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LETTERS TO THE EDITOR

A SIMPLIFIED MODEL OF A DAMAGED, VIBRATING STRUCTURAL ELEMENT BASED ON THE STRAIN ENERGY METHOD

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1. INTRODUCTION

In an excellent paper [1], Cornwell *et al.* have developed a damage-detection algorithm for plate-like structures characterized by two-dimensional curvature. Their method constitutes a significant extension of the approach developed by Stubbs *et al.* [2, 3] which is limited to structures characterized by one-dimensional curvature (in other words, curvature that is uniquely a function of one independent spatial variable). The referenced techniques constitute applications of the "strain energy damage-detection method".

The present study constitutes a modest extension of the approach presented in reference [1] based on reference [4]. It deals with the vibrating clamped or simply supported circular plate, originally isotropic. It is assumed that due to certain physical circumstances (fatigue, wear, corrosion, etc.) a central concentric circular portion of the structural element is damaged while the annular plate subdomain remains in a virgin, isotropic state [4]. Furthermore, it is assumed that the central core possesses polarly anisotropic characteristics [5] (Figure 1). The fundamental frequency coefficient of the damaged structural element is then obtained as a function of the intervening physical and geometric parameters D_r/D; D_θ/D_r, μ, μ_θ and b/a by means of the optimized Rayleigh–Ritz method.

Admittedly, the present analysis constitutes a first order, rather crude approximation since it is assumed that cracks have not appeared yet in the vibrating element or that they are taken into account in the polar orthotropic characteristics of the inner core.

2. APPROXIMATE ANALYTICAL SOLUTION

In the case of normal modes of transverse vibration of the system shown in Figure 1 the problem is governed by the combined energy functional [5]

$$J(W) = \iint_{P_2}\left[D_r W''^2 + D_\theta\left(\frac{W'}{\bar{r}}\right)^2 + 2D_r\nu_\theta\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r}\,\mathrm{d}\theta$$

$$+ D\iint_{P_1}\left[\left(W'' + \frac{W'}{\bar{r}}\right)^2 - 2(1-\nu)\frac{W'W''}{\bar{r}}\right]\bar{r}\,\mathrm{d}\bar{r}\,\mathrm{d}\theta$$

$$- Da\int_0^{2\pi}\left[W''(a) + \nu\frac{W'(a)}{a}\right]W'(a)\,\mathrm{d}\theta - \rho h\omega^2\iint_P W^2\,\bar{r}\,\mathrm{d}\bar{r}\,\mathrm{d}\theta, \qquad (1)$$

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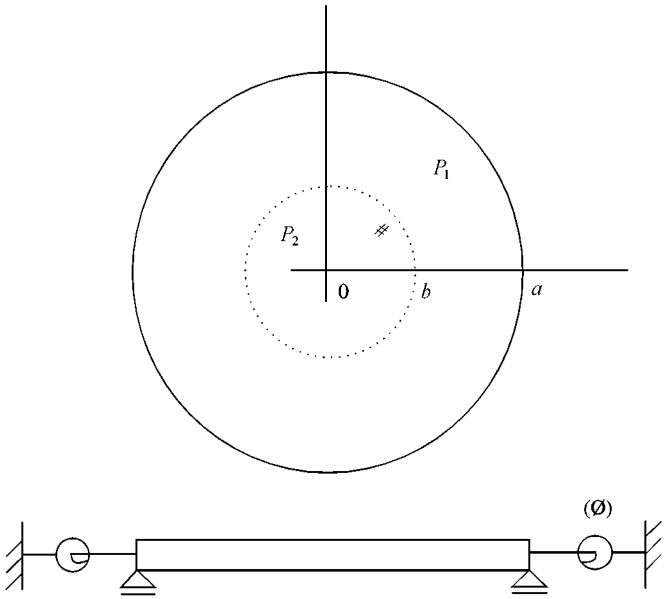

Figure 1. Vibrating damaged circular plate.

subject to the boundary conditions

$$W(a) = 0, \qquad W'(a) = -\phi D\left[W''(a) + \nu \frac{W'(a)}{a}\right]. \tag{2a, b}$$

The flexibility coefficient ϕ is defined by means of equation (2b) (if the plate is clamped: $\phi \to 0$ and when the plate is simply supported: $\phi \to \infty$).

Introducing the dimensionless variable $r = \bar{r}/a$ into equation (1) one obtains

$$\frac{a^2}{2\pi D} J(W) = \int_0^{r_b} \left[\frac{D_r}{D} W''^2 + \frac{D_\theta}{D}\frac{W'^2}{r^2} + 2\frac{D_r}{D}\nu_\theta \frac{W'W''}{r}\right] r\,\mathrm{d}r$$

$$+ \int_{r_b}^{1}\left[\left(W'' + \frac{W'}{r}\right)^2 - 2(1-\nu)\frac{W'W''}{r}\right] r\,\mathrm{d}r$$

$$- [W''(1) + \nu W'(1)]W'(1) - \Omega^2 \int_0^l W^2 r\,\mathrm{d}r, \tag{3}$$

where $\Omega^2 = (\rho h a^4/D)\omega^2$ and $r_b = b/a$.

The boundary conditions become

$$W(1) = 0, \qquad W'(1) = -\phi'[W''(1) + \nu W'(1)], \qquad \phi' = \phi D/a. \tag{4a, b}$$

The following approximating expression has been used:

$$W \cong W_a = \sum_{j=1}^{N} C_j \varphi_j(r) = \sum_{j=1}^{N} C_j(a_j r^{p+j-1} + b_j r^{j+1} + 1), \tag{5}$$

where the a_j's and b_j's are obtained substituting each co-ordinate function $\varphi_j(r)$ in conditions (4) and p is the Rayleigh's optimization parameter.

Substituting equation (5) into equation (3) and making use of Ritz's minimization procedure one obtains the following linear system of equations in the C_j's:

$$\frac{a^2}{4\pi D}\frac{\partial J}{\partial C_i} = \left\{ \sum_j \int_0^{r_b} \left[\frac{D_r}{D}\varphi_j''\varphi_i'' + \frac{D_\theta}{D}\frac{\varphi_j'\varphi_i'}{r^2} + \frac{D_r}{D}\nu_\theta \frac{\varphi_j''\varphi_i' + \varphi_j'\varphi_i''}{r} \right] r\,\mathrm{d}r \right.$$

$$+ \int_{r_b}^{1} \left[\left(\varphi_j'' + \frac{\varphi_j'}{r}\right)\left(\varphi_i'' + \frac{\varphi_i'}{r}\right) - \frac{1-\nu}{r}(\varphi_j''\varphi_i' + \varphi_j'\varphi_i'') \right] r\,\mathrm{d}r$$

$$- \frac{1}{2}[\varphi_j'(1)(\varphi_i''(1) + \nu\varphi_i'(1)) + (\varphi_j''(1) + \nu\varphi_j'(1))\varphi_i'(1)]$$

$$\left. - \Omega^2 \int_0^l \varphi_j\varphi_i r\,\mathrm{d}r \right\} C_j = 0. \tag{6}$$

The non-triviality condition yields a determinantal equation whose lowest root is the fundamental frequency coefficient $\Omega_1 = \sqrt{\rho h/D}\,\omega_1 a^2$.
Since

$$\Omega_1 = \Omega_1(p), \tag{7}$$

by minimizing it with respect to p one is able to optimize the value of the fundamental eigenvalue.

3. NUMERICAL RESULTS

The numerical determinations have been performed making $N = 5$, $D = D_r$ and $\nu = \nu_\theta = 0{\cdot}3$.

Table 1 depicts values of Ω_1 for simply supported and clamped circular plates for several values of D_θ/D_r and r_b.

TABLE 1

Fundamental frequency coefficients $\Omega_1 = \sqrt{(\rho h/D)}\, a^2\omega_1$ of vibrating, circular, damaged plates

D_θ/D_r	$r_b = 0$	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
SS 0·50	4·935	4·920	4·871	4·794	4·698	4·590	4·477	4·366	4·260	4·163	4·075
0·75	4·935	4·928	4·907	4·874	4·832	4·784	4·732	4·680	4·630	4·583	4·542
1	4·935	4·935	4·935	4·935	4·935	4·935	4·935	4·935	4·935	4·935	4·935
1·25	4·935	4·941	4·957	4·983	5·018	5·060	5·105	5·152	5·199	5·242	5·281
1·50	4·935	4·946	4·975	5·023	5·088	5·165	5·252	5·343	5·433	5·518	5·593
1·75	4·935	4·950	4·991	5·057	5·148	5·257	5·381	5·513	5·645	5·770	5·879
C 0·50	10·216	10·183	10·077	9·928	9·764	9·610	9·485	9·397	9·347	9·327	9·324
0·75	10·216	10·201	10·156	10·091	10·081	9·946	9·886	9·843	9·818	9·808	9·806
1	10·216	10·216	10·216	10·216	10·216	10·216	10·216	10·216	10·216	10·216	10·216
1·25	10·216	10·229	10·263	10·316	10·378	10·442	10·498	10·540	10·565	10·576	10·577
1·50	10·216	10·239	10·302	10·399	10·516	10·637	10·745	10·828	10·879	10·901	10·904
1·75	10·216	10·249	10·335	10·470	10·635	10·809	10·967	11·090	11·167	11·200	11·205

Note: The outer subdomain conserves the original properties of the virgin plate while the inner core possesses polarly orthotropic characteristics.

When $D_\theta/D_r > 1$ one may assume that a strain hardening effect has taken place in the tangencial direction. For $D_\theta/D_r = 1$ the eigenvalues are in excellent agreement with the exact values. For $D_\theta/D_r \neq 1$ and $r_b = 1$, the results are in good agreement with values previously published in the open literature.

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