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SKEWING OF THE ACOUSTIC WAVE ENERGY VECTOR IN STACKED 1–3 ANISOTROPIC MATERIAL SYSTEMS

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1. INTRODUCTION

Anisotropic material systems occur naturally (such as wood, tissues, etc.), or are engineered as in the case of fibrous composites (such as fiber-glass, graphite-epoxy, ceramics matrix, etc.), or can be due to the manufacturing process; like for instance metals with preferred grain orientations (rolled metal, columnar cast stainless steel, etc.). Anisotropy in solid materials has a tendency to change the direction of the acoustic wave energy (group) vector along preferred orientations. Thus, the wave energy vector (also sometimes referred to as power density vector, Poynting vector, power flow vector) is not necessarily along the wave vector direction [1–3]. This effect is often called skewing. In anisotropic materials, the skewing angle, which is the angle between the energy vector and the wave vector, equals zero only when the wave vector is along one of the directions of material symmetry.

Since engineered material systems can be designed and fabricated to specifications, further discussions in this paper will be limited to stratified fiber-reinforced composite materials and more specifically to graphite fiber–epoxy resin systems. It has been previously demonstrated that the fiber direction in fiber reinforced composites plays a critical role in the direction of propagation of the acoustic energy [4, 5]. The basic anisotropy of a unidirectional fiber-reinforced laminate can be assumed to be transversely isotropic, and hence be characterized using five independent elastic material constants. In many practical instances transversely isotropic layers oriented at different angles with respect to each other can be combined to form multilayered material systems. In the past, such multilayered structures were considered to be orthotropic, mono-clinic, and tri-clinic using effective constants methods [6].

Acoustic plane waves when obliquely incident upon a plane interface between any two dissimilar materials (isotropic or anisotropic) will cause some of the incident energy to be reflected back to the incident medium while the remaining portion will be transmitted at a refracted angle based on Snell's law. Assuming a lossless medium, the sum of the reflected and the transmitted energy should be equal to the incident energy. But, due to mode conversion, other wave modes are generated which alters this equation, particularly when the mode-converted wave is a guided wave which travels along the structure [7]. The guided waves may also leak energy to the surrounding media. During the generation of guided modes, especially plate wave modes, the specularly reflected energy approaches a minimum value thus acting as reflection factor filter (changing the equation between transmission and reflection based on the Kramer coincidence principle [1–3]). Hence, by

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selecting the correct material thickness, the reflection/transmission of acoustic energy can be controlled. This is a well-known effect, but is limited to a very narrow frequency bandwidth and/or a small number of incident angles. This is a limitation, particularly, when the goal is to design panels with reflection/transmission response over a wide range of frequencies and angles.

In an isotropic media, the bulk wave modes (longitudinal and transverse) are pure with displacements in orthogonal directions. In contrast, the influence of the anisotropy on acoustic wave propagation will be to modify the mode conversion process to produce impure modes types such as quasi-longitudinal and quasi-shear. The more important influence of concern here is the angle between the direction of energy propagation and the wave vector orientation. In isotropic media, the wave vector is always normal to the wavefront and the energy of the wave is along the direction of the wave vector. In anisotropic media, these two directions do not coincide and there is a deviation from the wave vector and is often referred as the beam skewing phenomena (the angle between the energy vector and the wave vector is the beam skewing angle) [1].

In an anisotropic lossless medium, the energy propagation direction is along the preferred "group" velocity direction and can be determined through the analysis of the slowness surface (inverse of velocity profile) [1]. The energy propagation direction at any "phase" velocity angle (wave vector direction) is oriented along the normal to the slowness surface measured at that "phase" angle. The energy propagation direction may be significantly influenced by the preferred direction of the anisotropic material, but practically

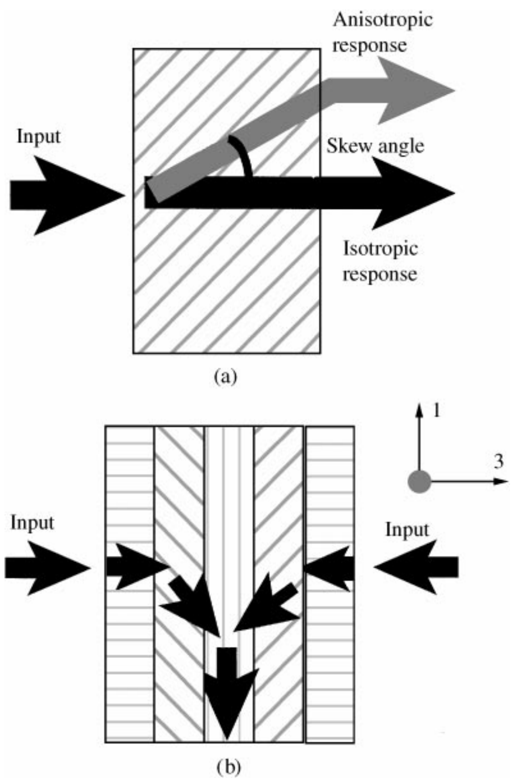

Figure 1. Schematic representation of (a) acoustical waveguide effect of single-layer fiber-reinforce composite material panel with fiber orientation at an angle with the panel, and (b) the channeling of the incident acoustical energy along the structure using a multi-layered wave guide.

there may be a limited degree of flexibility in controlling/designing the anisotropic property of a single layer. In order to increase the skewing, multi-layered panels, constructed using anisotropic layers, have also been selected for further analysis. Here, the materials are anisotropic along the 1–3 plane, i.e., the fibers are along the 1–3 plane. The angle between axis 3 and the incident wave vector is called as the angle of incidence (θ) and the angle between axis 1 and the projection of the incident wave vector on the 1–2 plane is called the azimuthal angle (ϕ). In Figure 1(a), the schematic represents a normally incident wave, impinging on a single layered structure, and two different paths for propagation of the wave for two cases: (1) isotropic media and (2) anisotropic media. In the case of the isotropic media, the wave propagation direction does not change, and the energy propagation vector is along the incident wave vector. In contrast, for the anisotropic material, the skew effect changes the energy propagation path. In Figure 1(b), this effect is hypothetically illustrated on a particular multi-layered anisotropic structure where subsequent skewing will guide the wave to travel along the structure, thus reducing the transmission of the acoustical energy through the structure.

Thus conceptually, it should be possible to design anisotropic waveguides which will significantly increase or decrease the reflection and/or the transmission of a specific mono-chromatic acoustic wave incident at a particular angle. In order for this waveguide to be widely applicable, it must be designed to uniformly improve/reduce the reflection/transmission of acoustic waves over a wide band of frequencies and over a large range of angles of impingement.

2. BACKGROUND

The acoustic Poynting theorem which is used here to define the wave energy behavior is derived in a procedure similar to the electromagnetic Poynting theorem described in Harrington [8]. Musgrave [9] investigated the propagation of plane wave through a crystalline medium and obtained the equations for the energy (group) direction. The reflected and transmission coefficients are obtained by using a model for multi-layered visco-elastic material system with generally anisotropic behavior (21 independent elastic constants) based on the plane wave transfer matrix method [10–14]. The traditional technique has certain limitations due to the occurrence of numerical instabilities when analyzing thick, multi-layered structures and intricate anisotropic orientations and has been extensively documented. A delta-operator technique has been suggested as a remedy [15]. In this paper, a numerical truncation algorithm was used, instead of the delta-operator, to modify the transfer matrix technique and avoid the numerical instabilities for the thick and relatively complex ply-lay-up case studies which were considered [16].

Considering the plane wave propagation in a generally anisotropic multi-layered structure, the displacement vector (u_i) can be assumed to be in the form

$$u_i = AU_i \mathrm{e}^{\mathrm{j}(k_i x_i - \omega t)}, \tag{1}$$

where $i = 1, 2, 3$; A is the wave amplitude; U_i is the particle displacement unit vector; k_i are the wave number's cosine components; ω is the circular frequency and t is time. The particle velocity (v_i) is computer by applying a derivative with respect to 't':

$$v_j = -\mathrm{j}\omega u_i. \tag{2}$$

Applying continuity of stresses and displacements boundary conditions on each interface, the displacements and stresses of the top interface ($l = 0$) and the bottom interface

($l = n + 1$) of the multi-layered laminate can be related as

$$X_0 = M_G X_{n+1}, \tag{3}$$

where M_G is a global transfer matrix for the entire n layer structure, and the field vector at the upper substrate (incident medium represented by 0) and the lower substrate (transmitted medium represented by $n + 1$) respectively are represented in the transformed matrix form:

$$X_0 = \lfloor u_1^0 \;\; u_2^0 \;\; u_3^0 \;\; \sigma_{33}^0 \;\; \sigma_{23}^0 \;\; \sigma_{13}^0 \rfloor^T, \quad X_{n+1} = [u_1^{n+1} \;\; u_2^{n+1} \;\; u_3^{n+1} \;\; \sigma_{33}^{n+1} \;\; \sigma_{23}^{n+1} \;\; \sigma_{13}^{n+1}]^T. \tag{4}$$

The power flow (energy reflection and energy transmission vector obtained from Poynting's theorem and hence also called Poynting vector) is defined as [1, 8, 13]

$$P_i = \tfrac{1}{2}\sigma_{ij} v_j^*, \tag{5}$$

where v_i^* is the complex conjugate of the particle velocity. The power flow for each partial wave mode (represented by superscript α) is provided by computing the expression

$$P_i^\alpha = \tfrac{1}{2}\sigma_{ij}^\alpha v_j^\alpha{}^*. \tag{6}$$

The reflection and transmission coefficients of wave energy are then defined by

$$R^\alpha = -\,p_3^\alpha / p_3^I \tag{7}$$

and

$$D^\alpha = +\,p_3^\alpha / p_3^I, \tag{8}$$

where the subscript I represents incident wave and is often considered to be equal to unity, and the subscript 3 indicates the vertical component. The reflection coefficient (R) of the power flow (energy) is negative showing energy flow in the opposite direction to the incidence direction while the transmission coefficient (D) of energy is in the positive direction.

3. NUMERICAL RESULTS AND DISCUSSION

The case study considered in this paper involves three fiber-reinforced anisotropic graphite-epoxy composite plate structures embedded in between two semi-infinite media. The first case represents fiber orientations along the normal direction (0°), while the second case represents fibers oriented along the tangential direction (90°). Here, the fiber directions are measured from the x_3-axis. The third case is the multi-layered composite with a 11-layer unsymmetrical lay-up represented as (0/10/20/30/40/50/60/70/80/90/100) with layers at 10° increments. In all the above cases the total thickness of the composite plate was kept constant (0·12 m) and the material properties used in the computation are provided in Table 1. The incident acoustic wave was a harmonic longitudinal wave and the reflection and transmission coefficients for the acoustical energy were studied. The coefficients are plotted as a function of incident angle with respect to the normal to the surface (x_3-axis) or as a function of the input frequency of the acoustic plane wave.

The first case study involved water as the two semi-infinite substrates (media 1 and 2) and the transmission factor is plotted as a function of wave frequency at a normal angle of incidence ($\theta = 0$) in Figure 2(a). It can be observed that the 11-layer composite has a lower energy transmission coefficient when compared with the 0-layer composite. This is

TABLE 1

Materials constants used in the theoretical analysis

Elastic constant (GPa)	Plexiglass	Composite laminate (0° lay-up)
C_{11}	10·19	110·7
C_{22}	10·19	14·05
C_{33}	10·19	14·05
C_{44}	1·57	3·37
C_{55}	1·57	6·09
C_{66}	1·57	6·09
C_{12}	7·05	7·48
C_{13}	7·05	7·48
C_{16}	0·0	0·0
C_{23}	7·05	7·31
C_{26}	0·0	0·0
C_{36}	0·0	0·0
C_{45}	0·0	0·0
Density ρ (kg/m^3)	2700	1550

particularly true for frequencies higher than 10 kHz. Also it can be seen that the 0 composite exhibits the expected periodic behavior. The average transmission coefficient for the 0 composite was 0·60 while for the 11-layer composite it was 0·19. This could manifest into a 10 db change in transmitted amplitude. This effect can be explained by the fact that even at normal incidence, the group velocity angle is given by the normal vector to the slowness curve of the solid material. For the 0 composite, this vector remains normal, while for the 11 layer case, the skewing effect is observed. The dependency of the transmission factor on the incidence angle is represented in Figure 2(b). It can be noticed that the 11-layer case exhibits a relatively low transmission factor when compared with the 0-layer case for most angles. The average transmission factor difference again translates to a 12 dB change in the transmission factor.

In Figure 3, the magnitude and the direction of the partial wave power flow (energy) vector within the 11-layer composite is plotted, as a function of distance from the incident surface ($l = 1$), for the three transmitted acoustic bulk wave modes: (a) quasi-longitudinal partial wave mode (D^{ll}), (b) fast quasi-transverse partial wave mode (D^{tf}) and (c) slow quasi-transverse partial wave mode (D^{ts}). Here, the superscripts ll, tf and ts represent $\alpha = 4$, 5, and 6 respectively. Since the particle displacement vectors are no longer in an ortho-normal relationship with the wave vector, these modes are generalized as quasi-modes. This specific case looks at an angle of incidence (θ) of 10 at a frequency of 20 kHz. In Figure 3(a), it can be observed that only D^{ll} and D^{tf} exist while the D^{ts} has zero magnitude within the composite. It can be observed that the D^{ll} is more prominent when compared with the D^{tf}. In Figure 3(b), the propagation direction of the power flow vector (in degs) is plotted with depth (distance from the incident surface) and is measured from the normal to upper surface. Since, the longitudinal partial wave mode (D^{ll}) is the dominant mode, it will influence the energy propagation more significantly when compared with the other partial wave modes. Also since the composite panel is immersed in water, which only supports the longitudinal mode, we will focus the analysis on D^{ll} for this case study.

It can be observed that the D^{ll} mode does not change the angle of energy propagation (ϕ) in the first two layers. Then in the next four layers the energy deviates considerably and

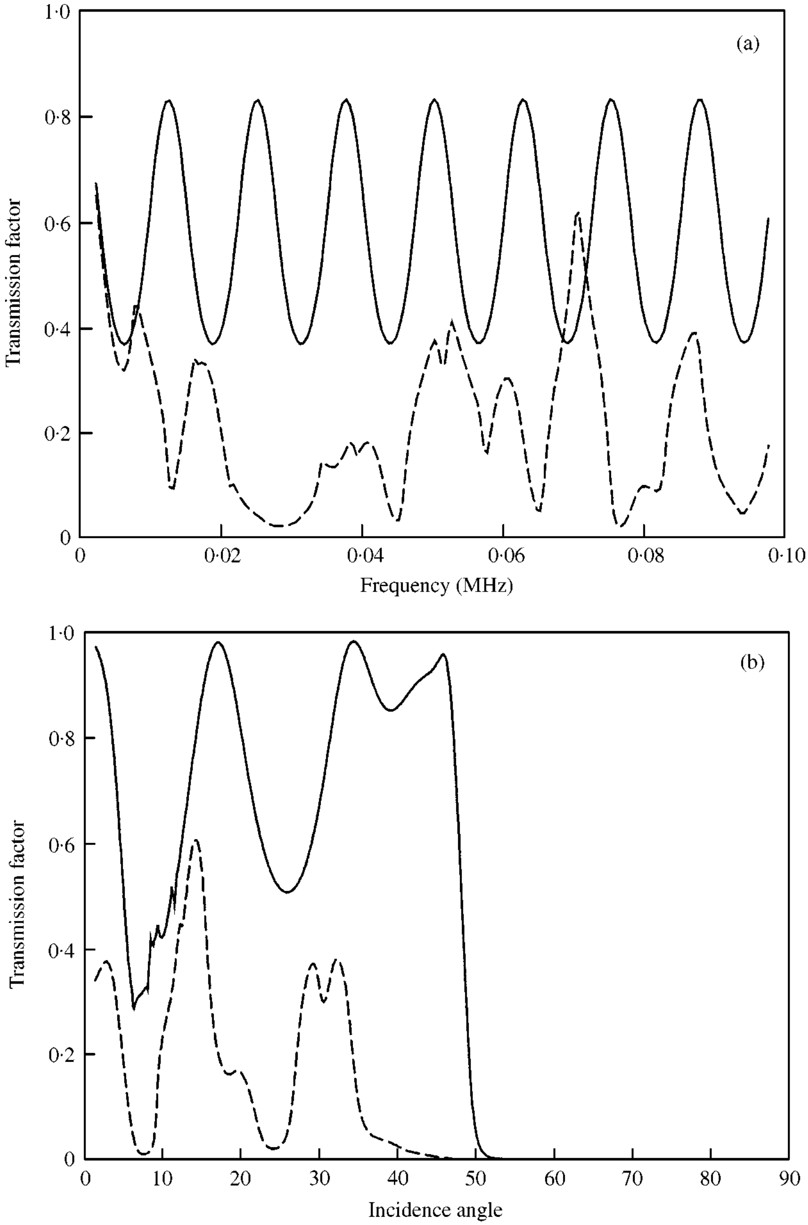

Figure 2. The energy transmission factor response, for the composite waveguide immersed in water with the acoustic energy incident at the water-composite interface, as (a) function of frequency for normal incidence, and (b) function of incidence angle for 10 kHz wave frequency: (—) 0°-layer case; (– – –) multi-layer case.

propagates at almost 30° to the normal ($\phi = 30°$). Then in the next five layers, the energy direction initially deviates back to nearly 0 and then undergoes significant change in the propagation path until in the final two layers the energy propagates nearly along the structure ($\phi = 90°$).

When the angle of incidence was kept constant and the effect of frequency on the power flow was studied, an interesting conclusion was derived. By keeping $\theta = 10°$ but decreasing

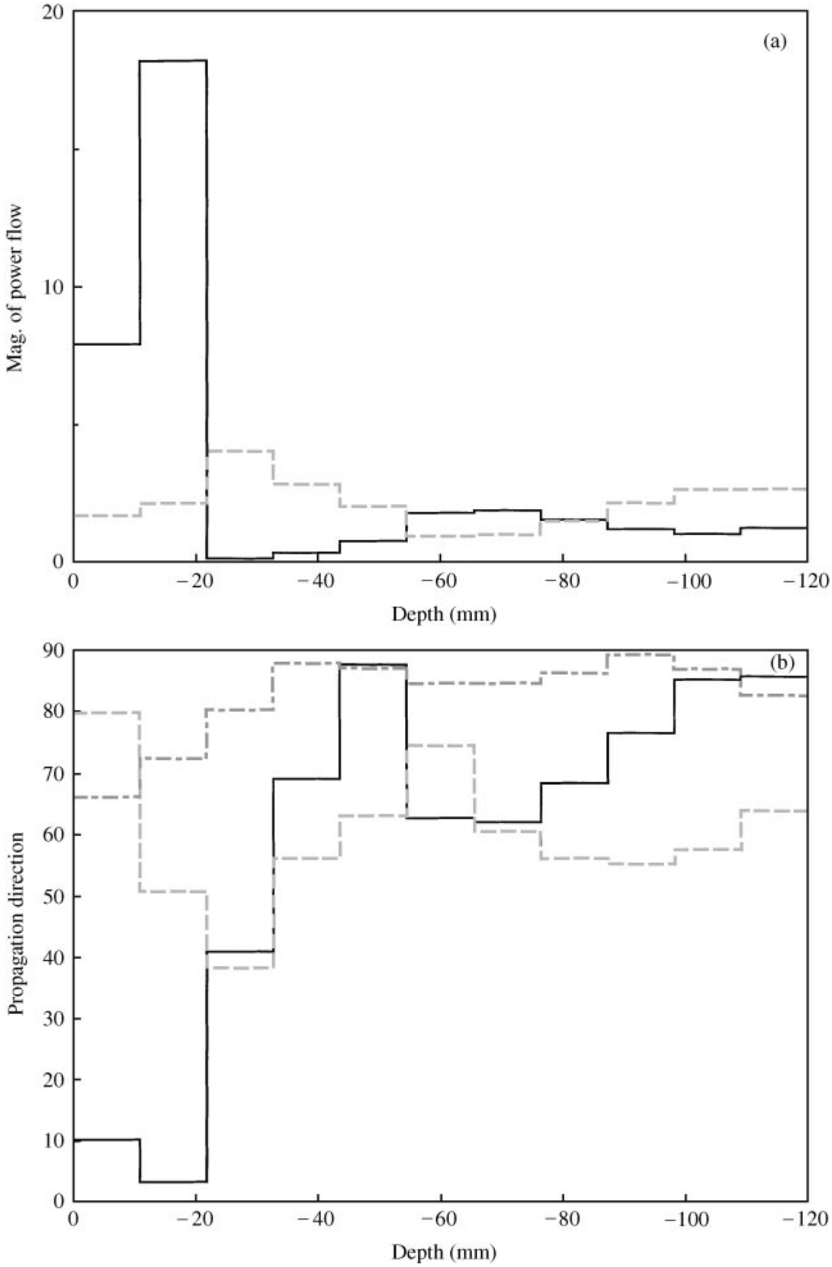

Figure 3. The distribution of power flow as a function of depth for the 11-layer waveguide immersed in water studied at an angle of incidence of 10° and a frequency of 20 kHz: (a) shows the average magnitude of the power flow vector, and (b) shows the direction of energy propagation: (—) the D^{ll} mode; (– – –) D^{tf} mode (–· –·) D^{ts} mode.

the frequency to 10 kHz, it was observed that the relative magnitude distribution of the power flow among the different layers (relative to each other) did not change significantly, with the D^{ll} mode again dominant in the bottom two layers. However, the absolute values were found to be different. When comparing the propagation direction, it was found that the wave deviation pattern did not change with frequency. This result demonstrates that the

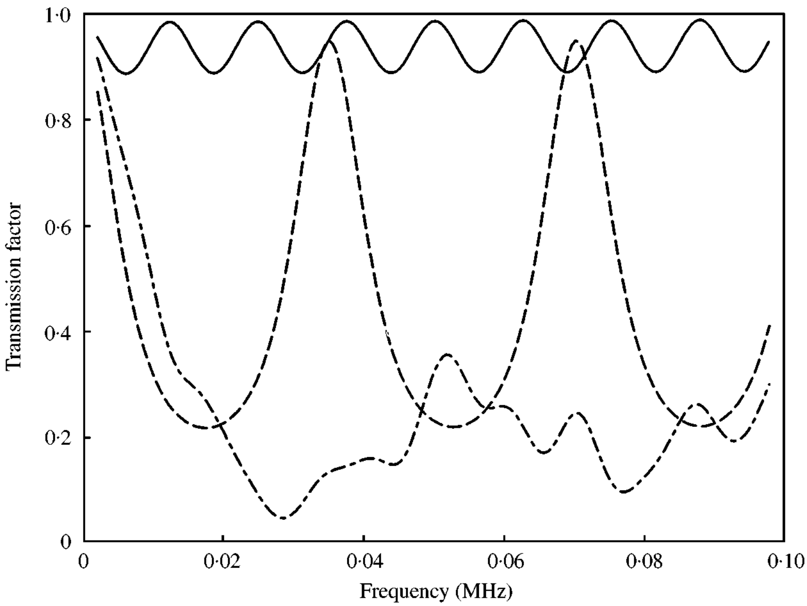

Figure 4. The transmission factor for the three composite cases embedded in between two semi-infinite plexiglass elastic solid media at normal incidence as a function of frequency: (—) 0°-layer case; (– – –) 90°-layer case; (–·–·) multi-layer case.

propagation direction is not dependent on the angle of incidence which is useful when designing optimum waveguide panels. The above results also illustrates that the 11-layer case chosen in this study is not optimized for minimum transmission of acoustic energy as was initially intended, but performed significantly better than the single-layered cases.

In Figures 4 and 5, the response of the transmission (D^{ll}) coefficients, for the three composite panels which are embedded between two semi-infinite plexiglass media (solid), as a function of angle and acoustic frequency are plotted. From the plot (normal incidence as a function of frequency – Figure 8), it can be observed that the 0° layer has the highest transmission and consequently low reflection over the frequency range up to 100 kHz. The 11-layer case displays a low transmission factor and an average reflection factor of around 0·4. The exception to this is at very low frequency below 10 kHz. The 90-layer case illustrates a transmission factor which varies from 0·9 at the resonance peaks to 0·2 in the valley. Consequently, the transmission coefficient values were high. Thus, in the case of the multi-layered composite, it is again demonstrated that the energy is not transmitted due to the waveguide effect of the anisotropy of the fibers, even when the two semi-infinite media are both elastic solids.

In Figure 5, similar results for the two semi-infinite plexiglass media were plotted, this time for an acoustic frequency of 50 kHz and the response was studied as a function of angle of incidence. Here again, the results illustrate similar behavior for all the three cases. Except, the average transmission coefficient for 11-layer case is still consistently smaller than the other two cases. This result again illustrates the anisotropic waveguide effect of a multi-layered composite plate.

These results theoretically demonstrate the feasibility of customizing/designing the reflection and the transmission of acoustic energy using anisotropic plate/panel waveguides. The potential for application in several areas including passive noise control, aero-acoustics, etc., is significant. But before that, the concept must be validated through

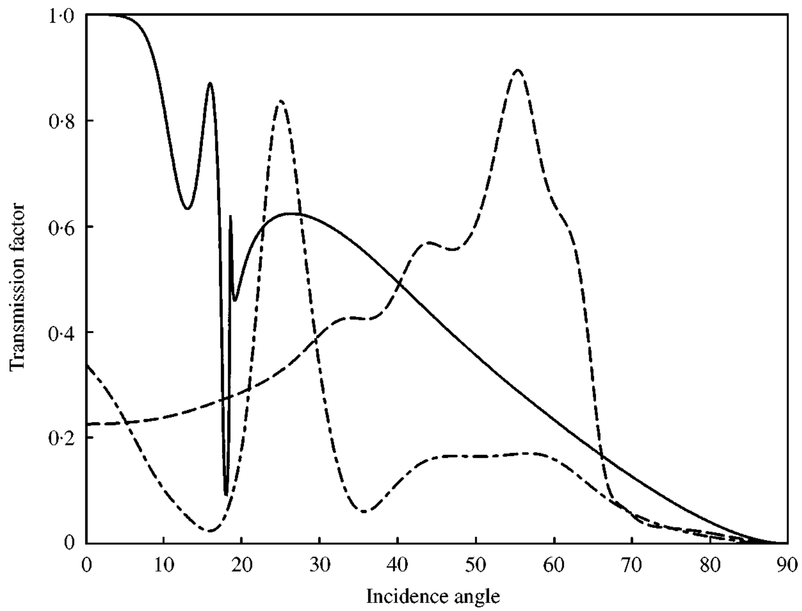

Figure 5. The transmission factor results for 50 kHz acoustic frequency at normal incidence for the 0°-layer (—), the 90°-layer (– – –), and the multi-layer (–·–), embedded in between two semi-infinite plexiglass elastic solid media.

experiments and a criteria for selection of the different anisotropic and geometrical parameters must be well defined.

Fabrication of such structures would also be challenging. Resin-infused composites from woven/braided, and/or stitched, preforms can be used to obtain panels with fibers aligned in the 1–3 plane. [17]. It must be noted that the model used here is for unbounded plane waves incident on solid layers, which were considered flat. Further modelling work must be conducted to verify if similar skewing effects can be predicted using bounded beam and spherical wave models. Also, other structural shapes must be examined.

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