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COMMENTS ON "DIRECT TREATMENT AND DISCRETIZATIONS OF NON-LINEAR SPATIALLY CONTINUOUS SYSTEMS"

M. PAKDEMIRLI

Department of Mechanical Engineering, Celal Bayar University, 45140, *Muradiye, Manisa, Turkey*

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I have read the paper by Lacarbonara [1]. The author did a good job. To clarify some of the issues, additional comments are given.

First of all, the non-linear operator notation used throughout the text [1] has been previously developed and used in a number of papers [2–7]. When I was working with Professor Nayfeh as a post-doc on the comparison of direct versus discretization–perturbation methods, we were treating special problems. I thought that it would be better to make the comparisons on a general equation having arbitrary quadratic and cubic non-linearities. For this reason, I developed an operator notation which is suitable for perturbative calculations. The first paper using that notation appeared in Mechanics Research Communications [2]. The paper was a single-mode comparison of direct versus discretization–perturbation methods for arbitrary quadratic and cubic non-linearities. The second paper [3], which is also referenced by the author, was an infinite mode comparison between the methods in the absence of internal resonances. Although the author refers to this paper by saying "However, in their analysis, one of the fundamental results was postulated instead of proved", all conclusions presented in the paper were clearly shown and justified by calculations. The general operator notation was further used for the subharmonic, superharmonic and combination resonance cases [4]. The case of arbitrary odd non-linearities were considered by using the same notation [5, 6]. Finally, the notation has been developed to express and solve coupled systems of equations with arbitrary quadratic and cubic non-linearities [7].

In the comparison of direct perturbation and discretization–perturbation methods, two papers are worth mentioning. All papers dealing with this comparison issue treated non-linear problems. In fact, the problem arises also in linear equations and the direct-perturbation method yields more accurate results for finite mode truncations [8]. The comparison of both methods in a gyroscopic system revealed another interesting result [9]. For such systems, researchers usually discretize the equations first and then apply perturbations. If travelling string eigenfunctions are used (better convergence properties than stationary string eigenfunctions), the equation of motion should be cast into a convenient first order form since the eigenfunctions do not have orthogonality properties. However, it is shown that, such transformations are unnecessary when using direct-perturbation methods, and the original equation of motion can be treated directly by perturbations.

REFERENCES

1. W. LACARBONARA 1999 *Journal of Sound and Vibration* **221**, 849–866. Direct treatment and discretizations of non-linear spatially continuous systems.

0022-460X/00/350874 + 03 $35.00/0

2. M. PAKDEMİRLİ 1994 *Mechanics Research Communications* **21**, 203–208. A comparison of two perturbation methods for vibrations of systems with quadratic and cubic nonlinearities.
3. M. PAKDEMİRLİ and H. BOYACI 1995 *Journal of Sound and Vibration* **186**, 837–845. Comparison of direct-perturbation methods with discretization–perturbation methods for non-linear vibrations.
4. M. PAKDEMİRLİ and H. BOYACI 1996 *Applied Mechanics and Engineering* **1**, 445–463. Vibrations of continuous systems having arbitrary quadratic and cubic nonlinearities.
5. H. BOYACI and M. PAKDEMİRLİ 1997 *Journal of Sound and Vibration* **204**, 595–607. A comparison of different versions of the method of multiple scales for partial differential equations.
6. M. PAKDEMİRLİ, H. BOYACI and M. YILMAZ 1997 *Mathematical and Computational Applications* **2**, 85–90. Continuous systems with odd nonlinearities: a general solution procedure.
7. M. PAKDEMİRLİ and H. BOYACI 1997 *Mathematical and Computational Applications* **2**, 141–154. A generalized approach to coupled nonlinear vibrations of continuous systems.
8. M. PAKDEMİRLİ and H. BOYACI 1997 *Journal of Sound and Vibration* **199**, 825–832. The direct-perturbation method versus the discretization–perturbation method: linear systems.
9. M. PAKDEMİRLİ and A. G. ULSOY 1997 *Journal of Sound and Vibration* **203**, 815–832. Stability analysis of an axially accelerating string.

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AUTHOR'S REPLY

W. LACARBONARA

Dipartimento di Ingegneria Strutturale e Geotecnica, University of Rome La Sapienza, via A. Gramsci, 53 Rome 00197, Italy

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The author appreciates the opportunity to comment on the letter to the editor by Professor M. Pakdemirli.

The first issue discussed is the use of general operator notation with the method of multiple scales. While it seems to be a well-established practice to use abstract operator notation in theoretical mechanics, in the framework of the method of multiple scales, Professor Pakdemirli invaluably envisioned the importance of proposing a general non-linear operator notation [1]. In fact, the latter allows for a broader generality of the relevant obtained results. It is also worth mentioning that such a notation had been previously used, among others, by Simmons [2] in a pedestrain expansion constructed to determine general resonance conditions for weak wave interactions as Professor Pakdemirli himself pointed out in reference [1].

The second issue regards the comparison between the full-basis Galerkin discretization procedure (using the eigenbase of the associated linear undamped unforced problem) and the direct perturbation approach to non-linear vibrations of continuous systems with quadratic and cubic non-linearities. Referring to the paper by Pakdemirli and Boyaci [3], my statement in reference [4]—"However, in their analysis, one of the fundamental results was postulated instead of proved"—requires some additional clarifying comments.

To show that the approximate solutions obtained with the two approaches are equivalent, in reference [3] they constructed second order expansions of the displacement fields with both methods thereby "directly" concluding that the second order spatial shape functions obtained with the direct approach are the converged forms of the infinite series obtained with discretization. In an earlier work, to show this result in the particular case of