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ON FRICTION-DRIVEN VIBRATIONS IN A MASS BLOCK–BELT–MOTOR SYSTEM WITH A LIMITED POWER SUPPLY

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1. INTRODUCTION

The current engineering systems present a wide range of non-linear phenomena. The knowledge of their dynamic characteristics is an important step in systems design and control. A simple non-linear system of single degree of freedom may present multiple solutions, jumps in the response, limit cycles and intermittence (chaos) [1].

A non-linear phenomenon present in many engineering mechanisms and machines is the dry friction among contact surfaces. The characteristics of the friction produce two effects in mechanical systems: the energy dissipation and self-excitation effects. Examples of self-excitation appear in the motion of linear guides and articulations of positioning mechanisms, in many brake systems and couplings for friction. The self-excitation occurs in many engineering systems where the friction forces have significant influence on the system operation, hence the dry friction has been the object of many experimental investigations. Mathematical models have been proposed to study the static and dynamic properties of dry friction [2–7].

A class of systems which presents an interaction phenomenon due to dry friction is the motion or power transmission in machines which use flexible elements such as tracks or belts. The belt transmits the motion between pulleys or interacts with other mechanisms through the contact of its surface. An example of this class of systems is a mass block–belt–motor system. The analysis of the non-linear dynamics of such systems became fundamental for the solution of the engineering problems which have to do with the control of vibrations for functionality purposes or structural integrity or for environmental comfort due to noise emission. Therefore, several scientific investigations on the interaction

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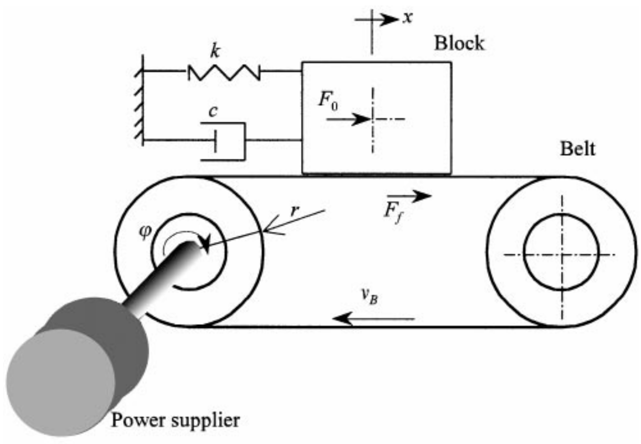

Figure 1. Mechanical system with energy source: mass block–belt–motor system.

phenomenon have been accomplished with the objective to develop new design and control techniques [3–7].

In the present work, a mass block–belt–motor system is analyzed. Simulation results of two problems of self-excited oscillations by an interaction force are presented and compared. The first considers the so-called ideal problem and the other the non-ideal problem [8]. In the ideal problem, the excitation source is assumed to be a constant or a time function whereas in the non-ideal problem the self-excited system is assumed to be dependent on the properties of its power supply. The latter may be described as an autonomous system. In both problems, the dry friction force is the force responsible for the self-excitation effect. The dry friction model adopted considers the interaction phenomenon from the point of view of engineering application.

2. SELF-EXCITED VIBRATING SYSTEM

Consider the mass block–belt system shown in Figure 1. The transference of energy between the contact of the belt and an oscillating block is due to the friction. The oscillating mass block has its motion damped or excited by the friction force. In the steady state regime, if only the dynamic friction relative to the sliding is considered the motion presents three different situations which are dependent on the relative velocity between the mass block and the belt: first, the block motion is in the opposite direction of the belt motion, when the effect of damping takes place, second, the direction is the same, but the mass block moves faster than the belt and again the effect of damping takes place and, finally, the mass block moves slower than the belt and the self-excitation effect takes place. The mass block oscillates with limited amplitude as the amount of energy drawn from the oscillating system for the friction and the amount of energy given to the oscillating system (self-excitation effect) are equivalent during a cycle [2]. When the mass block velocity, during the oscillating cycle, becomes equal to the belt velocity, the phenomenon of static friction force in the contact called no-sliding or stick occurs. The alternation between the sliding and no-sliding modes is a characteristic of the system motion which generates a complex interaction belt–mass block. When the effect of the static and dynamic friction is considered and the velocity of the mass block is the same as the belt, the phenomenon of no-sliding or stick occurs and the oscillating system is excited or damped by the static friction with larger intensity, because the static friction force can reach larger values than the dynamic friction force. As the effect of damping is associated with a motion opposing force, the sliding (dynamic mode) is the main mode during the motion damping, which involves smaller values of the friction force than when in the non-sliding mode (static mode). On the other hand, the excitation effect presents the contribution of both friction forces for a longer time, in the sliding and non-sliding modes.

A number of works on the theory of oscillations with self-excitation exists in the literature. The self-excitation appears in cases when the oscillator possesses a non-linear behavior dependent on damping forces, that is, the damping force tends to increase the amplitude of the oscillations when they present a small amplitude and tends to decrease them when they present a large amplitude. A possible steady state motion is found when the system gains energy during part of the cycle and loses energy during the remaining part of the cycle, in such a way that at the end of each cycle the net energy is null [9]. Several authors studied this type of problem. In Nayfeh and Mook [10], and Nayfeh [11], self-excited oscillations were studied by perturbation methods. A good revision of this problem class can be found in Schmidt and Tondl [12]. An older text, even so of good quality and comprehensible, is found in Stoker [13].

2.1. SELF-EXCITED SYSTEMS WITH A LIMITED POWER SUPPLY

In engineering problems, in general, the possible influence of the oscillating system motion on its power supply or external excitation is disregarded. However, in many practical problems of engineering it was observed that the excitation or its source of energy is influenced by the response of the system. This invalidates the traditional formulation of the theory of oscillations and a more realistic formulation that takes into account the interaction among the state variables of the source of energy or external excitation and the state variables of the mechanical system is needed [8,14]. The traditional model dynamic system is a denominated system with an ideal source of energy, where the existence of the interaction phenomenon between the dynamical system and source of energy is not considered. In this way, the adoption of another model based on the concept of non-ideal dynamic system, that is, a dynamic system with a source of energy of limited power (non-ideal source) is required.

The non-ideal machine is a concept that depends fundamentally on some of its characteristics, for example, the structure that supports it. A classic example of a non-ideal system is a flexible structure (for example, an embedded beam or a simple portico) on which is mounted a motor with an unbalanced shaft, with an energy source of limited power. In the mentioned case, the motion of the system structure due to its own flexibility, affects the working conditions of the machine. In consequence of the interaction between the flexible structure and the excitation source in this type of dynamic system the following effects may be observed: (1) discontinuities or abrupt jumps in the amplitude versus frequency curve and different curves of amplitude versus frequency when increasing or decreasing the motor velocity not predicted by the theory of oscillation; (2) dependence between the effects mentioned above and the characteristics of the motor.

Therefore, it has been noticed that the non-ideal dynamic systems possess additional degrees of freedom depending on the number of sources of limited power interacting in the system, when compared to the corresponding ideal system. A system with a limited power supply is characterized by the inability to operate in close velocities to the critical velocities (resonance) and, also, it has difficulty in accelerating and/or decelerating during the passage through the critical velocity (resonance), when a large amplitude in the transient response of the system occurs. This kind of problem was considered in Yamakawa and Murakani [15]; an optimization method of the operation curves of rotating shaft machine with source of a limited power was presented. This phenomenon, known in the literature as the Sommerfeld effect, in honor of the first researcher to observe it, is also described in Kononenko [14]. The vibrations control of a non-ideal dynamic system during the passage through resonance was achieved, experimentally, by Dimentberg *et al.* [16], through momentary alteration of the rigidity of the system and for Balthazar *et al.* [17] using an optimization technique. A complete and comprehensive review of different approaches to non-ideal problems, up to 1979, is given in Nayfeh and Mook [10] and more recently in Balthazar *et al.* [8]. The topological aspects of the parametric and non-parametric vibrations of some models given in Kononenko [14] were analyzed in Balthazar *et al.* [17].

3. THE ANALYZED PROBLEM

The first observations made on the properties of the dry friction indicated that the friction force was proportional to the normal contact force. Coulomb proposed the concept of a limit value for the static friction force, which says that the external forces applied to a body in relative rest will not cause sliding until the limit value is surpassed. The limit value for static friction force is larger than the maximum value of the dynamic friction force that occurs during the relative sliding over the contact surfaces. From this were defined the static friction and dynamic friction coefficients which relate the value of the friction force developed to a function of the normal contact force. The consideration that the friction force during the sliding is almost independent of the relative velocity of sliding is known as the Coulomb friction law. For many situations, this representation is considered a good model for the phenomenon of friction during sliding. In special cases the Coulomb friction model presents limitations [2–5].

3.1. SYSTEM EQUATIONS FOR THE NON-IDEAL PROBLEM

In the present work, the analyzed non-ideal problem is described by the mass block–belt dynamical system and the rotational motion equations.

The motion equation of the mass block–belt system is

$$m\ddot{x} + c\dot{x} + kx + F_f(x, \dot{x}, v_{\mathrm{Re}l}) = F_0 \cos(\omega_E t), \tag{1}$$

where x and $\dot{x}$ are the mass block displacement and velocity, respectively; m the block mass; c the viscous damping coefficient; k the elastic constant; F_0 and ω_E the amplitude and the frequency of the external excitation force, respectively; $v_{\mathrm{Re}l} = v_B - \dot{x} = r\dot{\varphi} - \dot{x}$ the relative velocity between mass block and belt; v_B the belt velocity; r the radius of the belt pulley or transmission rate; $\dot{\varphi}$ the angular velocity of the DC motor shaft; F_f the friction force interaction function which represents the static and dynamic friction effects and is as in [2]. After some manipulations, one obtains

$$\ddot{x} + (2\xi\omega_N)\dot{x} - (\omega_N)^2 x - \left(\frac{F_f}{m}\right) = f_0 \cos(\omega_E t), \tag{2}$$

where $\xi = c/c_{Cr} = c/2m\omega_N$ is the damping ratio; $\omega_N = \sqrt{k_1/m}$ the natural frequency of the mass–spring system; and $f_0 = F_0/m$ the amplitude of the external excitation force normalized by the block mass.

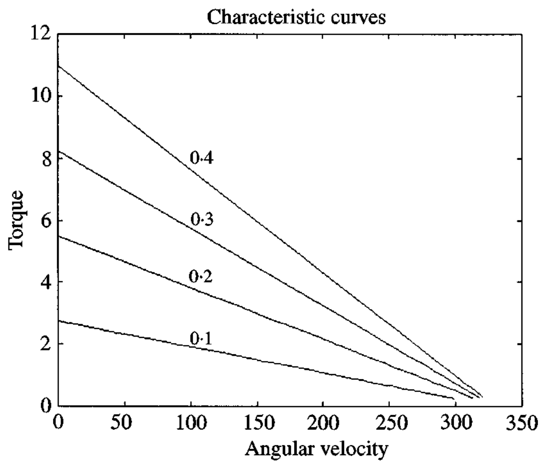

Figure 2. Torque characteristic curves of the DC motor for a given applied voltage, for torque constant $K_T = 0{\cdot}1$; 0·2; 0·3 and 0·4.

The rotational motion equation which describes the interaction between the power source and the friction force is

$$I\ddot{\varphi} = T_{Motor} - rF_{Atrito} \tag{3}$$

where I is the inertia moment of the system rotate part; $\ddot{\varphi}$ the angular acceleration of the power source (motor); T_{Motor} the mechanical torque, described by the characteristic curves given in Figure 2.

Defining the state variables $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \varphi$, $x_4 = \dot{\varphi}$, the non-ideal system model may be thus represented by the state space equation system

$$\frac{\mathrm{d}x_1}{\mathrm{d}\tau} = \dot{x} = x_2, \qquad \frac{\mathrm{d}x_2}{\mathrm{d}\tau} = \ddot{x} = -(2\xi\omega_N)x_2 - (\omega_N)^2 x_1 + \frac{F_f}{m} + f_0 \cos(\omega_E \tau),$$

$$\frac{\mathrm{d}x_3}{\mathrm{d}\tau} = \dot{\varphi} = x_4, \qquad \frac{\mathrm{d}x_4}{\mathrm{d}\tau} = \ddot{\varphi} = \frac{(T_{motor} - rF_{atrito})}{I} \tag{4}$$

3.2. SYSTEM EQUATIONS FOR THE IDEAL PROBLEM

In the ideal problem analyzed, the system of equations is formed considering equilibrium between the motor torque and the required torque for the oscillating block–belt system. However, the angular acceleration is null and the angular velocity is constant. Then the ideal system model may be represented by the state space equation system

$$\frac{\mathrm{d}x_1}{\mathrm{d}\tau} = \dot{x} = x_2, \qquad \frac{\mathrm{d}x_2}{\mathrm{d}\tau} = \ddot{x} = -(2\xi\omega_N)x_2 - (\omega_N)^2 x_1 + \frac{F_f}{m} + f_0 \cos(\omega_E \tau) \tag{5}$$

4. NUMERICAL SIMULATION RESULTS

In this section, we shall analyze the dynamical system represented by equations (4). The state variables are defined as x_1 = displacement, x_2 = velocity, x_3 = motor angular

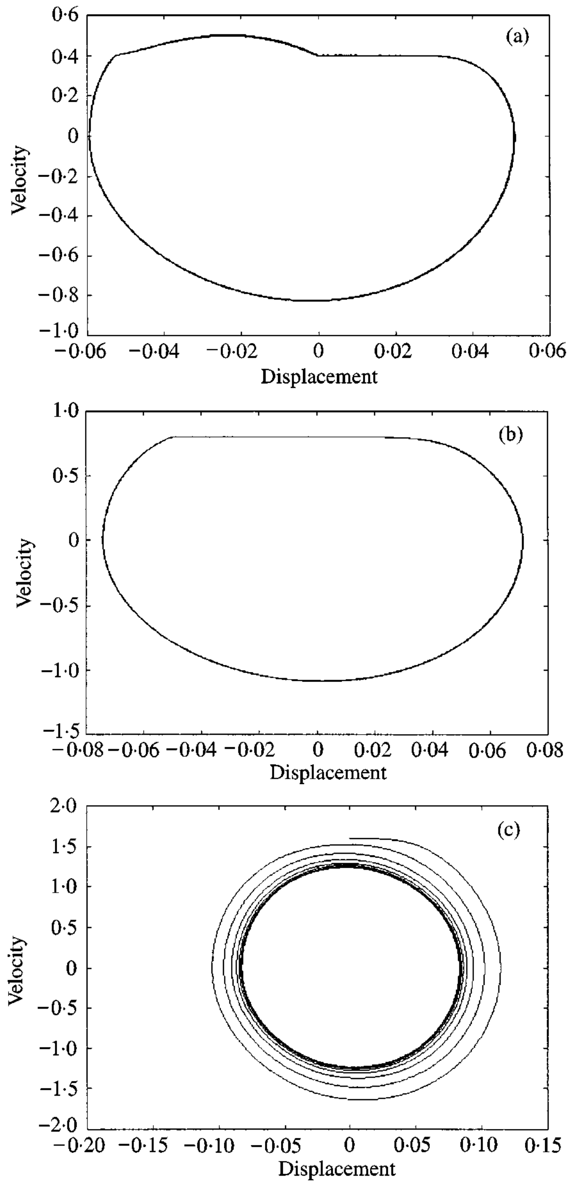

Figure 3. Phase portrait x_2–x_1 of the ideal case without external excitation ($f_0 = 0$) for (a) $v_B = 0{\cdot}4$ m/s, (b) $v_B = 0{\cdot}8$ m/s and (c) $v_B = 1{\cdot}6$ m/s.

displacement, x_4 = motor angular velocity. The numerical simulation results presented were obtained using the Matlab-Simulink™ from Mathworks®.

In Figure 3, results of the self-excited ideal mass block–belt system are presented for the phase portrait velocity–displacement of the block x_2–x_1 for the different belt velocity values $v_B = 0{\cdot}4$, $0{\cdot}8$, $1{\cdot}6$ m/s. For belt velocities from 0·4 to 0·8 m/s a transition path can be observed between the sliding (slip) and the no-sliding (stick) modes. The occurrence of the no-sliding (stick) between the belt and the mass block is evidenced by the horizontal straight line in the phase portrait.

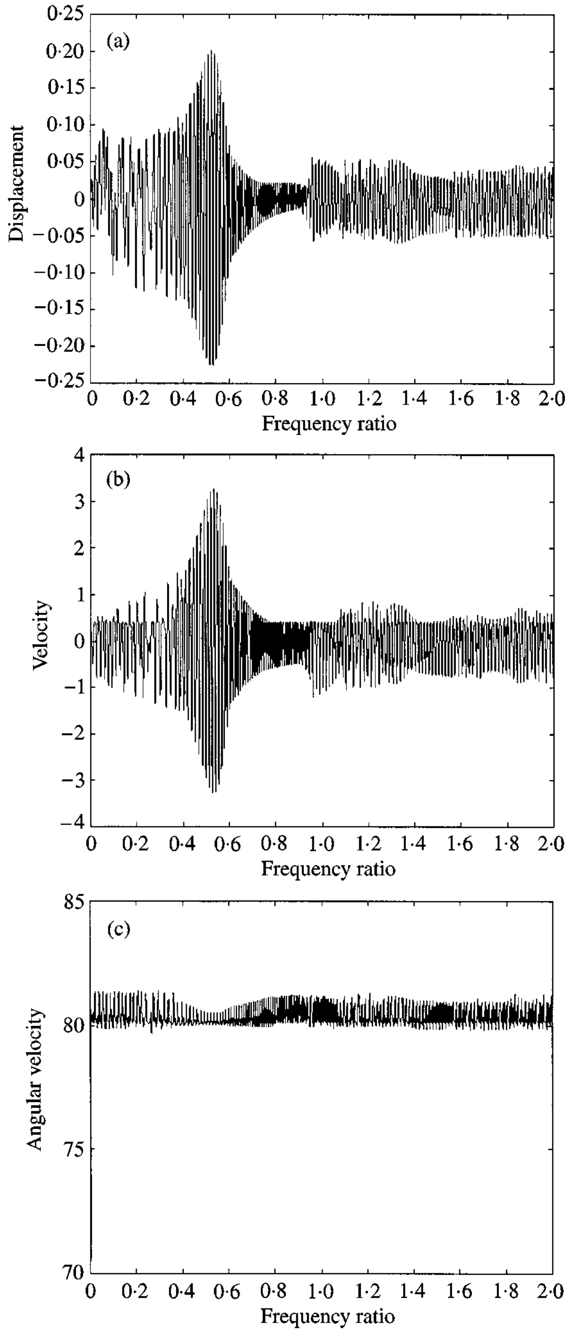

Figure 4. Frequency responses for the non-ideal case with external excitation for $K_T = 0{\cdot}4$, $v_B \approx 0{\cdot}4$ m/s and $f_0 = 10$ N/kg. (a) Displacement versus frequency ratio, (b) velocity versus frequency ratio and (c) angular velocity versus frequency ratio.

For values of the belt velocity below 0·8 m/s, the relative velocity v_{Rel} between mass block and belt change of sign and the transition response is affected by the sliding mode (slip) and the no-sliding mode (stick). When the belt velocity surpasses the value 0·8 m/s, the dominant mode is sliding (slip) and the system undergoes a limit cycle as shown in Figure 3(c).

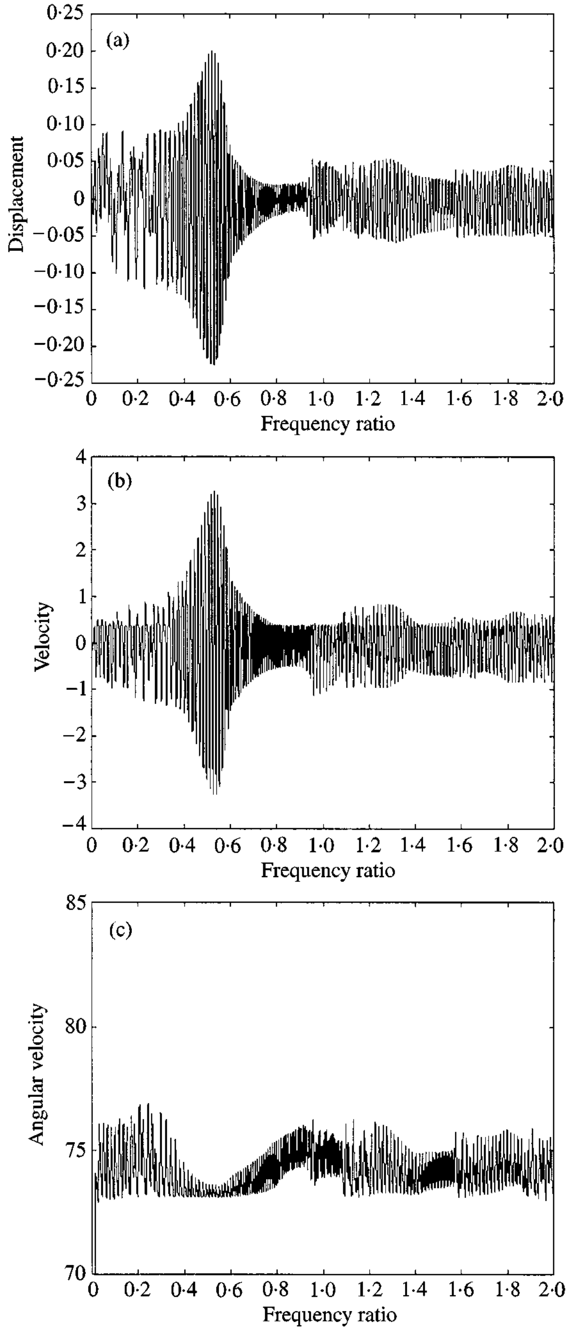

Figure 5. Frequency responses for the non-ideal case with external excitation for $K_T = 0{\cdot}1$, $v_B \approx 0{\cdot}4$ m/s and $f_0 = 10$ N/kg. (a) Displacement versus frequency ratio, (b) velocity versus frequency ratio and (c) angular velocity versus frequency ratio.

To analyze the behavior of the non-ideal dynamical system, when the oscillating mass block is excited by an harmonic force, two set of results were obtained, one for a torque constant $K_T = 0{\cdot}4$ and another for torque constant $K_T = 0{\cdot}1$. In both cases, the same applied voltage values are used. In each set frequency response results for the oscillating mass block displacement and velocity, the motor angular velocity, are shown. The results for the torque constant $K_T = 0{\cdot}4$ are presented in Figure 4 and the results for the torque constant $K_T = 0{\cdot}1$ in Figure 5. The results obtained for a constant torque $K_T = 0{\cdot}4$, representing an energy source with a high power to velocity range. The results showed that

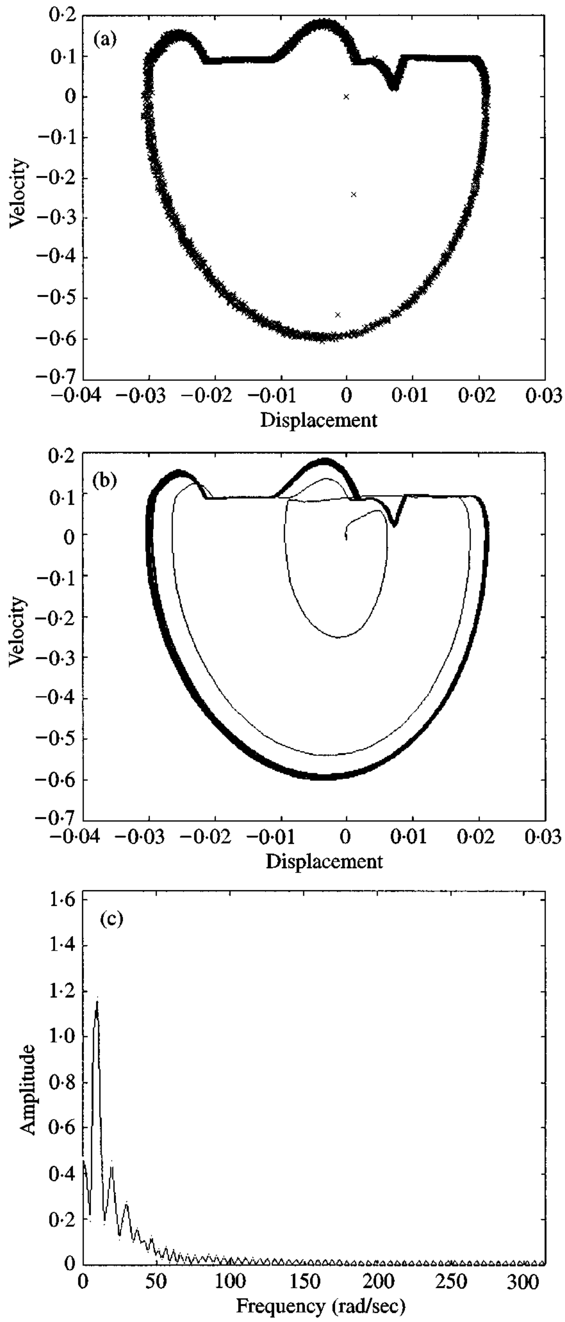

Figure 6. (a) Poincaré section x_2–x_1, (b) phase portrait x_2–x_1 and (c) associated FFT spectrum, (d) Poincaré section x_4–x_1 and (e) phase portrait of the angular velocity versus displacement (x_4–x_1) for $\omega_E/\omega_N = 1{\cdot}87$, $K_T = 0{\cdot}1$, $v_B \approx 0{\cdot}1$ m/s, $f_0 = 7$ N/kg and 4280 excitation periods.

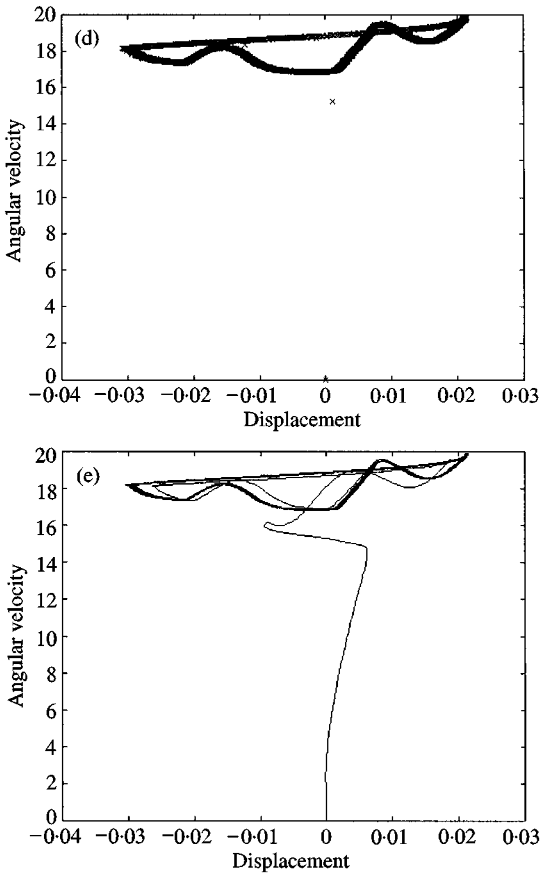

Figure 6. Continued.

for a constant torque $K_T = 0{\cdot}1$, the oscillating system suffers more influence from the energy source. The interaction between the motor and the oscillating system is evidenced by a non-constant angular velocity, as shown in Figures 4(c) and 5(c).

Poincaré sections and associated FFT spectra [18] were obtained to illustrate the complexity of the system behavior under certain conditions. In Figures 6(a–c) are shown the motion Poincaré sections of the system obtained for a case with 4280 excitation periods, the associated phase portrait and FFT spectrum for a frequency ratio $\omega_E/\omega_N = 1{\cdot}87$. Figures 6(d–e) show the Poincaré section for the displacement and motor angular velocity and the associated phase portrait.

Finally, we note the non-periodic behavior of the system for given parameters for a frequency ratio $\omega_E/\omega_N = 1{\cdot}87$, by the Poincaré sections and the FFT spectra shown in Figure 6.

5. SOME CONCLUDING REMARKS

This paper analyzes the behavior of a non-ideal self-excited vibrating system. The main results obtained are on the interaction between the system power supply and friction-driven vibrations.

The occurrence of no-sliding (stick) between the belt and the mass block is evidenced by horizontal straight lines in the phase portrait. When the belt velocity surpasses a certain value, the dominant mode is the sliding (slip) and the system undergoes a limit cycle.

The dynamical influence of the motor on the oscillating system is evidenced by the angular velocity response. From the investigations carried out it was possible to observe the power supply influence on the vibrating system along with non-periodic motions with chaotic characteristic.

The results obtained here are the first ones in the study of non-ideal friction-driven vibration problems. The research in progress involves the study of other engineering aspects of the problem which shall be presented later.

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