Journal of Sound and Vibration (2000) **234**(4), 611–624
doi:10.1006/jsvi.1999.2892, available online at http://www.idealibrary.com on IDEAL®

DYNAMIC ABSORBERS FOR AN EXTERNALLY EXCITED PENDULUM

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(*Received* 9 *March* 1999, *and in final form* 20 *December* 1999)

A dynamic absorber, which can move in the transverse or longitudinal direction, is added to an externally excited pendulum. The quenching efficiency of the two different systems is studied and compared. The absorber can be highly efficient for slightly damped systems if it is correctly tuned. This is especially true when only small absorber mass can be used.

1. INTRODUCTION

The dynamic absorber is an important tool for vibration quenching. In most cases where vibration needs to be reduced the body or structure (basic subsystem) performs a rectilinear motion. The motion of the added dynamic absorber (absorber subsystem) can be rectilinear when it consists of a mass and a spring, but it can also be rotational when dealing with a swinging pendulum. In the first case (rectilinear motion for both subsystems) the theory is well known (see, e.g., reference [1]), while the second case (rectilinear motion of the body and rotational motion of the pendulum) is less well understood.

This second problem has been studied extensively by one of the authors, and the results obtained can be summarized as follows. There are two principal possibilities for the absorber: the equilibrium axis of the pendulum is either perpendicular to or coincident with the vibration direction of the basic subsystem. In the first case the theory also includes the case of a rotating pendulum [2], while in the second, the system belongs to the class of autoparametric systems [3–6].

The present paper deals with two new systems, firstly where the basic subsystem performs a swinging motion and secondly where the absorber executes rectilinear motion relative to it. Such systems can be found in different fields of applied engineering. In Naval Architecture, for example, the first system is used to model the roll motion of a ship stabilized by means of passive tanks [7], while the second system is used to model the roll motion of a floating offshore structure equipped with passive dampers [8].

2. DIFFERENTIAL EQUATIONS OF MOTION, BASIC ANALYSIS

Two different mechanical systems consisting of a basic subsystem, a pendulum of mass M and length l, and an absorber of mass m elastically suspended to the pendulum are considered. The systems differ from each other only in the elastic mounting of the absorber with respect to the basic subsystem. For System I the absorber mass can move transversally in the direction perpendicular to the pendulum axis (see Figure 1), while for System II the absorber mass can move longitudinally along the pendulum axis (see Figure 2).

Here, φ is the angular deflection of the pendulum and u is the displacement of the absorber mass from its equilibrium position. At rest, mass m is usually not at the centre of gravity C of the pendulum, but can be shifted by a length l_0, which can either be positive (m below C) or negative (m above C). The stiffness of the elastic mounting of the absorber is denoted as k in both cases.

2.1. SYSTEM I

By considering the system in Figure 1, the kinetic and potential energies are given in the following terms:

$$T = \tfrac{1}{2}Ml^2\dot{\varphi}^2 + \tfrac{1}{2}m\{[(l + l_0)\dot{\varphi} + \dot{u}]^2 + u^2\dot{\varphi}^2\},$$

$$U = Mgl(1 - \cos\varphi) + mg[(l + l_0)(1 - \cos\varphi) + u\sin\varphi] + \tfrac{1}{2}ku^2, \tag{1}$$

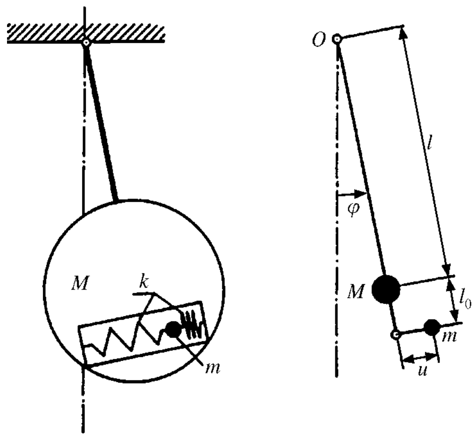

Figure 1. Schematic representation of System I. The absorber moves transversally to the pendulum axis.

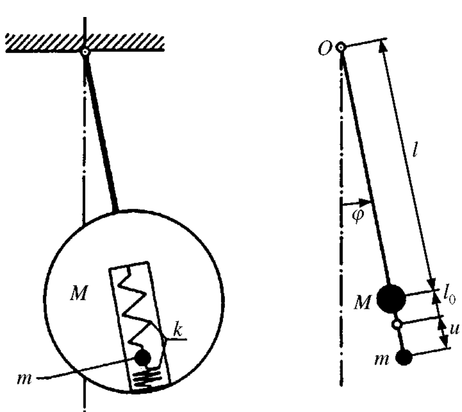

Figure 2. Schematic representation of System II. The absorber moves longitudinally along the pendulum axis.

where g is the acceleration due to gravity. Using Lagrangian equations and taking into account the effects of linear viscous damping, the following differential equations of motion are obtained:

$$[Ml^2 + m(l + l_0)^2]\ddot{\varphi} + b_0\dot{\varphi} + [Ml + m(l + l_0)]g\sin\varphi$$

$$+ m[u^2\ddot{\varphi} + (l + l_0)\ddot{u} + 2u\dot{u}\dot{\varphi} + gu\cos\varphi] = Pl\cos\omega t,$$

$$m\ddot{u} + b\dot{u} + ku + m[(l + l_0)\ddot{\varphi} - u\dot{\varphi}^2 + g\sin\varphi] = 0. \quad (2)$$

Here, it is assumed that an external excitation moment is acting on the basic subsystem.

Denoting ω_0 as $= \sqrt{g/l_c}$ the natural frequency of the basic subsystem, and using the time transformation $\tau = \omega_0 t$, equations (2) can be transformed into the dimensionless form:

$$\varphi'' + \kappa_0\varphi' + \sin\varphi$$

$$+ \bar{\mu}[w^2\varphi'' + (1 + \alpha)w'' + 2ww'\varphi' + q^2 w\cos\varphi] = \varepsilon\cos(\eta\tau + \psi),$$

$$w'' + \kappa w' + Q^2 w + (1 + \alpha)\varphi'' - w\varphi'^2 + q^2\sin\varphi = 0, \quad (3)$$

where $l_c = [Ml^2 + m(l + l_0)^2]/[Ml + m(l + l_0)]$ is the reduced length of the pendulum and $w = u/l, \alpha = l_0/l, \mu = m/M, \bar{\mu} = \mu/[1 + \mu(1 + \alpha)^2], q^2 = g/l\omega_0^2 = l_c/l = [1 + \mu(1 + \alpha)^2]/[1 + \mu(1 + \alpha)], Q^2 = k/m\omega_0^2, \eta = \omega/\omega_0, \kappa_0 = b_0/\omega_0 Ml^2[1 + \mu(1 + \alpha)^2], \kappa = b/\omega_0 m, \varepsilon = P/Ml\omega_0^2[1 + \mu(1 + \alpha)^2]$. In the first equation (3) the phase shift ψ between excitation and the steady state response is introduced in order to simplify the further analysis. The equations of motion are mutually coupled by linear, third and higher order terms.

Approximating the steady state response by the following expressions:

$$\varphi = R\cos\eta\tau, \qquad w = A\cos\eta\tau + B\sin\eta\tau, \quad (4)$$

after substituting equation (4) in equation (3) and using the harmonic balance method, the following algebraic equations are obtained:

$$(1 - \eta^2)R + \bar{\mu}fA = \varepsilon\cos\psi, \qquad -\kappa_0\eta R + \bar{\mu}fB = -\varepsilon\sin\psi,$$

$$(Q^2 - \eta^2)A + \kappa\eta B = -fR, \qquad -\kappa\eta A + (Q^2 - \eta^2)B = 0, \quad (5)$$

where

$$f = q^2 - (1 + \alpha)\eta^2, \quad (6)$$

and third and higher order terms have been neglected.

By solving the previous system, equation (5), the oscillation amplitude of the absorber is

$$r = \sqrt{A^2 + \mathrm{B}^2} = \frac{|f|}{\sqrt{(Q^2 - \eta^2)^2 + \kappa^2\eta^2}}R, \quad (7)$$

and for the basic subsystem

$$R = \frac{\varepsilon}{\sqrt{(1 - \eta^2 + \bar{\mu} f F_1)^2 + (\kappa_0 \eta - \bar{\mu} f F_2)^2}}, \tag{8}$$

where

$$F_1 = -f \frac{Q^2 - \eta^2}{(Q^2 - \eta^2)^2 + \kappa^2 \eta^2}, \qquad F_2 = -f \frac{\kappa \eta}{(Q^2 - \eta^2)^2 + \kappa^2 \eta^2}. \tag{9}$$

In particular, for $Q = \eta = 1$,

$$(R)_{Q=\eta=1} = \frac{\varepsilon}{|k_0 + \bar{\mu} f/\kappa|}, \tag{10}$$

from which follows

$$\lim_{\kappa \to 0} (R)_{Q=\eta=1} = 0. \tag{11}$$

This confirms the rule of optimal tuning at $Q = \eta$ and minimal damping of the absorber, as in the case where both the motions of the basic body and the absorber mass are rectilinear [1].

2.2. SYSTEM II

The kinetic and potential energies for the system in Figure 2 can be expressed as follows:

$$T = \tfrac{1}{2} M l^2 \dot{\varphi}^2 + \tfrac{1}{2} m[\dot{u}^2 + (l + l_0 + u)^2 \dot{\varphi}^2],$$

$$U = Mgl(1 - \cos\varphi) + mg(l + l_0 + u)(1 - \cos\varphi) + \tfrac{1}{2} k u^2. \tag{12}$$

By means of Lagrangian procedure and considering both the effects of linear viscous damping and external excitation on the basic subsystem, the following equations are obtained:

$$[Ml^2 + m(l + l_0)^2]\ddot{\varphi} + b_0 \dot{u} + [Ml + m(l + l_0)] g \sin\varphi$$

$$+ m\{[2(l + l_0) + u] u \ddot{\varphi} + 2(l + l_0 + u)\dot{u}\dot{\varphi} + g u \sin\varphi\} = Pl \cos\omega t,$$

$$m\ddot{u} + b\dot{u} + ku - m(l + l_0 + u)\dot{\varphi}^2 + mg(1 - \cos\varphi) = 0. \tag{13}$$

Using again the time transformation $\tau = \omega_0 t$ and the previous notations for the system parameters, the following equations of motion in dimensionless form result:

$$\varphi'' + \kappa_0 \varphi' + \sin\varphi + \bar{\mu}\{[2(1 + \alpha) + w] w \varphi''$$

$$+ 2(1 + \alpha + w)w'\varphi' + q^2 w \sin\varphi\} = \varepsilon \cos(\eta\tau + \psi),$$

$$w'' + \kappa w' + Q^2 w - (1 + \alpha + w)\varphi'^2 + q^2(1 - \cos\varphi) = 0. \tag{14}$$

The equations of motion (14) differ from equations (3) due to the fact that the coupling terms are of second and higher order, i.e., the linear coupling terms are missing. There is a certain similarity with the autoparametric systems presented in reference [5], although equations (14) have no semi-trivial solution. In any case, the internal reasonance 1 : 2 can be expected and therefore considered as important for the behaviour of the system.

When the system is tuned into or close to the internal resonance, the steady state solution can be approximated in the form

$$\varphi = R\cos\eta\tau, \qquad w = w_0 + A\cos 2\eta\tau + B\sin 2\eta\tau. \tag{15}$$

Using the harmonic balance method, and neglecting the terms of the third and higher order, the following algebraic equations are obtained after substituting equation (15) into equation (14):

$$(1 - \eta^2)R + \tfrac{1}{2}\bar{\mu}h_+ AR + \bar{\mu}h_- w_0 R = \varepsilon\cos\psi,$$

$$-\kappa_0\eta R + \tfrac{1}{2}\bar{\mu}h_+ BR = -\varepsilon\sin\psi, \qquad (Q^2 - 4\eta^2)A + 2\kappa\eta B = -\tfrac{1}{4}h_+ R^2,$$

$$-2\kappa\eta A + (Q^2 - 4\eta^2)B = 0, \qquad Q^2 w_0 = -\tfrac{1}{4}h_- R^2, \tag{16}$$

where

$$h_- = q^2 - 2(1 + \alpha)\eta^2, \qquad h_+ = q^2 + 2(1 + \alpha)\eta^2. \tag{17}$$

By solving the previous system (16), it is possible to obtain for the absorber

$$w_0 = H_0 R^2, \qquad r = \sqrt{A^2 + B^2} = \frac{1}{4}\frac{|q^2 + 2(1 + \alpha)\eta^2|}{\sqrt{(Q^2 - 4\eta^2)^2 + 4\kappa^2\eta^2}}\, R^2, \tag{18}$$

and for the basic subsystem

$$\bar{\mu}^2\left[\frac{1}{4}h_+^2(H_1^2 + H_2^2) + h_+ h_- H_0 H_1 + h_-^2 H_0^2\right]R^6 + \bar{\mu}\{h_+[(1 - \eta^2)H_1 - \kappa_0\eta H_2]$$

$$+ 2h_-(1 - \eta^2)H_0\}R^4 + [(1 - \eta^2)^2 - \kappa_0^2\eta^2]R^2 - \varepsilon^2 = 0, \tag{19}$$

where

$$H_0 = -\frac{1}{4Q^2}h_-, \qquad H_1 = -\frac{1}{4}h_+\frac{Q^2 - 4\eta^2}{(Q^2 - 4\eta^2)^2 + 4\kappa^2\eta^2},$$

$$H_2 = -\frac{1}{4}h_+\frac{2\kappa\eta}{(Q^2 - 4\eta^2)^2 + 4\kappa^2\eta^2}. \tag{20}$$

Once again, it can be shown that the optimal tuning condition occurs at $Q = 2\eta = 2$. Moreover,

$$\lim_{\kappa \to 0} (R)_{Q=2\eta=2} = 0, \tag{21}$$

from which follows the need for little absorber damping.

A system similar to the one here considered, but with elastic mounting of the pendulum, has been analyzed in references [9,10].

3. ENGINEERING APPLICATIONS

Two practical applications from the field of Naval Architecture will be here considered: the case of a seagoing ship equipped with antirolling tanks as an example of System I, and a floating offshore structure with passive dampers as an example of System II.

3.1. SYSTEM I

To improve successful exploitation and mission effectiveness of a ship in rough sea it is necessary to increase the seakeeping performance of the vessel by reducing motions and related undesirable effects. Due to relatively small wave-induced excitations, the roll motion is the most easily controlled from a point of view of stabilization. There are many types of stabilizers and other tools to control rolling [11], e.g., bilge keels, gyroscopic stabilizers, movement of weight, rudder action, jet flaps, passive and active roll tanks, stabilizing fins. Some of the above devices are not used at present mainly because of economic considerations. Others, although not the most effective, are technically well developed and suitable for practical use.

Passive antirolling tanks are usually left alone on board except for tuning operations, which are needed for a change in the ship loading conditions. The sophisticated control unit required in an active system is not needed in a passive system, and this makes the latter more attractive than its active counterpart. It is also relatively simple to design and build a resonant antirolling tank system. Three types of antirolling tanks have been used in practical applications [12], namely, the free surface tank, the U-tube tank and the external tank. Apart from their technical differences, it has been shown [7] that any ship–tank system used as a passive motion stabilizer is equivalent to a compound pendulum, having the two pendula of the same length in order to make their periods close to each other.

A vessel fitted with antirolling U-tanks, having the following main characteristics [13] is now considered: length between perpendiculars $L = 49{\cdot}10$ m, beam $B = 9{\cdot}00$ m, draft $T = 3{\cdot}68$ m, mass displacement $M = 936$ t, centre of gravity and metacentre above keel at $KG = 3{\cdot}69$ m and $KM = 4{\cdot}32$ m respectively. The hydrodynamic computations give $I = 12655$ t m^2 for the moment of inertia of the vessel about its central longitudinal axis (taking into account the added moment of inertia) and $T_0 = 8{\cdot}63$ s for the period of rolling in still water. In the model this corresponds to a physical pendulum with same mass, same moment of inertia about the oscillation axis and reduced length equal to the metacentric height ($GM = KM - KG$), i.e., $l = 0{\cdot}73$ m.

To tune the U-tanks with the natural roll oscillations ($\omega_0 = 2\pi/T_0 = 0{\cdot}728$ rad/s), the location, the dimensions and the shape of the reservoirs have to be appropriately chosen. This is a typical ship design problem [11,12], which cannot be considered explicitly. However, according to engineering experience, it is convenient to install the tanks in the central part of the vessel slightly forward of the middle frame by filling them with fresh/salt water. As a result, the solution obtained [13] indicates, for the liquid ballast in reservoirs, the mass displacement $m = 28{\cdot}8$ t and the centre of gravity above keel at $KG_B = 1{\cdot}56$ m, which corresponds to $l_0 = KG - KG_B = 2{\cdot}13$ m.

In the worst navigation condition, which corresponds to a regular beam sea with synchronous waves (wavelength $\lambda = 116$ m), the exciting moment intensity is $M_W = 163$ kNm for unitary wave height $H_W = 1{\cdot}0$ m. Moreover, for this type of ship, the roll damping usually varies between 5 and 10 per cent of the critical damping, while the tank damping can be properly adjusted [14] over a wide range of values by means of flow regulation in the connecting water/air canal.

In the analogy with System I, the previous ship–tank system corresponds to $\mu = m/M = 0{\cdot}031$, $\alpha = l_0/l = 2{\cdot}9$ and $\varepsilon = M_W/I\omega_0^2 = 0{\cdot}024$.

3.2. SYSTEM II

Tension leg platforms (TLP) can be considered as one of the most promising offshore floating systems intended for oil exploitation in deep-water scenarios. The TLP displays large amplitude motions mainly due to wave loads which increase considerably by increasing the severity of sea conditions. In particular, large heave amplitudes appear as one of the most deleterious effects to the structural safety and integrity of the vessel, mainly for the critical components: tethers and raisers together with their links and connections. In more recent designs [8], the passive/active control of heave motion is made feasible by means of reaction masses counteracting the movements of the floating hull. Besides minimizing stress levels and related fatigue problems, the TLP heave motion control allows both the production performance and service life of the whole system to be improved. Examples of engineering conceptions for active, semi-active and passive control of floating structures may be found in literature [15,16].

The active/passive control of TLP heave motion consists of several tuned mass dampers installed within the vertical columns of the hull. The dampers are able to move in the vertical direction and their total mass must be feasible to install, with values between 0·2 and 1 per cent of the total mass [8]. In particular, suitable tuning of the dampers could allow their use also for the control of roll and/or pitch motion during installation or de-commissioning operations, when the platform is freely floating without any influence from the mooring lines.

A TLP design with four circular columns of 21·0 m diameter and the following main characteristics is considered: breadth in longitudinal and transverse directions $L \times B = 86{\cdot}0\ \text{m} \times 86{\cdot}0$ m, draft $T = 28{\cdot}0$ m, mass displacement $M = 0{\cdot}348 \times 10^5$ t, centre of gravity above baseline at $KG = 34{\cdot}8$ m. The hydrostatic and hydrodynamic computations give the metacenter above baseline at $KM = 53{\cdot}6$ m and the moment of inertia about central longitudinal axis (added moment of inertia included) $I = 0{\cdot}434 \times 10^8$ t m^2. For the natural period of rolling without tethers one obtains $T_0 = 16{\cdot}4$ s (natural frequency $\omega_0 = 0{\cdot}383$ rad/s). The corresponding physical pendulum has the same mass, same moment of inertia and reduced length $l = GM = 18{\cdot}8$ m.

A suitable mass for passive motion control equals $m = 348$ t. Since the absorbers are assumed to be distributed among the four columns with their centre of gravity above baseline at $KG_B = 3{\cdot}00$ m, this corresponds to $l_0 = KG - KG_B = 31{\cdot}8$ m.

To obtain the excitation term, a synchronous regular wave (wavelength $\lambda = 420$ m) broadside to the platform can be considered. For unitary wave height $H_W = 1{\cdot}0$ m this corresponds to an exciting moment intensity of $M_W = 0{\cdot}432 \times 10^5$ kNm.

In the analogy with System II, the previous TLP–damper system corresponds to $\mu = m/M = 0{\cdot}010$, $\alpha = l_0/l = 1{\cdot}69$ and $\varepsilon = M_W/I\omega_0^2 = 0{\cdot}0068$. In this case, the roll damping usually varies between 10 and 30 per cent of the critical damping, while the damping of the absorbers can be changed substantially by proper regulation.

4. RESULTS

Due to the wide variation of system parameters in different fields of engineering applications, two hypothetical systems will be considered here. However, their parameters will be chosen to be close to those in the previous examples.

In all the cases presented the values $\varepsilon = 0{\cdot}02, \mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$ and $\kappa = 0{\cdot}02$ are the same for both systems. The results of the analytical and numerical investigations are presented in figures showing the response amplitudes $R(\eta)$ and $r(\eta)$ for different values of Q and α or κ marked directly in the figures. To investigate the sensitivity to parameter changes, a fairly large interval of values has been considered. The actual restrictions depend on the particular problem under consideration.

4.1. SYSTEM I

The first objective has been to analyze the influence of the absorber position with respect to the pendulum, expressed by the parameter $\alpha = l_0/l$. Figure 3 shows the dependence of the oscillation amplitudes R and r, given by equations (8) and (7), on excitation frequency η for $Q = 1$ (the absorber is exactly tuned to the natural frequency of the basic subsystem). It can be seen that for $\alpha = 0$, i.e., when the absorber is located at the centre of gravity of the pendulum subsystem, the absorber practically does not influence its vibration amplitude. For $\alpha \neq 0$ both the $R(\eta)$ and $r(\eta)$ curves have a double-peak form, and the distance between peaks grows by increasing the absolute value of α. The greater the distance of the absorber equilibrium position from the centre of gravity of the basic subsystem the better is the quenching effect. It can be shown that for higher values of μ the distance between peaks is increasing, while the oscillation amplitudes of the basic system near $\eta = 1$ decrease. Similar results are also obtained when $\alpha < 0$, i.e., for the equilibrium position of the absorber above the centre of gravity of the pendulum.

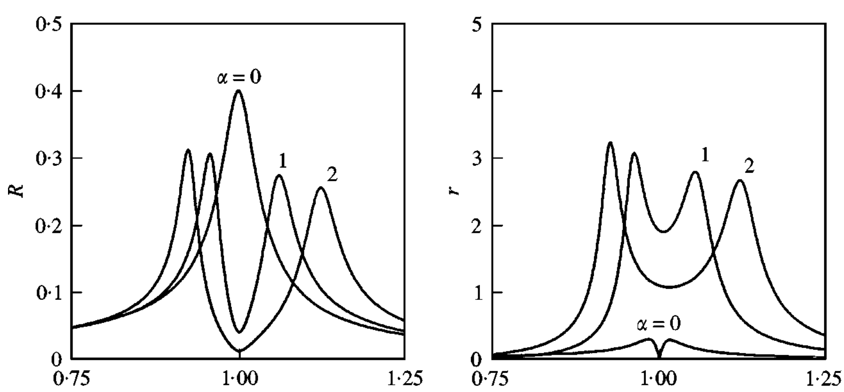

Figure 3. Oscillation amplitudes R and r versus excitation frequency η for System I in case $Q = 1$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

The effect of the absorber damping is shown in Figure 4 for $\alpha = 2$ and three values of κ, i.e., $\kappa = 0{\cdot}02$, $0{\cdot}10$ and $0{\cdot}20$. It can be seen that when η is very close to 1, the smaller the value of κ, the better the quenching efficiency. When the interval of excitation frequencies is greater, only a certain level of damping has a favourable effect on the basic subsystem. Another influence is shown in Figures 5 and 6 and where the oscillation amplitudes R and r are shown to be dependent on η for two values of Q not exactly tuned in resonance, i.e., $Q = 0{\cdot}9$ and $1{\cdot}1$. At $\eta = Q$ the amplitude R is substantially reduced but the quenching effect at resonance is not significant.

4.2. SYSTEM II

The results are presented by showing the dependence of the oscillation amplitudes R, r and the constant displacement w_0 on excitation frequency η, as obtained from equations (19) and (18). The influence of the absorber position, determined by the parameter α, is considered in Figure 7 for $Q = 2$ (the absorber is exactly tuned into the internal resonance).

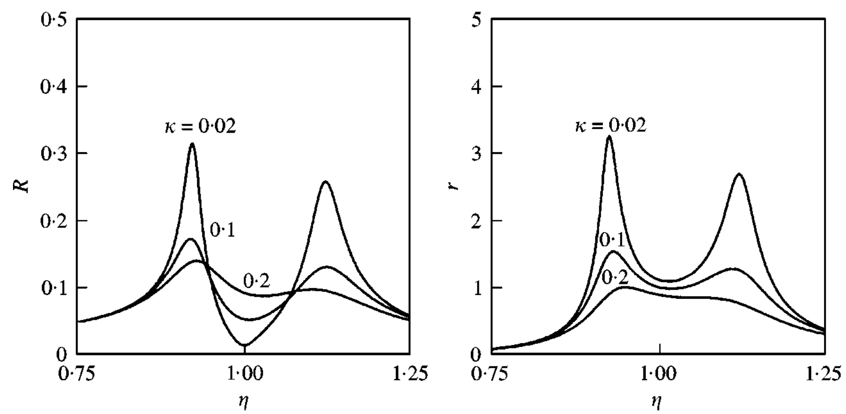

Figure 4. Oscillation amplitudes R and r versus excitation frequency η for System I in case $Q = 1$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\alpha = 2$, and $\kappa = 0{\cdot}02$, $0{\cdot}10$, $0{\cdot}20$.

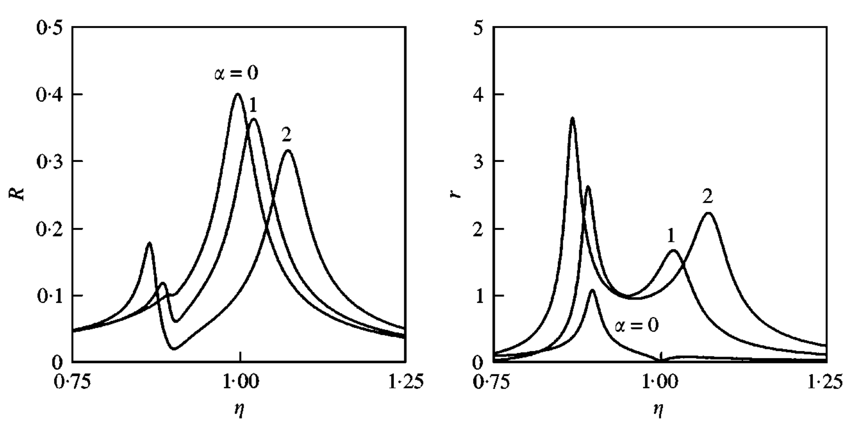

Figure 5. Oscillation amplitudes R and r versus excitation frequency η for System I in case $Q = 0{\cdot}9$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

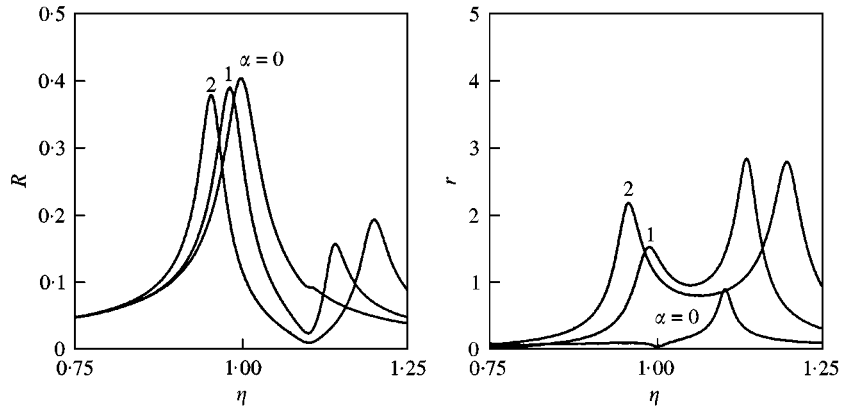

Figure 6. Oscillation amplitudes R and r versus excitation frequency η for System I in case $Q = 1{\cdot}1$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

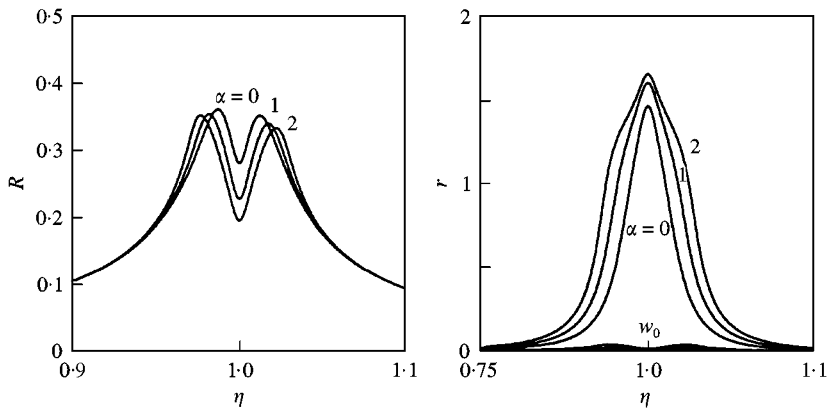

Figure 7. Oscillation amplitudes R, r and constant displacement w_0 versus excitation frequency η for System II in case $Q = 2$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

It can be seen that the effect of α differs from that of the previous system because the absorber yields only slight quenching ability for the basic subsystem vibration. Moreover, it follows that the effect of non-linear coupling terms asymmetry is negligible because w_0 is very small. The double-peak form can be seen for $R(\eta)$ curves only, while $r(\eta)$ curves present a single maximum. The quenching diminishes for $\alpha < 0$ and becomes zero for $\alpha = -1$, i.e., for the absorber located in the suspension point.

The important effect of the absorber damping is shown in Figure 8 for $\alpha = 2$ and three values of κ, i.e., $\kappa = 0{\cdot}02$, $0{\cdot}10$ and $0{\cdot}20$. The effect of the absorber detuning can be seen in Figures 9 and 10, for $Q = 1{\cdot}95$ and $2{\cdot}05$ respectively.

4.3. VALIDATION

The results of analytical analysis have been supplemented by direct numerical solution of differential equations (3) and (13), without neglecting the third and higher order terms.

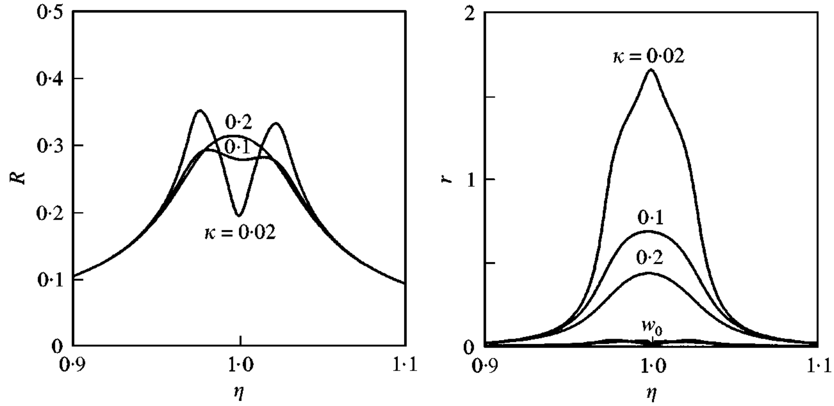

Figure 8. Oscillation amplitudes R, r and constant displacement w_0 versus excitation frequency η for System II in case $Q = 2$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\alpha = 2$, and $\kappa = 0{\cdot}02$, $0{\cdot}10$, $0{\cdot}20$.

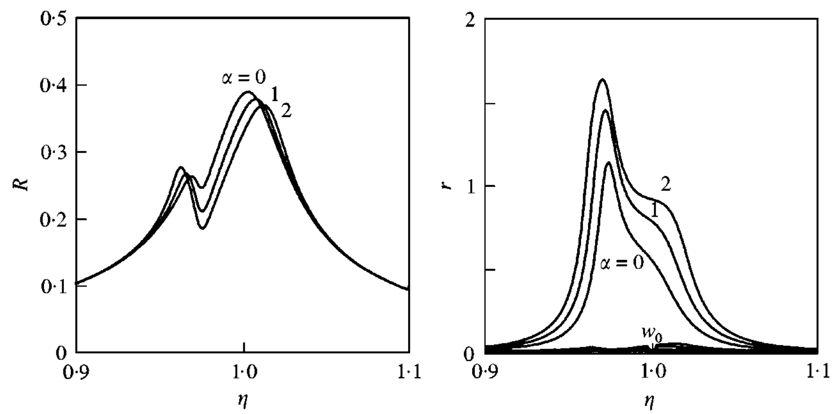

Figure 9. Oscillation amplitudes R, r and constant displacement w_0 versus excitation frequency η for System II in case $Q = 1{\cdot}95$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

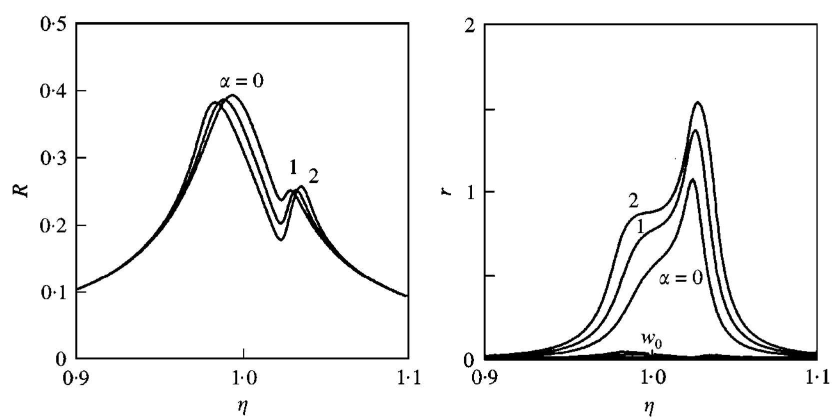

Figure 10. Oscillation amplitudes R, r and constant displacement w_0 versus excitation frequency η for System II in case $Q = 2{\cdot}05$, $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, and $\alpha = 0, 1, 2$.

Although in systems characterized by non-linear coupling, a non-periodic or even chaotic response of the harmonic excitation can occur (for autoparametric systems see, e.g., reference [5]), for the time being only periodic or quasi-periodic vibrations have been found by numerical solution of the differential equations of motion.

Thus, after the transient had died out, the extreme values (maxima and minima) of the oscillation amplitudes of φ and w, denoted as $[\varphi]$ and $[w]$, were recorded and stored for further consideration. Figures 11 and 12 show the numerically obtained extrema (dots) during steady state vibration together with the analytically predicted amplitudes (solid lines) obtained from equations (4) and (14) respectively. The effect of detuning from the internal resonance has been explicitly considered for the case $\alpha = 1$ and the values of parameters $Q = 0{\cdot}9$, $1{\cdot}1$ (System I) and $Q = 1{\cdot}95$, $2{\cdot}05$ (System II).

For System I the oscillations of both subsystems are symmetric with respect to the initial equilibrium position, while for System II the oscillations of the pendulum are symmetric and those of the dynamic absorber asymmetric. The agreement between analytical

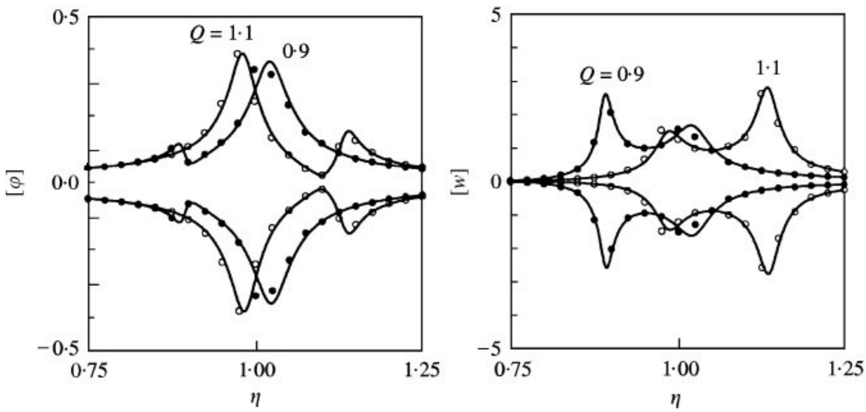

Figure 11. Extreme values $[\varphi]$ and $[w]$ of oscillation amplitudes versus excitation frequency η for System I in case $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, $\alpha = 1$, and $Q = 0{\cdot}9$, $1{\cdot}1$. Comparison between analytical predictions (solid lines) and numerical simulation (dots).

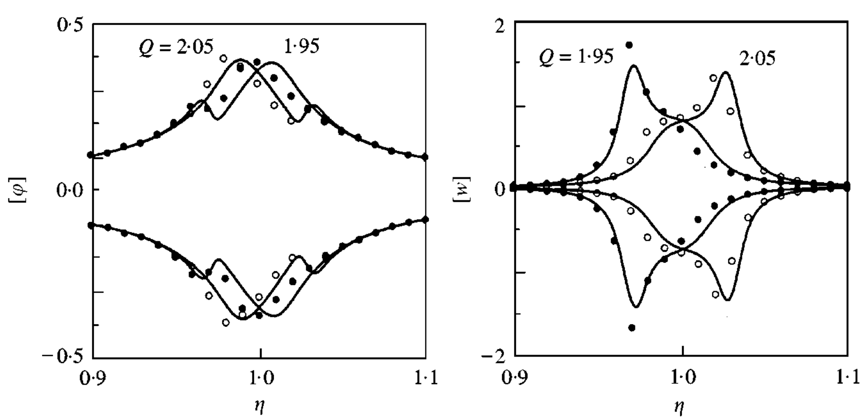

Figure 12. Extreme values $[\varphi]$ and $[w]$ of oscillation amplitudes versus excitation frequency η for System II in case $\varepsilon = 0{\cdot}02$, $\mu = 0{\cdot}01$, $\kappa_0 = 0{\cdot}05$, $\kappa = 0{\cdot}02$, $\alpha = 1$, and $Q = 1{\cdot}95$, $2{\cdot}05$. Comparison between analytical predictions (solid lines) and numerical simulation (dots).

predictions and numerical simulation is fairly good for both systems, even if the theoretical maxima are slightly underpredicted and shifted towards higher frequencies. Such a result is mainly related to the fact that, for both systems, analytical solutions neglect terms of order three and higher. The soft character of the pendulum, which becomes significant for oscillation amplitudes of 15–20° (0·25–0·30 rad), is certainly the factor most responsible for limitations of the theory.

5. CONCLUSIONS

In both systems considered, the effect of changing the position of the absorber with respect to the basic system has been examined. For System I the absorber has practically no effect on the behaviour when it is situated at the centre of gravity of the basic subsystem, but its efficiency is increased when displaced far from the centre of gravity. For System II the absorber has a zero or slight quenching effect when it is situated in the suspension point or close to it respectively.

Both systems differ as far as the optimal tuning is concerned. In particular, for System I it occurs in the main resonance, while for System II the absorber natural frequency should be double the resonance frequency of the basic subsystem. Vibration quenching ability is higher for System I; hence System II can be used only for a slightly damped system when the absorber can be correctly tuned. Moreover, it was found that the quenching efficiency increases for more intensive excitations and for higher mass ratios.

For the application of the absorbers considered the character of excitation is also important. When the excitation is harmonic with constant frequency and when the absorber can be correctly tuned then a small damping of the absorber is convenient for both systems. When the excitation is periodic with changing frequency or stochastic it is necessary to tune the absorber in resonance, i.e., the natural frequency of the absorber should be equal (System I) or double (System II) the natural frequency of the basic subsystem. In this case a certain level of absorber damping is convenient for System I.

ACKNOWLEDGMENT

This work has been partly supported by contract no. 101/98/0972 of the Grant Agency of the Czech Republic and by the Italian Ministry of University and Scientific and Technological Research (MURST).

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