2. M. PAKDEMİRLİ 1994 *Mechanics Research Communications* **21**, 203–208. A comparison of two perturbation methods for vibrations of systems with quadratic and cubic nonlinearities.
3. M. PAKDEMİRLİ and H. BOYACI 1995 *Journal of Sound and Vibration* **186**, 837–845. Comparison of direct-perturbation methods with discretization–perturbation methods for non-linear vibrations.
4. M. PAKDEMİRLİ and H. BOYACI 1996 *Applied Mechanics and Engineering* **1**, 445–463. Vibrations of continuous systems having arbitrary quadratic and cubic nonlinearities.
5. H. BOYACI and M. PAKDEMİRLİ 1997 *Journal of Sound and Vibration* **204**, 595–607. A comparison of different versions of the method of multiple scales for partial differential equations.
6. M. PAKDEMİRLİ, H. BOYACI and M. YILMAZ 1997 *Mathematical and Computational Applications* **2**, 85–90. Continuous systems with odd nonlinearities: a general solution procedure.
7. M. PAKDEMİRLİ and H. BOYACI 1997 *Mathematical and Computational Applications* **2**, 141–154. A generalized approach to coupled nonlinear vibrations of continuous systems.
8. M. PAKDEMİRLİ and H. BOYACI 1997 *Journal of Sound and Vibration* **199**, 825–832. The direct-perturbation method versus the discretization–perturbation method: linear systems.
9. M. PAKDEMİRLİ and A. G. ULSOY 1997 *Journal of Sound and Vibration* **203**, 815–832. Stability analysis of an axially accelerating string.

doi:10.1006/jsvi.1999.2709, available online at http://www.idealibrary.com on IDEAL®

AUTHOR'S REPLY

W. LACARBONARA

Dipartimento di Ingegneria Strutturale e Geotecnica, University of Rome La Sapienza, via A. Gramsci, 53 Rome 00197, Italy

(*Received* 5 *October* 1999)

The author appreciates the opportunity to comment on the letter to the editor by Professor M. Pakdemirli.

The first issue discussed is the use of general operator notation with the method of multiple scales. While it seems to be a well-established practice to use abstract operator notation in theoretical mechanics, in the framework of the method of multiple scales, Professor Pakdemirli invaluably envisioned the importance of proposing a general non-linear operator notation [1]. In fact, the latter allows for a broader generality of the relevant obtained results. It is also worth mentioning that such a notation had been previously used, among others, by Simmons [2] in a pedestrain expansion constructed to determine general resonance conditions for weak wave interactions as Professor Pakdemirli himself pointed out in reference [1].

The second issue regards the comparison between the full-basis Galerkin discretization procedure (using the eigenbase of the associated linear undamped unforced problem) and the direct perturbation approach to non-linear vibrations of continuous systems with quadratic and cubic non-linearities. Referring to the paper by Pakdemirli and Boyaci [3], my statement in reference [4]—"However, in their analysis, one of the fundamental results was postulated instead of proved"—requires some additional clarifying comments.

To show that the approximate solutions obtained with the two approaches are equivalent, in reference [3] they constructed second order expansions of the displacement fields with both methods thereby "directly" concluding that the second order spatial shape functions obtained with the direct approach are the converged forms of the infinite series obtained with discretization. In an earlier work, to show this result in the particular case of unforced undamped finite-amplitude vibrations of a beam resting on an elastic foundation with quadratic and cubic non-linearities, Nayfeh *et al.* [5] expanded the second order shape functions obtained with the direct approach in infinite series of the relevant eigenfunctions and obtained the same series of the discretization procedure.

In reference [4] this result is shown to hold in a general and systematic fashion for self-adjoint continuous systems with either geometric and inertia quadratic and cubic non-linearities subject to either primary- or subharmonically resonant excitations with or without internal resonances. The simplicity of the analysis relies on the idea of showing that the infinite series obtained with discretization are solutions of the same boundary-value problems governing the second order shape functions obtained with the direct approach. In addition, contrary to the involved computations in reference [3], the perturbation schemes in reference [4] take into account the influence of the directly excited mode only at first order in the absence of internal resonances or that of the internally resonant modes only in the presence of internal resonances. In fact, it is known *a priori* that the influences of all of the other modes decay at steady state due to action of the damping.

Furthermore, in reference [4] the equivalence is established between the full-basis discretization or direct approach and a novel low order-rectified Galerkin procedure [6]. The computational characteristics of the three analytical strategies are comparatively shown as to possibly clarify advantages and drawbacks.

REFERENCES

1. M. Pakdemirli 1994 *Mechanics Research Communications* **21**, 203–208. A comparison of two perturbation methods for vibrations of systems with quadratic and cubic nonlinearities.
2. W. F. Simmons 1969 *Proceedings of the Royal Society A* **309**, 551–575. A variational method for weak resonant wave interactions.
3. M. Pakdemirli and H. Boyaci 1995 *Journal of Sound and Vibration* **186**, 837–845. Comparison of direct-perturbation methods with discretization–perturbation methods for non-linear vibrations.
4. W. Lacarbonara 1999 *Journal of Sound and Vibration* **221**, 849–866. Direct treatment and discretizations of non-linear spatially continuous systems.
5. A. H. Nayfeh, S. A. Nayfeh and M. Pakdemirli 1995 *Nonlinear Dynamics and Stochastic Mechanics* (W. Kliemann and N. Sri Namachchivaya, editors), 175–200. On the discretization of weakly nonlinear spatially continuous systems.
6. A. H. Nayfeh 1998 *Nonlinear Dynamics* **16**, 105–125. Reduced-order models of weakly nonlinear spatially continuous systems.