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PARAMETRIC STABILITY OF NON-LINEARLY ELASTIC COMPOSITE PLATES BY LYAPUNOV EXPONENTS

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The dynamic stability of non-linearly elastic composite plates subjected to periodic in-plane loading is investigated. Infinitely wide plates made of resin matrix composite are considered. The non-linearly elastic behavior of the resin matrix is modelled by the generalized Ramberg–Osgood representation. The effect of the matrix non-linearity on the overall response of the composite is predicted by the micromechanical method of cells. The dynamic stability analysis is performed by evaluating the largest Lyapunov exponent, the sign of which indicates whether the system is stable or not. It is shown that this approach forms a convenient tool for predicting parametric stability of non-linear composite structures.

1. INTRODUCTION

Parametric instability of plates arises when a periodic in-plane loading induces transverse vibrations of increasing amplitudes. It turns out that this occurs for certain relationships between the loading frequency and the natural frequencies of the structure.

Numerous problems of dynamic instability in elastic structures were studied by Bolotin [1] where the dynamic instability regions were constructed through the investigation of the Mathieu equation by Fourier analysis. In those works, the influence of damping on the dynamic stability was studied. Non-linear effects occurring either due to geometric non-linearities (e.g., the non-linear von-Karman strain–displacement relations), or due to the non-linear behavior of the material (e.g., non-linear stress–strain relations) were also included. A review and a monograph including further results and extensive bibliography was written by Evan-Iwanowski [2, 3].

Analysis of parametric instability of anisotropic plates was first carried out by Bennet [4]. Composite cylindrical shells were considered by Bogdanovich [5] where the integro-differential equation, resulting from the equations of motion by the application of series expansion and Laplace transform, was investigated. Geometric non-linearity and viscoelastic effects were taken into account. The parametric excitation of linear viscoelastic homogeneous as well as laminated plates was investigated by Aboudi *et al.* [6] and Cederbaum *et al.* [7] using the Lyapunov exponents. This approach was further employed by Touati and Cederbaum [8] to study the dynamic instability of non-linear viscoelastic homogeneous plates.

Lyapunov exponents are numbers which reflect global properties of the attractors of a dynamical system. Their sign indicates whether the correlation between two initially close

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trajectories will be kept or lost, namely whether the motion is stable or not [9, 10]. This method is specially useful for defining the loss of stability of non-linear structures. In linear structures, the resonance parametric vibrations are of unrestrictedly increasing amplitude. Hence, the evolution of an unbounded response can be the criterion for loss of stability. In contrast, the response of non-linear structures can be unstable but yet bounded. For such a case, a different criterion for instability is needed.

In the present paper, the method of Lyapunov exponents is employed to predict whether a non-linearly elastic composite plate which is subjected to periodic in-plane loading is stable or not. The considered non-linearity is due to the non-linear behavior of the resin matrix which is modelled by the Ramberg–Osgood representation. Relying on the behavior of the matrix and the fibers, the effective constitutive relations for the anisotropic non-linearly elastic composite is established by the micromechanical method of cells [12]. Due to the existence of non-linear effects, an incremental procedure is necessary for the analysis of the response. This is performed in conjunction with an incremental procedure according to which the Lyapunov exponent is determined at every time step.

Results that show the effect of the material non-linearity are given for infinitely wide plates made of initially isotropic and homogeneous non-linearly elastic material as well as for non-linearly elastic composite plates.

2. PROBLEM FORMULATION

Consider a rectangular non-linearly elastic anisotropic plate of an infinite width in the y direction. The plate is uniformly supported along the edges $x = 0, L$ which are subjected to a uniform normal in-plane periodic load. Neglecting the in-plane inertia, the response of the plate is governed by the following equations [11]:

$$N_{xx,x} = 0, \qquad N_{xy,x} = 0, \qquad M_{xx,xx} + N_{xx} w_{,xx} = I\ddot{w}. \tag{1}$$

Here w is displacement in the transverse direction z, $I = \int_{-h/2}^{h/2} \rho \, dz$ where ρ is the material effective density and h is the plate thickness, and dots denote differentiation with respect to time t. The stress resultants and moment N_{xx}, N_{xy}, M_{xx} are given by

$$N_{xx} = \int_{-h/2}^{h/2} \sigma_{xx} \, dz, \qquad N_{xy} = \int_{-h/2}^{h/2} \sigma_{xy} \, dz, \qquad M_{xx} = \int_{-h/2}^{h/2} z\sigma_{xx} \, dz. \tag{2}$$

For a simply supported plate the boundary conditions at $x = 0, L$ are given by

$$N_{xx} = N(t), \qquad N_{xy} = 0, \qquad w = 0, \qquad M_{xx} = 0. \tag{3}$$

The strain–displacement relations for the present cylindrical bending situation are

$$\varepsilon_{xx} = \varepsilon_{xx}^0 - z w_{,xx}, \qquad \varepsilon_{yy} = 0, \qquad \varepsilon_{xy} = \varepsilon_{xy}^0, \tag{4}$$

where ε_{xx}^0, ε_{xy}^0 are the strains of the midplane of the plate.

Let the plate layers consist of unidirectional linearly elastic anisotropic fibers reinforcing non-linearly elastic resin matrix. The linearly elastic fiber material behavior is ruled by

$$\sigma_{ij}^{(f)} = C_{ijkl}^{(f)} \varepsilon_{kl}^{(f)}, \tag{5}$$

where $C_{ijkl}^{(f)}$ is the elastic stiffness tensor of the fiber material.

The non-linearly elastic behavior of the elastic matrix is modelled by the Ramberg–Osgood representation, according to which the uniaxial strain–stress relations in the x_1 direction are of the form

$$\varepsilon_{11}^{(m)} = \frac{\sigma_{11}^{(m)}}{E^{(m)}} + \frac{\sigma_0}{E^{(m)}}\left(\frac{\sigma_{11}^{(m)}}{\sigma_0}\right)^n, \tag{6}$$

where $E^{(m)}$ is Young's modulus of the matrix, and σ_0 and n are parameters characterizing the matrix material non-linearity. A generalization of equation (6) leads to following multiaxial constitutive relations:

$$\varepsilon_{ij}^{(m)} = \frac{1+\nu^{(m)}}{E^{(m)}}\sigma_{ij}^{(m)} - \frac{\nu^{(m)}}{E^{(m)}}\sigma_{kk}^{(m)}\delta_{ij} + \frac{3}{2E^{(m)}}\hat{\sigma}_{ij}^{(m)}\left(\frac{\check{\sigma}^{(m)}}{\sigma_0}\right)^{n-1}, \tag{7}$$

where $\nu^{(m)}$ is the Poissons ratio, $\hat{\sigma}_{ij}^{(m)} = \sigma_{ij}^{(m)} - \frac{1}{3}\sigma_{kk}^{(m)}\delta_{ij}$ is the deviatoric stress (δ_{ij} is the Kronecker delta) and $\check{\sigma}^{(m)} = \sqrt{\frac{3}{2}\hat{\sigma}_{ij}^{(m)}\hat{\sigma}_{ij}^{(m)}}$.

The overall response of the two-phase composite is obtained by the micromechanical method of cells [12]. This micromechanical analysis predicts the non-linear anisotropic effective constitutive relations for the composite, relying on the material behavior of its constituents. By adopting an incremental formulation in conjunction with the micromechanical method of cells analysis, the instantaneous effective stiffness tensor $\bar{\mathbf{C}}^{\mathbf{I}}$ can be established such that the instantaneous response of the non-linear composite is given by

$$\Delta\bar{\boldsymbol{\sigma}} = \bar{\mathbf{C}}^{\mathbf{I}}\Delta\bar{\boldsymbol{\varepsilon}}, \tag{8}$$

where $\Delta\bar{\boldsymbol{\varepsilon}}$ and $\Delta\bar{\boldsymbol{\sigma}}$ are the increments of strains and stresses, respectively, and all tensors are referred to the material axes, one of which coincides with the fibers directions. By the application of the standard transformation of co-ordinates, the instantaneous effective constitutive law (8) becomes

$$\Delta\boldsymbol{\sigma} = \mathbf{C}^{\mathbf{I}}\Delta\boldsymbol{\varepsilon}, \tag{9}$$

where all tensors are related to the plate co-ordinates (x, y, z).

For a state of plane stress the increment of the stress at a point in the plate is related to the increment of strain through the following relations:

$$\begin{Bmatrix}\Delta\sigma_{xx}\\ \Delta\sigma_{yy}\\ \Delta\sigma_{xy}\end{Bmatrix} = \begin{bmatrix} Q_{11}^I & Q_{12}^I & Q_{16}^I\\ Q_{12}^I & Q_{22}^I & Q_{26}^I\\ Q_{16}^I & Q_{26}^I & Q_{66}^I\end{bmatrix}\begin{Bmatrix}\Delta\varepsilon_{xx}\\ \Delta\varepsilon_{yy}\\ 2\Delta\varepsilon_{xy}\end{Bmatrix}, \tag{10}$$

where $\mathbf{Q}^{\mathbf{I}}$ is the instantaneous reduced stiffness matrix which depends on the current state of elastic field.

As a result of the above formulation of the instantaneous behavior of the composite, the incremental constitutive relations of the non-linearly elastic plate under cylindrical bending can be expressed as follows:

$$\begin{aligned}\Delta N_{xx} &= A_{11}^I\Delta\varepsilon_{xx}^0 + A_{16}^I 2\Delta\varepsilon_{xy}^0 - B_{11}^I\Delta w_{,xx},\\ \Delta N_{xy} &= A_{61}^I\Delta\varepsilon_{xx}^0 + A_{66}^I 2\Delta\varepsilon_{xy}^0 - B_{61}^I\Delta w_{,xx},\\ \Delta M_{xx} &= B_{11}^I\Delta\varepsilon_{xx}^0 - B_{16}^I 2\Delta\varepsilon_{xy}^0 - D_{11}^I\Delta w_{,xx},\end{aligned} \tag{11}$$

where the instantaneous extension, coupling and bending plate stiffnesses are given by

$$(A^I_{ij}, B^I_{ij}, D^I_{ij}) = \int_{-h/2}^{h/2} Q^I_{ij}(1, z, z^2)\,\mathrm{d}z.$$

The first two equations (1) imply that the inplane stress resultants are independent of x such that taking into account the boundary conditions (3) their increments are given by

$$\varDelta N_{xx} = \varDelta N, \qquad \varDelta N_{xy} = 0. \tag{12}$$

Through a comparison between expressions (12) and equations (11), the increment of the midplane strains can be written in the following form:

$$\varDelta\varepsilon^0_{xx} = \frac{A^I_{66}\varDelta N + (B^I_{11}A^I_{66} - B^I_{16}A^I_{16})\varDelta w_{,xx}}{A^I_{11}A^I_{66} - A^{I^2}_{16}},$$

$$2\varDelta\varepsilon^0_{xy} = -\frac{A^I_{16}\varDelta N + (B^I_{11}A^I_{16} - B^I_{16}A^I_{11})\varDelta w_{,xx}}{A^I_{11}A^I_{66} - A^{I^2}_{16}}. \tag{13}$$

By substituting the latter relation (13) into the third equation (11), the expression for $\varDelta M_{xx}$ becomes

$$\varDelta M_{xx} = K_1\varDelta N + K_2\varDelta w_{,xx}, \tag{14}$$

where

$$K_1 = \frac{B^I_{11}A^I_{66} - B^I_{16}A^I_{16}}{A^I_{11}A^I_{66} - A^{I^2}_{16}},$$

$$K_2 = \frac{B^I_{11}(B^I_{11}A^I_{66} - B^I_{16}A^I_{16}) - B^I_{16}(B^I_{11}A^I_{16} - B^I_{16}A^I_{11})}{A^I_{11}A^I_{66} - A^{I^2}_{16}} - D^I_{11}. \tag{15}$$

In order to investigate the parametric stability of the plate, we consider a time-dependent in-plane load which is the result of the following strain imposed to the edges $x = 0, L$:

$$\varepsilon^0_{xx} = \varepsilon_s + \varepsilon_d\cos(\theta t). \tag{16}$$

In equation (16), ε_s and ε_d are constants and θ is the load frequency. The increment of the corresponding inplane load can be determined by the first equation (13).

Having established the incremental constitutive equations (10) for the non-linearly elastic plate, the third equation (1) can be transformed into an incremental form. Consequently, the variation of the displacement $\varDelta w$ within a time increment $\varDelta t$, $t^{(l-1)} \leqslant t \leqslant t^{(l)}$ is governed by

$$R + \varDelta M_{xx,xx} + N^{(l-1)}\varDelta w_{,xx} + \varDelta N(w^{(l-1)}_{,xx} + \varDelta w_{,xx}) = I\varDelta\ddot{w}, \tag{17}$$

where

$$R = M^{(l-1)}_{xx,xx} + N^{(l-1)}w^{(l-1)}_{,xx}$$

with

$$w^{(l)} = w(x, t^{(l)}), \quad N^{(l)} = N(t^{(l)}), \ldots\ .$$

Using the separation of variables, the displacement w is assumed to have the following form which satisfies the simply supported boundary conditions

$$w(x, t) = \sum_m W_m(t) \sin(m\pi x/L), \tag{18}$$

namely

$$\Delta w = \sum_m \Delta W_m(t) \sin(m\pi x/L). \tag{19}$$

It should be noted that the boundary condition on M_{xx} is satisfied as long as the initial configuration of the plate is symmetric with respect to $z = 0$. In such cases, due to the fact that the loading is constant throughout the plate thickness, B^I_{ij} at the edges remain zero.

By employing the Galerkin method in conjunction with equations (14) and (19), equation (17) is reduced to the following set of ordinary non-linear differential equations:

$$R_k - \Delta N\alpha_k(W^{(l-1)} + H_k) - (N^{(l-1)} + \Delta N)\alpha_k \Delta W_k + \alpha_k \sum_m \frac{m^2\pi^2}{L^2} J_{km}\Delta W_m = \Delta \ddot{W}_k, \tag{20}$$

where

$$R_k = \frac{2}{LI}\int_0^L R \sin\frac{k\pi x}{L}\mathrm{d}x, \qquad H_k = \frac{2}{L}\int_0^L K_1 \sin\frac{k\pi x}{L}\mathrm{d}x,$$

$$J_{km} = \frac{2}{L}\int_0^L K_2 \sin\frac{k\pi x}{L}\sin\frac{m\pi x}{L}\,\mathrm{d}x, \qquad \alpha_k = \frac{k^2\pi^2}{L^2 I}, \quad m, k = 1, \ldots, M.$$

3. PARAMETRIC STABILITY ANALYSIS

In order to investigate the stability of non-linearly elastic plates under periodic in-plane loads, the concept of Lyapunov exponent is employed. Lyapunov stability analysis of a dynamical system consists of the evaluation of a corresponding set of characteristic numbers (e.g., see Reference [9]). The negative values of these characteristic numbers are known as Lyapunov exponents. According to Lyapunov, the motion is asymptotically stable if all the exponents are negative. A positive Lyapunov exponent indicates an exponential separation between two initially close trajectories, namely instability of the system [10]. The system is stable if the largest Lyapunov exponent is not greater than zero. Consequently, it is sufficient to evaluate the largest Lyapunov exponent in order to characterize the behavior of a dynamical system.

According to Goldhirsch *et al.* [13], the Lyapunov exponents can be determined through the following procedure. Consider the system of ordinary non-linear differential equations

$$\dot{\mathbf{v}} = \mathbf{F}(\mathbf{v}). \tag{21}$$

The stability equation is defined to be

$$\dot{\mathbf{y}} = \mathbf{G}\mathbf{y}, \tag{22}$$

where

$$G_{ij} = \left.\frac{\partial F_i}{\partial v_j}\right|_{\mathbf{v} = \mathbf{v}(t)}$$

and $\mathbf{y}$ can be regarded as a small perturbation $\delta\mathbf{v}$. Within the time increment $0 < t < t^{(1)}$, system (22) with $\mathbf{G}(t = 0)$ is solved numerically for the normalized initial conditions

$$\|\mathbf{y}(0)\| = 1,$$

where $\|.\|$ is the Euclidean norm. This yields $\mathbf{y}(t^{(1)})$.

Equations (22) with $\mathbf{G}(t = t^{(1)})$ and with the following initial conditions,

$$\mathbf{y}(t^{(1)}) = \mathbf{z}(t^{(1)}),$$

where

$$\mathbf{z}(t^{(1)}) = \frac{\mathbf{y}(t^{(1)})}{\|\mathbf{y}(t^{(1)})\|}$$

are then solved within the second time interval $t^{(1)} < t < t^{(2)}$ yielding $\mathbf{v}(t^{(2)})$. The process is repeated for n time intervals while correspondingly, system (21) is solved to provide the values of $\mathbf{v}(t^{(l)})$ needed for the evaluation of $\mathbf{G}$. Namely, the incremental procedure is simultaneously used to get the non-linear response and the Lyapunov exponents.

For the n'th time interval, define the value of the parameter μ_n as follows:

$$\mu_n = \frac{\sum_{l=1}^{n} \ln\|\mathbf{y}(t^{(l)})\|}{t^{(n)}}. \tag{23}$$

It has been shown [13] that for finite large time, the value of μ_n approaches to the value of the Lyapunov exponent.

Let us employ the above procedure to investigate the dynamic stability of a non-linearly elastic infinitely wide plate. For the present problem, equation (21) is obtained by reducing equation (20) to a set of first order differential equations. The matrix $\mathbf{G}$ in the stability equations (22) can thus be explicitly written as follows:

$$\mathbf{G} = \begin{bmatrix} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} & \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \\ \begin{bmatrix} \alpha_1^2 I^2 J_{11} - N\alpha_1 & \alpha_1^2 I^2 J_{12} & \cdots & \alpha_1^2 I^2 J_{1M} \\ \alpha_2^2 I^2 J_{21} & \alpha_2^2 I^2 J_{22} - N\alpha_2 & \cdots & \alpha_2^2 I^2 J_{2M} \\ \vdots & \vdots & & \vdots \\ \alpha_M^2 I^2 J_{M1} & \alpha_M^2 I^2 J_{M2} & \cdots & \alpha_M^2 I^2 J_{MM} - N\alpha_M \end{bmatrix} & \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \end{bmatrix}.$$

Note that the matrix $\mathbf{G}$ is a function of the instantaneous stiffnesses A_{11}^I, B_{11}^I, D_{11}^I, which depend on the current state of stress. The latter is the solution of equation (20) which (when reduced to a set of first order equations) is the relevant special case of equation (21). Hence equation (20), has to be solved simultaneously with the progressing of the stability analysis. Within a time interval $t^{(l-1)} < t < t^{(l)}$, the numerical integration (high order Range–Kutta) of equations (20), with the initial conditions

$$\Delta W_k = 0, \qquad \Delta \dot{W}_k = \dot{W}_k \quad \text{at } t = t^{(l-1)} \tag{25}$$

yields the displacement field at $t^{(l)}$. On the basis of this, the stress field and the elements of the instantaneous stiffness tensor at every point of the plate can be re-evaluated. The coefficients of the stability equations G_{ij} are updated accordingly, and the stability analysis is carried on.

4. APPLICATION

The previously described approach is applied to investigate the parametric stability of an infinitely wide plate made of non-linearly elastic material. Two kinds of plate are considered: (a) Homogeneous isotropic plate made of epoxy having the following material properties: $E = 6$ GPa, $\nu = 0{\cdot}35$, $\sigma_0 = 160$ MPa, $n = 3$, $\rho = 1400\ \mathrm{kg/m^3}$. Note that for the presently used n, the material behavior has the same nature in both the positive and negative stress ranges. Namely, the stress–strain curve decreases in slope as deformation (negative or positive) increases. (b) Cross-ply plates the laminae of which are made of the previously described epoxy reinforced by fibers having the following properties: $E^{(f)} = 73$ GPa, $\nu^{(f)} = 0{\cdot}22$, $\rho^{(f)} = 2540\ \mathrm{kg/m^3}$. A fiber volume fraction $v_f = 0{\cdot}3$ is considered.

The thickness ratio of the plate is $L/h = 40$. One mode approximation with $M = 1$ is conducted. The inplane load is the result of the edges strain constraints equation (16) with

$$\varepsilon_s = \beta\varepsilon^*, \qquad \varepsilon_d = 2\eta(1+\beta)\varepsilon^*, \qquad \theta = 2\Omega\omega,$$

where $\varepsilon^* = \pi^2 D_{11}^{(0)}/A_{11}^{(0)}$ is the axial strain corresponding to the buckling load of the corresponding linearly elastic plate, $\omega = (\pi^2/L^2)\sqrt{(D_{11}^{(0)}/\rho h)}\sqrt{1+\beta}$ and $\beta = -0{\cdot}98$.

4.1. HOMOGENEOUS ISOTROPIC PLATE

In general, the non-linear material behavior induces anisotropy even in an initially isotropic material. Moreover, due to the dependence of the constitutive law on the deformation state, the structure has to be considered as a non-homogeneous one. Hence, the previously presented general formulation is to be used for initially homogeneous isotropic plates.

The variations of the largest Lyapunov exponent under periodic loads of various parameters are shown in Figures 1–3. In order to demonstrate the effect of non-linearity of the elastic material, the Lyapunov exponents of the corresponding linearly elastic plate are also shown.

As is shown in Figure 1, for $\Omega = 0{\cdot}55$, $\eta = 0{\cdot}5$, the Lyapunov exponent approaches zero for both the linear and non-linear plates, hence both reveal a stable behavior.

Figure 2 displays the variation of μ versus $1/t$ for $\Omega = 0{\cdot}5$, $\eta = 0{\cdot}1$. Under these loading conditions the linearly elastic plate is unstable, since its Lyapunov exponent approaches a positive value. On the other hand, the Lyapunov exponent of the non-linearly elastic plate tends to zero indicating the stability of that system.

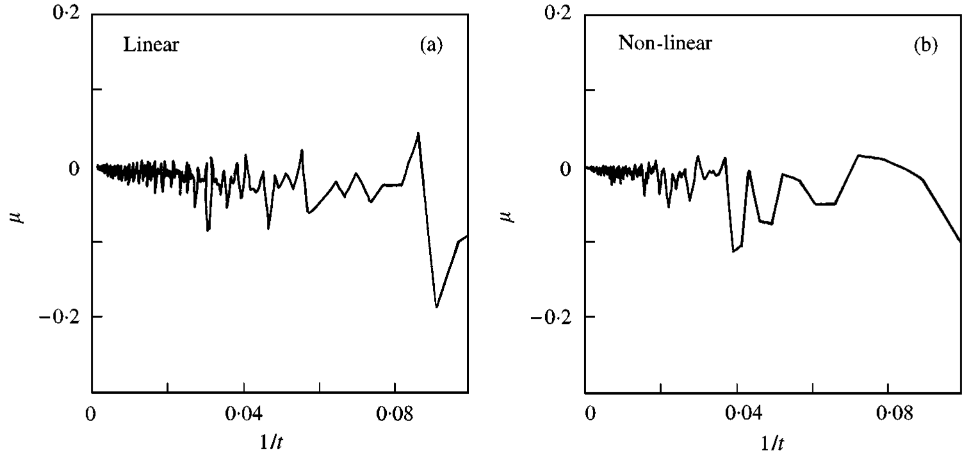

Figure 1. Lyapunov exponent for a homogeneous plate $\Omega = 0{\cdot}55$, $\eta = 0{\cdot}5$.

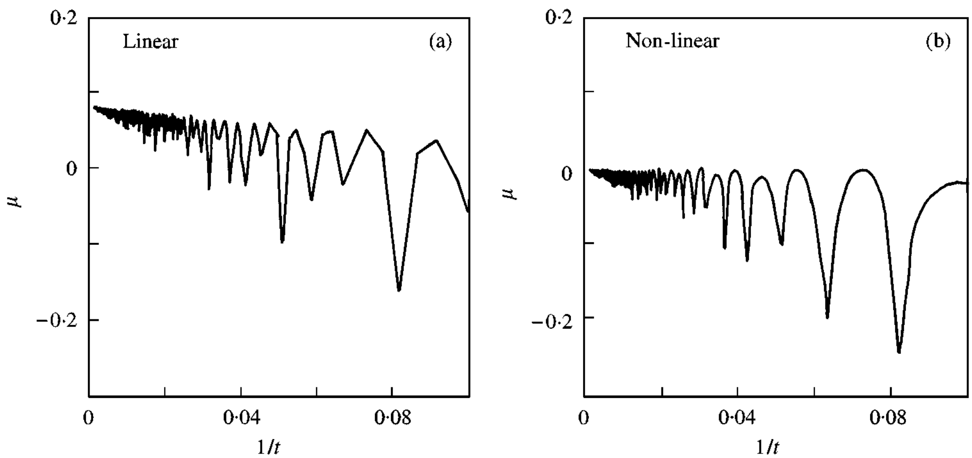

Figure 2. Lyapunov exponent for a homogeneous plate $\Omega = 0{\cdot}5$, $\eta = 0{\cdot}1$.

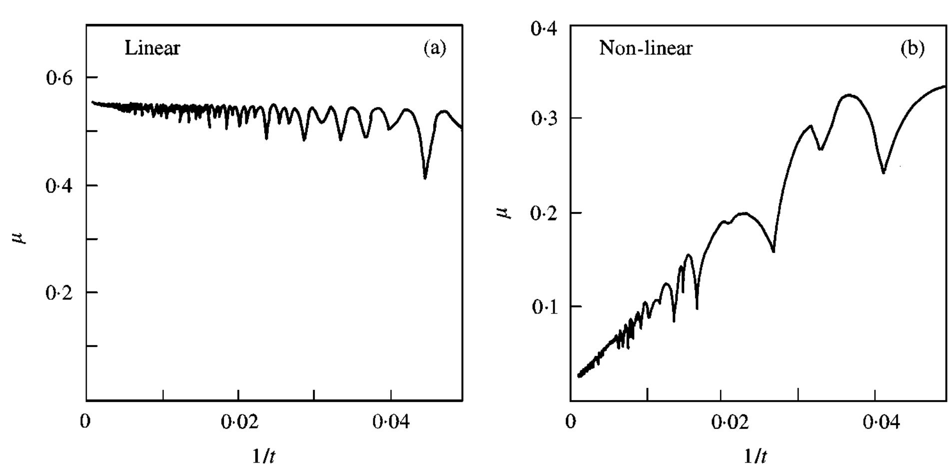

Figure 3. Lyapunov exponent for a homogeneous plate $\Omega = 1{\cdot}0$, $\eta = 0{\cdot}05$.

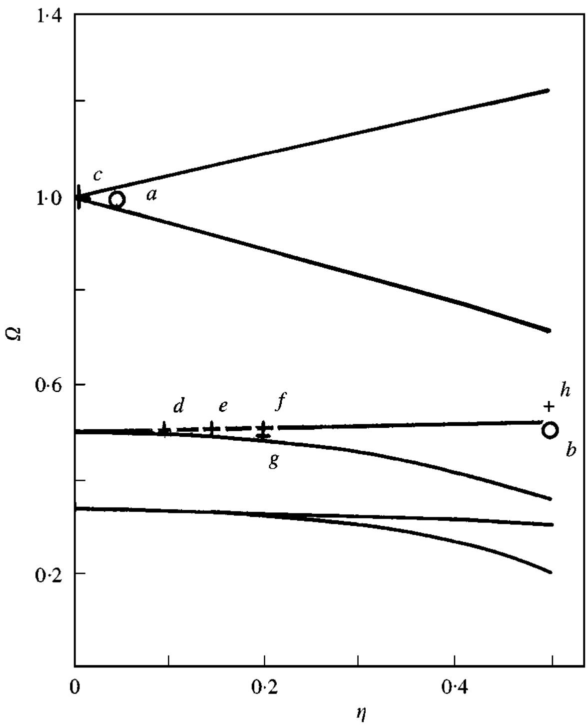

Figure 4. Stability regions for the linearly elastic problem as predicted by Bolotin [1]. Circles and crosses indicate correspondingly stable and unstable non-linearly elastic cases.

For $\Omega = 1$, $\eta = 0{\cdot}05$ both the linear and non-linear plates exhibit an instable behavior. This is deduced from the results presented in Figure 3, according to which the two corresponding Lyapunov exponents approach positive values. However, the value of the Lyapunov exponents of the linearly elastic plate is the highest.

In Figure 4 several non-linearly elastic cases (including the previously discussed ones) are indicated on the map of linearly elastic stability regions as drawn by Bolotin [1]. A circle is used for indicating a stable non-linearly elastic behavior, while a cross indicates instability. It can be observed that the non-linearity seems to reduce the region of instability in a way resembling the effect of viscoelasticity [1, 6]. This observation is in accordance with the conclusion of Touati and Cedrebaum [8], that material non-linearity tends to increase the stability of non-linearly elastic plates. However, no negative Lyapunov exponent was obtained for any of the examined cases. Namely, in contrast to the behavior of viscoelastic plates, none of the presently examined non-linearly elastic plates exhibit damping effect which yields asymptotically stable (negative Lyapunov exponents) behavior.

4.2. CROSS-PLY PLATE

The effect of the non-linear matrix behavior on the stability of unidirectional plates subjected to a parametric loading of $\Omega = 0{\cdot}5$, $\eta = 0{\cdot}1$, is demonstrated in Figures 5 and 6 for plates with lamination angle $\theta = 0^\circ$ and 90° respectively. While the material non-linearity stabilizes the behavior of the [0°] plate, its effect on the behavior of the [90°] plate is less

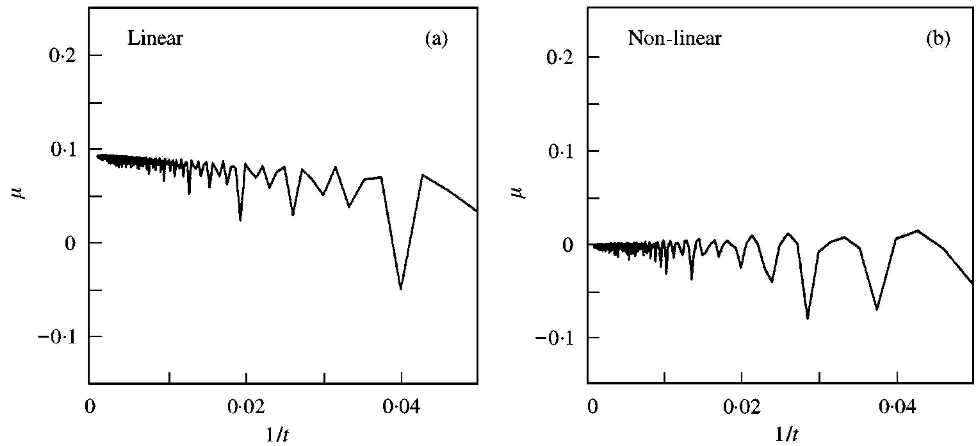

Figure 5. Lyapunov exponent for a unidirectional [0°] plate $\Omega = 0{\cdot}5$, $\eta = 0{\cdot}1$.

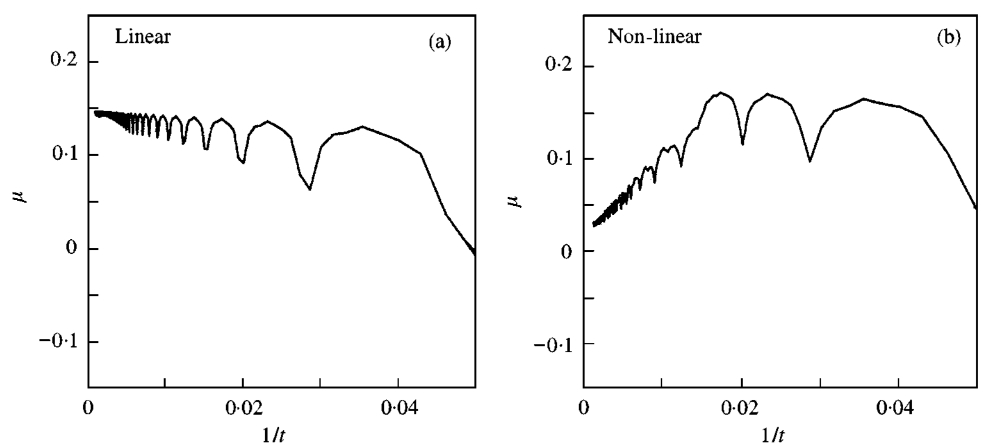

Figure 6. Lyapunov exponent for a unidirectional [90°] plate $\Omega = 0{\cdot}5$, $\eta = 0{\cdot}1$.

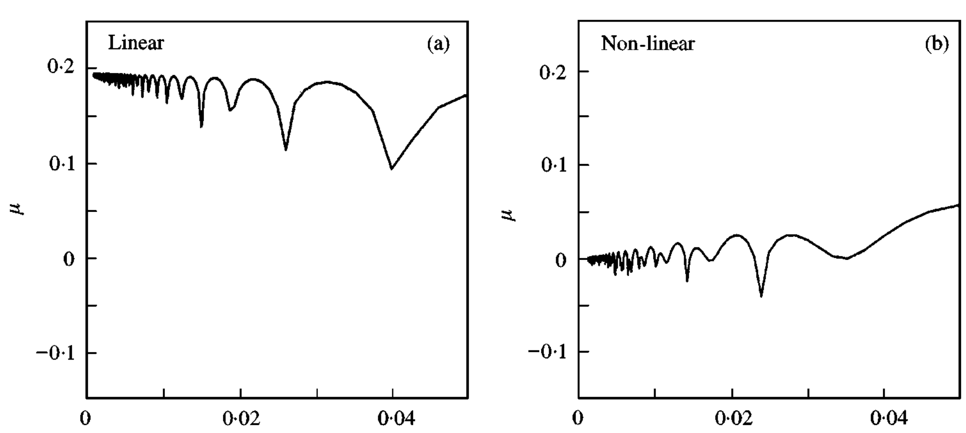

Figure 7. Lyapunov exponent for a unidirectional [0°/90°/0°] plate $\Omega = 0{\cdot}1$, $\eta = 0{\cdot}01$.

pronounced. The largest Lyapunov exponent of the non-linearly elastic [90°] plate, like that of the corresponding linear plate, approaches a positive value which indicates an unstable response.

The variation of μ versus $1/t$ for a [0°/90°/0°] plate under loading conditions having $\Omega = 0{\cdot}1$, $\eta = 0{\cdot}01$ is shown in Figure 7. For this configuration a stable behavior is predicted due to the inclusion of non-linear effects.

5. CONCLUSIONS

The parametric stability analysis of an infinitely wide non-linearly elastic composite plates has been performed by the method of Lyapunov exponents. This method is specially useful for defining the loss of stability of non-linear structures exhibiting an unstable bounded response.

The non-linearly elastic behavior of the matrix material has been modelled by the generalized Ramberg–Osgood representation. The effective behavior of the composite has been expressed by means of the instantaneous stiffness tensor, established by the micromechanical method of cells.

The non-linear effects due to the material behavior have been found to increase the stability of the non-linearly elastic composite plate.

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