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RANDOM EXCITATION FOR MODAL TESTING OF ROTATING MACHINERY: USE OF MODULATION TECHNIQUE

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One of the most popular excitation methods for modal testing of rotating machinery is known to be the uncorrelated isotropic excitation, which requires two independent random sources with equal power for generation of a pair of uncorrelated excitation signals. In this work, a new excitation method, the modulated random excitation, is proposed such that a pair of random signals with equal power for modal testing of anisotropic rotors can be effectively generated by modulating the random signal from a single random source with two harmonic carriers of a frequency with 90° phase difference. The real as well as complex approaches are taken to illustrate the effectiveness of the modulation technique. Finally, digital data processing technique is discussed in relation to the implementation of the modulation technique in the discrete time domain. © 2000 Academic Press

1. INTRODUCTION

In general, a rotor-bearing system consists of rotor and stator parts, which may have some degree of non-axisymmetric properties. According to the non-axisymmetric properties, a rotor-bearing system may be classified as follows [1–3]: *isotropic* (*symmetric*) *rotor system*—both the rotor and the stator are axisymmetric; *anisotropic rotor system*—the rotor is axisymmetric but the stator is not; *asymmetric rotor system*—the stator is axisymmetric but the rotor is not; *general rotor system*—neither the rotor nor the stator is axisymmetric. The accidental or intended presence of asymmetry and/or anisotropy in a rotor system, if not too small, can significantly alter its dynamic characteristics, such as the unbalance response, critical speeds and stability, from the ideal isotropic (symmetric) rotor. Thus, accurate identification of such asymmetric and anisotropic properties becomes essential in gaining an adequate physical understanding of the dynamic behaviors of practical rotors.

The complex modal testing method, which has been recently developed for rotor systems, utilizes the so-called directional frequency response functions (dFRFs) between complex inputs and outputs for effective modal parameter identification [1–8]. It gives not only the directivity of the backward and forward modes but also completely separates those modes in the frequency domain so that effective modal parameter identification is possible. Two kinds of co-ordinate system have been used to define the dFRFs associated with general

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rotors [1, 2]: stationary and rotating co-ordinates which are commonly adopted for the identification of anisotropic and asymmetric rotor systems respectively. The dFRFs defined in the stationary (rotating) co-ordinate system have been known to be effectively used as a diagnosis tool for anisotropy (asymmetry) in a rotor [1, 2]. Thus, two independent procedures for identification of the asymmetry and the anisotropy of a general rotor are needed for calculation of dFRFs based on two different co-ordinate systems. On the other hand, it has been proven [3] that the dFRFs defined in the stationary co-ordinate system can also be used for identification of the asymmetry in an asymmetric rotor system. It should be noted that it is easier to deal with the dFRFs defined in the stationary co-ordinate than in the rotating co-ordinate, since the excitations and responses are usually measured with respect to the stationary co-ordinate system.

Because the complex modal testing method requires the use of dFRFs between complex inputs and outputs, the excitation and measurement techniques are quite different from the conventional ones. For unbiased estimation of dFRFs associated with anisotropic rotor systems, Lee *et al.* [5–7] proposed the bidirectional random excitation technique, which requires the simultaneous (bidirectional) excitations in two directions at right angle and perpendicular to the rotation axis. The widely used bidirectional random excitation is known to be the uncorrelated isotropic random excitation, requiring two independent uncorrelated random sources with equal power, which may be a burden for precise data generation. As the system anisotropy becomes null, i.e., for isotropic (symmetric) rotors, only a unidirectional excitation suffices, with the response measurements along the two perpendicular directions. For asymmetric rotors, Lee and Joh [1, 2] proposed a similar bidirectional excitation technique, which converts the measured input and output signals in the stationary co-ordinate system to those in the rotating co-ordinate system. Later, based on stationary co-ordinate formulation, Lee *et al.* [3] also developed a unidirectional random excitation technique to estimate the dFRFs of asymmetric rotor systems using the modulated relationship between the two complex input models.

The main objective of this work is to propose a new excitation method based on modulation technique for complex modal testing of anisotropic rotor systems, which requires only one stationary random source so that signal generation becomes straightforward. From one stationary random source, two real random signals for modal testing of rotors are obtained by modulating the single stationary random signal with two sinusoidal carriers which have the same frequency but a phase difference of 90°. This work adds two new practical findings to the previous work. One shows that the severe uncorrelatedness property of two excitation signals, required for unbiased estimation of dFRFs of anisotropic rotors, can be easily released by use of the proposed modulation technique. This fact contradicts with the common understanding of modal testing of non-rotating structures, where modulated signals for multi-input excitation are not recommended for use. Another is that the modulation technique naturally enhances the practicality in modal testing of anisotropic rotors by requiring only a single random signal source and yet producing statistically identical results. To investigate the effectiveness and practicality of the proposed modulation technique, the real and complex approaches are taken and the digital analysis is carried out.

2. DIRECTIONAL FREQUENCY RESPONSE FUNCTIONS

Using the stationary co-ordinate system, the $N \times 1$ complex response and input vectors, $\mathbf{p}(t)$ and $\mathbf{g}(t)$, are defined by the real response vectors, $\mathbf{y}(t)$, and the real input vectors, $\mathbf{f}_y(t)$

and $\mathbf{f}_z(t)$ respectively, as [1–9]

$$\mathbf{p}(t) = \mathbf{y}(t) + \mathrm{j}\mathbf{z}(t), \quad \bar{\mathbf{p}}(t) = \mathbf{y}(t) - \mathrm{j}\mathbf{z}(t),$$

$$\mathbf{g}(t) = \mathbf{f}_y(t) + \mathrm{j}\mathbf{f}_z(t), \quad \bar{\mathbf{g}}(t) = \mathbf{f}_y(t) - \mathrm{j}\mathbf{f}_z(t). \tag{1}$$

Here j means the imaginary number and the bar denotes the complex conjugate.

For anisotropic rotor systems where the rotor is axisymmetric but the stator is not, the directional frequency response matrices (dFRMs) between complex inputs and outputs are defined as [1, 2, 4–7]

$$\mathbf{P}(\omega) = [\mathbf{H}_{gp}(\omega) \quad \mathbf{H}_{\hat{g}p}(\omega)] \begin{bmatrix} \mathbf{G}(\omega) \\ \hat{\mathbf{G}}(\omega) \end{bmatrix}, \tag{2}$$

where $\mathbf{P}(\omega)$, $\mathbf{G}(\omega)$ and $\hat{\mathbf{G}}(\omega)$ are the Fourier transforms of $\mathbf{p}(t)$, $\mathbf{g}(t)$ and $\bar{\mathbf{g}}(t)$ respectively. Here $\mathbf{H}_{gp}(\omega)$ is referred to as the *normal* dFRM whereas $\mathbf{H}_{\hat{g}p}(\omega)$, is referred to as the *reverse* dFRM of anisotropic rotor, [1, 2, 4–7]. The reverse dFRM, $\mathbf{H}_{\hat{g}p}(\omega)$, represents the degree of anisotropy in an anisotropic rotor [1, 2].

3. ESTIMATION OF dFRFs

The key feature of the complex modal testing of an anisotropic rotor system is the estimation of dFRFs using the two complex input and single complex output model in equation (2) [1, 2, 5, 6]. Unless the directional coherence function (dCOH), $\gamma^2_{g\hat{g}}(\omega)$, between $g(t)$ and $\bar{g}(t)$, is unity, the estimates of dFRFs, $H_{gp}(\omega)$ and $H_{\hat{g}p}(\omega)$, of anisotropic rotor become [1, 2, 5, 6]

$$H_{gp}(\omega) = \frac{S_{gp}(\omega)}{S_{gg}(\omega)} \frac{1 - \dfrac{S_{\hat{g}p}(\omega)S_{g\hat{g}}(\omega)}{S_{gp}(\omega)S_{\hat{g}\hat{g}}(\omega)}}{1 - \gamma^2_{g\hat{g}}(\omega)},$$

$$H_{\hat{g}p}(\omega) = \frac{S_{\hat{g}p}(\omega)}{S_{\hat{g}\hat{g}}(\omega)} \frac{1 - \dfrac{S_{gp}(\omega)S_{\hat{g}g}(\omega)}{S_{\hat{g}p}(\omega)S_{gg}(\omega)}}{1 - \gamma^2_{g\hat{g}}(\omega)}, \tag{3}$$

where

$$\gamma^2_{g\hat{g}}(\omega) = \frac{|S_{g\hat{g}}(\omega)|^2}{S_{gg}(\omega)S_{\hat{g}\hat{g}}(\omega)}. \tag{4}$$

Here, $S_{ik}(\omega)$, $i = g, \hat{g}$ and $k = p, g, \hat{g}$, are the two-sided directional auto- (for $i = k$) and cross- (for $i \neq k$) spectral density functions (dPSDs and dCSDs) between the complex time signals, $i(t)$ and $k(t)$. In the ideal case of uncorrelated complex input signals, $g(t)$ and $\bar{g}(t)$, that is, $\gamma^2_{g\hat{g}}(\omega) = 0$, equation (3) reduces to [1, 2, 5, 6]

$$H_{gp}(\omega) = \frac{S_{gp}(\omega)}{S_{gg}(\omega)},$$

$$H_{\hat{g}p}(\omega) = \frac{S_{\hat{g}p}(\omega)}{S_{\hat{g}\hat{g}}(\omega)}. \tag{5}$$

4. EXCITATION METHODS FOR COMPLEX MODAL TESTING

For unbiased estimation of dFRFs associated with anisotropic rotor systems, the bidirectional random excitation method has been widely used, which essentially utilizes a complex random input signal, $g(t)$, and its conjugate random input signal, $\bar{g}(t)$, satisfying the relation given by [1, 2, 5, 6]

$$R_{g\bar{g}}(\tau) = R_{\bar{g}g}(\tau) = 0,$$

or equivalently, since $R_{g\bar{g}}(\tau) = R_{f_y f_y}(\tau) - R_{f_z f_z}(\tau) - \mathrm{j}\{R_{f_y f_z}(\tau) + R_{f_z f_y}(\tau)\}$,

$$R_{f_y f_y}(\tau) = R_{f_z f_z}(\tau), \quad R_{f_y f_z}(\tau) = -R_{f_z f_y}(\tau), \tag{6}$$

where the correlation functions are defined by

$$R_{ik}(\tau) = \mathrm{E}[\bar{i}(t)k(t+\tau)] = \lim_{T\to\infty} \frac{1}{T}\int_0^T \bar{i}(t)k(t+\tau)\,\mathrm{d}t, \quad i, k = g, \bar{g}, \text{ or } f_y, f_z.$$

The relation (6) can be re-expressed, in the frequency domain, as

$$S_{g\hat{g}}(\omega) = S_{\hat{g}g}(\omega) = 0,$$

or

$$S_{f_y f_y}(\omega) = S_{f_z f_z}(\omega) \text{ and } \mathrm{Re}\{S_{f_y f_z}(\omega)\} = 0, \tag{7}$$

where the spectral density functions are defined by

$$S_{ik}(\omega) = \int_{-\infty}^{\infty} R_{ik}(\tau)\,\mathrm{e}^{-\mathrm{j}\omega\tau}\,\mathrm{d}\tau, \quad i, k = g, \hat{g}, \text{ or } f_y, f_z.$$

The excitation methods satisfying condition (7) may be classified according to property of the imaginary part of $S_{f_y f_z}(\omega)$: one method with $\mathrm{Im}\{S_{f_y f_z}(\omega)\} = 0$ and another with $\mathrm{Im}\{S_{f_y f_z}(\omega)\} \neq 0$. Two practical excitation methods have already been suggested to estimate the dFRFs: for one method with $\mathrm{Im}\{S_{f_y f_z}(\omega)\} \neq 0$, the directional (or bidirectional rotating) random excitation is suggested, which satisfies $S_{f_y f_z}(\omega) = -\mathrm{j}S_{f_y f_y}(\omega)$ or $S_{f_y f_z}(\omega) = \mathrm{j}S_{f_y f_y}(\omega)$; for another method with $\mathrm{Im}\{S_{f_y f_z}(\omega)\} = 0$, the uncorrelated isotropic (or bidirectional stationary) random excitation is suggested [1–3, 5, 6].

In case of directional random excitation, the condition (7) with $\mathrm{Im}\{S_{f_y f_y}(\omega)\} \neq 0$, that is, $S_{f_y f_z}(\omega) = \mathrm{j}S_{f_y f_y}(\omega)$ or $S_{f_y f_z}(\omega) = -\mathrm{j}S_{f_y f_y}(\omega)$, imposed on generation of directional random excitation signals may be too severe to be easily realized in practice [6, 7]. Another critical drawback of this method is that the estimation of the normal and reverse dFRFs, $H_{gp}(\omega)$ and $H_{\hat{g}p}(\omega)$, requires two subsequent modal testings, one with the forward rotating excitation and the other with backward rotating excitation [6, 7]. Because of the drawbacks of directional random excitation, the uncorrelated isotropic random excitation has been widely used in practice [1, 5, 6, 9].

In this work, a new random excitation method based on modulation technique is proposed such that a pair of modulated random excitation signals with equal power satisfying condition (7) with $\mathrm{Im}\{S_{f_y f_z}(\omega)\} \neq 0$ can be generated effectively by using a stationary random signal from a single random source, whereas the widely used uncorrelated isotropic random excitation requires two independent uncorrelated random sources with equal power. The new technique will be referred to as the modulated random excitation.

4.1. MODULATED RANDOM EXCITATION

The correlation and spectral relations between a complex input signal and its modulated signal with the rotational frequency for modal testing of asymmetric rotors have been well discussed in [3]. In this section, the relationships are extended to the complex input signals, which are modulated with an arbitrary carrier frequency for modal testing of anisotropic rotors. Now, consider the stationary real random process, $\{f(t)\}$, and the corresponding amplitude modulated complex process, $\{g(t)\}$, with a carrier frequency of ω_0, such that

$$g(t) = f(t)\mathrm{e}^{\mathrm{j}\omega_0 t} = f(t)\cos\omega_0 t + \mathrm{j} f(t)\sin\omega_0 t = f_y(t) + \mathrm{j} f_z(t). \tag{8}$$

Now it will be proved that a pair of complex input processes, $\{g(t)\}$ and $\{\bar{g}(t)\}$, become individually stationary but jointly non-stationary. First, consider the double time cross-correlation function, $R_{g\bar{g}}(t_1, t_2)$, which is derived as [10]

$$\begin{aligned}\mathscr{R}_{g\bar{g}}(\tau, t) &= R_{g\bar{g}}(t_1, t_2) = E[\bar{g}(t_1)\bar{g}(t_2)] \\ &= R_{ff}(t_2 - t_1)\mathrm{e}^{-\mathrm{j}\omega_0(t_1+t_2)} = R_{ff}(\tau)\mathrm{e}^{-\mathrm{j}2\omega_0 t},\end{aligned} \tag{9}$$

where a different correlation structure is defined by the transformation given by $t_1 = t - \tau/2$, $t_2 = t + \tau/2$ and $\mathscr{R}$ is used in place of R to distinguish planes (τ, t) from planes (t_1, t_2). Here $R_{ff}(\tau)$ is the auto-correlation function of $f(t)$. Because the cross-correlation function, $R_{g\bar{g}}(t_1, t_2)$, is dependent on the absolute time t as shown in equation (9), the complex input processes, $\{g(t)\}$ and $\{\bar{g}(t)\}$, remain jointly non-stationary unless the carrier frequency, ω_0, is zero. The double-frequency dCSD, $S_{g\hat{g}}(\omega_1, \omega_2)$, can be derived from the double-time cross-correlation function, $R_{g\bar{g}}(t_1, t_2)$, as [3, 9]

$$\mathscr{S}_{g\hat{g}}(\omega, \chi) = S_{g\hat{g}}(\omega_1, \omega_2) = 2\pi S_{ff}\left(\frac{\omega_1 + \omega_2}{2}\right)\delta_1(\omega_2 - \omega_1 + 2\omega_0), \tag{10}$$

with the relationship

$$S_{ik}(\omega_1, \omega_2) = \mathscr{S}_{ik}(\omega, \chi) = \iint \mathscr{R}_{ik}(\tau, t)\mathrm{e}^{-j(\omega\tau + \chi t)}\,\mathrm{d}\tau\,\mathrm{d}t, \quad i, k = g, \hat{g}, \tag{11}$$

where a different spectral structure is defined by the transformation given by $\omega_1 = \omega - \chi/2$, $\omega_2 = \omega + \chi/2$ and $\mathscr{S}$ is used in place of S to distinguish plane (ω, χ) from plane (ω_1, ω_2). Here $S_{ff}(\omega)$ is the PSD of $f(t)$ and $\delta_1(\omega)$ is the finite delta function defined by

$$\delta_1(\omega) = \begin{cases} \dfrac{T}{2\pi}, & \left(-\dfrac{\pi}{T}\right) < \omega < \left(\dfrac{\pi}{T}\right), \\ 0, & \text{otherwise}. \end{cases} \tag{12}$$

Since the dFRFs are estimated on the line $\omega_1 = \omega_2 = \omega$ in the (ω_1, ω_2) plane, condition (7) can be satisfied for a sufficiently long record length $T > \pi/\omega_0$ as, from equation (10),

$$S_{g\hat{g}}(\omega, \omega) = TS_{g\hat{g}}(\omega) = 0, \quad \text{or equivalently, } S_{\hat{g}g}(\omega, \omega) = TS_{\hat{g}g}(\omega) = 0. \tag{13}$$

It means that a single random source incorporated with the modulation technique is sufficient to generate a pair of real input signals satisfying the condition (7) required for modal testing of anisotropic rotors.

Similarly, the double-time auto-correlation functions, $R_{gg}(t_1, t_2)$ and $R_{\bar{g}\bar{g}}(t_1, t_2)$, and the double-frequency dPSDs, $S_{gg}(\omega_1, \omega_2)$ and $S_{\hat{g}\hat{g}}(\omega_1, \omega_2)$, for $\omega_1 = \omega_2 = \omega$, can be derived as

$$R_{gg}(t_1, t_2) = E[\bar{g}(t_1)g(t_2)] = R_{ff}(t_2 - t_1)e^{j\omega_0(t_2 - t_1)} = R_{ff}(\tau)e^{j\omega_0\tau},$$

$$R_{\bar{g}\bar{g}}(t_1, t_2) = E[g(t_1)\bar{g}(t_2)] = R_{ff}(t_2 - t_1)e^{-j\omega_0(t_2 - t_1)} = R_{ff}(\tau)e^{-j\omega_0\tau},$$

$$S_{gg}(\omega, \omega) = TS_{gg}(\omega) = TS_{ff}(\omega - \omega_0),$$

$$S_{\hat{g}\hat{g}}(\omega, \omega) = TS_{\hat{g}\hat{g}}(\omega) = TS_{ff}(\omega + \omega_0). \tag{14}$$

It implies that the dPSDs, $S_{gg}(\omega)$ and $S_{\hat{g}\hat{g}}(\omega)$, can be obtained from the dPSD of $f(t)$, $S_{ff}(\omega)$, simply by a frequency shift of ω_0. In practice, this frequency shift may cause an aliasing problem unless an anti-aliasing filter is properly used, or a sufficiently higher sampling frequency than normally required may be needed in the process of data acquisition. In addition, due to the frequency shift, the carrier frequency, ω_0, may have to be carefully selected in consideration of the excitation frequency bandwidth, which will be further discussed later. Note that the relations (14) confirm that the complex input processes, $\{g(t)\}$ and $\{\bar{g}(t)\}$, are individually stationary, although they are not jointly stationary as discussed before.

4.2. REALIZATION OF MODULATION TECHNIQUE

For better understanding of physical realization of the condition (7) using a single stationary random source, real formulation of the proposed modulation technique is examined. A similar problem has been discussed in communication application [11, 12], for a complex modulated signal, $[f(t) + jh(t)]e^{j\omega_0 t} = f_y(t) + jf_z(t)$, where $h(t)$ is also real. It is concluded in [11, 12] that, for the modulated random processes, $\{f_y(t)\}$ and $\{f_z(t)\}$, to be wide-sense stationary (WSS) satisfying the condition (7), $S_{f_yf_y}(\omega) = S_{f_zf_z}(\omega)$ and $S_{f_yf_z}(\omega) = -S_{f_zf_y}(\omega)$, the original random processes, $\{f(t)\}$ and $\{h(t)\}$, should be wide-sense stationary with zero mean, satisfying $S_{ff}(\omega) = S_{hh}(\omega)$ and $S_{fh}(\omega) = -S_{hf}(\omega)$. On the other hand, the proposed modulation technique deals with a single random source, that is $h(t) = 0$, and thus the processes, $\{f_y(t)\}$ and $\{f_z(t)\}$, are not WSS. The modulated excitation signals in equation (8) can be re-written in the real domain as

$$f_y(t) = f(t)\cos\omega_0 t = \frac{f(t)}{2}\{e^{j\omega_0 t} + e^{-j\omega_0 t}\}, \quad f_z(t) = f(t)\sin\omega_0 t = \frac{f(t)}{2j}\{e^{j\omega_0 t} - e^{-j\omega_0 t}\}. \tag{15}$$

Thus, the double time auto-correlation function, $\mathscr{R}_{f_y f_y}(\tau, t)$, and the double frequency dPSD, $\mathscr{S}_{f_y f_y}(\omega, \chi)$, can be expressed as

$$R_{f_y f_y}(t_1, t_2) = \mathscr{R}_{f_y f_y}(\tau, t) = E[\bar{f}_y(t_1) f_y(t_2)] = \frac{1}{4} R_{ff}(\tau)\{\mathrm{e}^{\mathrm{j}2\omega_0 t} + \mathrm{e}^{-\mathrm{j}2\omega_0 t} + \mathrm{e}^{\mathrm{j}\omega_0 \tau} + \mathrm{e}^{-\mathrm{j}\omega_0 \tau}\}, \tag{16}$$

and

$$\mathscr{S}_{f_y f_y}(\omega, \chi) = \frac{\pi}{2}\delta_1(\chi - 2\omega_0)\mathscr{S}_{ff}(\omega) + \frac{\pi}{2}\delta_1(\chi + 2\omega_0)S_{ff}(\omega) + \frac{\pi}{2}\delta_1(\chi)[S_{ff}(\omega - \omega_0) + S_{ff}(\omega + \omega_0)]. \tag{17}$$

Because only the frequency line $\omega_1 = \omega_2 = \omega$ or $\chi = 0$ in plane $(\mathrm{j}\omega_1, \mathrm{j}\omega_2)$ is used for estimation of dFRFs, equation (17) becomes, for a sufficiently long record length $T > \pi/\omega_0$.

$$\mathscr{S}_{f_y f_y}(\omega, 0) = S_{f_y f_y}(\omega, \omega) = T S_{f_y f_y}(\omega) = \frac{T}{4} S_{ff}(\omega - \omega_0) + \frac{T}{4} S_{ff}(\omega + \omega_0), \tag{18}$$

Similarly, we can easily derive the relations

$$\mathscr{R}_{f_z f_z}(\tau, t) = \frac{1}{4} R_{ff}(\tau)\{-\mathrm{e}^{\mathrm{j}2\omega_0 t} - \mathrm{e}^{-\mathrm{j}2\omega_0 t} + \mathrm{e}^{\mathrm{j}\omega_0 \tau} + \mathrm{e}^{-\mathrm{j}\omega_0 \tau}\},$$

$$\mathscr{S}_{f_z f_z}(\omega, 0) = S_{f_z f_z}(\omega, \omega) = T S_{f_z f_z}(\omega) = \frac{T}{4} S_{ff}(\omega - \omega_0) + \frac{T}{4} S_{ff}(\omega + \omega_0), \tag{19}$$

$$\mathscr{R}_{f_y f_z}(\tau, t) = -\frac{\mathrm{j}}{4} R_{ff}(\tau)\{\mathrm{e}^{\mathrm{j}2\omega_0 t} - \mathrm{e}^{-\mathrm{j}2\omega_0 t} + \mathrm{e}^{\mathrm{j}\omega_0 \tau} - \mathrm{e}^{-\mathrm{j}\omega_0 \tau}\},$$

$$\mathscr{S}_{f_y f_z}(\omega, 0) = S_{f_y f_z}(\omega, \omega) = T S_{f_y f_z}(\omega) = -\mathrm{j}\frac{T}{4} S_{ff}(\omega - \omega_0) + \mathrm{j}\frac{T}{4} S_{ff}(\omega + \omega_0), \tag{20}$$

$$\mathscr{R}_{f_z f_y}(\tau, t) = -\frac{\mathrm{j}}{4} R_{ff}(\tau)\{\mathrm{e}^{\mathrm{j}2\omega_0 t} - \mathrm{e}^{-\mathrm{j}2\omega_0 t} - \mathrm{e}^{\mathrm{j}\omega_0 \tau} + \mathrm{e}^{-\mathrm{j}\omega_0 \tau}\},$$

$$\mathscr{S}_{f_z f_y}(\omega, 0) = S_{f_z f_y}(\omega, \omega) = T S_{f_z f_y}(\omega) = \mathrm{j}\frac{T}{4} S_{ff}(\omega - \omega_0) - \mathrm{j}\frac{T}{4} S_{ff}(\omega + \omega_0). \tag{21}$$

Note that, from equations (18)–(21),

$$S_{f_y f_y}(\omega) = S_{f_z f_z}(\omega), \quad S_{f_y f_z}(\omega) = -S_{f_z f_y}(\omega),$$

and

$$S_{f_y f_z}(\omega) = 0 \quad \text{for} \quad S_{ff}(\omega - \omega_0) = S_{ff}(\omega + \omega_0) \tag{22}$$

holds. The ideal white noise case with $S_{ff}(\omega - \omega_0) = S_{ff}(\omega + \omega_0)$, rarely occurs in practice. This means that the uncorrelatedness property of two excitation signals, that is, $S_{f_y f_z}(\omega) = 0$, required for unbiased estimation of dFRFs of anisotropic rotors [5, 6] can be easily released. In that sense, the modulated complex random excitation is different from the previous uncorrelated isotropic random excitation. It can be concluded here that a single stationary real random source, utilizing the proposed modulation technique, is sufficient to provide a pair of random signals satisfying the condition (7) for modal testing of anisotropic rotors. The proposed modulation technique is better than the uncorrelated isotropic random excitation and far superior to the directional random excitation int terms of practicality.

4.3. DIGITAL IMPLEMENTATION

For digital implementation of the proposed modulation technique, we need to discuss the finite discrete Fourier transform (DFT) defined as [10]

$$I(\omega_k) = \Delta t \sum_{n=0}^{N-1} i(n\Delta t)\,\mathrm{e}^{(-\mathrm{j}2\pi kn/N)}, \quad \text{for } i = g, \bar{g}, p, \tag{23}$$

with

$$\omega_k = \frac{2\pi k}{T} = \frac{2\pi k}{N\Delta t} = k\Delta\omega, \quad k = -\frac{N}{2}+1,\ -\frac{N}{2}+2, \ldots, \frac{N}{2}-1, \frac{N}{2}, \tag{24}$$

where Δt is the sampling interval, N is the even number of samples, $\Delta\omega$ is the frequency resolution and $I(\omega)$ is the finite discrete Fourier transform of $i(t)$. Then the DFT of the modulated complex signal, $g(t) = f(t)\mathrm{e}^{\mathrm{j}\omega_0 t}$, can be written as

$$G(\omega_k) = \Delta t \sum_{n=0}^{N-1} g(n\Delta t)\mathrm{e}^{-\mathrm{j}\omega_k n\Delta t} = \Delta t \sum_{n=0}^{N-1} f(n\Delta t)\mathrm{e}^{-\mathrm{j}(\omega_k - \omega_0)n\Delta t}$$

$$= \Delta t \sum_{n=0}^{N-1} f(n\Delta t)\mathrm{e}^{-\mathrm{j}2\pi(k-\ell)n/N} = F(\omega_{k-\ell})$$

$$k = -\frac{N}{2}+1,\ -\frac{N}{2}+1, \ldots, \frac{N}{2}-1, \frac{N}{2}, \tag{25}$$

where $\omega_0 = \ell\Delta\omega$ is the carrier frequency. Here, for simplicity, ℓ is assumed to be an integer satisfying $\ell < N/2$ and $F(\omega)$ is the Fourier transform of $f(t)$. Similarly, the discrete Fourier transform of $\bar{g}(t) = f(t)\mathrm{e}^{-\mathrm{j}\omega_0 t}$ can be obtained as

$$\hat{G}(\omega_k) = \Delta t \sum_{n=0}^{N-1} \bar{g}(n\Delta t)\mathrm{e}^{-\mathrm{j}\omega_k n\Delta t} = \Delta t \sum_{n=0}^{N-1} f(n\Delta t)\mathrm{e}^{-\mathrm{j}(\omega_k+\omega_0)n\Delta t}$$

$$= \Delta t \sum_{n=0}^{N-1} f(n\Delta t)\mathrm{e}^{-\mathrm{j}2\pi(k+\ell)n/N} = F(\omega_{k+\ell}),$$

$$k = -\frac{N}{2}+1,\ -\frac{N}{2}+2,\ldots,\frac{N}{2}-1,\frac{N}{2}. \tag{26}$$

Now, assume that $f(t)$ behaves similar to a band-limited white noise over the frequency resolution bandwidth $\Delta\omega = 2\pi/T$. It then follows that for any two discrete frequencies, $\omega_1 = k_1\Delta\omega$ and $\omega_2 = k_2\Delta\omega$ for $k_1, k_2 = -N/2+1, -N/2+2, \ldots N/2-1, N/2$, expected value operations on $\bar{F}(\omega_1)$ and $F(\omega_2)$ will give [10]

$$\frac{1}{T}E[\bar{F}(\omega_1)F(\omega_2)] = \begin{cases} 0 & \text{for } k_1 \neq k_2, \\ S_{ff}(\omega_1) \neq 0 & \text{for } k_1 = k_2. \end{cases} \tag{27}$$

It means that the finite Fourier transform of $f(t)$, $F(\omega)$, has uncorrelated frequency structure in neighboring frequencies, say, ω_1 and ω_2, as long as the frequencies, ω_1 and ω_2, are apart from each other at least by the resolution bandwidth $\Delta\omega$. From equations (25)–(27), it follows that

$$S_{g\hat{g}}(\omega_k) = \frac{1}{T}E[\bar{G}(\omega_k)\hat{G}(\omega_k)] = \frac{1}{T}E[\bar{F}\{\omega_{(k-\ell)}\}F\{\omega_{(k+\ell)}\}] = 0, \quad \text{for } \ell \neq 0. \tag{28}$$

The condition for equation (13), that is, $T > \pi/\omega_0$, can now be re-interpreted from equations (27) and (28) as

$$|\omega_0| > \frac{\Delta\omega}{2} \quad \text{or } |\ell| > 0.5. \tag{29}$$

Note here that ℓ is not necessarily an integer in practice. The above relation holds true when there is no spectral leakage problem. In practice, the leakage problem occurs in analysis of truncated data which is not periodic of period T. This effect can be analyzed by treating the time domain truncation as weighting the original data by a rectangular weighting function [10]. Among others, Hanning window function is commonly used to suppress the leakage, particularly for random data. For example, the sampled time record $i_w(t)$ can be considered to be the product of two functions as

$$i_w(t) = u_h(t)i(t), \quad i = f, g, \hat{g}, p, \tag{30}$$

where $i(t)$ is the unlimited time history and $u_h(t)$ is the Hanning window function. It follows that the Fourier transform of $i_w(t)$ is the convolution of the Fourier transforms of $u_h(t)$ and $i(t)$, i.e.,

$$I_w(\omega) = \int_{-\infty}^{\infty} U_h(\alpha)I(\omega-\alpha)\,\mathrm{d}\alpha. \tag{31}$$

Here $U_h(\omega)$ is the Fourier transform of $u_h(t)$ which is given by [10]

$$U_h(\omega) = \frac{1}{2} U_r(\omega) - \frac{1}{4} U_r\left(\omega - \frac{2\pi}{T}\right) - \frac{1}{4} U_r\left(\omega + \frac{2\pi}{T}\right), \tag{32}$$

where

$$U_r(\omega) = T\left(\frac{\sin(\omega/2)T}{(\omega/2)T}\right)\mathrm{e}^{-\mathrm{j}(\omega/2)T}$$

is the Fourier transform of the rectangular window function. Although Hanning window, $U_h(\omega)$, has a broader main lobe, it has side lobes in frequency domain of lower amplitude than those of rectangular window function, $U_r(\omega)$, implying that Hanning window produces less leakage [10]. The broader main lobe in Hanning window affects the neighboring discrete frequencies, ω_1 and ω_2, which are apart from each other by the resolution bandwidth, $\Delta\omega$, resulting in some correlation whereas its effects to distant discrete frequencies are decreased significantly [10]. Therefore, in practice, considering the relation (28) and the window effect in equation (32), the carrier frequency, ω_0, should be designed sufficiently higher than the resolution bandwidth of $\Delta\omega$.

Now, suppose that the discrete Fourier transform of $f(t)$, $F(\omega_k)$, exists only within the Nyquist frequency, that is,

$$F(\omega)\begin{cases} \neq 0, & \omega_{-N/2+1} \leqslant \omega \leqslant \omega_{N/2} \\ = 0, & \text{otherwise.} \end{cases}$$

Then the frequency bandwidth of the modulated signal, $g(t) = f(t)\mathrm{e}^{\mathrm{j}l\Delta\omega t}$, ℓ being a positive integer, will be shifted to $\omega_{-N/2+\ell+1} \leqslant \omega_k \leqslant \omega_{N/2+\ell}$. It means that the discrete Fourier transform $G(\omega_k)$ of the modulated signal, $g(t)$, does not exist over $\omega_{-N/2+1} \leqslant \omega_k \leqslant \omega_{-N/2+\ell}$, and, in turn, it exists over the region $\omega_{N/2+1} \leqslant \omega_k \leqslant \omega_{N/2+\ell}$ beyond the positive Nyquist frequency, violating the sampling theorem and being folded into $\omega_{-N/2+1} \leqslant \omega_k \leqslant \omega_{-N/2+l}$. Thus, it can be concluded that the effective frequency range for estimation of normal dFRFs based on the modulation technique becomes $\omega_{-N/2+l+1} \leqslant \omega_k \leqslant \omega_{N/2}$ for a positive integer l. Similarly, the effective frequency range for estimation of reverse dFRFs can be derived from the modulated signal, $\bar{g}(t) = f(t)\mathrm{e}^{-\mathrm{j}\ell\Delta\omega t}$, as $\omega_{-N/2+1} \leqslant \omega_k \leqslant \omega_{N/2-\ell}$.

Note that there is trade-off between the condition (28) considering window effect in equation (32) and the effective frequency range for estimation of dFRFs. A rule of thumb is that the carrier frequency is recommended to be about three times the frequency resolution, i.e., $\omega_0 \approx 3\Delta\omega$ for the number of data $N > 100$.

5. NUMERICAL EXAMPLE

In this section, in order to demonstrate the effectiveness of the proposed modulated random excitation method in estimation of dFRFs for anisotropic rotors, numerical simulations are performed and compared with the widely used uncorrelated isotropic random excitation method.

To simulate the modulated random excitation, input force data, $g(t) = f_y(t) + \mathrm{j}f_z(t)$, were numerically calculated from a stationary random signal, $f(t)$, using the relations $f_y(t) = f(t)\cos\omega_0 t$ and $f_z(t) = f(t)\sin\omega_0 t$, where the carrier frequency is $\omega_0 = \ell\Delta\omega$. The real

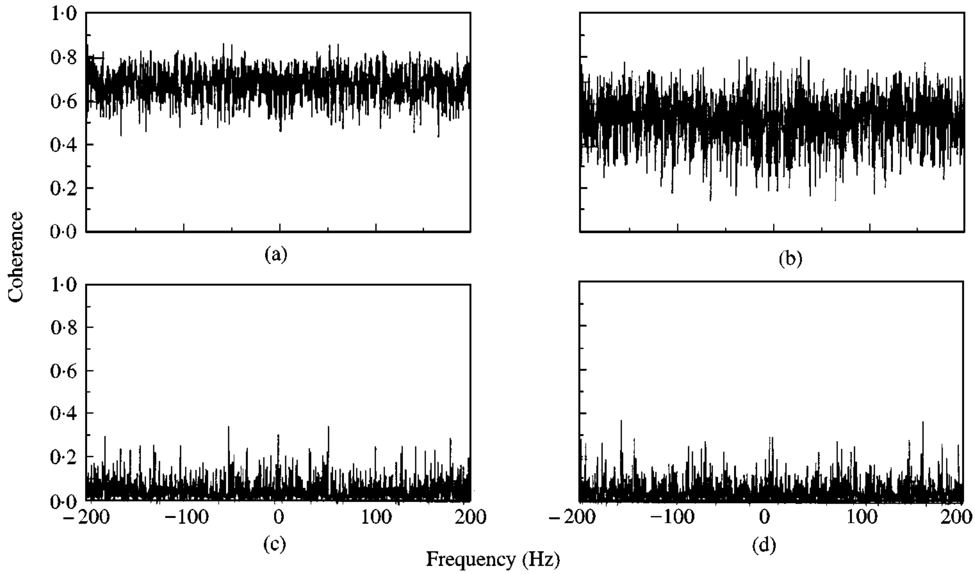

Figure 1. Coherence function of complex input signals, $\gamma^2_{g\hat{g}}(\omega)$, with n_d (number of averaging) = 20: $g(t) = f(t)e^{j\ell\Delta\omega t}$ with (a) $\ell = 0.01$, (b) 0.5, (c) 3.0, (d) 100.

random signal, $f(t)$, was generated from a Gaussian random process with time interval of 2.5 ms, and then modulated with a carrier frequency for $\ell = 0.01$, 0.5, 3.0 and 100. Figure 1 compares the coherence functions, $\gamma^2_{g\hat{g}}(\omega)$, which were calculated with 20 ensemble averagings (n_d) of the 2048 point of FFT using Hanning window. The results indicate that $\ell \geqslant 3$ suffices to nearly meet the requirement $\gamma^2_{g\hat{g}}(\omega) = 0$ for the effective generation of excitation signal for modal testing of rotors. It was shown in reference [13] that the coherence functions, $\gamma^2_{g\hat{g}}(\omega)$, which were calculated by the two different methods, that is, modulated random excitation with a carrier frequency of $\omega_0 = 3\Delta\omega$ and isotropic random excitation, equally tend to vanish as n_d (the number of averagings) increases. It confirms that the two methods can be equally and effectively used for excitation of anisotropic rotors.

Even though the above two methods show a similar behavior in satisfying the condition (7), it normally holds $\mathrm{Im}\{S_{f_y f_z}(\omega)\} \neq 0$ for the modulated random excitation technique unlike the uncorrelated isotropic random excitation [13].

As an illustrate example, we treat a simple rotor whose equation of motion, using a complex displacement, $p(t)$, and a complex input, g(t), is given by [8]

$$m\ddot{p}(t) + (c - j\Omega_p)\dot{p}(t) + kp(t) + \Delta k\bar{p}(t) = g(t), \tag{33}$$

where m, c and Ω_p indicate the mass, damping and gyroscopic effect respectively, and k and Δk correspond to the mean and deviatoric stiffnesses, the latter indicating the degree of anisotropy. The normal and reverse dFRFs associated with equation (33) can be expressed theoretically, by introducing the conjugate form of equation (33), as [2]

$$H_{gp}(\omega) = \frac{-m\omega^2 + jc\omega - \Omega_p\omega + k}{(-m\omega^2 + jc\omega + \Omega_p\omega + k)(-m\omega^2 + jc\omega - \Omega_p\omega + k) - \Delta k^2},$$

$$H_{\hat{g}p}(\omega) = \frac{\Delta k}{(-m\omega^2 + jc\omega + \Omega_p\omega + k)(-m\omega^2 + jc\omega - \Omega_p\omega + k) - \Delta k^2}. \tag{34}$$

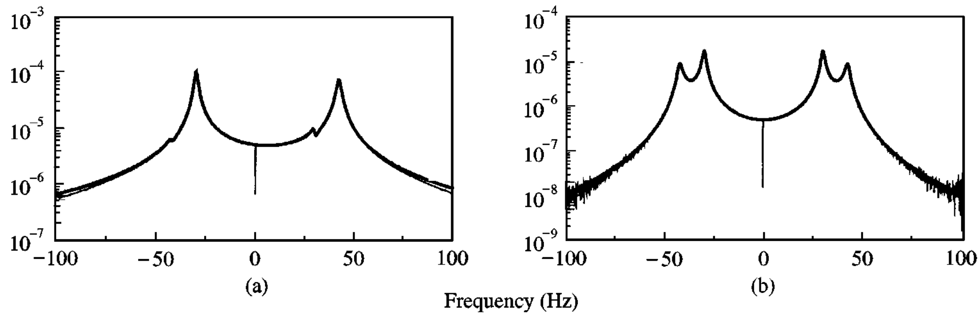

Figure 2. Magnitude plot of estimated dFRFs: (a) $H_{gp}(\omega)$, (b) $H_{\hat{g}p}(\omega)$, with $\omega_0 = 3\Delta\omega$: $\omega_0 = 3.7$ rad/s (0.59 Hz). ——, Theoretical, ——, estimated.

In the simulations, the following numerical values have been used: $m = 4$ kg, $c = 50$ N s/m, $k = 2 \times 10^5$ N/m, $\Delta k = -2 \times 10^4$ N/m, and $\Omega_p = 300$ N s/m.

To estimate dFRFs using the modulation technique, the real random signal, $f(t)$, was generated from a Gaussian random process with the time interval of 2.5 ms and digitally band-pass-filtered with the filter frequency of 100 Hz. The modulated input force, $g(t) = f_y(t) + \mathrm{j} f_z(t)$, was numerically calculated from a filtered random signal, $f(t)$, with the carrier frequency of $\omega_0 = 3.7$ rad/s (0.59 Hz, $3\Delta\omega$). The equation of motion was numerically integrated with the time interval of 2.5 ms using the Runge–Kutta integration method to compute the complex response, $p(t) = y(t) + \mathrm{j}z(t)$ of the simple rotor. Then, the dFRFs are obtained by downward decimating the input and response data such that Nyquist frequency is reduced to 100 Hz. The dFRFs were estimated with 100 ensemble averagings of the 2048 point FFT using Hanning window. Gaussian-distributed random measurement noises were also added to response signals so that the rms ratio of noise to signal was kept to be 0.01. Figure 2 shows the magnitude plots of dFRFs obtained by modulated excitation technique. Note the theoretical and estimated dFRFs are in good agreement, which shows that modulated signals for modal testing of anisotropic rotors are effective. Here, the violation of sampling theorem due to modulation is negligible because only three discrete spectral lines ($l = 3$) out of 2048 are shifted beyond Nyquist frequency [13].

6. CONCLUSIONS

A new efficient excitation technique for modal testing of anisotropic rotors, the modulated random excitation, is proposed. It features that, two real random signals used for excitation devices can be obtained from a single stationary random source using modulation technique. It is shown that the proposed method differs from the widely used uncorrelated isotropic random excitation in that the two directional random excitation signals may be correlated with each other. Finally, it is recommended as a rule of thumb that the carrier frequency of about 3 times the frequency resolution is optimal, maximizing the effective frequency range for dFRF estimates and the modulation effect.

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