



## ACOUSTIC COUPLING EFFECTS IN ST PAUL'S CATHEDRAL, LONDON

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*(Received 20 July 1999, and in final form 8 November 1999)*

In St Paul's Cathedral there are many arches, columns and cornices which enable the internal space to be divided into subspaces. The subspaces may be considered to be acoustically coupled via areas which connect the rooms. Two of the most acoustically important subspaces in the Cathedral are the choir and the space under the dome. The choir, the space within the wooden choir stalls, has more sound absorption than the rest of the building, which is mostly marble and Portland stone. In the model of coupled subspaces an acoustic energy balance equation, applied to a diffuse field, is derived for each subspace. In St Paul's Cathedral the internal space is divided into 70 acoustical subspaces. The initial-value problem which is formulated by the system of 70 acoustic energy balance equations with initial conditions has been reduced to the eigenvalue problem. The decay of sound energy density has been obtained for different locations in the Cathedral and for different positions of the sound source. Experimentally obtained sound decay curves are in good agreement with numerical results. Both the experimental and numerical results demonstrate the fine structure of reverberation.

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### 1. INTRODUCTION

The aim of this paper is the study of the fine structure of reverberation in St Paul's Cathedral, London. In particular, the intention is to investigate the early stage of the sound decay which is the part that contributes mostly to the subjective perception of speech and music [1–8]. Non-exponential sound decay may be associated with different phenomena such as direct sound, presence or lack of early reflections, two-dimensional reverberation, etc. A general classification of the different shapes of early decay curves, including cliff-type, plateau-type and sagging-type, has been presented by Barron [7]. Two of the most significant objective factors characterizing auditoria, early decay time EDT and reverberation time RT, are also features of fine structure of the reverberation [7, 9–15]. They define a two-stage sound decay which is an approximation of the non-exponential sound decay.

Non-exponential sound decays can also result from coupling between different subspaces of an enclosure. The character of the interior of St Paul's Cathedral raises the expectation that coupling effects should occur. It is this aspect of fine structure of reverberation which is investigated in this work.

Rooms which are coupled together by a small open area are known to exhibit some interesting acoustical phenomena. Acoustic coupling between two rooms has been investigated both experimentally and theoretically by Eyring [16]. Subsequently, acoustic coupling was studied in detail by Cremer and Müller [4]. The most dramatic evidence of

coupling occurs when one of the two rooms contains a large amount of acoustic absorption and the second room is more reverberant. If only one source exists, and that source is in the room with large absorption, the decay of the sound energy density level for the source room exhibits a "sagging" appearance [4]. The curve consists of two distinctive parts which refer to the early and late stages of the sound decay. The coupling phenomenon is most noticeable when there is a small coupling area between the rooms; in this case, the coupling is referred to as weak or loose. If the coupling area between the rooms is large, the coupling phenomenon is less evident and it is possible to regard the two rooms as one combined room.

Recordings of decay curves which show coupling effects are not often quoted in the literature. The phenomena of coupling may be observed in churches where there are side chapels or aisles [17] or a dome [18] or where the choir is connected to the nave by a chancel arch. Raes and Sacerdote [17] have shown the results of measurements of sound decay in two Roman basilicas. They have reported the phenomenon of acoustic coupling between the central and lateral naves of the church of St John Lateran in Rome.

In order to detect the effect of acoustic coupling from the sound decay curves, careful measurements have to be made in the early stage of the sound decay. Some evidence of coupling may have been recorded in St Paul's Cathedral, when the decay of sound was measured in the choir with the source in the pulpit (close to the boundary between the choir and the crossing) [19].

The interior of St Paul's Cathedral is divided into many acoustical subspaces or rooms by columns, arches and cornices. The presence of statues, curved surfaces and rich ornamentation on the walls, columns and ceilings contributes to a high diffusivity of the sound field in the subspaces, according to the classification based on the work of Haan and Fricke and given in Beranek's book [9]. As encouraging results have been obtained with a model of coupled subspaces applied to a large interior [20], it was decided to use the same model to search for coupling effects in St Paul's Cathedral. In the earlier work [20], it was found that the behaviour of sound in the system of many coupled rooms was more complicated than in the simple system of two coupled rooms [4] and it was concluded that the effect of coupling strongly depends on the spatial distribution of both the equivalent absorption areas and the sound sources. The results obtained from the coupled subspaces model, as applied to St Paul's Cathedral, are compared with experiment.

## 2. DESCRIPTION OF ST PAUL'S CATHEDRAL

The characteristics of St Paul's Cathedral which influence its acoustics have been described previously [19]. In this study, the interior of St Paul's is treated as a system of 70 acoustical rooms (subspaces) formed by arches, columns and cornices.

A plan of the Cathedral is shown in Figure 1. In most cases, the way in which the cathedral is divided into the subspaces is fairly clear from the plan. For example, in the nave a typical subspace is formed by the main columns in the nave and the arches to the aisles so that there are four columns at the corners of the subspace. Just above the arches to the aisles there are cornices which run along the entire length of the nave at a height of about 16.5 m. The cornices overhang the nave by almost 2 m and can be used to divide the nave into lower and upper rooms. Thus, the nave is divided into six lower and six upper subspaces. The total height of the nave is about 28.5 m. The aisles on either side of the nave, which are of lower height, are divided into six rooms each. The two side chapels, of St Dunstan and of St Michael and St George, are two separate subspaces. The way in which subspaces are divided is illustrated schematically in Figure 2 in which the dividing arches and cornices are

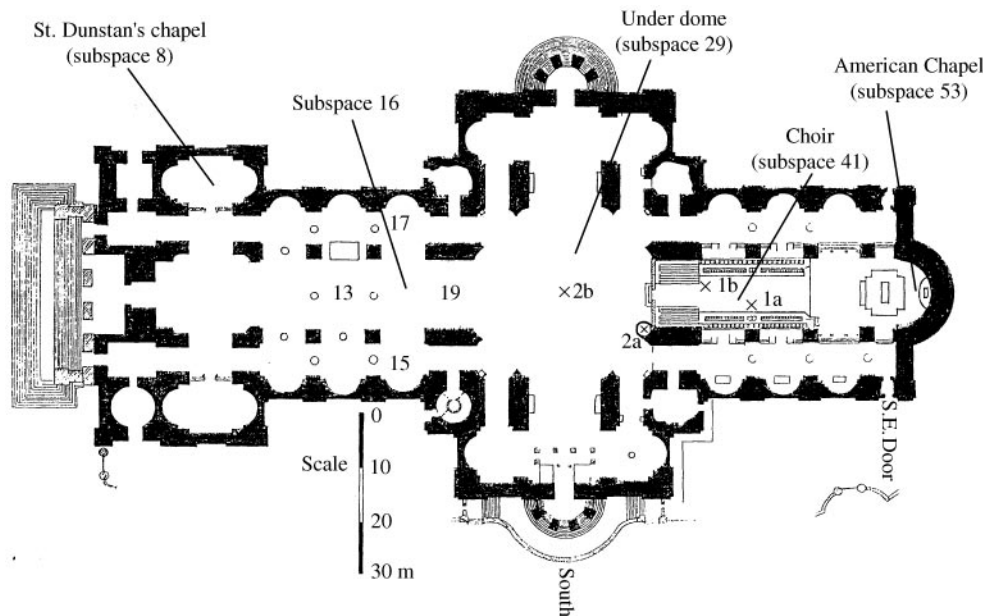


Figure 1. Plan of interior of St Paul's Cathedral. Notations: 1a and 1b refer to positions of sound source and microphone, respectively, during experimental measurements in the choir. Similarly, notations 2a and 2b denote locations of the sound source (in the pulpit) and microphone (1.4 m above the floor) during experimental measurements under the dome.

shown without any details or ornamentation. In this isometric diagram the subspace 16 in the nave is seen to be coupled by arches to subspaces 15 and 17, through columns to 13 and 19, and via cornices to 58 above.

A photograph of part of the nave, looking towards the dome area and chancel, is shown in Figure 3. The arches and cornices which divide the subspaces, shown schematically in Figure 2, can be seen in the photograph. For example, the arch leading to subspace 17 is located next to the right of the Wellington monument which is on the lower left of the picture.

Even though the space beneath the dome is huge, it was only possible to create three acoustical subspaces in this region. The lowest space (subspace 29) is formed between the floor and the overhanging whispering gallery; the horizontal cross-section of this subspace forms an area of a circle inscribed within the eight great piers supporting the dome. A second subspace is essentially a cylinder and is formed between the whispering gallery and a circular cornice below the painted ceiling of the dome. In this subspace the surfaces are stone blocks, stone statues and glazed windows. The part of the dome above the cornice forms the third subspace which is essentially a hemisphere, the surface of which is painted plaster on brick. The volumes of the subspaces under the dome are much larger than the volumes of the other subspaces in the Cathedral.

The chancel has an important role in the acoustics of the Cathedral. The choir, the part of the chancel formed by the wooden choir stalls and the organ cases, is relatively small but has a larger mean sound absorption coefficient than the rest of the Cathedral. The choir and the high altar can be seen in the background towards the lower right of Figure 3. The choir stalls are elaborately carved and contribute not only to extra absorption but also to increasing the diffusivity of the sound. The upper parts of the choir stalls and the organ cases overhang the choir subspace, which extends to a height of 6 m. The space above the

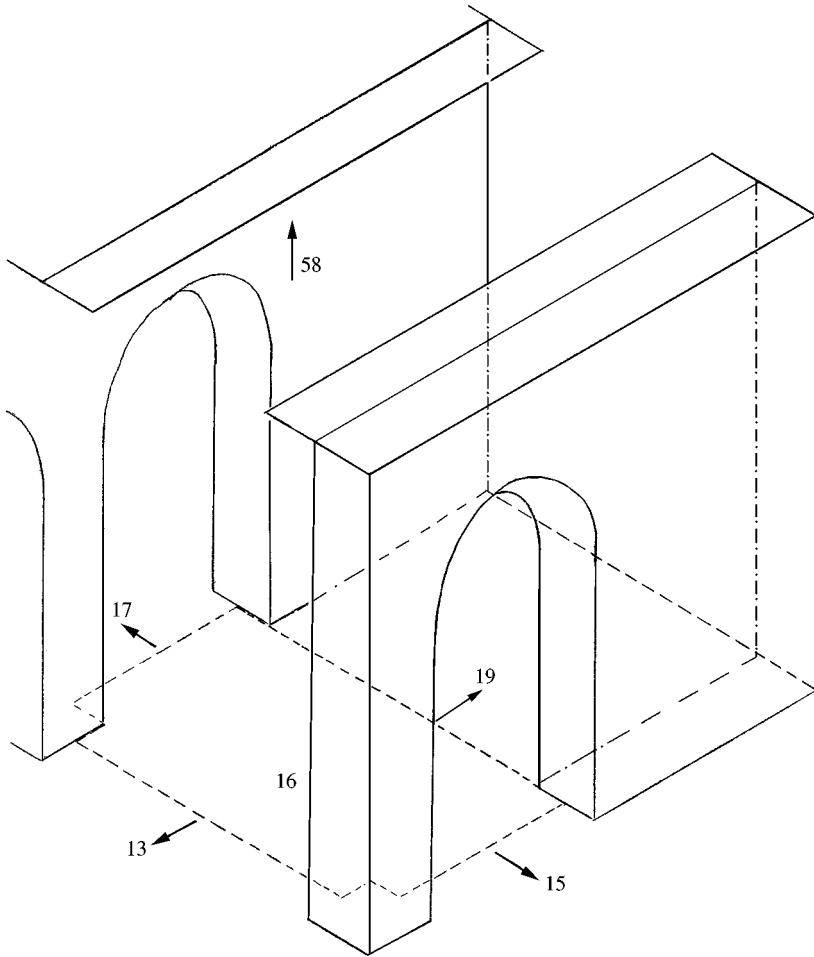


Figure 2. Schematic diagram of coupled rooms in the nave.

choir is divided into two subspaces. Subspaces are formed in the rest of the chancel according to similar principles which have been described for the nave, although there are some complexities around the high altar.

The transepts are divided into subspaces in a manner similar to that adopted for the nave. The central parts of the transepts have a height almost equal to that of the nave and are divided by a cornice into lower and upper subspaces.

In the Cathedral the columns and piers are of Portland stone. The eight piers which support the dome are very large with a cross-section  $11 \text{ m} \times 3 \text{ m}$ . In the nave, the arches and most of the decorative features are also of Portland stone, although the ribs in the ceiling are of a brown stone. The surface material of the vaulting in the nave is primarily plaster. The choir is structurally similar to the nave, but is much more richly decorated, particularly by mosaics on the ceiling. The tesserae of the mosaics in the chancel are made of glass set in cement. The choir stalls are made of oak and lime wood which provide more sound absorption than the stone, marble and plaster. In a space as large as St Paul's the absorption of sound in the air plays a large part, particularly at the higher frequencies. In the dome, where the volume is large compared with the surface area, the air absorption is



Figure 3. Photograph of the nave, looking towards the dome and high altar.

comparatively large even at 1 kHz. The role of air absorption in the Cathedral has been discussed previously [19]. Some theoretical results have been obtained with the Cathedral occupied, in which case the Cathedral was assumed to be filled with 2500 people who were distributed in the dome area, transepts and nave.

### 3. MODEL OF THE COUPLED SUBSPACES

A model of coupled subspaces (rooms) can be applied to any system of subspaces provided that in each room the sound field is diffuse. Obviously, this type of assumption can be fulfilled in reality only with certain approximation. However, the degree of diffusivity is believed to be high in all subspaces specified in St Paul's Cathedral. It is assumed that the sound energy density is constant within the individual subspace, but can change from one subspace to the next.

The non-stationary processes of the reverberant sound energy decay are considered, following either steady state or impulse excitations. The sound energy decay in the whole interior, divided into 70 acoustical subspaces, is described by a system of 70 sound energy balance equations:

$$V_k(dE_k/dt) = -cA_kE_k/4 + \sum_s cS_{s,k}(E_s - E_k)/4, \quad (1)$$

where  $k = 1, \dots, m (= 70)$ ,  $c$  is the sound speed,  $E_k$  denotes the average sound energy density in the  $k$ th subspace,  $V_k$  is the volume of the  $k$ th subspace and  $A_k$  is the equivalent absorption area of the  $k$ th subspace. The term "average", used in this paper, means the running time average, see references [21, 22]. The area of coupling between subspace  $k$  and adjacent subspace  $s$  is denoted by  $S_{s,k}$ . The way in which the subspaces are divided in the nave is shown in Figure 2.

The system of linear differential equations (1) can be presented in matrix form:

$$d\{E\}/dt = -[M]\{E\}, \quad (2)$$

where  $[M]$  is the square matrix defined by the coefficients of system (1) and  $\{E\}$  is a vector such that

$$\{E\} = [E_1, \dots, E_k \dots E_m]^T.$$

Since all eigenvalues of the matrix  $[M]$  are distinct, the general solution of equation (2) is represented by the expression

$$\{E\} = \sum_{i=1}^m C_i \exp(-\lambda_i t) \{Y\}_i,$$

where  $\{Y\}_i$  ( $i = 1, \dots, m$ ) denote the eigenvector belonging to the appropriate eigenvalue  $\lambda_i$ . The constants  $C_i$  are determined from initial conditions which refer either to the steady state or impulse excitations.

### 4. RESULTS

The coupling phenomenon in St Paul's Cathedral can be detected by analyzing the sound decay curves for different locations in the interior. The cases calculated for this paper deal

with a single sound source located either in the choir or in the space under the dome, apart from one example where the sound source is in the American Chapel. The choice of the locations of the sound sources was justified by the fact that during services or concerts the musicians are usually placed either in the chancel or under the dome close to the chancel. The sound decay curves refer to the decay of sound energy density which is represented by the sound energy density level. The sound energy density level is defined in, for example, the  $k$ th subspace as  $10 \log[E_k(t)/E_r(0)]$ , where  $E_r(0)$  is the average sound energy density at  $t = 0$  in the reference  $r$ th subspace (room) and  $E_k(t)$  denotes the average sound energy density in the  $k$ th subspace at time  $t$ . In the considered cases, the reference subspace is the sound source room.

The theoretically modelled sound energy density decay following steady state excitation is shown in Figure 4 for the octave bands of centre frequencies of 1 and 2 kHz at different locations in the Cathedral. The sound source is located in the choir (subspace 41) (see Figure 1). The sound decay (see Figure 4) is analyzed in the choir, in the subspace which is under the dome and below the whispering gallery (subspace 29), in the main nave (subspace 16) and in St Dunstan's Chapel (subspace 8), as indicated in Figure 1. The theoretical sound decay curves shown in Figure 4 exhibit the characteristic two-stage structure, caused by the acoustic coupling. For the early stage of sound decay the rate of the decay changes with time. The late stage is achieved when the sound decays with a constant rate. The duration of the early stage of sound decay is denoted as the "equalization time",  $t_{eq}$  [20].

Two sets of theoretical impulse response curves obtained in the unoccupied and the occupied St Paul's Cathedral are presented in Figure 5. The impulse sound source with an octave band of centre frequency 1 kHz is located in the dome area (subspace 29). The sound

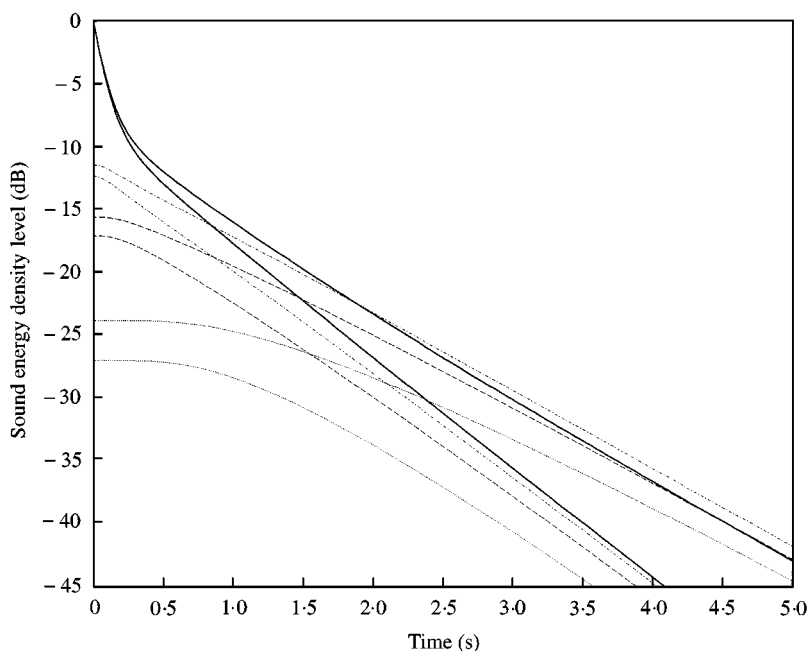


Figure 4. Decay with time of sound energy density level in different subspaces after discontinuing the steady state excitation; sound source in the choir (subspace 41); Cathedral unoccupied; octave band centre frequency 1 kHz — upper family of curves; octave band centre frequency 2 kHz — lower family of curves; receivers in: ....., subspace 8; - · - · -, subspace 16; - - - -, subspace 29; —, subspace 41.

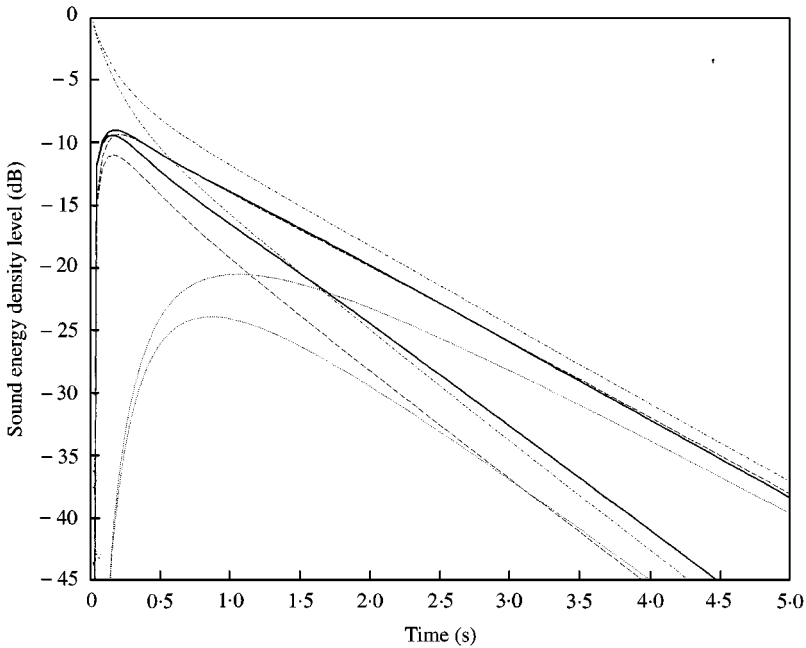


Figure 5. Decay with time of sound energy density level in different subspaces after impulse excitation; sound source in the subspace below the dome (29); octave band centre frequency 1 kHz; Cathedral unoccupied — upper family of curves; Cathedral occupied — lower family of curves; receivers in: ....., subspace 8; ----, subspace 16; - · - · -, subspace 29; —, subspace 41.

decay parts of the impulse responses exhibit the similar two-stage appearance associated with acoustic coupling. The sound field in the Cathedral depends upon the distribution of the sound sources, as illustrated in Figure 6 where two sets of theoretical impulse response curves for different locations of the sound sources are compared. The lower family of curves refers to the sound source in the choir and the upper family of curves is obtained for the sound source located under the dome, below the whispering gallery.

The rate of decay of the sound energy density varies in space and time, as shown in Figures 5 and 6. Since the standard definition of reverberation time fails to be a useful characteristic of the sound decay when the effect of coupling is not negligible, it is desirable to define a “local” or running reverberation time  $T_r$ , represented by the slope of the sound decay curve and based on a 60 dB sound decay. This concept is useful in determination of the early stage of sound decay when  $T_r$  changes with time [20]. The running reverberation time  $T_r$  as a function of time is presented in Figure 7 for frequencies of 1 and 2 kHz for the unoccupied Cathedral. Both the sound source and receiver are in the choir (subspace 41) and the initial excitation was steady state. The plateaux of the curves refer to the standard reverberation time for the late stage of the sound decay.

In order to provide confirmation for the theory some experimental results were obtained in the Cathedral. The sound source used was a Brüel & Kjaer type 4205 which is a loudspeaker fed from a noise generator via octave filters. The receiver was a sound level meter, Brüel & Kjaer type 2231 with octave filters. The sound level meter was mounted on a tripod so that the microphone was 1.4 m above the floor. When the sound source was switched off the decay of the sound pressure was recorded on a portable computer via



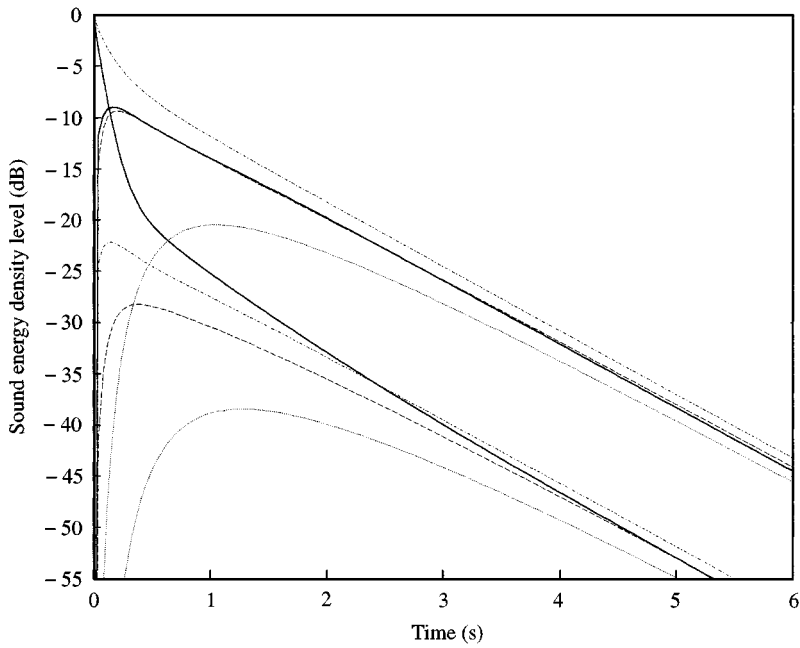


Figure 6. Decay with time of sound energy density level in different subspaces after impulse excitation; octave band centre frequency 1 kHz; Cathedral unoccupied; sound source in the choir (subspace 41) — lower family of curves; sound source in the subspace below the dome (29) — upper family of curves; receivers in: ..... , subspace 8; ----, subspace 16; - · - ·, subspace 29; —, subspace 41.

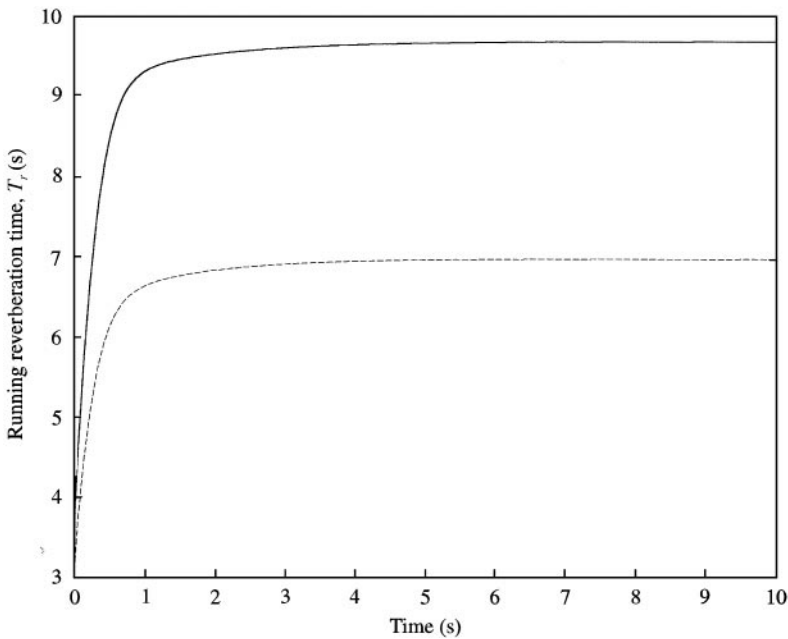


Figure 7. Running reverberation time  $T_r$  as a function of time for octave bands of centre frequencies 1 kHz (—) and 2 kHz (---); sound source and receiver in choir (subspace 41); initially steady state excitation.

a sound card. The experimental results presented refer to the sound decay after discontinuing the steady state excitation.

A comparison of the experimental sound decay curve with the theoretical curve for the unoccupied Cathedral is shown in Figure 8 for sound in the octave frequency band of 1 kHz. In order to obtain the experimental results the sound source and the microphone are both located in the choir at positions 1a and 1b, respectively (see Figure 1). In Figure 8, good agreement is exhibited between the theoretical and experimental results. Similar agreement is shown in Figure 9 when the sound source remains positioned in the choir (position 1a) but the microphone is located near the floor under the dome centre, position 2b. In this case (see Figure 9) the sound decay curve for the space under the dome is almost a straight line, since the space is more reverberant than the adjacent sound source space (choir). The other set of results for a frequency of 1 kHz is shown in Figure 10 where theory and experiment are compared when both the sound source and the receiver are in the subpace under the dome (29). In the experiment, the sound source was placed in the pulpit, (position 2a in Figure 1) and the microphone at position 2b. In the case of the sound source located under the dome (see Figure 10) the effect of coupling is not significantly marked for the sound decay in the source subpace. Similarly, as for previous results, agreement between theory and experiment has been achieved.

Comparison of the experimental and theoretical results are also presented in Figures 11 and 12 for the case of sound at 2 kHz and the sound source located in the choir. The sound decay curves apply to the receiver located either in the choir (Figure 11) or in the subpace under the dome, below the whispering gallery, subpace 29 (Figure 12). Figures 11 and 12 are comparable to Figures 8 and 9, but for a different frequency. The final comparison, shown in Figure 13, is for the case of both the sound source and receiver located under the dome (compare with Figure 10).

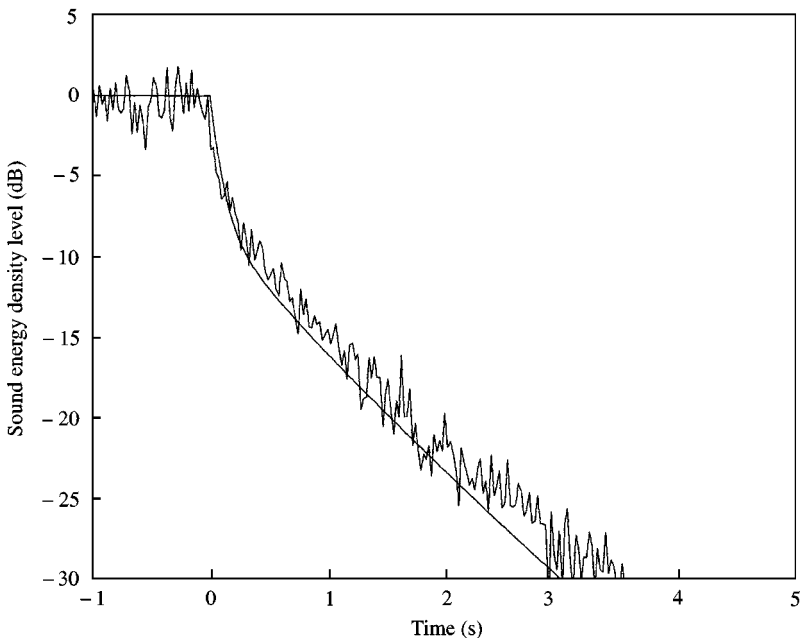


Figure 8. Comparison of experimental and theoretical results for octave band centre frequency 1 kHz; sound source and receiver in the choir; Cathedral unoccupied; initially steady state excitation.

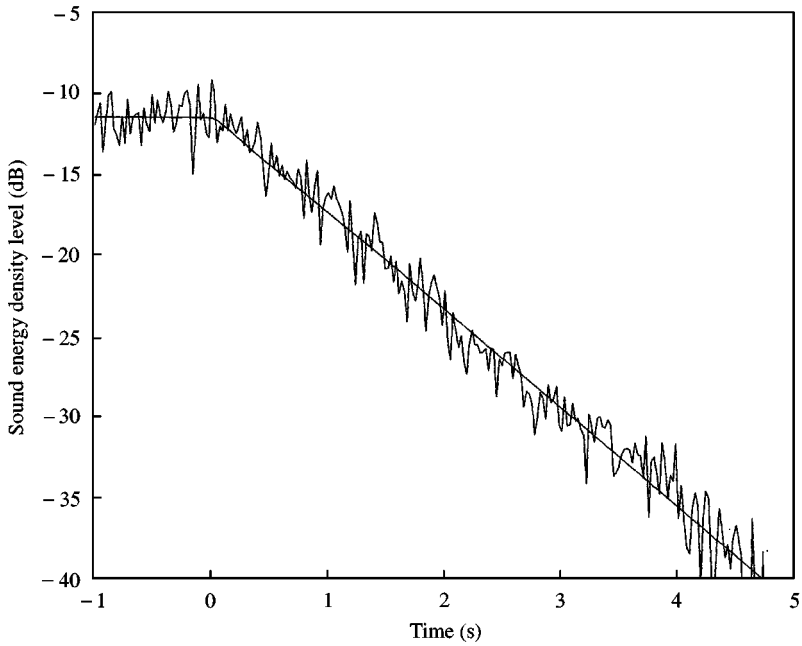


Figure 9. Comparison of experimental and theoretical results for octave band centre frequency 1 kHz; sound source in the choir and receiver in the subspace below the dome (29); Cathedral unoccupied; initially steady state excitation.

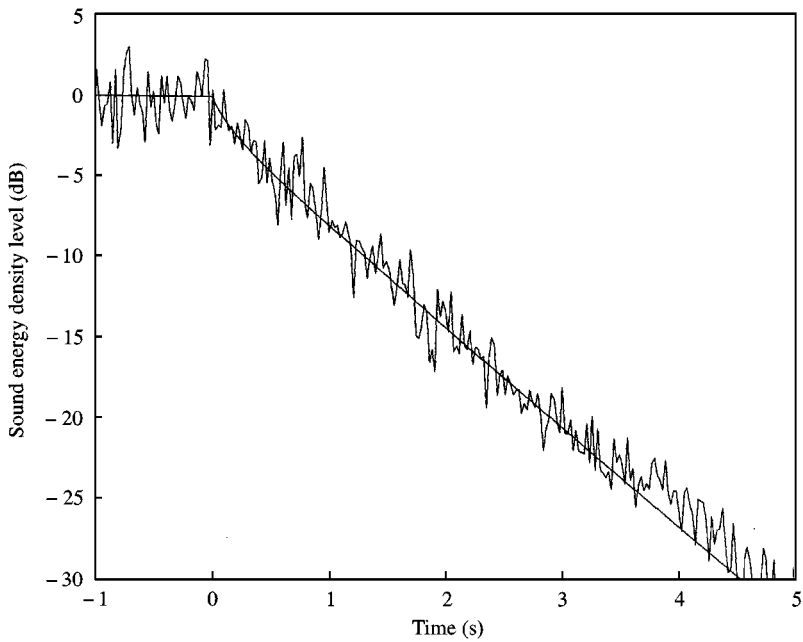


Figure 10. Comparison of experimental and theoretical results for octave band centre frequency 1 kHz; sound source and receiver in the subspace below the dome (29); Cathedral unoccupied; initially steady state excitation.

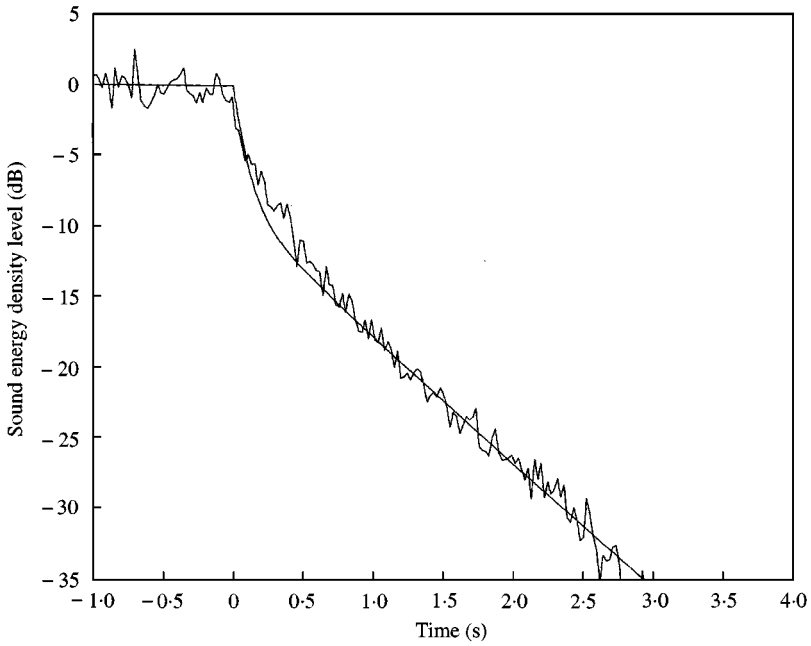


Figure 11. Comparison of experimental and theoretical results for octave band centre frequency 2 kHz; sound source and receiver in the choir; Cathedral unoccupied; initially steady state excitation.

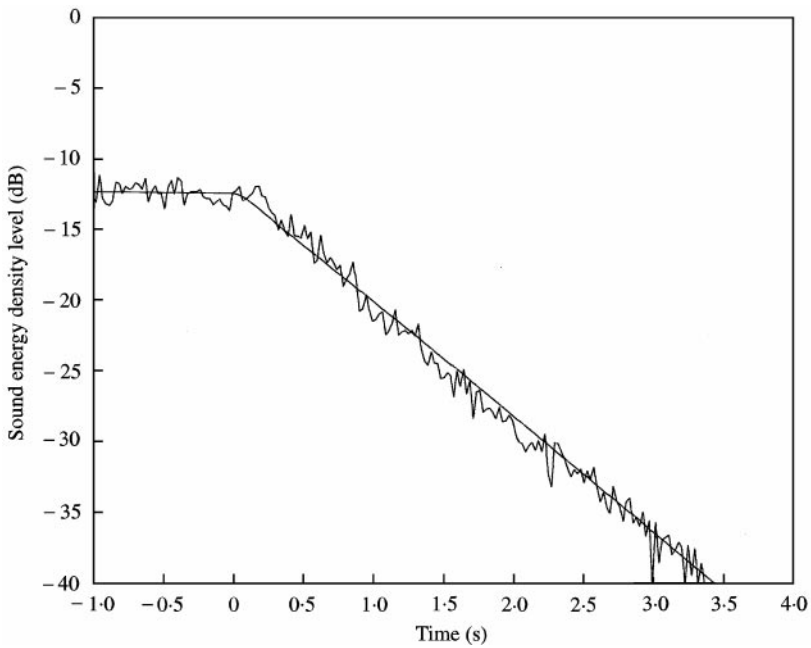


Figure 12. Comparison of experimental and theoretical results for octave band centre frequency 2 kHz; sound source in the choir and receiver in the subspace below the dome (29); Cathedral unoccupied; initially steady state excitation.

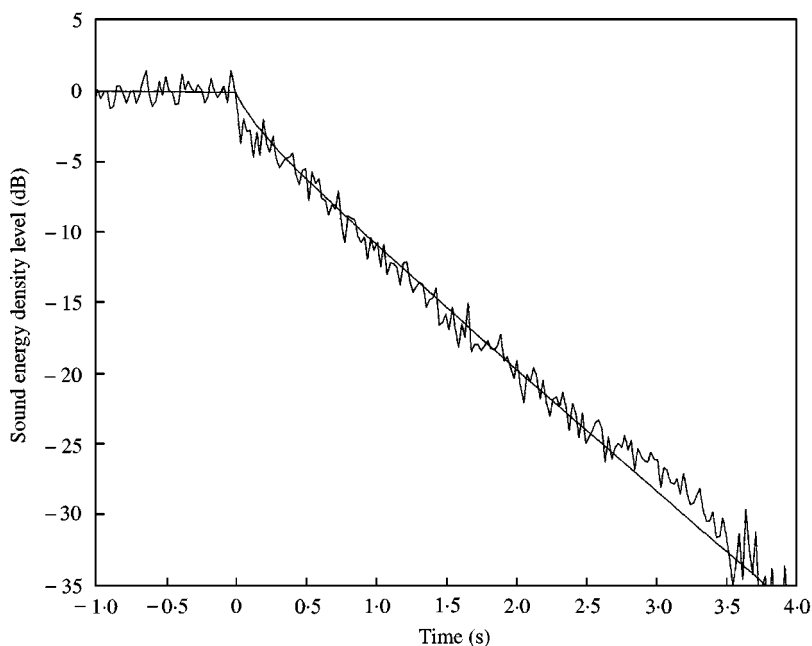


Figure 13. Comparison of experimental and theoretical results for octave band centre frequency 2 kHz; sound source and receiver under the dome (29); Cathedral unoccupied; initially steady state excitation.

## 5. DISCUSSION

Ray tracing and the coupled subspaces model are two approaches that can be applied to a large building such as St Paul's Cathedral. Both simplify significantly the physics of sound propagation in interiors and also both have limitations in practical applications. The ray-tracing method has been applied to St Paul's Cathedral, but has achieved limited success in predicting experimental results, particularly in the determination of the fine structure of reverberation [23]. The hybrid ray-tracing and image source model ODEON does not appear to be able to predict well the reverberation in coupled spaces [24]. However, ray tracing has been applied to the simple case of two coupled rooms by Ricol and Junker [25] with apparently more success. The ray-tracing model is more applicable to the investigation of the huge space under the dome than the coupled rooms model which is reserved for diffusive sound fields. The space under the dome can only be divided into three subspaces and also the specific properties of the whispering gallery cannot be explained in the model of the coupled subspaces. On the other hand, in the case of a building like St Paul's Cathedral with so many sound diffusing architectural elements there is a problem in setting up the proper geometrical model for ray tracing [23]. The model of the coupled subspaces can incorporate to a certain extent architectural elements, as well as the distribution of absorption around the interior space of the cathedral, by the appropriate choice of coupled subspaces with their equivalent absorption areas. It also represents computational simplicity in comparison to models based on ray tracing.

The applicability of the model of coupled rooms is always associated with the question of the diffusivity of the sound fields within the rooms. The diffusivity of a sound field decreases *a priori* near the absorbing walls of an enclosure and near the subspaces coupling areas where the sound energy flux density vector is not equal to zero.

The sound field under a dome with its whispering gallery presents a specific challenge to the diffuse sound field model. However, Joyce [26] found that applying the Kuttruff integral equation [27–29], based on the geometrical ray model of sound propagation, to a uniformly absorbing spherical enclosure (randomizing enclosure) with weak absorptivity leads to results which confirm the validity of the Sabine formula. A similar conclusion was formulated by Gilbert [30] who introduced an iterative procedure to solve Kuttruff's integral equation for an auditorium which comprised a specularly reflecting dome over a diffusely reflecting floor. In this model, the hemispherical dome reflects sound specularly, forming a whispering gallery. For low values of the loss parameters in the auditorium the reverberation time agrees with Sabine. Since individual subspaces in St Paul's Cathedral are relatively large another question arises as to what extent the assumption of constant sound energy density in an individual subspace is justified. Support for this assumption can be found in the work of Beranek and Hidaka [31], where the applicability of Sabine's model for the prediction of reverberation times of large spaces, such as concert halls, is confirmed. Although the architecture of the interior of St Paul's Cathedral is very different from an average concert hall, there are many surface irregularities which lead to the expectation that the sound fields in the individual subspaces of the cathedral are approximated well by a diffuse field.

The comparison of the theoretical and experimental results in Figures 8–13, results in the conclusion that the model of the coupled spaces can be successfully applied to the investigation of the acoustics of St Paul's Cathedral. Particularly, coupling effects can be detected and the fine structure of reverberation in different locations of the Cathedral can be explained.

The most pronounced two-stage structure is observed for the sound decay in the choir when the sound source is also in the choir (see Figure 6). However, the two-stage sound decay curves are also detected in that part of the dome space which is below the whispering gallery (subspace 29), for the case when the sound source is in the same subspace. The theoretical sound decay curves for subspace 29 (with the sound source) for the unoccupied and occupied Cathedral are shown in Figures 5 and 14, where deviations from straight lines can be seen.

The sagging appearance of the experimental curves can also result from the lack of diffusion of the sound field [2, 7, 32]. However, in the case of St Paul's Cathedral the rich carving and ornamentation, as well as the relatively low absorption of the enclosure surfaces, contribute to the high diffusivity of the individual sound fields in the subspaces.

Significant coupling effects can be predicted in other parts of the Cathedral, e.g., in other parts of the chancel. Theoretical results are shown in Figure 15 for the case when the sound source is located in the American Chapel (subspace 53) (see Figure 1).

In a system of coupled rooms the acoustic reciprocity principle [21] in relation to sources and receivers holds. This principle can be illustrated by considering Figure 6, for example. The curve “—” (marked by the continuous line) of the upper family of curves, obtained for the case of the sound source located under the dome in subspace 29 and the receiver positioned in the choir (subspace 41), is identical in shape with curve “-.-.-” (chain-dotted line) of the lower family of curves which refers to the case of the sound source in the choir and the receiver in subspace 29 under the dome. The considered curves do not overlap each other because their sound energy density levels are obtained by different scaling. The reference sound energy densities for both curves are different since the sound sources are located in different subspaces. Similarly, to demonstrate the principle of reciprocity in the case of steady state excitation, the chain-dotted line of the upper family of curves in Figure 4 (the sound source in the choir (41) and the receiver under the dome in subspace 29) can be compared with the continuous curve of the upper family of curves in Figure 14 (the sound source in subspace 29 and the receiver in the choir).

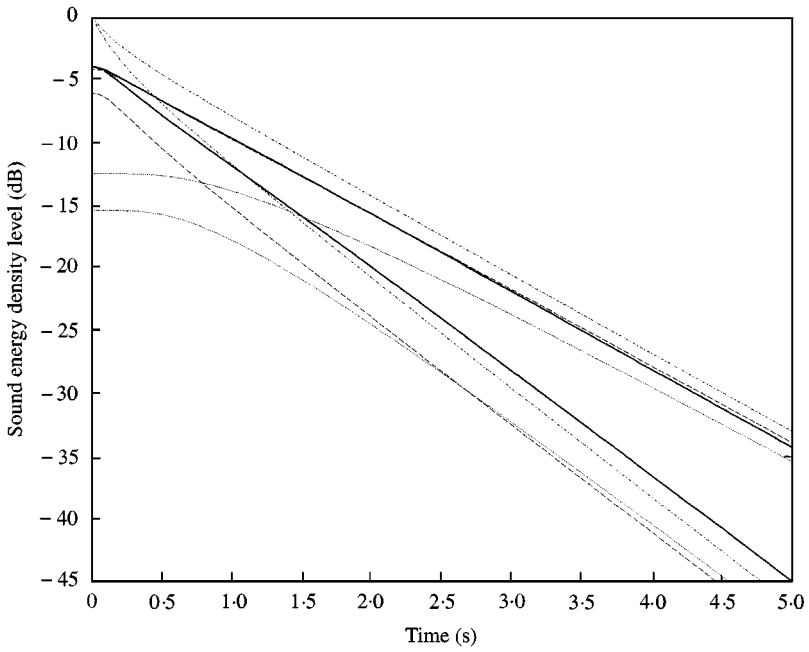


Figure 14. Decay with time of sound energy density level in different subspaces after discontinuing the steady state excitation; sound source in the subspace below the dome (29); octave band centre frequency 1 kHz; Cathedral unoccupied — upper family of curves; Cathedral occupied — lower family of curves; receivers in: ..... , subspace 8; ----, subspace 16; - · - · -, subspace 29; —, subspace 41.

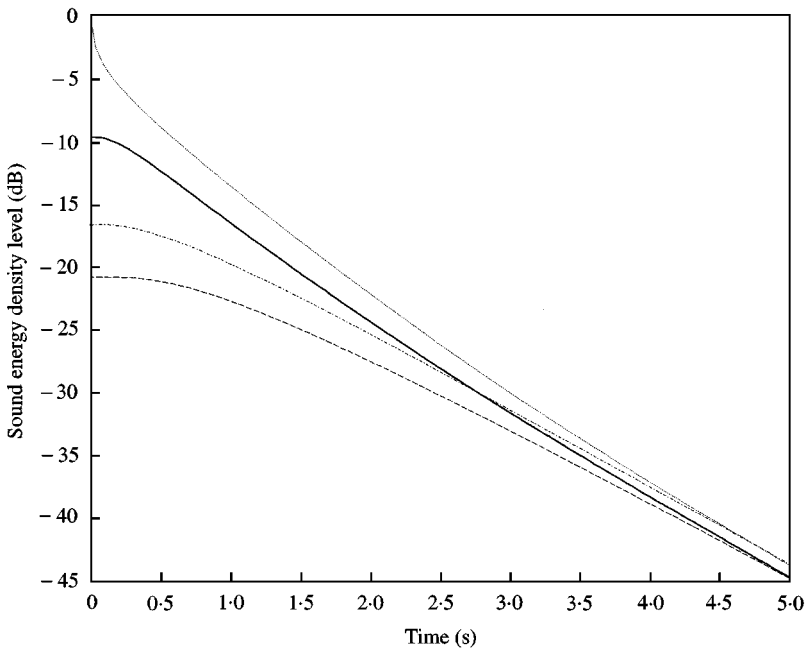


Figure 15. Decay with time of sound energy density level in different rooms after discontinuing the steady state excitation; octave band centre frequency 1 kHz; sound source in the American chapel (subspace 53); receivers in: ----, subspace 16; - · - · -, subspace 29; —, subspace 41; ..... , subspace 53.

## 6. CONCLUSION

A numerical model of St Paul's Cathedral, London, has been constructed by dividing the interior space into 70 acoustical subspaces which are coupled together by the interchange of sound energy. The model is able to explain experimental results which show a two-stage structure of sound decay in different locations of the Cathedral. With all its limitations the model of the coupled subspaces can be applied usefully to buildings with highly diffuse sound fields such as St Paul's Cathedral.

## ACKNOWLEDGMENT

The authors wish to thank the Dean and Chapter of St Paul's Cathedral for their permission to carry out the tests described herein.

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