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REFLECTION OF PLANE SOUND WAVE FROM A MICROPOLAR GENERALIZED THERMOELASTIC SOLID HALF-SPACE

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The present study is concerned with the reflection and refraction of plane sound wave at an interface between a liquid half-space and a micropolar generalized thermoelastic solid half-space. The numerical results are calculated in terms of amplitude ratios for water/aluminium-epoxy composite model for L-S (Lord and Shulman) and G-L (Green and Lindsay) theories. The comparison between these theories reveals the effect of second thermal relaxation time taken by Green and Lindsay. The results are also compared with those without thermal effect.

1. INTRODUCTION

Jeffereys [1] and Gutenberg [2] considered the reflection of elastic plane waves at a solid half-space. Chadwick and Sneddon [3] and Lockett [4] studied the propagation of thermoelastic plane waves. Knot [5] derived the general equations for reflection and refraction at plane boundary.

In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. To overcome this contradiction, the coupled theory of thermoelasticity has been extended by including the thermal relaxation time in constitutive relations by Lord and Shulman [6] and Green and Lindsay [7]. Some problems on reflection in thermoelastic solid have been discussed by Deresiewicz [8], Sinha and Sinha [9] and Sharma [10].

A theory of micropolar continua was proposed by Eringen and Suhubi [11] and Eringen [12] to explain the continuum behaviour of materials possessing microstructure. The propagation of plane waves in an infinite micropolar elastic solid has been discussed by Parfitt and Eringen [13], Ariman [14] and Smith [15]. Parfitt and Eringen [13] have shown that four basic waves (a longitudinal displacement wave, two sets of coupled waves and a longitudinal microrotational wave) propagate in an infinite micropolar elastic solid.

The linear theory of micropolar thermoelasticity was developed by extending the theory of micropolar continua to include thermal effect by Eringen [16] and Nowacki [17] and is known as micropolar coupled thermoelasticity. Dost and Tabarrok [18] have presented the generalized micropolar thermoelasticity by using Green–Lindsay theory. Kumar and Singh [19] have also presented the generalized micropolar thermoelasticity with stretch by using Lord–Shulman and Green–Lindsay theories. Wave propagation in a micropolar

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generalized thermoelastic body with stretch has been studied by Kumar and Singh [19]. Singh and Kumar [20, 21] have discussed some problems on reflection of plane waves from flat boundary of a micropolar generalized thermoelastic half-space. Singh and Kumar [22] have also proposed a generalized thermo-microstretch elastic solid and have discussed the reflection of plane waves from the free surface of a generalized thermo-microstretch elastic solid.

In the present paper, a problem of reflection and refraction of plane sound wave has been studied at an interface between a thermally conducting liquid half-space and a micropolar generalized thermoelastic solid half-space.

2. FORMULATION OF THE PROBLEM

We consider a homogeneous micropolar generalized thermoelastic solid and thermally conducting liquid which occupy lower and upper half-spaces respectively. We assume that heat sources, external force loading and body forces are absent and consider a fixed rectangular Cartesian co-ordinate system (**x**, **y**, **z**). We consider that the two semi-infinite media are in contact at a plane interface ($\mathbf{z} = 0$) and suppose that the plane sound wave impinges on the interface from above which we take as first medium, the negative **z**-axis lying inside the solid half-space (second medium). We take the plane wave motion in the **xz**-plane (i.e., $\partial/\partial\mathbf{y} = 0$). Following Eringen [16], Lord and Shulman [6] and Green and Lindsay [7], the constitutive and field equations of micropolar generalized thermoelastic solid without body forces and body couples can be written as (Figure 1)

$$\sigma_{kl} = \lambda u_{r,r}\delta_{kl} + \mu(u_{k,l} + u_{l,k}) + \kappa(u_{l,k} - \varepsilon_{klr}\phi_r) - \nu(\theta + t_1\dot{\theta})\delta_{kl}, \tag{1}$$

$$m_{kl} = \alpha\phi_{r,r}\delta_{kl} + \beta\phi_{k,l} + \gamma\phi_{l,k}, \tag{2}$$

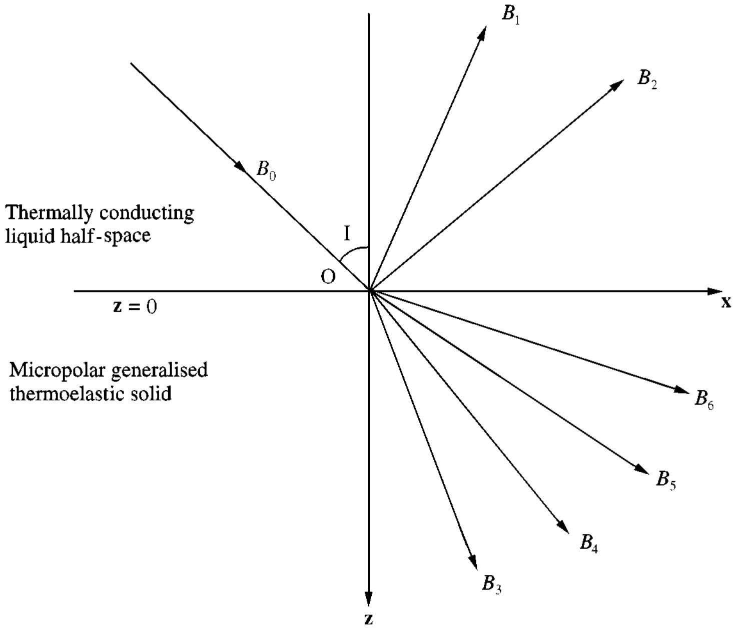

Figure 1. Geometry of the problem.

$$(c_1^2 + c_3^2)\,\nabla(\nabla\cdot \mathbf{u}) - (c_2^2 + c_3^2)\,\nabla\mathbf{x}(\nabla\mathbf{x}\mathbf{u}) + c_3^2\,\nabla\mathbf{x}\boldsymbol{\phi} - \bar{v}\nabla(\theta + t_1\dot{\theta}) = \ddot{\mathbf{u}}, \tag{3}$$

$$(c_4^2 + c_5^2)\,\nabla(\nabla\cdot \boldsymbol{\phi}) - c_4^2\,\nabla\mathbf{x}(\nabla\mathbf{x}\boldsymbol{\phi}) + \omega_0^2\,\nabla\mathbf{x}\mathbf{u} - 2\omega_0^2\boldsymbol{\phi} = \ddot{\boldsymbol{\phi}}, \tag{4}$$

$$\rho C^*(\dot{\theta} + t_0\ddot{\theta}) + v\theta_0[\dot{u}_{i,i} + \Delta t_0\ddot{u}_{i,i}] = K^*\nabla^2\theta, \tag{5}$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho, \qquad c_2^2 = \mu/\rho, \qquad c_3^2 = \kappa/\rho,$$
$$c_4^2 = \gamma/\rho j, \qquad c_5^2 = (\alpha + \beta)/\rho j, \qquad \omega_0^2 = c_3^2/j = \kappa/\rho j, \tag{6}$$

where symbols have their usual meanings. A superposed dot denotes differentiation with respect to time and a comma followed by a subscript denotes partial differentiation with respect to the corresponding co-ordinate. The use of symbol Δ, in equation (5) makes these fundamental equations possible for the two different theories of the generalized thermoelasticity.

For the L–S (Lord–Shulman) theory $t_1 = 0$, $\Delta = 1$ and for G–L (Green–Lindsay) theory $t_1 > 0$ and $\Delta = 0$. The thermal relaxations t_0 and t_1 satisfy the inequality $t_1 \geqslant t_0 \geqslant 0$ for the G–L theory only.

We define the angle of incidence (I) as the angle between the propagation of plane sound wave and normal to the boundary of the first medium.

3. SOLUTION OF THE PROBLEM

To solve the problem in second medium, we decompose the displacement and microrotation vectors as

$$\mathbf{u} = \nabla\phi + \nabla\mathbf{x}\mathbf{U}, \qquad \nabla\cdot\mathbf{U} = 0, \tag{7}$$

$$\boldsymbol{\phi} = \nabla\xi + \nabla\mathbf{x}\boldsymbol{\Phi}, \qquad \nabla\cdot\boldsymbol{\Phi} = 0, \tag{8}$$

Using equations (7) and (8), equations (3)–(4) reduce as

$$(c_1^2 + c_3^2)\,\nabla^2\phi = \ddot{\phi} + \bar{v}(\theta + \mathrm{t}_1\dot{\theta}). \tag{9}$$

$$(c_2^2 + c_3^2)\,\nabla^2\mathbf{U} + c_3^2\,\nabla\mathbf{x}\boldsymbol{\Phi} = \ddot{\mathbf{U}} \tag{10}$$

$$c_4^2\,\nabla^2\boldsymbol{\Phi} - 2\omega_0^2\boldsymbol{\Phi} + \omega_0^2\,\nabla\mathbf{x}\mathbf{U} = \ddot{\boldsymbol{\Phi}} \tag{11}$$

$$(c_4^2 + c_5^2)\,\nabla^2\xi - 2\omega_0^2\xi = \ddot{\xi}. \tag{12}$$

From equations (9) to (12), we see that the longitudinal displacement wave (LD wave) is affected due to the thermal wave, the coupled transverse and microrotational waves (CD I and CD II waves) and longitudinal microrotational wave (LM wave) remain unaffected.

From equation (9), we have

$$\theta = (V_1^2\,\nabla^2\phi - \ddot{\phi})/\bar{\gamma}, \tag{13}$$

where

$$V_1^2 = c_1^2 + c_3^2, \qquad \bar{\gamma} = \bar{v}[1 + t_1(\partial/\partial t)]. \tag{14}$$

Eliminating θ from equations (5) and (13), we get

$$\nabla^4\phi - \left[\frac{C^*}{\bar{K}^*}\left\{\left(1 + t_0\frac{\partial}{\partial t}\right) + \varepsilon\left(1 + t_1\frac{\partial}{\partial t}\right)\left(1 + \Delta t_0\frac{\partial}{\partial t}\right)\right\} + \frac{1}{V_1^2}\frac{\partial}{\partial t}\right]\frac{\partial}{\partial t}\nabla^2\phi$$

$$+ \frac{C^*}{\bar{K}^*}\frac{1}{V_1^2}\left(1 + t_0\frac{\partial}{\partial t}\right)\frac{\partial^3\phi}{\partial t^3} = 0, \tag{15}$$

where

$$\varepsilon = \bar{v}\theta_0/V_1^2C^*. \tag{16}$$

We assume the solution of equation (15) in the form

$$\phi = f(\mathbf{z})\exp[\mathrm{i}k(ct - \mathbf{x})] \quad (c > V_1). \tag{17}$$

With the help of equation (17), equation (15) reduces to

$$\frac{\mathrm{d}^4 f(\mathbf{z})}{\mathrm{d}\mathbf{z}^4} + \mathrm{A}\frac{\mathrm{d}^2 f(\mathbf{z})}{\mathrm{d}\mathbf{z}^2} + Bf(\mathbf{z}) = 0, \tag{18}$$

where

$$A = k^2\left(\frac{c^2}{V_1^2} - 2\right) - kc(C^*/\bar{K}^*)[(\mathrm{i} - t_0kc) + \varepsilon(\mathrm{i} - t_1kc)(1 + \mathrm{i}kct_0\Delta)], \tag{19}$$

$$B = k^4\left(1 - \frac{c^2}{V_1^2}\right) + ck^3(C^*/\bar{K}^*)\Bigg[(\mathrm{i} - t_0kc) + \varepsilon(\mathrm{i} - t_1kc)(1 + \mathrm{i}kct_0\Delta),$$

$$- \frac{c^2}{V_1^2}(\mathrm{i} - t_0kc)\Bigg]. \tag{20}$$

The solution of equation (18) is of the form

$$f(\mathbf{z}) = \mathrm{A}_1\exp(m_1\mathbf{z}) + A_2\exp(-m_1\mathbf{z}) + A_3\exp(m_2\mathbf{z}) + A_4\exp(-m_2\mathbf{z}), \tag{21}$$

where

$$m_1 = [\{(A^2 - 4B)^{1/2} - A\}/2]^{1/2}, \tag{22}$$

$$m_2 = [-\{(A^2 - 4B)^{1/2} + A\}/2]^{1/2}, \tag{23}$$

correspond to the thermal and modified LD waves, respectively, and A_1, A_2, A_3, A_4 are arbitrary constants.

Making use of equation (8) in equation (10), we get

$$\phi_2 = \frac{1}{c_3^2}\frac{\partial^2\psi}{\partial t^2} - \left(\frac{c_2^2}{c_3^2} + 1\right)\nabla^2\psi, \tag{24}$$

where

$$\phi_2 = (-\boldsymbol{\phi})_{\mathrm{y}} = \frac{\partial\Phi_{\mathrm{x}}}{\partial\mathbf{z}} - \frac{\partial\boldsymbol{\Phi}_{\mathrm{x}}}{\partial\mathbf{x}}. \tag{25}$$

Using equation (24) in equation (11) and then the final solution is in the form

$$\psi = [A_5 \exp(m_3\mathbf{z}) + A_6 \exp(-m_3\mathbf{z}) + A_7 \exp(m_4\mathbf{z}) + A_8 \exp(-m_4\mathbf{z})][\exp \mathrm{i}(\omega t - k\mathbf{x})], \quad (26)$$

where

$$m_3 = \{(1/2)[(C^2 - 4D)^{1/2} - C]\}^{1/2} \quad \text{and} \quad m_4 = \{(-1/2)[(C^2 - 4D)^{1/2} + C]\}^{1/2}, \quad (27)$$

correspond to CDI- and CDII-waves and

$$C = k^2\left(\frac{c^2}{c_2^2 + c_3^2} + \frac{c^2}{c_4^2} - 2\right) + \frac{\omega_0}{c_4^2}\left(\frac{c_3^2}{c_2^2 + c_3^2} - 2\right), \quad (28)$$

$$D = k^4\left(1 - \frac{c^2}{c_2^2 + c_3^2} - \frac{c^2}{c_4^2} + \frac{c^4}{c_4^2(c_2^2 + c_3^2)}\right) - k^2\frac{\omega_0^2}{c_4^2}\left(\frac{c_3^2}{c_2^2 + c_3^2} + \frac{2c^2}{c_2^2 + c_3^2} - 2\right), \quad (29)$$

and A_5, A_6, A_7, A_8 are arbitrary constants.

If we assume $\mu = \kappa = \alpha = \beta = \gamma = 0$, we see that the longitudinal wave in a thermally conducting liquid medium is affected due to the presence of a thermal wave. In this case, there is no existence for other waves. We consider the variables with primes in the thermally conducting liquid medium.

The appropriate potentials for two media {after dropping the exponential term $\mathrm{i}k(ct - x)$} are as follows:

$$\phi' = B_0 \exp((m_2'\mathbf{z}) + B_1 \exp(-m_1'\mathbf{z}) + B_2 \exp(-m_2'\mathbf{z}), \quad (30)$$

$$\theta' = (1/\bar{\gamma}_0')[b_2' B_0 \exp(m_2'\mathbf{z}) + b_1' B_1 \exp(-m_1'\mathbf{z}) + b_2' B_2 \exp(-m_2'\mathbf{z})], \quad (31)$$

$$\phi = B_3 \exp(m_1\mathbf{z}) + B_4 \exp(m_2\mathbf{z}), \quad (32)$$

$$\theta = (1/\bar{\gamma}_0)[b_1 B_3 \exp(m_1\mathbf{z}) + b_2 B_4 \exp(m_2\mathbf{z})], \quad (33)$$

$$\psi = B_5 \exp(m_3\mathbf{z}) + B_6 \exp(m_4\mathbf{z}), \quad (34)$$

$$\phi_2 = b_3 B_5 \exp(m_3\mathbf{z}) + b_4 B_6 \exp(m_4\mathbf{z}), \quad (35)$$

where B_i $(i = 0, 1, 2, \ldots, 6)$ are arbitrary constants, and

$$b_{1,2} = k^2(c^2 - V_1^2) + m_{1,2}^2 V_1^2, \quad (36)$$

$$b_{1,2}' = k^2(c^2 - \alpha_1^2) + m_{1,2}'^2 \alpha_1^2, \quad (37)$$

$$b_{3,4} = k^2\{(1 + (c_2^2/c_3^2) - (c^2/c_3^2)\} - m_{3,4}^2\{(1 + (c_2^2/c_3^2)\}, \quad (38)$$

$$\bar{\gamma}_0 = \bar{v}(1 + \mathrm{i}\omega t_1), \qquad \bar{\gamma}_0'(1 + \mathrm{i}\omega t_1'), \quad (39)$$

where α_1 is velocity of sound wave and m_1' and m_2' correspond to thermal wave and modified longitudinal wave in liquid medium and are obtained from equations (22) and (23), if we let $\mu = \kappa = \alpha = \beta = \gamma = 0$.

Here we assume that the boundary conditions at the interface $\mathbf{z} = 0$ are independent of $\mathbf{x}$ and t, so the values of the phase velocity and wave number in $\boldsymbol{\phi}$, ψ, θ, ϕ_2 must be same as those in $\boldsymbol{\phi}'$ and θ'. We consider the continuity of stresses and displacements at the interface $\mathbf{z} = 0$ as

$$\sigma_{\mathbf{zz}} = \sigma_{\mathbf{zz}}', \qquad \sigma_{\mathbf{zx}} = 0, \qquad u_3 = u_3',$$
$$m_{\mathbf{zy}} = 0, \qquad \theta = \theta', \qquad K^*(\partial\theta/\partial\mathbf{z}) = K^{*\prime}(\partial\theta'/\partial\mathbf{z}). \quad (40)$$

Making use of the potentials given by equations (30)–(35) in boundary conditions (40), after using equations (1), (2), (7) and (8), we get a system of six non-homogeneous equations which can be written as

$$\sum_j^6 a_{ij} Z_j = b_i \qquad (i = 1, 2, \dots, 6), \tag{41}$$

where

$$a_{11} = \lambda'(m_1'^2 - k^2) - \rho' b_1', \qquad a_{12} = \lambda'(m_2'^2 - k^2) - \rho' b_2',$$

$$a_{13} = -(\lambda + 2\mu + \kappa)m_1^2 + \lambda k^2 + \rho b_1, \qquad a_{14} = -(\lambda + 2\mu + \kappa)m_2^2 + \lambda k^2 + \rho b_2,$$

$$a_{15} = -\mathrm{i}(2\mu + \kappa)m_3 k, \qquad a_{16} = -\mathrm{i}(2\mu + \kappa)m_4 k,$$

$$a_{21} = 0 = a_{22}, \quad a_{23} = -\mathrm{i}(2\mu + \kappa)m_1 k, \quad a_{24} = -\mathrm{i}(2\mu + \kappa)m_2 k,$$

$$a_{25} = \mu k^2 + (\mu + \kappa)m_3^2 - \kappa b_3, \quad a_{26} = \mu k^2 + (\mu + \kappa)m_4^2 - \kappa b_4,$$

$$a_{31} = m_1', \quad a_{32} = m_2', \quad a_{33} = m_1, \quad a_{34} = m_2, \quad a_{35} = \mathrm{i}k = a_{36},$$

$$a_{41} = a_{42} = a_{43} = a_{44} = 0, \quad a_{45} = m_3 b_3, \quad a_{46} = m_4 b_4,$$

$$a_{51} = b_1', \quad a_{52} = b_2', \quad a_{53} = -(\bar{\gamma}_0'/\bar{\gamma}_0)b_1, \quad a_{54} = -(\bar{\gamma}_0'/\bar{\gamma}_0)b_2,$$

$$a_{55} = a_{56} = 0,$$

$$a_{61} = m_1' b_1', \quad a_{62} = m_2' b_2', \quad a_{63} = (K^* \bar{\gamma}_0'/K^{*\prime} \bar{\gamma}_0) m_1 b_1,$$

$$a_{64} = (K^* \bar{\gamma}_0'/K^{*\prime} \bar{\gamma}_0) m_2 b_2, \quad a_{65} = a_{66} = 0,$$

and

$$b_1 = -a_{12} \quad b_2 = a_{22} \quad b_3 = a_{32} \quad b_4 = a_{42} \quad b_5 = -a_{52}, \quad b_6 = a_{62}, \tag{42}$$

and (Z_j) are the amplitude ratios for various reflected and refracted waves.

4. NUMERICAL RESULTS AND DISCUSSIONS

To explain the analytical procedure presented earlier, we now consider a numerical example. The results depict the variation of the angle of incidence with the modulus of the amplitude ratios in the context of water–aluminium-epoxy composite.

Physical constants for water

$$\rho' = 1{\cdot}0\ \mathrm{g/cm^3}, \quad \alpha_1 = 1{\cdot}439 \times 10^5\ \mathrm{cm/s},$$

$$K^{*\prime} = 0{\cdot}144\ \mathrm{cal/cm\,s\,^\circ C}, \quad C^{*\prime} = 1{\cdot}0\ \mathrm{cal/g\,^\circ C}.$$

Following Gauthier [23], the physical constants for aluminium-epoxy composite

$$\rho = 2{\cdot}19\ \mathrm{g/cm^3}, \qquad \lambda = 7{\cdot}59 \times 10^{11}\ \mathrm{dyn/cm^2},$$

$$\mu = 1{\cdot}89 \times 10^{11}\ \mathrm{dyn/cm^2}, \quad \kappa = 0{\cdot}0149 \times 10^{11}\ \mathrm{dyn/cm^2},$$

$$\gamma = 0{\cdot}0268 \times 10^{11}\ \mathrm{dyn}, \qquad j = 0{\cdot}0196\ \mathrm{cm^2},$$

$$K^* = 0{\cdot}48\ \mathrm{cal/cm\,s\,^\circ C}, \qquad C^* = 0{\cdot}206\ \mathrm{cal/g\,^\circ C}, \quad \theta_0 = 20^\circ\mathrm{C},$$

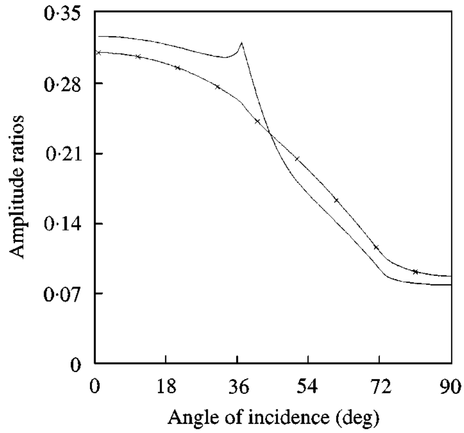

Figure 2. Variations of the amplitude ratios for reflected thermal waves with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory.

Nayfeh and Nasser [24] took $t_0 = 3K^*/\rho C^* \alpha^2$. We, therefore, take $t'_0 = 3K^{*\prime}/\rho' C^{*\prime} \alpha_1^2$ and $t_0 = 3K^*/\rho C^* V_1^2$. t'_1 and t_1 are considered to be of same order as that of t'_0 and t_0. Chadwick and Sneddon [3], Lockett [4], Lord and Shulman [6] and Nayfeh and Nasser [24] have considered $\varepsilon = \bar{v}^2 \theta_0 / \alpha^2 C^*$ as the thermoelastic coupling coefficient, where α is the velocity of longitudinal wave. All of them have considered it to be very small and have taken its value to be in the range of 0·03 and 0·073. We take this coupling coefficient to be 0·073.

The classical theory is believed to be inadequate for the treatment of deformations and motions of a material possessing granular structure. In particular, the effect of granular structure, or microstructure, becomes important in transmitting waves of small wavelength and/or high frequency. When the wavelength is comparable with the average grain size, the motion of the grains must be taken into account. This introduces new type of waves not encountered in the classical theory. Here, $\omega > \sqrt{2}\,\omega_0$ is the condition for existence of these new waves which implies that (ω^2/ω_0^2) should be greater than two. We have chosen $\omega^2/\omega_0^2 = 200$ arbitrarily to get results for the case of very high frequency. We can call ω^2/ω_0^2 as frequency ratio as ω and ω_0 are of same dimension.

For the above values of relevant physical constants, the system of equations (41) in reduced form for L–S theory, G–L theory and in absence of thermal effect has been solved for amplitude ratios by using the Gauss elimination method for different angle of incidence varying from 0 to 90°. The variations of the modulus of amplitude ratios for various reflected and refracted waves with the angle of incidence have been shown graphically in Figures 2–7 at a given excitation frequency, i.e., when $\omega^2/\omega_0^2 = 200$.

The variations of the amplitude ratios for reflected thermal waves with the angle of incidence have been shown in Figure 2 for L–S theory and G–L theory by solid line and solid line with centre symbols respectively. The amplitude ratios decrease with the increase in angle of incidence for both of L–S and G–L cases. The comparison between these two line curves shows the effect of second thermal relaxation time. Also, if we neglect the thermal effect, these thermal waves will disappear.

The amplitude ratios for reflected longitudinal wave for L–S theory, G–L theory shows the oscillatory behaviour. The variations for these amplitude ratios with the angle of incidence have been depicted in Figure 3. The solid curve in Figure 3 represents the variations for L–S theory whereas the solid curve with centre symbols represents the

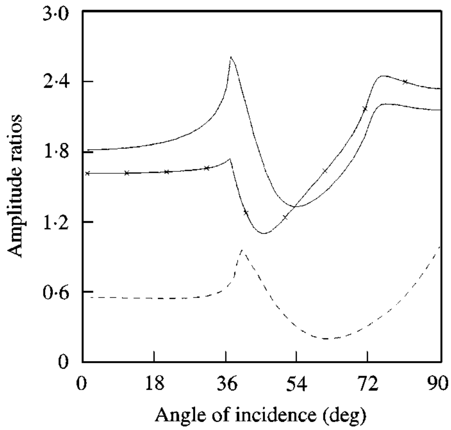

Figure 3. Variations of the amplitude ratios for reflected longitudinal waves with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory; – –, No thermal effect.

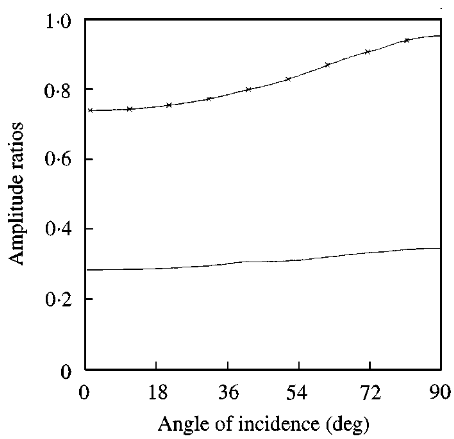

Figure 4. Variations of the amplitude ratios for refracted thermal waves with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory.

variations for G–L theory. Also, if thermal effect is neglected, then these variations reduce to those shown by dashed line in Figure 3.

The variations of the refracted thermal waves for L–S theory and G–L theory have been shown in Figure 4. If we compare the solid line with the solid line with centre symbol, we observe the significance of second thermal relaxation time taken by Green and Lindsay [7].

The amplitude ratio for refracted longitudinal displacement wave for L–S case first increases to its maxima and then decreases sharply whereas the amplitude ratios of LD wave for G–L case first increases to its maxima and then decreases slowly. The variations of these amplitude ratios for L–S case and G–L case have been shown in Figure 5 by solid and solid line with centre symbols respectively. The dashed line in Figure 5 represent the variations of the refracted LD wave with the angle of incidence in absence of thermal effect.

The variations of the amplitude ratios for two sets of refracted coupled waves (CD I and CD II waves) with the angle of incidence have been shown in Figures 6 and 7. These two sets of coupled wave behave alike. The comparison between solid line and solid line with centre

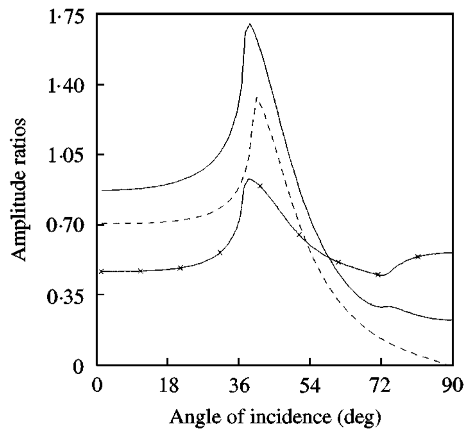

Figure 5. Variations of the amplitude ratios for refracted longitudinal displacement waves with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory; – –, No thermal effect.

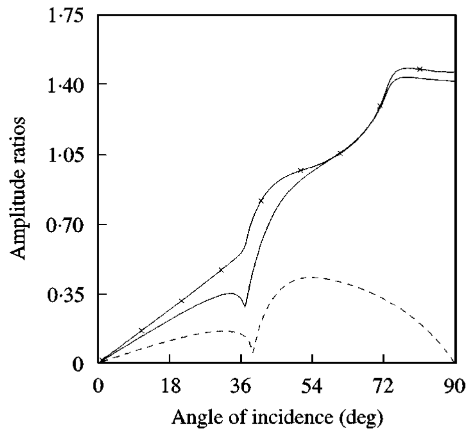

Figure 6. Variations of the amplitude ratios for refracted coupled waves (CD I waves) with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory; – –, No thermal effect.

symbols shows the importance of second thermal relaxation time. Also, if thermal effect is neglected, then these variations reduce to those shown by dashed lines in Figures 6 and 7.

The variations of these amplitude ratios with the excitation frequency at a given angle of incidence can be expressed in similar manner.

5. CONCLUSIONS

A problem of reflection and refraction of plane sound wave has been studied at an interface between a thermally conducting liquid and a generalized micropolar generalized thermoelastic solid. Both the theory and numerical results indicate the dependence of the amplitude ratios of various reflected and refracted waves upon the angle of incidence. The variations of the amplitude ratios for various reflected and refracted waves in G–L case are different from those in L–S case. The comparison between the amplitude ratios for L–S case

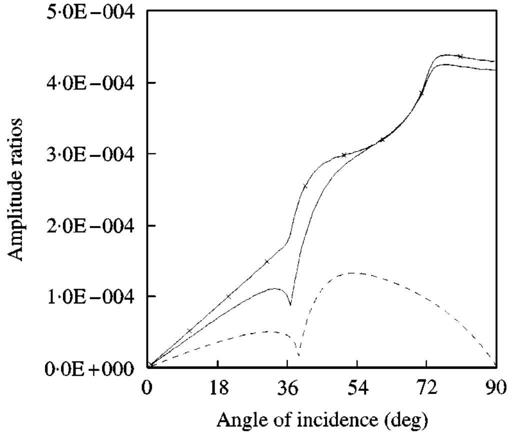

Figure 7. Variations of the amplitude ratios for refracted coupled waves (CD II waves) with the angle of incidence: ——, LS Theory; -×-×-×-×-, GL Theory; – –, No thermal effect.

and G–L case reveals the effect of second thermal relaxation time. The comparison of the variations of the amplitude ratios with those without thermal disturbances in figures shows the importance of thermal phenomenon in the problems of reflection and refraction. The final results also indicate that the problems of waves and vibrations become more important in the field of seismology, when we study the problem with additional parameters (e.g., thermal disturbance, microrotation, etc.). This problem introduces a more realistic model of the earth's crust. Such type of problems may provide some useful information about the presence of oil layers in earth's crust and may be of some use for experimental seismologists.

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APPENDIX A. NOMENCLATURE

c	apparent phase velocity on the surface
C^*	specific heat at constant strain
i	$=\sqrt{-1}$
j	microrotational inertia
k	wave number
K^*	coefficient of thermal conductivity
$\bar{K}^*$	$= K^*/\rho$
m_{kl}	components of the couple stress tensor
t_0, t_1	relaxation times
u_k	components of displacement vector $\mathbf{u}$
$\dot{u}_{i,j}$	$= \partial^2 u_i/\partial x_j \partial t$
$\ddot{u}_{i,j}$	$= \partial^3 u_i/\partial x_j \partial t^2$
$u_{k,l}$	$= \partial u_k/\partial x_1$
$\mathbf{u}, \boldsymbol{\phi}$	displacement and microrotation vector respectively
$\mathbf{U}, \boldsymbol{\Phi}$	vector potentials
x_k	components of the position vector ($x_1 = \mathbf{x}, x_2 = \mathbf{y}, x_3 = \mathbf{z}$)
α_t	coefficients of linear expansion
δ_{kl}	Kronecker delta
∇	del operator
ε	thermocoupling coefficient
ε_{klr}	alternate tensor
θ	temperature variable
θ_0	initial uniform temperature
$\lambda, \mu, \kappa, \gamma, \alpha, \beta$	micropolar material constants
v	thermal constants
v	$= (3\lambda + 2\mu + \kappa)\alpha_t$
$\bar{v}$	$= v/\rho$
ρ	density of micropolar generalized thermoelastic solid

σ_{kl}	components of the force stress tensor
ϕ_k	component of microrotation vector $\boldsymbol{\phi}$
ϕ_2	$= (-\boldsymbol{\phi})_y$
$\phi_{k,l}$	$= \partial\phi_k/\partial x_l$
ϕ, ξ	scalar potentials
ψ	$= (-\mathbf{U})_y$
$\omega\ (= kc)$	angular frequency
$'$	similar quantities in the liquid medium