Journal of Sound and Vibration (2000) **236**(1), 49–60
doi:10.1006/jsvi.2000.2981, available online at http://www.idealibrary.com on IDEAL®

NON-LINEAR VIBRATION METHOD FOR DETECTION OF FATIGUE CRACKS IN AIRCRAFT WINGS

S. L. TSYFANSKY AND V. I. BERESNEVICH

Research Laboratory "Nonlinear Phenomena of Vibrating Systems", Institute of Mechanics, Riga Technical University, 1 Kalku Street, Riga, LV-1658, Latvia

(*Received* 23 *April* 1999, *and in final form* 19 *January* 2000)

Some results of mathematical simulations of bending vibrations encountered in a cracked aircraft wing under external harmonic excitation are presented. It is shown that due to the influence of the elastic non-linearity of typical cracks, superharmonic vibration regimes appear in the system response. By utilizing specific features of these regimes a new vibration method for the detection of cracks is developed, and an original procedure for transformation of the non-linear elastic properties of the damaged cross-section is proposed. Unlike traditional procedures the vibration method proposed herein shows capability of high detection sensitivity and testing reliability.

1. INTRODUCTION

Wings are amongst the most critical structural units in any aeroplane and their physical condition determines to a considerable extent the aircraft's safety in flight. Dynamic loads acting on an aeroplane during flight encourage the onset of fatigue cracks in load-bearing elements such as wings. Immediate visual detection of these cracks is frequently a very difficult problem due to the thin-walled skin of the wing.

At the present time non-destructive testing of aircraft wings is often carried out by the use of traditional linear vibration techniques based on the mathematical representation of a crack by means of a local decrease of the flexural rigidity of the damaged cross-section of component [1, 2]. By using this linear approach the presence of cracks in a structure is detected through the monitoring of changes in its resonant frequencies [2, 3] or in its damping factors [4]. But these linear vibration procedures do not always come up to practical requirements because of their inherently low sensitivity to defects. For example, the presence of a crack which makes up about 15–20% of the undamaged cross-section area only reduces the natural frequencies of the component by 1–1·5% [3, 5]. Besides, natural frequencies and damping factors of real structures can not only change their values due to the generation of cracks, but also due to relaxation, wear, etc., and this latter change of parameters may be highly significant (up to 10–15%). Therefore, for the practical realization of linear vibration diagnostic methods it is necessary to determine initially the natural frequencies and damping characteristics of the structure in its undamaged condition and then to monitor the change of these parameters while it is in use. Such a procedure can be very laborious and can also require many working hours of trouble-shooting. In addition to this, high-precision, expensive, instrumentation has to be used because of the generally low sensitivity of simple linear vibrations detection and modelling techniques.

The disadvantages mentioned above are not inherent to non-linear methods of vibration monitoring [6–11] based on the utilization of non-linear vibration effects as a collection of diagnostic parameters in order to point to the appearance of a fault. Taking such an approach a non-linear model of an open and closed crack is used. At the present time various non-linear models of a crack are known distinguishable from each other by their different levels of mathematical complexity and their individual physical nature [5–15]. The most detailed description of stress distribution near a crack can be obtained from the use of finite element models [5, 12, 13]. Methods of analysis based on these models are of very wide general application and certainly can give an excellent solution to the structural problem considered in this paper.

But finite element crack models are usually too intricate and require high-speed computer facilities with specialized software and a large memory for their application. Therefore, in some particular applications it is acceptable to use the mathematically more simple piece-wise linear crack model [7–9, 16]. This model considers the opening and the closing of the crack edges as a momentary change in the flexural rigidity of the damaged cross-section of the component via the sign of the bend angle. In considering a bar for example [16], a piecewise linear crack model (in spite of its mathematical simplicity) ensures a sufficiently high accuracy in the dynamic analysis of beam-type structures with relatively small transverse cracks, whose value is not more than 20–25% of the cross-sectional area of the undamaged component. Structural dynamic problem of such type is considered in this paper. That is why it has been found expedient to use in the case studied the more simple piece-wise linear crack model.

High-sensitivity super-resonant vibration methods for crack detection in beam-type structural elements are proposed in references [7, 8]. These methods can be developed on the assumption of piece-wise linear crack elastic characteristic with a bend at the origin of the co-ordinates. However, such a model only holds true for components of relatively low weight (e.g., aerials of aeroplanes) or in cases of vertical positioning of the test component. If large, massive, components have to be tested (e.g., aircraft wings) it is necessary to take into account the influence of the component's own weight on the regular opening and closing of the crack edges during vibration. A corresponding revision to the procedure of super-resonant diagnostics is also necessary. Such a problem is considered in this paper.

2. DYNAMIC MODEL

The dynamic analysis is carried out with reference to the Russian aeroplane designated as the YAK-40. The wings on this aeroplane each comprise a solid-spar homogeneous viscoelastic beam component which resists the greater part of external loads. In accordance with the experimental data given by the manufacturers, flexural rigidity, EI, and distributed mass, μ, of this type of wings are predominantly determined by the spar parameters. Therefore, the most dangerous damage to the wing is that which occurs in the spar.

Taking account of these structural features of a wing its dynamic model may be defined by using a viscoelastic cantilever beam performing bending vibrations excited by a harmonic test force, $P \sin \omega t$ (Figure 1(a)). The reliability of this sort of dynamic model has been verified by experiments on real aircraft [3, 17]. Actual values of the flexural rigidity, $EI(x)$, and distributed mass, $\mu(x)$, of the cross-section of the wing vary with the co-ordinate x as shown in the experimental graphs presented in Figures 1(b) and 1(c). In the case studied here the ratio of the wing length l to its half-way depth is about $l/h \approx 40$, therefore the effects of lateral shear and rotary inertia of the beam cross-sections may be neglected [18].

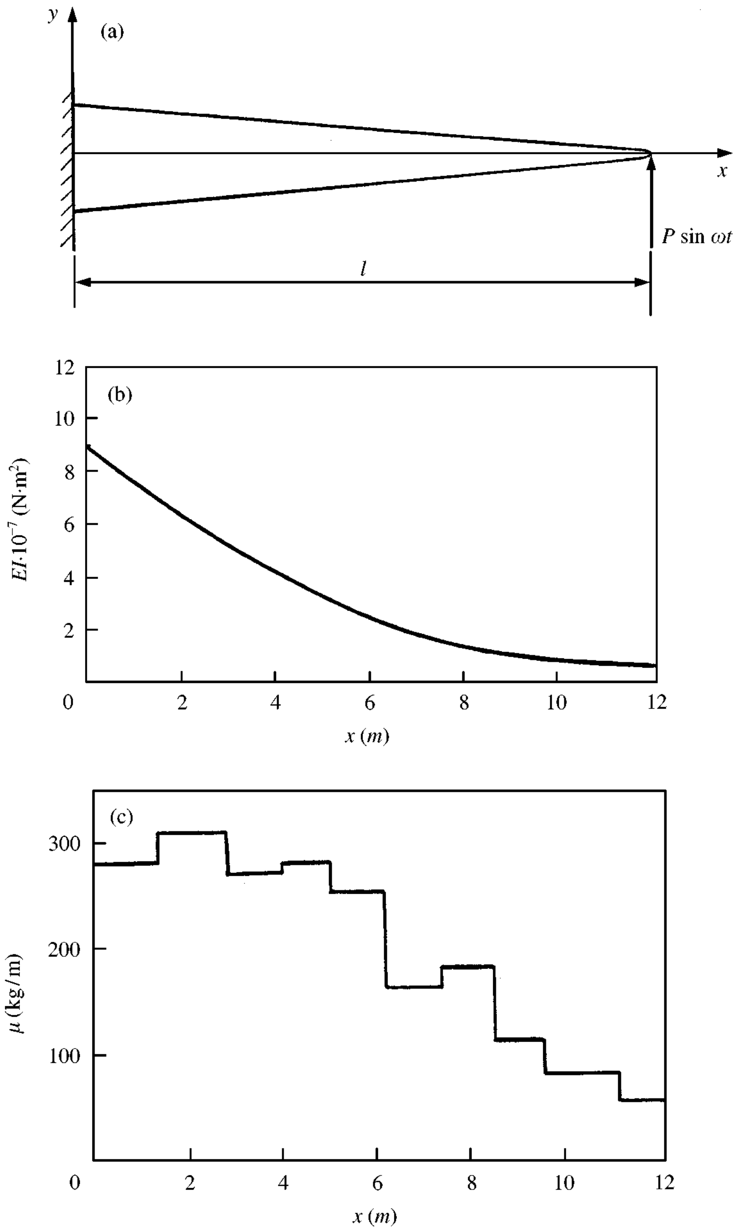

Figure 1. Schematic of the wing of the YAK-40 aeroplane (a) graphs of flexural rigidity EI (b) and distributed mass μ (c) of wing cross-section versus co-ordinate x.

Under the assumptions made the differential equation for bending vibrations of the wing can be stated as follows:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y}{\partial x^2}\right] + \mu(x)\frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left[bEI(x)\frac{\partial^3 y}{\partial t \partial x^2}\right] = q(x,t), \tag{1}$$

where $EI(x)$ and $\mu(x)$ are the flexural rigidity in bending and the distributed mass, respectively, of the wing cross-section with co-ordinate x; y is the lateral displacement of the wing cross-section measured from the static equilibrium position; and b is the coefficient of internal friction.

The intensity of the distributed load $q(x, t)$ is given by

$$q = P \sin \omega t \, \delta_1 (x - x_P), \tag{2}$$

where P and ω are the amplitude and the frequency of the external harmonic excitation; $\delta_1(x - x_P)$ is a Dirac delta function; and $x_P = l$ is the co-ordinate of the cross-section to which the external harmonic force is applied.

In the case of a cantilever beam spar-model the boundary conditions are as follows;

$$y(0, t) = 0, \quad \frac{\partial y}{\partial x}(0, t) = 0, \quad \frac{\partial^2 y}{\partial x^2}(l, t) = 0, \quad \frac{\partial^3 y}{\partial x^3}(l, t) = 0. \tag{3}$$

A fatigue crack occurring in a wing spar can be considered as an additional elastic non-linearity describing the cyclical process of opening and closing of the crack edges during vibration. This paper considers relatively small cracks whose value is not more than 20–25% of the undamaged cross-sectional area. In the case studied here (according to references [9, 16]) it is expedient to use a piece-wise linear crack model, which, in spite of its mathematical simplicity, ensures a sufficiently high accuracy for the dynamic analysis of many structures in which cracks occur.

Mathematically, the cyclical process of opening and closing of the crack edges during the vibration is simulated using a crack parameter, σ. The parameter σ is a step function describing the relationship between the flexural rigidity of the damaged cross-section and the bend angle $\varphi = \partial y / \partial x$. The mathematical form of this function is dependent on the location of the fatigue crack above or below the centreline along which x is measured, as shown in Figure 1.

In the case of an upper-half crack location the crack edges for the static equilibrium position of the system are initially opened due to the action of the wing's own weight. The elastic characteristic of the damaged cross-section in this case is as shown in Figure 2(a), and the crack parameter σ is described by the expression

$$\sigma = \begin{Bmatrix} 0, & \dfrac{\partial y}{\partial x} > \varphi_0 \\ \sigma_c, & \dfrac{\partial y}{\partial x} \leqslant \varphi_0 \end{Bmatrix}, \tag{4}$$

where $\sigma_c = (1 - I_d / I_0)$ is a measure of the relative crack value, I_d being the second moment of the damaged cross-section (on the condition that the crack is opened), and I_0 is the second moment of the undamaged cross-section, φ_0 is the threshold value of the bend angle, φ, corresponding to the instant of closing (or opening) of the crack edges.

If the crack is located in the lower-half of the beam spar the action of the wing's own weight causes an initial closing of the crack edges for the static equilibrium position of the system. The elastic characteristic of the damaged cross-section becomes as shown in Figure 2(b), and the crack parameter, σ, in this case is described by the expression

$$\sigma = \begin{Bmatrix} 0, & \dfrac{\partial y}{\partial x} \leqslant \varphi_0 \\ \sigma_c, & \dfrac{\partial y}{\partial x} > \varphi_0 \end{Bmatrix}, \tag{5}$$

The crack parameter, σ, defines the change in flexural rigidity of the damaged cross-section $EI(x = x_d)$ in accordance with the following mathematical expression

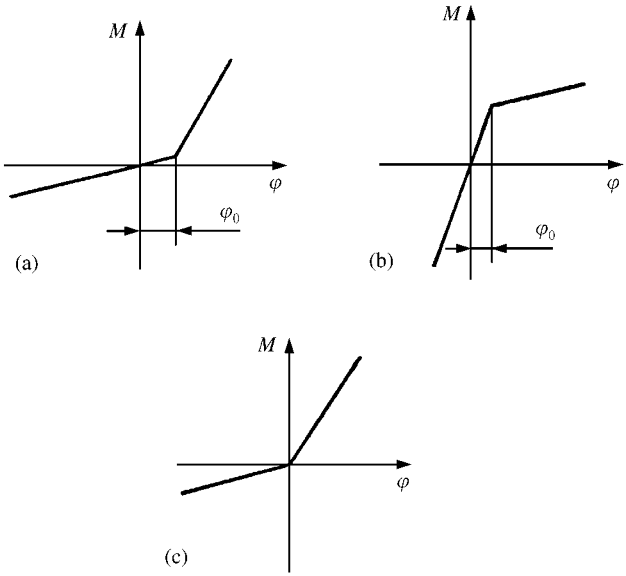

Figure 2. Elastic characteristic of the damaged cross-section of the wing: (a) case of crack location in the upper part of the beam spar (crack edges are initially opened); (b) case of crack location in the lower part of the beam spar (crack edges are initially closed); (c) crack model after optimization of the elastic properties of the damaged cross-section. Here M is the bending moment in the damaged cross-section of the wing.

[8, 9, 16]:

$$EI(x = x_d) = EI_0[1 - \sigma\delta(x - x_d)], \tag{6}$$

where EI_0 is the flexural rigidity in bending of the undamaged section, $\delta(x - x_d)$ is a Dirac delta function and x_d is the co-ordinate of the damaged cross-section of the wing.

Equation (1), subject to the expressions (2)–(6), was solved on an analogue-digital computer system predominantly set up for the solution of complex non-linear dynamics problems [19–21]. This method of dynamic analysis is aimed at some particular applications (like that considered in this paper). Other methods based on finite element crack model [5, 12–14] are of more wide general application and can also offer excellent solution to this structural problem.

The problem considered in this paper was solved assuming the parameters of equations (1)–(6) to be according to the measured inertial and elastic parameters of the aircraft wing (see Figures 1(b) and 1(c)). The location of the damaged cross-section was taken near restraint ($x_d = l/20$), where the origin of the fatigue crack is the most probable because of the maximal level of dynamic stresses at that point.

3. OPTIMIZATION OF THE ELASTIC CHARACTERISTICS OF THE DAMAGED CROSS-SECTION OF THE WING

As has been noted, a fatigue crack arising in the wing spar, initially in the static equilibrium position of the structure, is opened or closed (stressed) due to the action of the wing's own weight. Therefore, the regular process of opening and closing of the crack edges can only be realized in the case of rather high levels of wing-bending vibrations, when the maximal bend angle $\varphi = \partial y/\partial x$ exceeds the threshold value φ_0 (Figures 2(a) and 2(b)).

Excitation of such large vibrations becomes possible only under a harmonic test force of very high amplitude P.

From the results of mathematical simulation it follows that the non-linear regimes of wing-bending vibration caused by crack "breathing" occur only under the influence of high amplitude P, which is necessarily of the same order as the wing's own weight, G. For example, the second and third order superharmonic vibrations corresponding to the first flexural mode can be excited subject to the condition that $P \geqslant 0{\cdot}22G$, but half order subharmonic vibrations on the condition that $P \geqslant 0{\cdot}5G$. For lower values of P the amplitude of the wing-bending vibrations, over the frequency ranges of superharmonic ($\nu < 0{\cdot}85$) and subharmonic ($\nu > 1{\cdot}15$) responses, is insufficient for the initiation of closing and opening of the crack edges. In this case, the wing-bending vibrations only occur within the limits of the linear part of the elastic characteristic of the damaged cross-section, i.e. when $\partial y/\partial x < \varphi_0$ (Figures 2(a) and 2(b)). Therefore, the dynamic response of the system under the harmonic test excitation will stay within the initial linear properties, even if there is a crack in the wing.

Therefore, the practical problem of realizing superharmonic or subharmonic vibrations when doing non-destructive wing testing runs into the necessity to increase the amplitude P of the external excitation. However, excessive increase of P during vibration monitoring is extremely undesirable because it can cause a dangerous concentration of dynamic stresses within the damage zone and therefore accelerate the growth of the crack. In addition, the generation of a harmonic test force of very high amplitude (of the same order as the wing's own weight) calls for the use of especially powerful vibroexciters.

Fortunately, the problem under scrutiny here can be approached not only by such an excessive increase in the amplitude, P, of the external harmonic excitation, and a more preferable method is based on the idea of force-shifting the bending point, $\varphi = \varphi_0$, of the elastic characteristic of the damaged cross-section (see Figures 2(a) and (b)) into the equilibrium position ($\varphi_0 = 0$; see Figure 2(c)). The implementation of such a shift will make it possible to achieve the cyclical process of opening and closing of the crack edges at any amplitude, P, however small, of the harmonic test force [8]. On this basis, the zero position of the bending point, $\varphi = \varphi_0$, is considered to be optimal for an alternative non-linear damage diagnostics procedures. A simple way of attaining this is proposed next and is based on the insertion into the test structure of an additional pre-compressed elastic support, which has to be placed between the stationary base and the free end of the wing (Figure 3). The main function of the elastic support is to balance the influence of the wing's own weight with the position of the bend point, $\varphi = \varphi_0$, of the damaged wing's cross-section. The necessary conditions for such balancing have to be determined by means of dynamic analysis of the structure.

The bending vibrations of the wing when interacting with the additional elastic support are described by the partial differential equation (1), taking due account of the boundary conditions (3) and the non-linear crack functions (4)–(6). The influence of the elastic support on the dynamic behaviour of the system is taken into account by insertion of an additional term into the mathematical expression for the distributed load $q(x, t)$:

$$q(x, t) = P \sin \omega t \delta_1 (x - x_p) - k(y - y_{st}) \delta_1 (x - x_k), \tag{7}$$

where k and y_{st} are the stiffness coefficient and the initial static deformation in compression of the additional elastic support; $x_k = 0{\cdot}9l$ is the co-ordinate of the cross-section when interacting with the additional elastic support.

Solution of equation (1), taking into account expressions (3)–(7), shows that the most intensive super- and subharmonic bending vibrations in the wing correspond to the first flexural mode. Therefore, it is these responses that are used as the necessary theoretical basis

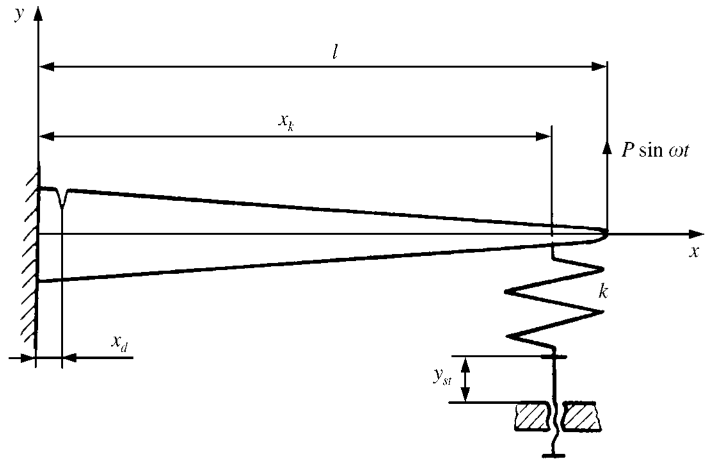

Figure 3. Schematic of the wing interacting with additional pre-compressed elastic support.

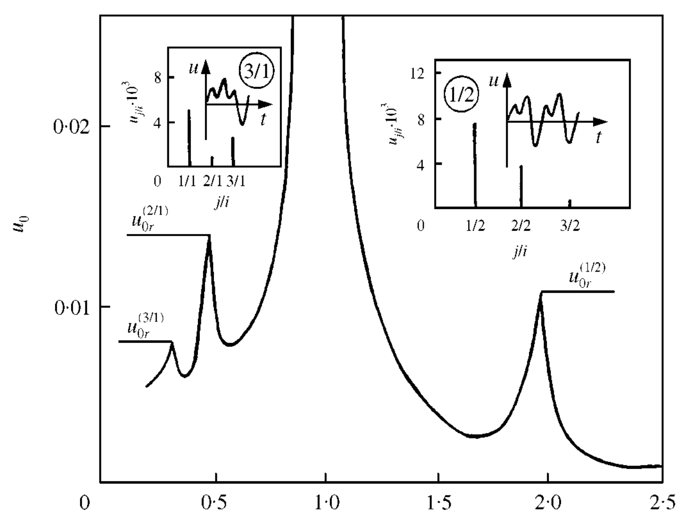

Figure 4. Amplitude–frequency characteristic for the first mode of bending vibrations of the cracked wing ($P = 0{\cdot}2G$; $\sigma_c = 0{\cdot}17$; $ky_{st}/G = 0{\cdot}34$).

for the development of this diagnostic procedure. As an example, Figure 4 shows the amplitude–frequency characteristics which graphically represent the relationship between the driving frequency, $v = \omega/\omega_1$, and the half-swing of vibrations, $u_0 = y_0/l$, where ω_1 is the first natural frequency of wing-bending vibrations in the undamaged state, and y_0 is the half-swing of bending vibrations of the free end of the wing ($x = l$). This characteristic has been plotted assuming $P = 0{\cdot}2G$, $\sigma_c = 0{\cdot}17$, and $ky_{st}/G = 0{\cdot}34$. As additional information the time responses, $u = f(t)$, and the spectrograms for third order superharmonic resonance and half order subharmonic resonance are presented (symbols j/i on the spectrograms denote the ordinal numbers of the harmonic components of the vibration spectrum).

From the mathematical simulations the static elastic action ky_{st} on the free end of the wing is seen to be directly connected to the co-ordinate of the bend point, $\varphi = \varphi_0$, of the

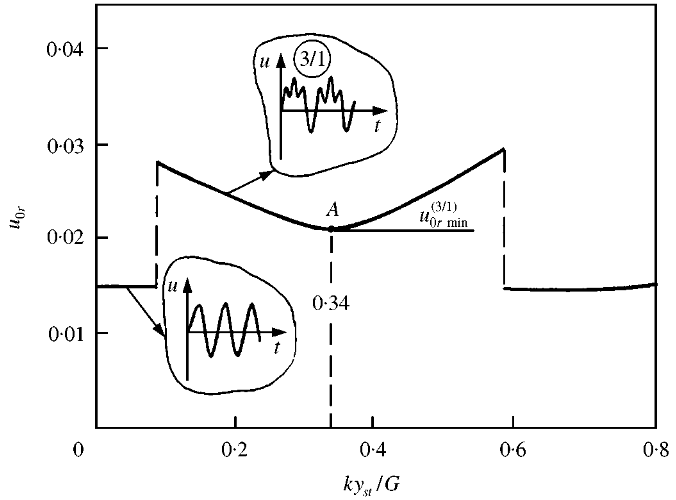

Figure 5. Relationship between the half-swing $u_0^{(3/1)}$ of third (3/1) order superharmonic resonant vibrations and the compression ratio, ky_{st}/G, of the additional elastic support ($P = 0{\cdot}2G$; $\sigma_c = 0{\cdot}17$).

elastic characteristic of the damaged cross-section. However, it is very difficult to measure the value of φ_0 directly within the damaged cross-section of the wing, therefore it has been found expedient to use an indirect method of vibration testing using the interconnections between parameter φ_0 and the characteristic features of the wing-bending vibrations.

In accordance with the problem as stated here it is necessary to determine only one zero position ($\varphi_0 = 0$) of the bend point, $\varphi = \varphi_0$, of the damaged cross-section. From the results of the simulation the condition $\varphi_0 = 0$ is satisfied when the half-swing $u_0^{(3/1)}$ of the third (3/1) order superharmonic vibrations reaches its negative peak value. Figure 5 shows in graphical form the relationship between the half-swing of the wing-bending vibrations $u_0^{(3/1)}$, measured on the third (3/1) order superharmonic resonance and compression ratio ky_{st}/G of the additional elastic support (the graph is plotted assuming $P = 0{\cdot}2G$). The local minimum of this graph corresponds to the zero position of the bend point, $\varphi = \varphi_0$, of the damaged cross-section, subject to the condition $ky_{st}/G = 0{\cdot}34$. But if the parameter ky_{st}/G falls outside the limits given by the inequality $0{\cdot}09 \leqslant ky_{st}/G \leqslant 0{\cdot}59$ then the cyclical process of opening and closing of the crack edges is terminated and as a result linear harmonic vibrations are established in the system. The amplitude, u_0, of these vibrations is not related to the static precompression ratio, ky_{st}/G, of the additional elastic support.

So, in order to transform the piecewise linear elastic characteristic of the fatigue crack into an optimal form in which the bend point, $\varphi = \varphi_0$, is in the zero position, it is enough to tune the testing system to the superharmonic resonance of third (3/1) order and to minimize the half-swing of vibrations $u_0^{(3/1)}$ by a smooth changing of the static compression value, ky_{st}, of the additional elastic support. The excitation of superharmonic vibrations in itself points to the presence of defects in the wing under test. Final conclusions to the problem of the existence of a crack can be made by detection of the value σ_c.

4. SUPER-RESONANT METHOD OF QUANTITATIVE VIBRATION TESTING

As has been established, so far superharmonic responses of wing-bending vibrations can be excited under substantially smaller crack values, σ_c, than in the case of subharmonic

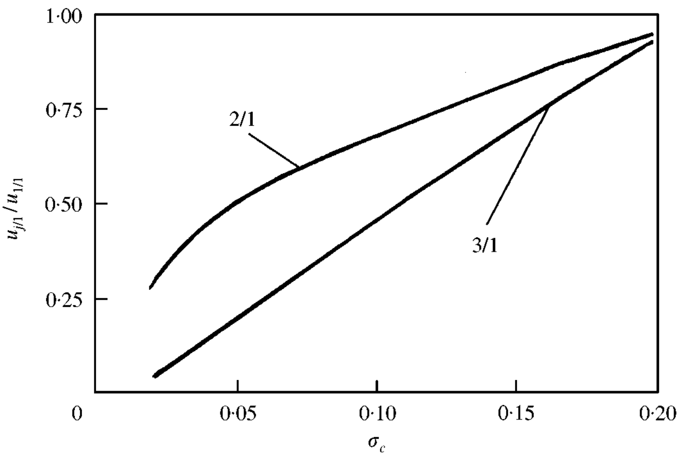

Figure 6. Graphs of spectral ratios $u_{2/1}/u_{1/1}$ and $u_{3/1}/u_{1/1}$ as functions of the crack parameter, σ_c, for second (2/1) order and third (3/1) order superharmonic regimes.

responses. For example, if a fatigue crack is located in the cross-section with co-ordinate $x_d = l/20$ then the excitation of superharmonic wing vibrations is possible for $\sigma_c \geqslant 0{\cdot}075$. Therefore, application of the superharmonic responses for vibration testing initially predetermines the lower threshold sensitivity, and that is the important advantage of a superharmonic approach.

In selection of the ordinal number $j/1$ of the superharmonic response for vibration testing it is necessary to take into account the intensity of the non-linear resonance, as well as the sensitivity required for recording the vibration response (diagnostic parameter) to the growth of a crack. Due to its intensity the most preferable condition is the second (2/1) order superharmonic regime (Figure 4). But from the standpoint of sensitivity of testing the superharmonic response of third (3/1) order is preferable, because in this case the system has a higher response with possible changes of crack value σ_c.

As an example, Figure 6 shows the relationship between the crack parameter, σ_c, and the spectral ratios $u_{2/1}/u_{1/1}$ and $u_{3/1}/u_{1/1}$, measured for the second (2/1) and the third (3/1) superharmonic resonances (here $u_{2/1}$ and $u_{3/1}$ are the amplitudes of the superharmonic components of the vibration spectrum of the second (2/1) and the third (3/1) superharmonic regimes, and $u_{1/1}$ is the amplitude of the fundamental harmonic). In spite of the relatively small intensity of the third (3/1) order superharmonic response, the variation of the spectral ratio $u_{3/1}/u_{1/1}$ with the increase of parameter σ_c is much higher in comparison with the variation of parameter $u_{2/1}/u_{1/1}$, measured for the second (2/1) order response. According to the analysis, as the crack value goes above 1% the spectral ratio, $u_{2/1}/u_{1/1}$, varies by 8–10%, whereas the ratio $u_{3/1}/u_{1/1}$ changes by 13–15%. In other words, the relative sensitivity of the diagnostic parameter $u_{3/1}/u_{1/1}$ to the growth of a fatigue crack is about 1·5–1·7 times higher in comparison with the sensitivity of diagnostic parameter $u_{2/1}/u_{1/1}$. Therefore, it has been proposed that the vibration testing should be carried out by recording the spectral ratio, $u_{3/1}/u_{1/1}$, on the third (3/1) order superharmonic resonance.

A schematic of the installation to implement the proposed super-resonant diagnostic vibration procedure is shown in Figure 7. The test component (the aircraft wing) is connected to the additional elastic support and adjustment of the static compression value, ky_{st}, of this elastic support is carried out by means of a special mechanism based on a ball joint, a screw, a stationary nut and a worm-gear. Due to the high gear ratio of the

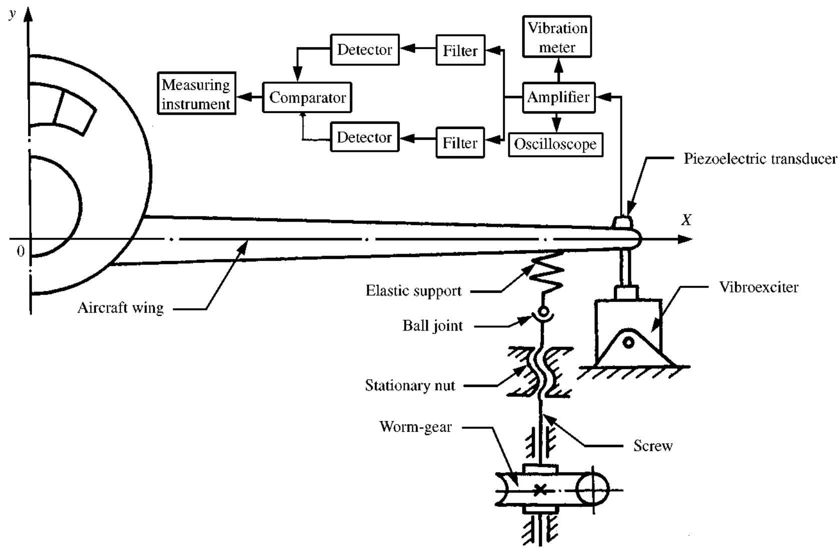

Figure 7. Schematic of the rig installation.

worm-gear (about 50 : 1) it is possible to make slow vertical displacements of the screw by relatively fast worm-handle rotation. By recourse to this simple system any required value for ky_{st} may be obtained to a high precision. Bending vibrations of the wing are excited by a vibroexciter and the vibration parameters of the wing are measured by use of standard electronic instrumentation.

Detection of fatigue cracks in aircraft wing by the super-resonant vibration method proposed in this paper may be carried out by the following operational procedure. Initially, the elastic support is mounted without any preliminary compression or extension ($y_{st} = 0$). Then the forced bending vibrations of the test wing are excited at a frequency which is three times smaller than the first natural frequency, ω_1, of the system. The Fourier spectrum of the wing's flexural vibrations is analyzed. The presence of a third (3/1) order harmonic component in the spectral response of the system indicates the potential origin of a fatigue crack in the component.

The next stage, of quantitative vibration monitoring, requires that the elastic characteristic of the damaged cross-section must be optimized so that the bend point is in the zero position. To this end the static compression value, ky_{st}, of the elastic support is continuously changed, until the half-swing of the vibrations, $u_0^{(3/1)}$, indicated by the vibration meter reaches the negative peak value (see Figure 5). After that the amplitudes of the harmonic components, $u_{3/1}$, and, $u_{1/1}$, in the frequency spectrum are measured. The size, σ_c, of the crack is evaluated by using the magnitude of the spectral ratio, $u_{3/1}/u_{1/1}$, from a pre-constructed calibration curve, $\sigma_c = f(u_{3/1}/u_{1/1})$. A typical curve is presented in Figure 6.

It is obviously very important that both the adjustment of the elastic characteristic of the damaged cross-section and the ensuing crack value-detection are carried out for the same 3/1 order superharmonic vibration response. The procedure of vibration testing is also sufficiently simplified and made robust due to the insensitivity of the spectral ratio, $u_{3/1}/u_{1/1}$, to possible changes of the amplitude, P, of the external harmonic excitation. The main advantage of this proposed super-resonant method for non-destructive testing lies in its detection sensitivity. This is about 10 times higher than that obtainable from familiar linear vibration procedures [2, 3].

5. CONCLUSIONS

The results presented in this paper define a new methodology for the detection of fatigue cracks in aircraft wings. The specific dynamic analysis described relates to wing vibrations in a YAK-40 aeroplane. Simulations show that the appearance of a fatigue crack has a stimulating influence on the excitation of superharmonic responses in wing-bending vibrations. Utilization of these superharmonic vibration effects has made it possible to develop a new, and highly sensitive vibration methodology for crack detection. An original procedure for optimization of the elastic characteristic of the damaged cross-section of the wing is proposed. It is shown that the detection sensitivity of the proposed super-resonant method for vibration testing is about 10 times higher than that obtainable from familiar linear vibration procedures.

REFERENCES

1. B. GRABOVSKI 1980 *Transactions of the American Society of Mechanical Engineers, Series B. Journal of Engineering for Industry* **102**, 140–148. The vibrational behaviour of a turbine rotor containing a transverse crack.
2. I. W. MAYES and W. G. DAVIES 1976 *Proceedings of the International Mechanical Engineering Conference, Cambridge, England*, 53–64. The vibrational behaviour of a rotating shaft system containing transverse crack.
3. A. B. MILOV, YU. N. NEVSKY, G. I. STRAKHOV and YU. G. UKHOV 1983 *The Problems of Dynamics and Strength*, Vol. **43**, 97–100. Riga: Zinatne. Utilization of dynamic properties for non-destructive testing of aviation structures (in Russian).
4. O. T. SIDOROV 1982 *Soviet Journal of Non-destructive Testing* **18**, 75–78. The influence of fatigue damages in built-up construction on damping.
5. I. IMAM, S. AZARRO, R. BANKERT and J. SCHEIBEL 1989 *Journal of Vibration, Acoustics, Stress and Reliability in Design* **3**, 241–250. Development of an on-line rotor crack detection and monitoring system.
6. S. TSYFANSKY and V. BERESNEVICH 1996 *Proceedings of the International Symposium "Analysis and Synthesis of Nonlinear Dynamical Systems in Mechanics"*, 140–147. Riga: Riga Technical University. Nonlinear vibrodiagnostics of engineering objects.
7. S. L. TSYFANSKY and V. I. BERESNEVICH 1998 *Journal of Sound and Vibration* **213**, 159–168. Detection of fatigue cracks in flexible geometrically non-linear bars by vibration monitoring.
8. S. L. TSYFANSKY, M. A. MAGONE and V. M. OZHIGANOV 1985 *Soviet Journal of Non-destructive Testing* **21**, 77–82. Utilization of nonlinear effects for detection of cracks in the elements of construction rods.
9. H. D. NELSON and C. NATARAJ 1986 *Transactions of the American Society of Mechanical Engineers, Series B. Journal of Engineering for Industry* **108**, 52–66. The dynamics of a rotor system with a cracked shaft.
10. A. D. DIMARAGONAS and C. A. PAPANDOPAULOS 1983 *Journal of Sound and Vibration* **91**, 583–593. Bending vibration of a cracked shaft.
11. K. WORDEN, G. MANSON and G. R. TOMLINSON 1993 *Proceedings of the Royal Society of London* **A 445**, 193–220. Nonlinearities in engineering structures.
12. B. O. DIRR and B. K. SCHMALHORST 1988 *Transactions of the American Society of Mechanical Engineers, Series B. Journal of Engineering for Industry* **110**, 158–164. Crack depth analysis of a rotating shaft by vibration measurement.
13. W. M. OSTACHOWICZ and M. KRAWCZUK 1990 *Computers and Structures* **36**, 245–250. Vibration analysis of a cracked beam.
14. M. KRAWCZUK and W. M. OSTACHOWICZ 1992 *Archives of Applied Mechanics* **62**, 463–473. Parametric vibrations of beam with crack.
15. M. I. FRISWELL and J. E. T. PENNY 1997 *Structural Damage Assessment Using Advanced Signal Processing Procedures. Proceedings of DAMAS '97, University of Sheffield, U.K.*, 351–362. Is damage location using vibration measurements practical?
16. R. GASCH, M. PERSON and B. WEITZ 1983 *Institute of Mechanical Engineering* **C 314/88**, 463–472. Dynamic behaviour of the Laval rotor with a cracked hollow shaft — a comparison of crack models.

17. S. L. Tsyfansky, V. M. Ozhiganov, A. B. Milov and Yu. N. Nevsky 1981 *The Problems of Dynamics and Strength*, Vol. **39**, pp. 3–10. Riga: Zinatne. On the method for detection of damages in aircraft wing by the analysis of its nonlinear vibrations (in Russian).
18. S. P. Timoshenko, D. H. Young and W. Jr. Weaver 1974 *Vibration Problems in Engineering*. New York: Wiley.
19. S. L. Tsyfansky, V. I. Beresnevich and A. B. Oks 1991 *Nonlinear and Parametric Oscillations of Technological Vibromachines*. Riga: Zinatne (in Russian).
20. S. Tsyfansky and V. Beresnevich 1996 *Proceedings of the 5th International Conference of Cranes and Textile Machines*, *Gdansk*, *Poland*, 257–260. Mathematical simulation of dynamics of textile machines on specialized analog–digital computational complex.
21. S. L. Tsyfansky 1979 *Electric Simulation of Oscillations in Complex Nonlinear Mechanical Systems*. Riga: Zinatne (in Russian).