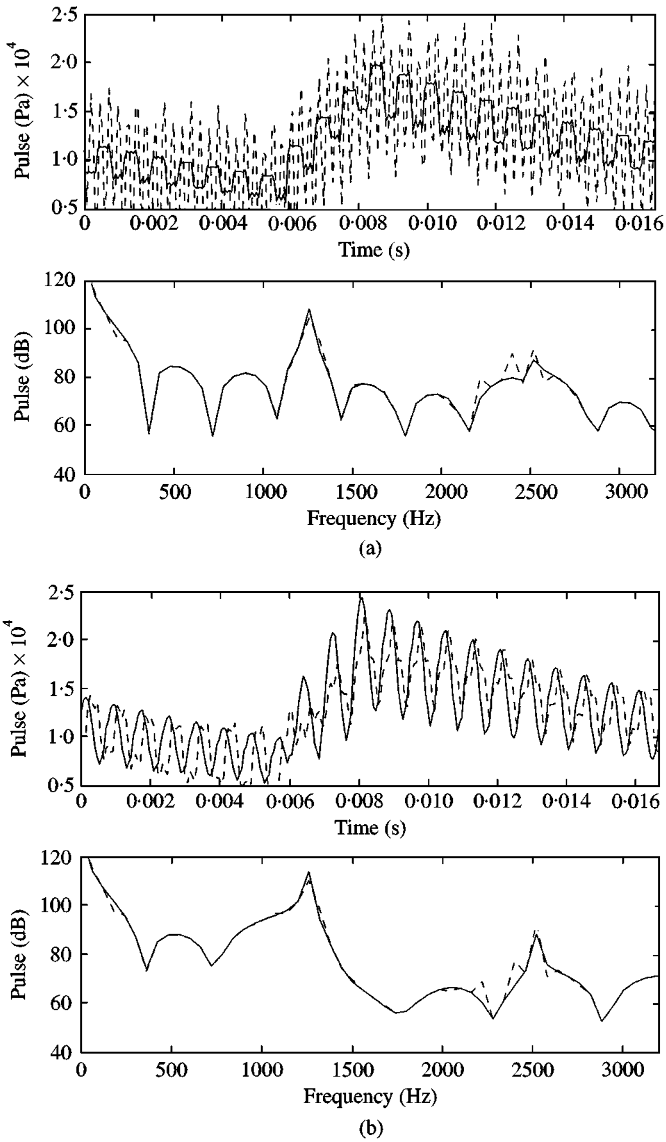

Figure 18. Compressor discharge pulsating pressure excited by (a) Squared wave input and (b) Saw-toothed wave input with $L_d = 2$ m and $Z = 0$. (—) Anechoic termination; (- - -) Non-anechoic termination.

- Without an anechoic termination, a single microphone or pressure transducer method cannot provide a consistent measurement in a practical environment, nor can the two-microphone method, which separates the incident and reflective waves. Both the total pulsating pressure and the incident pressure exhausted from a fixed volume machine are affected by the non-anechoic termination. As a matter of fact, the source impedance for a fixed volume machine is also influenced by different terminations in the exhaust piping line. Since the four pole parameters of this analytical model are available, this can be easily proven by the model.
- The three primary parameters in the exhaust line and adjacent apparatus are the pipe length, termination acoustic impedance, and damping. The parametric study was

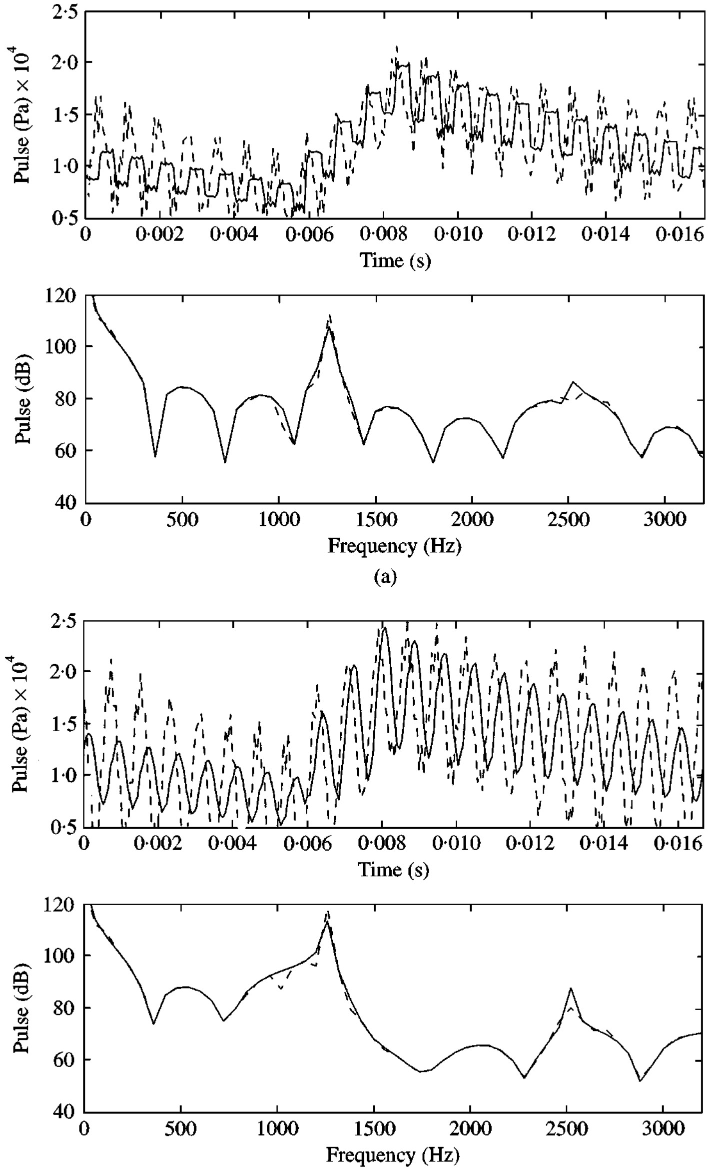

Figure 19. Compressor discharge pulsating pressure excited by (a) Squared wave input and (b) Saw-toothed wave input with $L_d = 1$ m and $Z \rightarrow \infty$. (—) Anechoic termination; (– – –) Non-anechoic termination.

conducted to demonstrate the impact of different parameters on the gas pulsation measurement.

- Implementing artificial damping materials to the exhaust line is one way to obtain consistent measurement without an anechoic termination. Even though a lot of application problems as addressed in this article need to be resolved, there is still possibility in the future.
- The peak-to-peak gas pulsation magnitude versus exhaust tube length and acoustic impedance at the termination (boundary) was investigated. The peak-to-peak value is

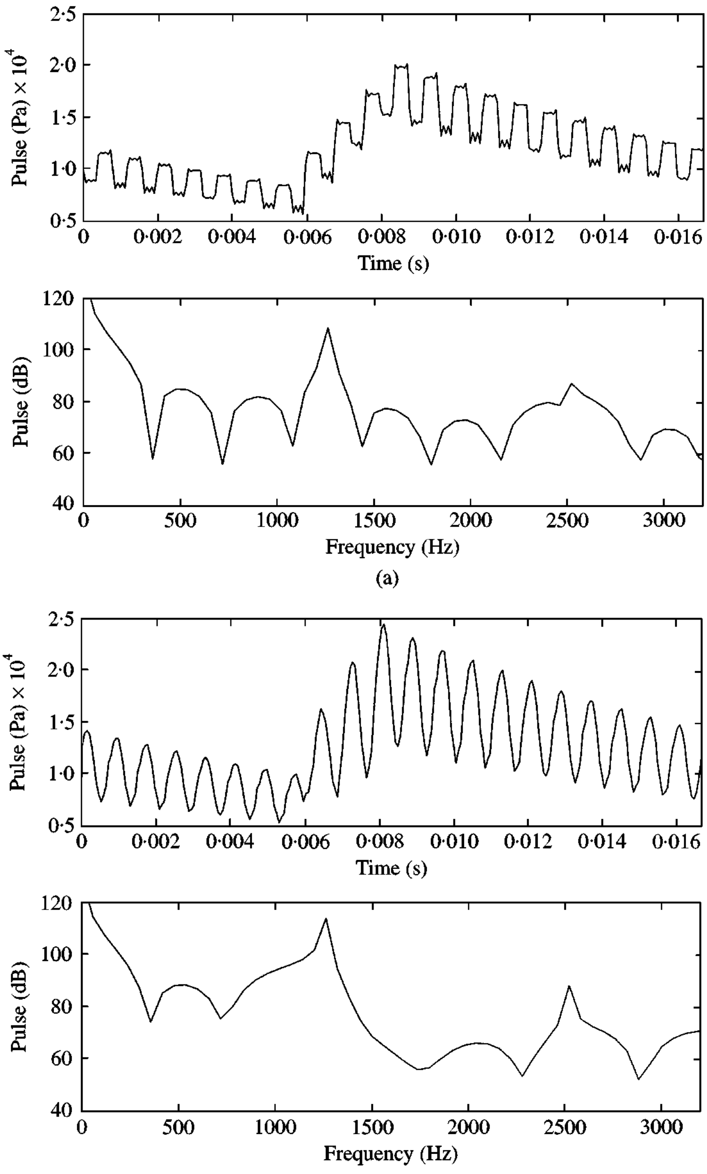

Figure 20. Compressor discharge pulsating pressure excited by (a) Squared wave input and (b) Saw-toothed wave input with $L_d = 1$ m and $Z = \rho c/S_d$, (—) Anechoic termination; (- - -) Non-anechoic termination.

sensitive to either the tube length or termination impedance such that an effort to optimize the exhaust piping length is impractical.

In sum, this step-by-step study has helped the physical understanding of the gas pulsation measurement in practical applications. It is difficult to make a consistent measurement without an anechoic termination boundary condition in the exhaust line due to the reflection wave. The anechoic termination can only be achieved by the dissipation of the wave propagation in the exhaust line. Both one- and two-microphone

TABLE 1

Peak-to-peak pulsating pressure (Pa) versus discharge line length (output impedance $Z = 0$)

Discharge line length (m)	Squared wave input	Saw-toothed wave input
0·6	4·3E + 04	7·2E + 04
0·9	4·0E + 04	5·9E + 04
1·0	1·3E + 04	1·5E + 04
1·2	1·8E + 04	2·3E + 04
1·5	1·1E + 04	1·1E + 04
1·8	1·4E + 04	1·7E + 04
2·0	2·6E + 04	1·8E + 04
3·0	2·1E + 04	2·7E + 04
5·0	3·9E + 04	5·9E + 04
10·0	1·2E + 04	1·5E + 04
Anechoic termination	1·4E + 04	1·9E + 04

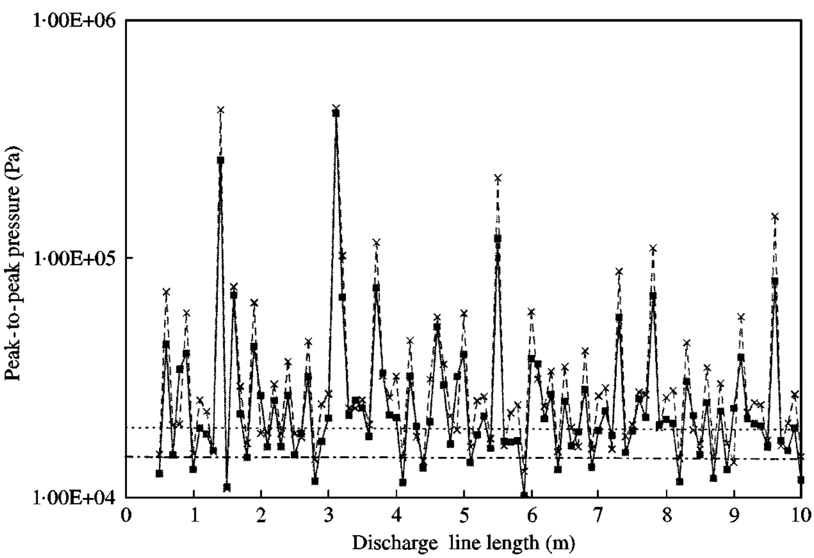

Figure 21. Peak-to-peak pulsating pressure excited by squared wave input and saw-toothed wave input versus discharge line lengths ($Z = 0$). (-■-) Squared wave input; (–·–·) Squared wave input with an anechoic termination (baseline); (-×-) Saw-toothed wave input; (···) Saw-toothed wave input with an anechoic termination (baseline).

measurement methods cannot serve as a standard measurement scheme without an anechoic termination.

ACKNOWLEDGMENTS

This study was supported for discussion of the future ISO gas pulsation measurement standard for the positive displacement machinery industries. The author would like to thank Mr Earl B. Muir and Dr Todd Rook (currently affiliated with BFGoodrich) of

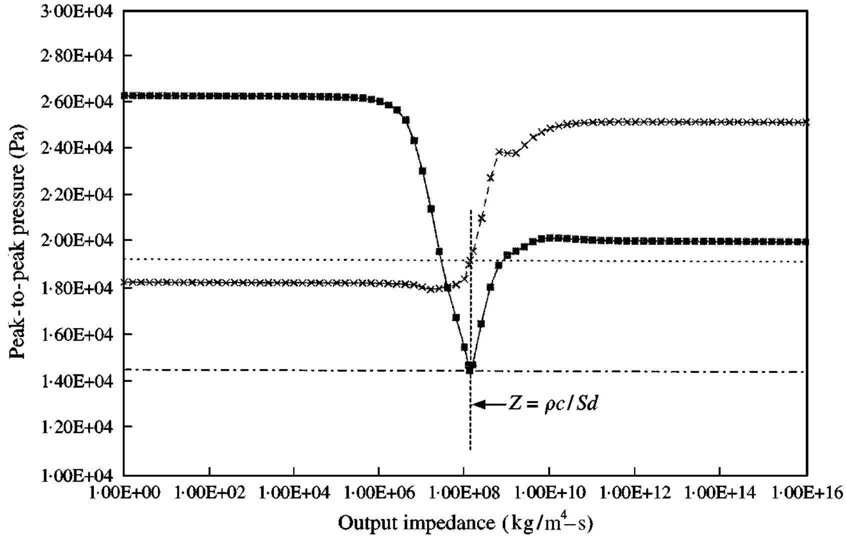

Figure 22. Peak-to-peak pulsating pressure excited by squared wave input and saw-toothed wave input with $L_d = 2$ m versus output impedances: (-■-) Squared wave input; (–·–·) Squared wave input with an anechoic termination (baseline); (-×-) Saw-toothed wave input; (···) Saw-toothed wave input with an anechoic termination (baseline).

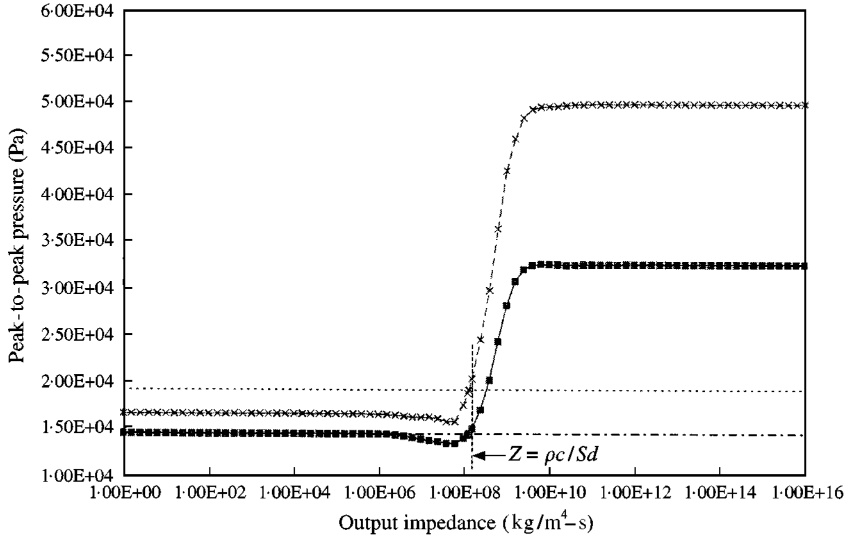

Figure 23. Peak-to-peak pulsating pressure excited by squared wave input and saw-toothed wave input with $L_d = 1{\cdot}8$ m versus output impedances: (-■-) Squared wave input; (–·–·) Squared wave input with an anechoic termination (baseline); (-×-) Saw-toothed wave input; (···) Saw-toothed wave input with an anechoic termination (baseline).

Copeland Corporation for allowing the publication of this work. Special thanks are due to Dr Robert J. Comparin and Dr Todd Rook of Copeland Corporation for discussion and reviewing this paper.

TABLE 2

Peak-to-peak pulsating pressure (Pa) versus output impedance (discharge line length L = 2 m)

Output impd (kg/m^4 s)	Squared wave input	Saw-toothed wave input
0	2·6E + 04	1·8E + 04
1·00E + 02	2·6E + 04	1·8E + 04
1·00E + 04	2·6E + 04	1·8E + 04
1·00E + 06	2·6E + 04	1·8E + 04
1·00E + 08	1·6E + 04	1·8E + 04
1·34E + 08	1·4E + 04	1·9E + 04
2·00E + 08	1·6E + 04	2·0E + 04
1·00E + 09	1·9E + 04	2·4E + 04
1·00E + 10	2·0E + 04	2·5E + 04
1·00E + 12	2·0E + 04	2·5E + 04
1·00E + 14	2·0E + 04	2·5E + 04
1·00E + 16	2·0E + 04	2·5E + 04
Anechoic termination	1·4E + 04	1·9E + 04

REFERENCE

1. R. J. COMPARIN 1997 *Personal communication.* Copeland Corporation, Sidney, Ohio; Copeland representative in Air-Conditioning and Refrigeration Institute (ARI), Arlington, Virginia; and ARI liaison to International Standard Organization (ISO), Genève, Switzerland.
2. 1989 Standard for Method of Measuring Sound and Vibration of Refrigerant Compressors 1989 *Technical Report* 530-89. Air-Conditioning and Refrigeration Institute, Arlington, VA.
3. Laboratory Method of Testing to Determine the Sound Power in a Duct 1995 Technical Report BSR/ASHRAE 68-1986R, American Society of Heating, Refrigeration and Air-Conditioning Engineerings, Inc., Atlanta, GA.
4. R. SINGH and W. SOEDEL 1978 *Journal of Sound and Vibration* **56**, 105–125. An efficient method of measuring impedances of fluid machinery manifolds.
5. H. MORIYAMA *et al.* 1987 *Transactions of the JSME*, Part C **53**, 3300–3308. Studies on noise generated by flow in expansion chamber type silencers.
6. T. MOREL, J. Morel and D. A. Blaser 1991 *SAE Technical Paper* 910072. Fluid dynamic and acoustic modeling of concentric-tube resonators/silencers.
7. W. NEISE and G. H. KOOPMANN 1991 *Journal of Vibration and Acoustics* **113**, 123–140. Active sources in the cutoff of centrifugal fans to reduce the blade tones at higher-order duct mode frequencies.
8. P. C.-C. LAI 1998 *Noise Control Engineering Journal* **46**, 109–119. Evaluation of several analytical methods on muffler acoustic modeling.
9. P. C.-C. LAI and W. SOEDEL 1997 *Proceedings of the* 1997 *ASME Design Engineering Technical Conferences*, DETC97/VIB-3847, 1–14. Two-dimensional gas pulsation modeling of compressor manifolds.
10. P. C.-C. LAI and W. SOEDEL *Journal of Vibration and Acoustics.* Acoustic analysis of a two-dimensional circular wave guide by modal expansion with special attention to scroll compressor head modeling (submitted).
11. P. C.-C. LAI and W. SOEDEL 1996 *Proceedings of Purdue Compressor Technology Conference*, pp. 814–819. On the anechoic termination assumption when modeling exit pipes.
12. P. C.-C. LAI and W. SOEDEL 1997 *Proceedings of the* 1997 *ASME International Mechanical Engineering Congress*, *Advanced Energy Systems Division*, AES-Vol. 37, 451–461. Approximating wave guide geometries in gas pulsation modeling.
13. J.-Y. CHUNG 1977 *General Motors Research Publication*, GMR-2617. Cross-spectral method of measuring acoustical intensity.
14. M. P. WASER and M. J. CROCKER 1984 *Noise Control Engineering Journal* **22**, 76–85. Introduction to the two-microphone cross-spectral method of determining sound intensity.
15. M. L. MUNJAL 1987 *Acoustics of Ducts and Mufflers with Application to Exhaust and Ventilation Design.* New York: Wiley–Interscience.

16. A. F. SEYBERT 1988 *Journal of Acoustical Society of America* **83**, 2233–2239. Two-sensor methods for the measurement of sound intensity and acoustic properties in ducts.
17. R. SINGH 1975 *Ph.D. thesis*, *Purdue University*. Modeling of multicylinder compressor discharge system.
18. W. SOEDEL 1978 *Gas Pulsations in Compressor and Engine Manifolds*, Short Course. Text Book of Purdue Compressor Technology Conference, Ray W. Herrick Laboratories, Purdue University.
19. P. C.-C. LAI and W. SOEDEL 1996 *Journal of Sound and Vibrations* **197**, 45–66. Gas pulsations in thin, curved or flat cavities due to multiple mass flow sources with special attention to multi-cylinder compressors.
20. L. KINSLER, A. FREY, A. COPPENS, and J. SANDERS 1982 *Fundamentals of Acoustics*. New York: Wiley, third edition.
21. E. SKUDRZYK 1980 *Journal of the Acoustical Society of America* **67**, 1105–1135. The mean-value method of predicting the dynamic response of complex vibrators.