Journal of Sound and Vibration (2000) 236(2), 259–276
doi:10.1006/jsvi.2000.2985, available online at http://www.idealibrary.com on IDEAL®

TRANSVERSE VIBRATIONS OF TENSIONED PIPES CONVEYING FLUID WITH TIME-DEPENDENT VELOCITY

HALİL RIDVAN ÖZ AND HAKAN BOYACI

Department of Mechanical Engineering, Celal Bayar University, 45140, Muradiye, Manisa, Turkey

(*Received* 26 *July* 1999, *and in final form* 12 *January* 2000)

In this study, the transverse vibrations of highly tensioned pipes with vanishing flexural stiffness and conveying fluid with time-dependent velocity are investigated. Two different cases, the pipes with fixed–fixed end and fixed–sliding end conditions are considered. The time-dependent velocity is assumed to be a harmonic function about a mean velocity. These systems experience a Coriolis acceleration component which renders such systems gyroscopic. The equation of motion is derived using Hamilton's principle and solved analytically by direct application of the method of multiple scales (a perturbation technique). The natural frequencies are found. Increasing the ratio of fluid mass to the total mass per unit length increases the natural frequencies. The principal parametric resonance cases are investigated in detail. Stability boundaries are determined analytically. It is found that instabilities occur when the frequency of velocity fluctuations is close to two times the natural frequency of the constant velocity system. When the velocity fluctuation frequency is close to zero, no instabilities are detected up to the first order of perturbation. Numerical results are presented for the first two modes.

1. INTRODUCTION

Due to their technological importance, the dynamics of axially moving continua was investigated by many researchers. The understanding of the vibrations of an axially moving continuous medium is important in the design of high-speed magnetic tapes, band-saws, power transmission chains and belts, textile and composite fibers, aerial cable tramways, paper sheets during processing, pipes and beams conveying fluid, etc. Especially, pipe lines are used in conveying gas, oil, water, dangerous liquids in chemical plants, cooling water in nuclear power plants, and in many other places. The vibrations occur due to many reasons. They are caused by compressors, ventilators, electrical power engines or internal combustion engines and also by valves, elbows, orifices, transition parts and abrupt area contractions. If these oscillations are not prevented, they can result in leakage, hazards and accidents. Ulsoy *et al.* [1] and Wickert and Mote [2] reviewed the relevant work up to 1978 and 1988 respectively. Older studies concentrated on the axial velocity-dependent natural frequency and existence of divergence instability at the critical velocity [3–6]. The natural frequencies decrease with increasing transport speed, and the translating continua experience divergence instability at a critical speed. The eigenfunctions are complex and speed-dependent due to a convective acceleration component in the equations of motion. The phases of the natural oscillations are not constant and propagate upstream at the phase propagation velocity. The periodic variation of total mechanical energy was studied [7], and also firstly, Miranker [8] derived the equations of motion for time-dependent axial velocity by using variational procedure. Wickert and Mote [9] showed that the energy flux at a fixed support is the product of the string tension and the convective component of a velocity. They [10] also investigated the transverse vibrations by complex modal analysis. An analogous case with a similar equation of motion exists in highly tensioned pipes conveying fluid with negligible flexural stiffness and fluid pressure effect. Wickert [11] analyzed free non-linear vibration of an axially moving, elastic, tensioned beam over sub- and supercritical speed ranges. Pakdemirli *et al.* [12] re-derived the equations of motion for an axially accelerating strip using Hamilton's principle and numerically investigated the stability of the response using Floquet theory. Pakdemirli and Batan [13] analyzed the constant acceleration-type motion. Pakdemirli and Ulsoy [14] obtained approximate analytical solutions by using the method of multiple scales and showed that direct-perturbation yields better results for higher order expansions with respect to the discretization-perturbation method. The direct-perturbation method does not require transformation of the equations or the selection of an orthogonal basis. The authors also investigated principal parametric resonances and combination resonances for any two modes for an axially accelerating strip. Öz *et al.* [15] studied the transition behaviour from string to beam for an accelerating material and presented an approximate analytical expression for the natural frequency and determined stability borders for variable velocity and studied principal parametric resonance case. Öz and Pakdemirli [16] investigated the transverse vibrations of an axially accelerating beam with simple supports. They considered six different flexural stiffness coefficients and solved the equations of motions by using the method of multiple scales and studied principal parametric resonances and combination resonances. They found that for velocity fluctuation frequency nearly twice any natural frequency, an instability region occurs whereas for the frequencies close to zero, no instabilities were detected. For combination resonances, instabilities occurred only for those of additive type. No instabilities were detected for difference-type combination resonances in agreement with references [14–16]. Öz *et al.* [17, 18] investigated linear and non-linear vibrations and performed a stability analysis.

Up to now in the studies related to fluid flow, the velocity was assumed as constant. Benjamin [19] neglected fluid friction effects. Nemat-Nasser *et al.* [20] and Gregory and Paidoussis [21] found the destabilizing effect of dissipation in a cantilevered, fourth order beam conveying fluids. Paidoussis and Li [22] and Lee and Mote [23, 24] neglected gravity and pressure effects. In references [23, 24] the energetics of translating one-dimensional uniform strings, highly tensioned pipes with vanishing bending stiffness and tensioned beams and flowing fluid are analyzed for fixed, free and damped boundary conditions. The fluid velocity was assumed to be constant. Natural frequencies are found by using phase-closure principle. It was found that the energies transferred at the different boundary supports were quantified by energy reflection coefficients which were determined completely by the boundary conditions.

In this study, the transverse vibrations of highly tensioned pipes conveying fluid are investigated. The pipe is assumed to have negligible flexural stiffness and it can be thought of as a string conveying fluid. A harmonically varying velocity function is chosen for the fluid flow. The ends of the pipe are on fixed supports in the first case and on fixed–sliding supports in the second case. The linear equations of motion are derived by using Hamilton's principle and solved analytically by means of direct application of the method of multiple scales. The natural frequencies are found analytically depending on fluid velocity and ratio of fluid mass to total mass per unit length and are calculated for the first two modes. Increasing the ratio of fluid mass to the total mass per unit length increases the natural frequencies. The natural frequencies for fixed–sliding tensioned pipe are lower than those of fixed–fixed tensioned pipe. The stability boundaries are determined analytically for principal parametric resonances. For velocity fluctuation frequency nearly twice any natural frequency, an instability region occurs whereas for frequencies close to zero, no instabilities are detected up to the first order of approximation. The stability borders shift to higher velocity fluctuation frequency values for the two cases. Stable–unstable regions shift to relatively lower velocity fluctuation frequency values for fixed–sliding end conditions compared with fixed–fixed end conditions.

2. EQUATIONS OF MOTION

For the fixed–fixed and fixed–sliding tensioned pipes conveying fluid in Figures 1(a) and 1(b), x^* and z^* are the spatial co-ordinates, u^* and w^* are longitudinal and transverse displacements respectively. υ^* is variable fluid velocity, ρ_f is the fluid density, A_f and A_p are the cross-sectional areas of the pipe and flow and assumed to be constant. Tension force in the pipe is $P(t^*)$. The length is L. The modulus of elasticity of the pipe is E_p. The transverse displacement is assumed to be small compared with span L and the tension force is assumed to be sufficiently large compared with the effects arising from elongation. The extensional stiffness is sufficiently large so that the longitudinal deformation resulting from the pretension is negligible. Variation of cross-sectional dimensions during vibration is not considered. In this study sub-critical region is considered. If gravity, pressure and fluid friction effects, and restoring flexural forces are neglected, then a pipe conveying fluid is considered to be a string conveying fluid. Let us denote the time by t^*, the derivatives with respect to the spatial variable by $(\ )'$ and the derivatives with respect to time by $(\dot{\ })$. Since only linear analysis is made the non-linear strains deriving from u^* and w^* are negligible. So the strain in the pipe is approximately

$$e \approx u^{*'}, \tag{1}$$

The total kinetic energy of the system is

$$T = \frac{1}{2}\rho_f A_f \int_0^L [(\upsilon^* + \dot{u}^* + u^{*'}\upsilon^*)^2 + (\dot{w}^* + w^{*'}\upsilon^*)^2]\,\mathrm{d}x^* + \frac{1}{2}\rho_p A_p \int_0^L \dot{w}^{*2}\,\mathrm{d}x^*, \tag{2}$$

where the first term denotes the kinetic energies of the fluid in the x^* and z^* directions, the last term is the kinetic energy of the pipe in the z^* direction. The elastic potential energy of the system

$$U = \frac{1}{2}E_p A_p \int_0^L e^2\,\mathrm{d}k^* + \int_0^L Pe\,\mathrm{d}x^* \tag{3}$$

relative to equilibrium derives from extension. The first term is the elastic potential energy of the pipe due to elongation, and the second term is due to the tensile force. The Lagrangian of the system is

$$£ = T - U. \tag{4}$$

The Hamilton's principle is

$$\delta \int_{t^*}^{t_2^*} £\,\mathrm{d}t^* = 0. \tag{5}$$

Substituting equation (1) into equation (3), and then substituting equations (2) and (3) into equation (4), and applying Hamilton's principle (5), the equations of motion for the

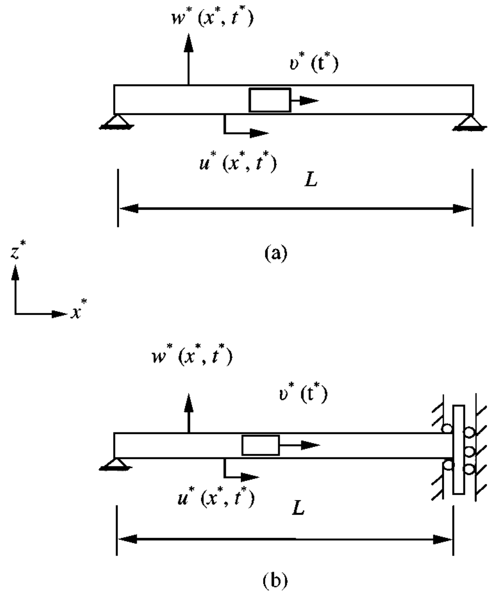

Figure 1. Schematics of second order translating continua in a tensioned pipe (a) with fixed–fixed supports, (b) with a sliding end at $x = L$.

transverse vibration and the boundary conditions are derived:

$$\ddot{w}^* + \frac{\rho_f A_f}{\rho_f A_f + \rho_p A_p}(2\dot{w}^{*'} \upsilon^* + w^{*'} \dot{\upsilon}^* + w^{*''} \upsilon^{*2}) = 0, \tag{6}$$

$$w^*(0, t^*) = w^*(L, t^*) = 0 \tag{7}$$

for fixed–fixed tensioned pipe. Following a similar way the boundary conditions for fixed–sliding pipe can be expressed as

$$w^*(0, t^*) = w^{*'}(L, t^*) = 0 \tag{8}$$

The following parameters are introduced to non-dimensionalize equations (6)–(8):

$$w = \frac{w^*}{L}, \qquad u = \frac{u^*}{L}, \qquad x = \frac{x^*}{L}, \qquad t = \frac{1}{L}\sqrt{\frac{P}{\rho_f A_f}}\,t^* \tag{9}$$

and the fluid velocity is non-dimensionalized by the critical flow velocity at which the pipe system experiences divergence instability

$$\upsilon = \frac{\upsilon^*}{\sqrt{P/\rho_f A_f}}. \tag{10}$$

One obtains non-dimensional linear equations of motion for the transverse vibration

$$\ddot{w} + \beta(2\dot{w}'\upsilon + w'\dot{\upsilon} + (\upsilon^2 - 1)w'') = 0 \tag{11}$$

with solutions satisfying the end conditions

$$w(0, t) = w(1, t) = 0 \tag{12}$$

for fixed–fixed tensioned pipe. Following a similar way the boundary conditions for fixed–sliding pipe can be expressed as

$$w(0,t) = w'(1,t) = 0 \tag{13}$$

and the ratio of the flowing fluid mass to the total mass per unit length is defined as

$$\beta = \frac{\rho_f A_f}{(\rho_f A_f + \rho_p A_p)}. \tag{14}$$

Several cases can be discussed for restricted parameter values. First, for the limiting case $\beta = 1$ the equation of motion for a travelling string with variable velocity is obtained as a special case of the second order fluid–pipe system. Second, for a stationary fluid, $\beta = 1$ and $\upsilon = 0$, the equation for linear stationary string is obtained. In equation (11) $\ddot{w}$ and $2\dot{w}'\upsilon$ denote local and Coriolis accelerations, respectively, $\dot{\upsilon}$ denotes variable fluid velocity, and $\upsilon^2 w''$ denotes centrifugal acceleration.

3. APPROXIMATE SOLUTION

Assuming that the velocity is harmonically varying about a constant mean velocity υ_0, one writes

$$\upsilon = \upsilon_0 + \varepsilon\upsilon_1 \sin \Omega t, \tag{15}$$

where ε is a small parameter and $\varepsilon\upsilon_1$ is also small, represents the amplitude of fluctuations. Ω is the fluctuation frequency. Substituting equation (15) into equation (11) and keeping terms up to the first order of approximation, one has

$$\ddot{w} + 2\beta\dot{w}'\upsilon_0 + \beta(\upsilon_0^2 - 1)w'' + \varepsilon\beta(\upsilon_1\Omega\cos\Omega t\, w' + 2\upsilon\sin\Omega t\, \dot{w}' + 2\upsilon_0\upsilon_1\sin\Omega t\, w'') = 0. \tag{16}$$

Direct-perturbation method will be applied to equation (16) in search of solutions. Using the method of multiple scales (a perturbation technique) [25, 26] and assuming a first order expansion, one writes

$$w(x,t;\varepsilon) = w_0(x, T_0, T_1) + \varepsilon w_1(x, T_0, T_1) + \cdots, \tag{17}$$

where w_0 and w_1 are the displacement functions at orders 1 and ε, $T_0 = t$ and $T_1 = \varepsilon t$ are the usual fast and slow time scales respectively. Now the time derivatives can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t} = D_0 + \varepsilon D_1 + \cdots, \qquad \frac{\mathrm{d}^2}{\mathrm{d}t^2} = D_0^2 + 2\varepsilon D_0 D_1 + \cdots, \tag{18}$$

where ($D_i = \partial/\partial T_i$). Substituting equations (17) and (18) into equation (16) and separating each order of approximation, one obtains

$$O(1) \quad D_0^2 w_0 + 2\upsilon_0\beta D_0 w_0' - \beta(1 - \upsilon_0^2)w_0'' = 0, \tag{19}$$

$$\begin{aligned} O(\varepsilon) \quad & D_0^2 w_1 + 2\beta\upsilon_0 D_0 w_1' - \beta(1 - \upsilon_0^2) w_1'' = \\ & - 2D_0 D_1 w_0 - 2\beta\upsilon_0 D_1 w_0' - 2\beta\upsilon_1 \sin\Omega T_0 D_0 w_0' \\ & - 2\beta\upsilon_0\upsilon_1 \sin\Omega T_0 w_0'' - \beta\upsilon_1\Omega\cos\Omega T_0 w_0'. \end{aligned} \tag{20}$$

The solution of equation (19) can be written as follows:

$$w_0(x, T_0, T_1) = \mathrm{e}^{\omega_n T_0} Y_n(x) + cc, \tag{21}$$

where n is mode number, ω_n is natural frequency and cc stands for complex conjugate. Substituting equation (21) into equation (19) one obtains

$$\beta(1 - \upsilon_0^2) Y_n'' - 2\beta\upsilon_0\omega_n Y_n' - \omega_n^2 Y_n = 0. \tag{22}$$

The solution of equation (22) is

$$Y_n(x) = C_{1n}\mathrm{e}^{k_1 x} + C_{2n}\mathrm{e}^{k_2 x}, \tag{23}$$

where k is wave number. Substituting equation (23) into equation (19), one obtains dispersive relation for the tensioned pipe conveying fluid

$$\beta(1 - \upsilon_0^2) k_j^2 - 2\upsilon_0\beta\omega_n k_j - \omega_n^2 = 0, \quad j = 1, 2. \tag{24}$$

Solving the polynomial (24) in terms of k_j and defining $k_u = k_1$ and $k_d = -k_2$,

$$k_d = \frac{\omega_n}{\beta\upsilon_0 + \sqrt{\beta^2\upsilon_0^2 + \beta(1 - \upsilon_0^2)}}, \qquad k_u = \frac{\omega_n}{\beta\upsilon_0 + \sqrt{\beta^2\upsilon_0^2 + \beta(1 - \upsilon_0^2)}}, \tag{25}$$

the downstream and upstream wave numbers, respectively, are found. The denominators of equation (25) are downstream and upstream phase velocities. Substituting equations (23) and (25) into equation (21) and applying fixed–fixed end conditions (12) one obtains

$$C_{2n} = -C_{1n}, \qquad \mathrm{e}^{-k_d} = \mathrm{e}^{k_u}. \tag{26}$$

In equation (26), $-k_d = k_u$ is trivial solution. For non-trivial solution the wave numbers must be complex:

$$k_d = \mathrm{i}\bar{k}_d, \qquad k_u = \mathrm{i}\bar{k}_u, \tag{27}$$

where i denotes complex number $\sqrt{-1}$, $\bar{k}_d$ and $\bar{k}_u$ are real parts of assumed wave numbers and the following relationship should hold,

$$\bar{k_u} = -\bar{k}_d + 2n\pi \tag{28}$$

to satisfy $\mathrm{e}^{-k_d} = \mathrm{e}^{k_u}$ condition, and of course from equation (25) the assumed natural frequency must be complex:

$$\omega_n = \mathrm{i}\bar{\omega}_n. \tag{29}$$

Substituting equations (27) and (29) into equation (25) and then together into equation (28), the natural frequency equation for fixed–fixed tensioned pipe conveying fluid is obtained as

$$\bar{\omega}_n = \frac{n\pi\beta(1 - \upsilon_0^2)}{\sqrt{\beta^2\upsilon_0^2 + \beta(1 - \upsilon_0^2)}}, \quad n = 1, 2, 3, \dots . \tag{30}$$

Following a similar derivation for fixed–sliding tensioned pipe conveying fluid, the relationship between wave numbers is obtained as

$$\bar{k}_u = -\bar{k}_d + 2\left(n - \frac{1}{2}\right)\pi, \quad n = 1, 2, 3, \ldots \tag{31}$$

and the natural frequency equation is obtained as

$$\bar{\omega}_n = \frac{(n-\frac{1}{2})\pi\beta(1-\upsilon_0^2)}{\sqrt{\beta^2\upsilon_0^2 + \beta(1-\upsilon_0^2)}}, \quad n = 1, 2, 3, \ldots. \tag{32}$$

Combining equations (25), (27), (29), and (30) for fixed–fixed tensioned pipe and equation (32) for fixed–sliding tensioned pipe and substituting into displacement function (16), one obtains

$$w_0 = C_n \mathrm{e}^{\mathrm{i}\bar{\omega}_n T_0}(\mathrm{e}^{-\mathrm{i}\bar{k}_d \mathrm{d}x} - \mathrm{e}^{\mathrm{i}\bar{k}_u x}) + cc \tag{33}$$

for fixed–fixed and fixed–sliding tensioned pipes conveying fluid with related natural frequencies and wave numbers. Substituting equation (33) into order ε equation one obtains

$$\begin{aligned} D_0^2 w_1 &+ 2\beta\upsilon_0 D_0 w_1' - \beta(1-\upsilon_0^2) w_1'' = \\ &- 2D_1 C_n (i\bar{\omega}_n Y_n + \beta\upsilon_0 Y_n') \mathrm{e}^{\mathrm{i}\bar{\omega}_n T_0} + 2D_1 \bar{C}_n (\mathrm{i}\bar{\omega}_n Y_n - \beta\upsilon_0 \bar{Y}_n') \mathrm{e}^{-\mathrm{i}\bar{\omega}_n T_0} \\ &- \bar{\omega}_n \beta\upsilon_1 [C_n Y_n' (\mathrm{e}^{\mathrm{i}(\bar{\omega}_n + \Omega)T_0} - \mathrm{e}^{\mathrm{i}(\bar{\omega}_n - \Omega)T_0} - \bar{C}_n \bar{Y}_n' (\mathrm{e}^{\mathrm{i}(\Omega - \bar{\omega}_n)T_0} - \mathrm{e}^{\mathrm{i}(\bar{\omega}_n + \Omega)T_0})] \\ &- \mathrm{i}\beta\upsilon_0\upsilon_1 [C_n Y_n'' (\mathrm{e}^{\mathrm{i}(\bar{\omega}_n + \Omega)T_0} - \mathrm{e}^{\mathrm{i}(\bar{\omega}_n - \Omega)T_0}) + \bar{C}_n \bar{Y}_n'' (\mathrm{e}^{\mathrm{i}(\Omega - \bar{\omega}_n)T_0} - \mathrm{e}^{\mathrm{i}(\bar{\omega}_n + \Omega)T_0})] \\ &- \frac{\Omega\beta\upsilon_1}{2} [C_n Y_n' (\mathrm{e}^{\mathrm{i}(\bar{\omega}_n + \Omega)T_0} + \mathrm{e}^{\mathrm{i}(\bar{\omega}_n - \Omega)T_0} + \bar{C}_n \bar{Y}_n' (\mathrm{e}^{\mathrm{i}(\Omega - \bar{\omega}_n)T_0} + \mathrm{e}^{-\mathrm{i}(\omega_n + \Omega)T_0})]. \end{aligned} \tag{34}$$

In the next section three different cases will be discussed for the fixed–fixed and fixed–sliding tensioned pipes.

4. PRINCIPAL PARAMETRIC RESONANCES

4.1. Ω IS AWAY FROM $2\bar{\omega}_n$ AND 0

In this case, for fixed–fixed end conditions, equation (34) becomes

$$D_0^2 w_1 + 2\beta\upsilon_0 D_0 w_1' - \beta(1-\upsilon_0^2) w_1'' = -2D_1 C_n (\mathrm{i}\bar{\omega}_n Y_n + \beta\upsilon_0 Y_n') \mathrm{e}^{\mathrm{i}\bar{\omega}_n T_0} + cc + NST, \tag{35}$$

where cc and NST stand for complex conjugates and non-secular terms respectively. The solution of equation (35) is as follows:

$$w_1(x, T_0, T_1) = \phi_n(x, T_1) \mathrm{e}^{\mathrm{i}\bar{\omega}_n T_0} + W(x, T_0, T_1) + cc. \tag{36}$$

The first term is related to secular terms and the second term is related to non-secular terms. If equation (36) is substituted into equation (35), ϕ_n satisfy the equations

$$\beta(1-\upsilon_0^2)\phi_n'' - 2\mathrm{i}\upsilon_0\beta\bar{\omega}_n\phi_n' + \bar{\omega}_n^2\phi_n = 2D_1 C_n (\mathrm{i}\bar{\omega}_n Y_n + \beta\upsilon_0 Y_n'), \tag{37}$$

$$\phi_n(0) = 0, \quad \phi_n(1) = 0. \tag{38}$$

The solvability condition requires (see reference [25] for details of calculating solvability conditions)

$$D_1 C_n = 0. \tag{39}$$

This means a constant amplitude solution up to the first order of approximation

$$C_n = C_0. \tag{40}$$

Following a similar way one obtains the solvability condition for fixed–sliding pipe as

$$D_1 C_n = 0. \tag{41}$$

This means a constant amplitude up to the first order of approximation

$$C_n = C_0. \tag{42}$$

4.2. Ω IS CLOSE TO 0

For this case, the nearness of fluctuation frequency to zero can be expressed as

$$\Omega = \varepsilon\sigma, \tag{43}$$

where σ is the detuning parameter. Substituting equations (33) and (43) into equation (34), the order ε equation is obtained as

$$\begin{aligned} &D_0^2 w_1 + 2\upsilon_0 \beta D_0 w_1' - \beta(1-\upsilon_0^2) w_1'' \\ &\quad = \Big\{ -2D_1 C_n (\mathrm{i}\bar{\omega}_n Y_n + \beta\upsilon_0 Y_n') - \bar{\omega}_n \beta \upsilon_1 C_n Y_n' (\mathrm{e}^{\mathrm{i}\sigma T_1} - \mathrm{e}^{-\mathrm{i}\sigma T_1}) \\ &\qquad + \mathrm{i}\beta\upsilon_0\upsilon_1 C_n Y_n'' (\mathrm{e}^{\mathrm{i}\sigma T_1} - \mathrm{e}^{-\mathrm{i}\sigma T_1}) - \frac{\Omega\beta\upsilon_1}{2} C_n Y_n' (\mathrm{e}^{\mathrm{i}\sigma T_1} + \mathrm{e}^{-\mathrm{i}\sigma\omega T_1}) \Big\}^{\mathrm{i}\bar{\omega}_n T_n} + cc + NST. \end{aligned} \tag{44}$$

Following a similar way to that in section 4.1 the solvability condition for fixed–fixed and fixed–sliding tensioned pipes is obtained as

$$D_1 C_n + (k_1 \cos\sigma T_1 + k_2 \sin\sigma T_1) C_n = 0, \tag{45}$$

where

$$\begin{aligned} k_1 &= \frac{\Omega\beta\upsilon_1 \int_0^1 Y_n' \bar{Y}_n \,\mathrm{d}x}{2\left(\mathrm{i}\bar{\omega}_n \int_0^1 Y_n \bar{Y}_n \,\mathrm{d}x + \beta\upsilon_0 \int_0^1 Y_n' \bar{Y}_n \,\mathrm{d}x\right)}, \\ k_2 &= \frac{\bar{\omega}_n \int_0^1 Y_n' \bar{Y}_n \,\mathrm{d}x + \upsilon_0 \int_0^1 Y_n'' \bar{Y}_n \,\mathrm{d}x}{\mathrm{i}\bar{\omega}_n \int_0^1 Y_n \bar{Y}_n \,\mathrm{d}x + \beta\upsilon_0 \int_0^1 Y_n' \bar{Y}_n \,\mathrm{d}x} \,\mathrm{i}\,\beta\upsilon_1. \end{aligned} \tag{46}$$

The solution of equation (45) is

$$C_n = C_0 \mathrm{e}^{(-(k_1/\sigma)\sin\sigma T_1 + (k_2/\sigma)\cos\sigma T_1)} \tag{47}$$

Since $-1 \leqslant \sin\sigma T_1 \leqslant -1$ and $-1 \leqslant \cos\sigma T_1 \leqslant 1$, there is no instability up to $O(\varepsilon)$.

4.3. Ω IS CLOSE TO $2\bar{\omega}_n$

The nearness of fluctuation frequency to twice any natural frequency can be expressed as

$$\Omega = 2\bar{\omega}_n + \varepsilon\sigma. \tag{48}$$

Substituting equations (33) and (43) into equation (34) the order ε equation for fixed–fixed and fixed–sliding tensioned pipes conveying fluid is obtained as

$$\begin{aligned} &D_0^2 w_1 + 2\upsilon_0 \beta D_0 w_1' - \beta(1-\upsilon_0^2) w_1'' \\ &= \Bigg\{ -2D_1 C_n(\mathrm{i}\bar{\omega}_n Y_n + \beta\upsilon_0 Y_n') \\ &\quad + \bar{C}_n \beta\upsilon_1 \mathrm{e}^{\mathrm{i}\sigma T_1}\left(\left(\bar{\omega}_n - \frac{\Omega}{2}\right)\bar{Y}_n' + \mathrm{i}\upsilon_0 Y_n''\right)\Bigg\} + cc + NST, \end{aligned} \tag{49}$$

where $\bar{C}_n$ is the complex conjugate of amplitude C_n. The solvability condition for this case is

$$D_1 C_n + k_0 \bar{C}_n \mathrm{e}^{\mathrm{i}\sigma T_1} = 0, \tag{50}$$

where

$$k_0 = \frac{\frac{1}{2}(\Omega - 2\bar{\omega}_n)\displaystyle\int_0^1 \bar{Y}_n' \bar{Y}_n \,\mathrm{d}x - \mathrm{i}\upsilon_0 \int_0^1 \bar{Y}_n'' \bar{Y}_n \,\mathrm{d}x}{2\left(\mathrm{i}\bar{\omega}_n \displaystyle\int_0^1 Y_n \bar{Y}_n \,\mathrm{d}x + \beta\upsilon_0 \int_0^1 \bar{Y}_n' \bar{Y}_n \,\mathrm{d}x\right)} \beta\upsilon_1. \tag{51}$$

The solution of equation (50) is assumed as

$$C_n = B_n \mathrm{e}^{\mathrm{i}\sigma T_1/2}, \qquad \bar{C}_n = \bar{B}_n \mathrm{e}^{\mathrm{i}\sigma T_1/2} \tag{52}$$

Substituting equation (52) into equation (50), one obtains

$$D_1 B_n + \frac{\mathrm{i}\sigma}{2} B_n + k_0 \bar{B}_n = 0, \tag{53}$$

where complex amplitudes are

$$B_n = b_n \mathrm{e}^{\lambda T_1}, \qquad \bar{B}_n = \bar{b}_n \mathrm{e}^{\lambda T_1} \tag{54}$$

and $b_n = b_n^R + \mathrm{i} b_n^I$, $k_0 = k_0^R + \mathrm{i} k_0^I$ where R and I denote real and imaginary parts respectively. Substituting equation (54) into equation (53) and separating real and imaginary parts, one obtains the matrix equation

$$\begin{bmatrix} \lambda + k_0^R & k_0^I - \dfrac{\sigma}{2} \\ k_0^I + \dfrac{\sigma}{2} & \lambda - k_0^R \end{bmatrix} \begin{Bmatrix} b_n^R \\ b_n^I \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}. \tag{55}$$

For non-trivial solution, the determinant of the coefficient matrix must be zero. This implies that

$$\lambda = \mp \frac{1}{2} \sqrt{-\sigma^2 + 4(k_0^{R^2} + k_0^{I^2})}. \tag{56}$$

Stable solution requires $\lambda = 0$. Then the stability boundaries can be written as

$$\sigma = \mp 2\sqrt{k_0^{R^2} + k_0^{I^2}}. \tag{57}$$

Substituting equation (57) into equation (48) the stability regions can be expressed as

$$\Omega = 2\bar{\omega}_n \mp 2\varepsilon\sqrt{k_0^{R^2} + k_0^{I^2}}. \tag{58}$$

The numerical solution for the two pipes will be given in the next section. The solutions differ in the integrals since the shape functions and natural frequencies are different for the two pipes.

5. NUMERICAL ANALYSIS

In this section, the numerical examples will be given for the vibrations of tensioned pipes conveying fluid. In Figures 2–5, the natural frequencies are plotted depending on mean

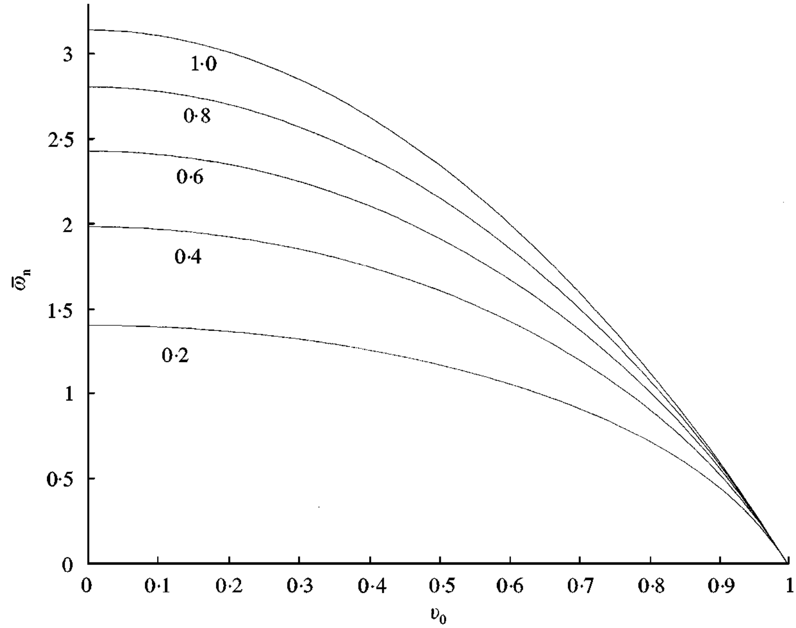

Figure 2. The natural frequency value versus mean velocity for different ratios of fluid mass to the total mass ($\beta = 0{\cdot}2$, 0·4, 0·6, 0·8, 1·0) for the first mode for fixed–fixed end conditions.

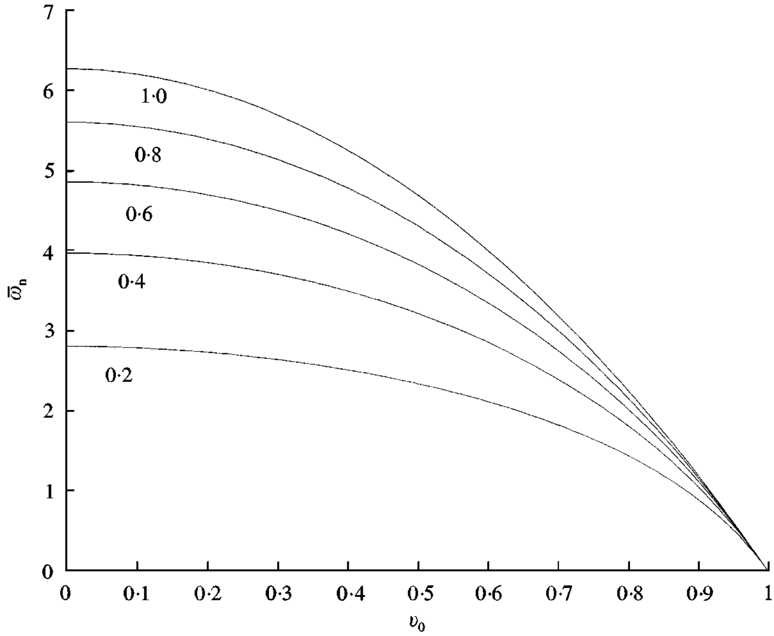

Figure 3. The natural frequency value versus mean velocity for different ratios of fluid mass to the total mass ($\beta = 0{\cdot}2$, 0·4, 0·6, 0·8, 1·0) for the second mode for fixed–fixed end conditions.

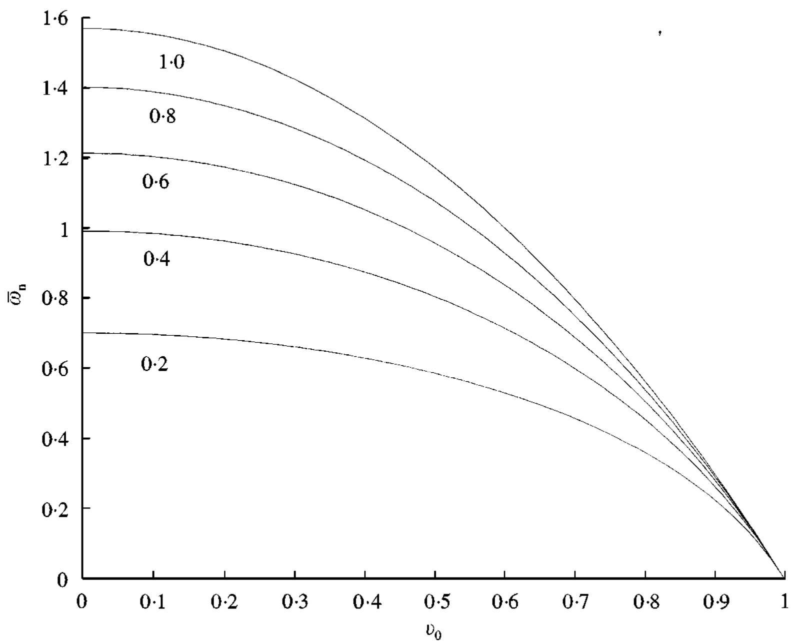

Figure 4. The natural value versus mean velocity for different ratios of fluid mass to the total mass ($\beta = 0{\cdot}2$, 0·4, 0·6, 0·8, 1·0) for the first mode for fixed–sliding end conditions.

velocity and the ratio of fluid mass to the total mass per unit length by using equation (30) for fixed–fixed tensioned pipe and equation (32) for fixed–sliding tensioned pipe for the first and second modes. Increasing the mean velocity decreases the natural frequency values. At

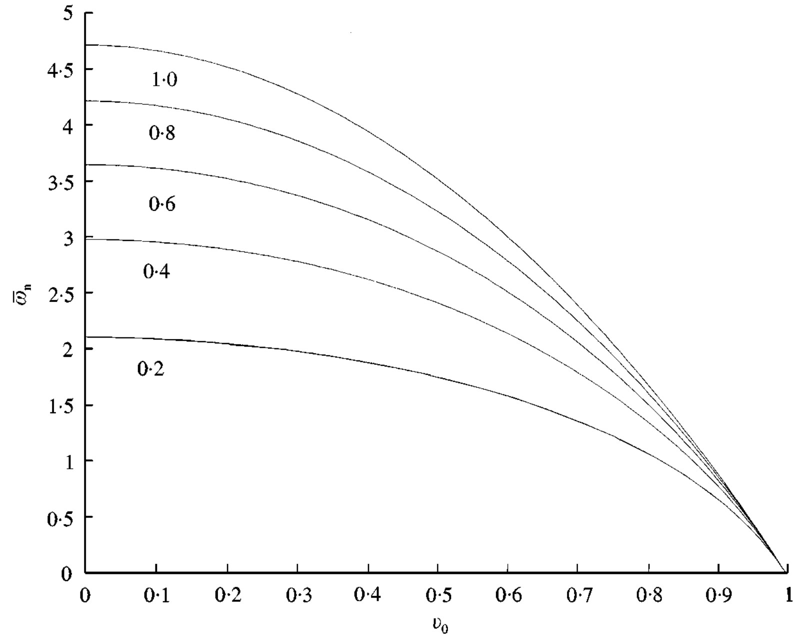

Figure 5. The natural frequency value versus mean velocity for different ratios of fluid mass to the total mass ($\beta = 0{\cdot}2$, 0·4, 0·6, 0·8, 1·0) for the second mode for fixed–sliding end conditions.

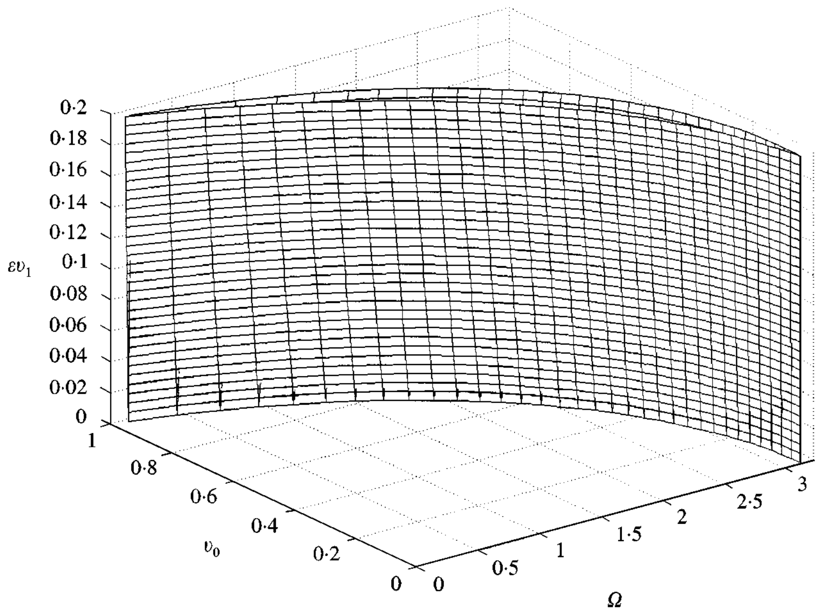

Figure 6. Stable and unstable regions for the principal parametric resonances for the first mode ($\beta = 0{\cdot}25$) for fixed–fixed end conditions.

the critical speed the frequency values vanish and divergence instability occurs. Increasing the ratio of fluid mass to the total mass per unit length, increases the natural frequencies. At the limiting value, $\beta = 1$, the results coincide with the results of axially moving string. The

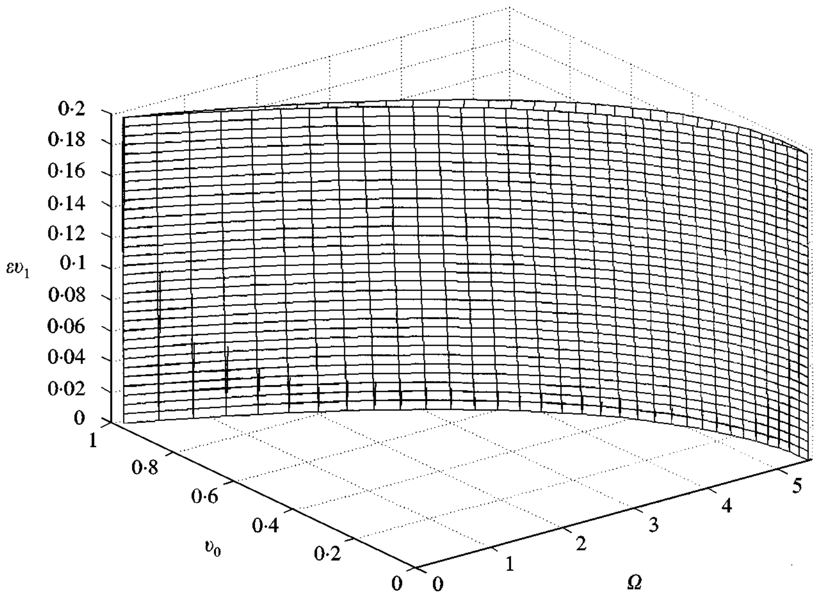

Figure 7. Stable and unstable regions for the principal parametric resonances for the first mode ($\beta = 0{\cdot}75$) for fixed–fixed end conditions.

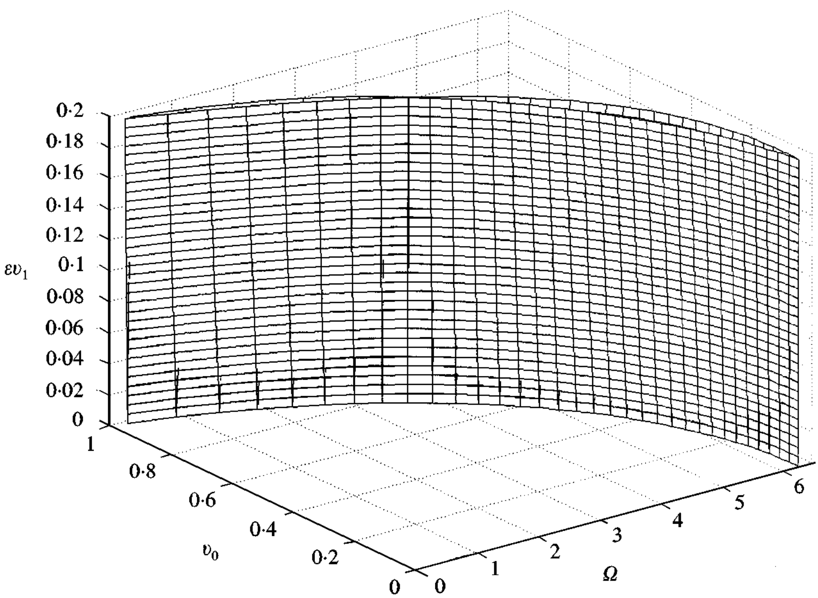

Figure 8. Stable and unstable regions for the principal parametric resonances for the second mode ($\beta = 0{\cdot}25$) for fixed–fixed end conditions.

natural frequencies for fixed–sliding tensioned pipe are lower than those of fixed–fixed tensioned pipe. At higher modes the natural frequency values and critical velocity values increase.

Stability analysis is made for the principal parametric resonance case. It is found that when the velocity fluctuation frequency is close to zero, no instabilities are detected up to

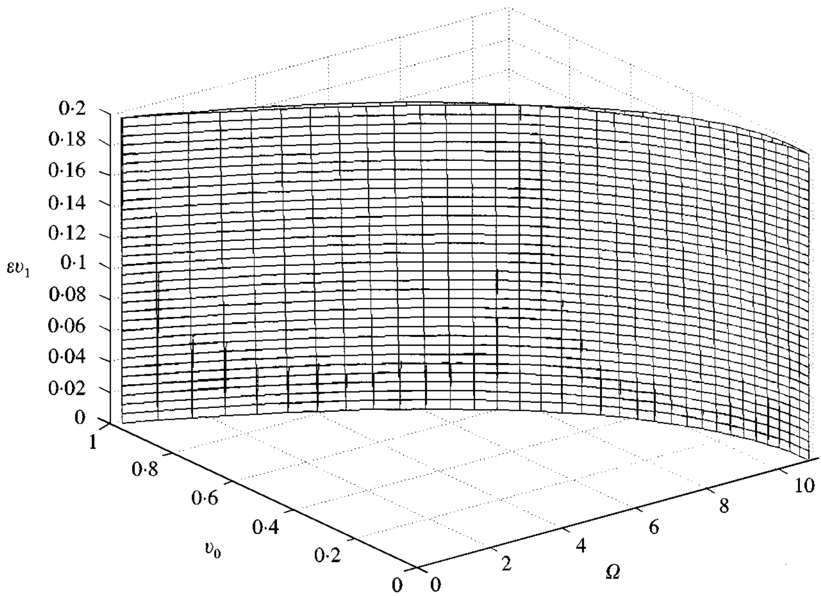

Figure 9. Stable and unstable regions for the principal parametric resonances for the first mode ($\beta = 0{\cdot}75$) for fixed–fixed end conditions.

the first order of perturbation. When the fluctuation frequency is away from zero and twice the natural frequency, the solutions are bounded and no instability exists. Instabilities occur when the frequency of velocity fluctuations is close to two times the natural frequency of the constant velocity system. The stable and unstable regions are plotted for the principal parametric resonance case by using equation (58) for fixed–fixed tensioned pipe and fixed–sliding tensioned pipe. In Figures 6 and 7, the stable and unstable regions are plotted for the principal parametric resonance case for different mean velocities and velocity fluctuation amplitudes ($\beta = 0{\cdot}25$, $0{\cdot}75$) for the first mode and in Figures 8 and 9 for the second mode ($\beta = 0{\cdot}25$, $0{\cdot}75$) for fixed–fixed end conditions. Increasing velocity fluctuation amplitude enlarges the stability regions. With increasing the mass ratio, the stability shift to higher Ω values. At the critical velocity values, the unstable regions widen. In Figures 10 and 11, the stable and unstable regions are plotted for the principal parametric resonance case for different mean velocities and velocity fluctuation amplitudes ($\beta = 0{\cdot}25$, $0{\cdot}75$) for the first mode and in Figures 12 and 13 for the second mode ($\beta = 0{\cdot}25$, $0{\cdot}75$) for fixed–sliding end conditions. The fixed–sliding tensioned pipe shows similar characteristics. Increasing velocity fluctuation amplitude enlarges the stability regions. With increasing the mass ratio, the stability regions shift to higher Ω values. At the critical velocity values, the unstable regions widen. The stability borders for fixed–sliding tensioned pipe have lower values those of fixed–fixed tensioned pipe. At higher modes the stability borders shift to higher Ω values for both cases.

In all figures (Figures 6–13), the regions in between the planar surfaces are unstable whereas the remaining regions are stable.

6. CONCLUDING REMARKS

In this study, the transverse vibrations of highly tensioned fixed–fixed and fixed–sliding supported pipes with vanishing flexural stiffness and transporting fluid with time-dependent

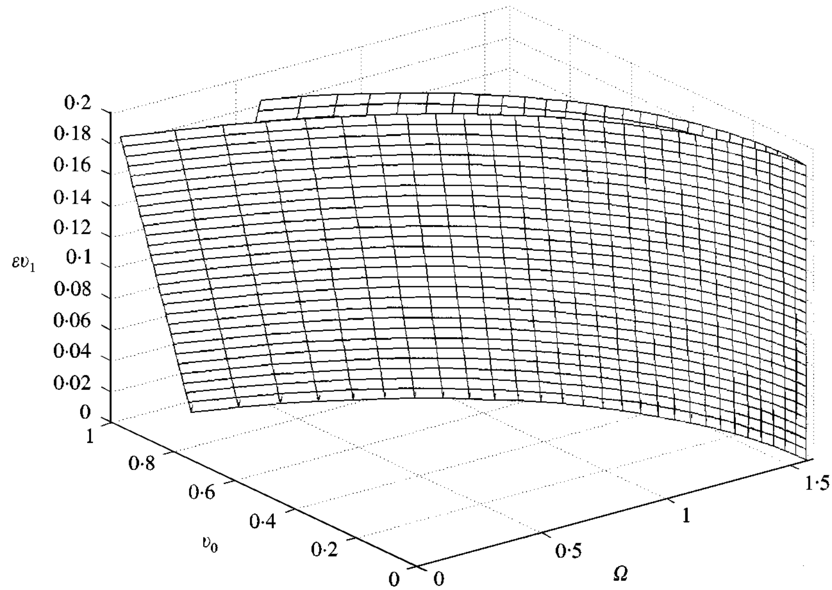

Figure 10. Stable and unstable regions for the principal parametric resonances for the first mode ($\beta = 0{\cdot}25$) for fixed–sliding end conditions.

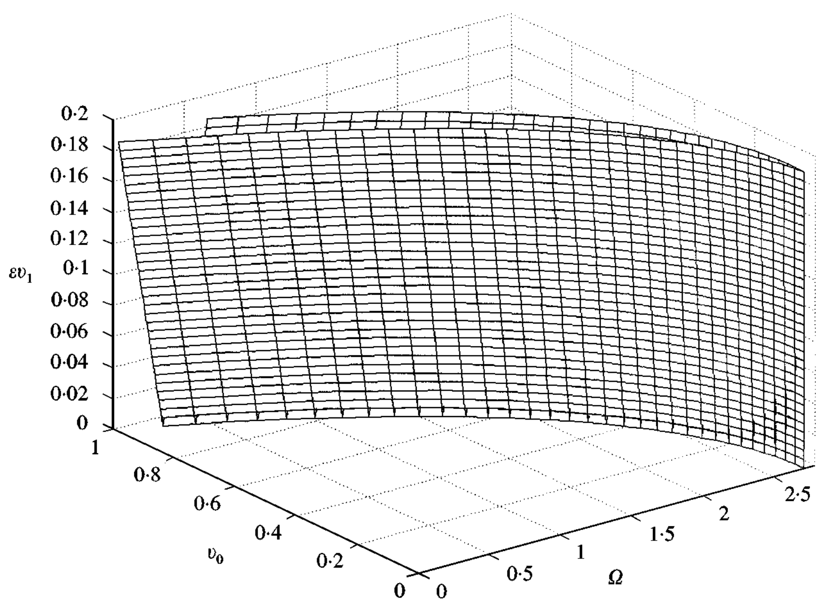

Figure 11. Stable and unstable regions for the principal parametric resonances for the first mode ($\beta = 0{\cdot}75$) for fixed–sliding end condtions.

velocity are investigated. The sliding end condition for tensioned pipes permits only transverse displacement but the slope is zero. The supports result in extension of the pipe during the vibration and hence introduce further non-linear terms to the equation of motion. The fluid velocity is assumed to be harmonically varying about a mean velocity. The equation of motion is solved analytically by direct application of the method of

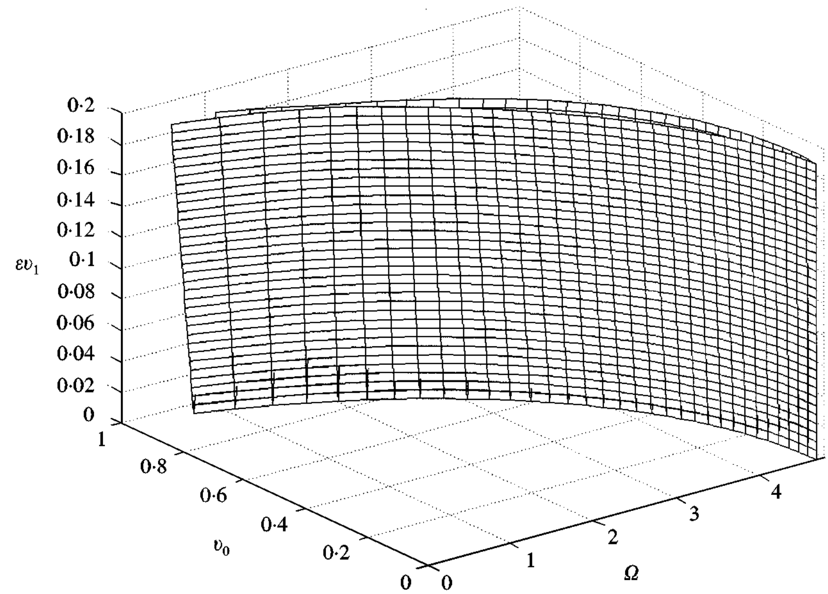

Figure 12. Stable and unstable regions for the principal parametric resonances for the second mode ($\beta = 0{\cdot}25$,) for fixed–sliding end conditions.

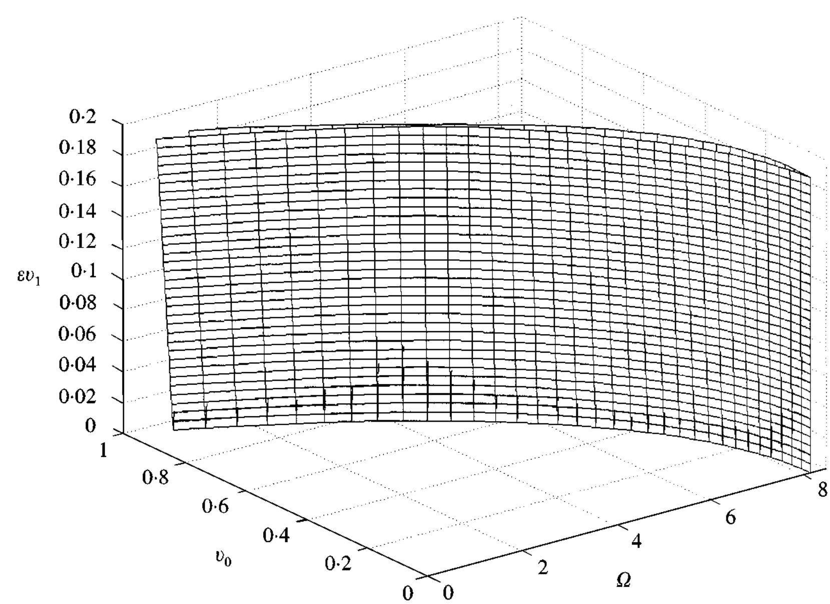

Figure 13. Stable and unstable regions for the principal parametric resonances for the second mode ($\beta = 0{\cdot}75$) for fixed–sliding end conditions.

multiple scales (a perturbation technique). The natural frequencies are found analytically depending on mean velocity and the ratio of fluid mass to the total mass per unit length. It is found that, increasing the mean velocity decreases the natural frequency values and at the critical speed the frequency values vanish and divergence instability occurs. Also increasing the ratio of fluid mass to the total mass per unit length, increases the natural frequencies.

The natural frequencies for fixed–sliding tensioned pipe are lower than those of fixed–fixed tensioned pipe. The principal parametric resonances are investigated in detail. Stability boundaries are determined analytically. It is found that instabilities occur when the frequency of velocity fluctuations is close to two times the natural frequency of the constant velocity system. When the velocity fluctuation frequency is close to zero, no instabilities are detected up to the first order of perturbation. When the fluctuation frequency is away from zero and twice the natural frequency, the solutions are bounded and no instability exists. Increasing velocity fluctuation amplitude enlarges the stability regions. With increasing the mass ratio, the stability regions shift to higher velocity fluctuation frequency values. Stable and unstable region shift to lower velocity fluctuation frequency values. Stable and unstable regions shift to lower velocity fluctuation frequency for fixed–sliding supports when compared with those of fixed–fixed supports.

REFERENCES

1. A. G. ULSOY, C. D. MOTE JR and R. SYZMANI 1978 *Holz als Roh-und Werkstoff* **36**, 273–280. Principal developments in band saw vibration and stability research.
2. J. A. WICKERT and C. D. MOTE JR 1988 *Shock and Vibration Digest* **20**, 3–13. Current research on the vibration and stability of axially moving materials.
3. R. A. SACK 1954 *British Journal of Applied Physics* **5**, 224–226, Transverse oscillation of travelling strings.
4. F. R. ARCHIBALD and A. G. EMSLIE 1958 *ASME*, *Journal of Applied Mechanics* **25**, 347–348. The vibration of a string having a uniform motion along its length.
5. R. D. SWOPE and W. F. AMES 1963 *Journal of the Franklin Institute* **275**, 36–55. Vibrations of a moving threadline.
6. C. D. MOTE JR 1965 *Journal of the Franklin Institute* **279**, 430–444. A study of band saw vibration.
7. T. CHUBACHI 1958 *Bulletin of Japanese Society of Mechanical Engineers* **1**, 24–29. Lateral vibration of axially moving wire or belt form materials.
8. W. L. MIRANKER 1960 *IBM Journal Research and Development* **4**, 36–42. The wave equation in a medium in motion.
9. J. A. WICKERT and C. D. MOTE JR 1989 *Journal of the Acoustical Society of America* **85**, 1365–1368. On the energetics of axially moving continua.
10. J. A. WICKERT and C. D. MOTE JR 1990 *ASME Journal of Applied Mechanics* **57**, 738–744. Classical vibration analysis of axially moving continua.
11. J. A. WICKERT 1992 *International Journal of Non-linear Mechanics* **27**, 503–517. Non-linear vibration of a travelling tensioned beam.
12. M. PAKDEMİRLİ, A. G. ULSOY and A. CERANOĞLU 1994 *Journal of Sound and Vibration* **169**, 179–196. Transverse vibration of an axially accelerating string.
13. M. PAKDEMİRLİ and H. BATAN 1993 *Journal of Sound and Vibration* **168**, 371–378. Dynamic stability of a constantly accelerating strip.
14. M. PAKDEMİRLİ and A. G. ULSOY 1997 *Journal of Sound and Vibration* **203**, 815–832. Stability analysis of an axially accelerating string.
15. H. R. ÖZ, M. PAKDEMİRLİ and E. ÖZKAYA 1998 *Journal of Sound and Vibration* **215**, 571–576. Transition behaviour from string to beam for an axially accelerating material.
16. H. R. ÖZ and M. PAKDEMİRLİ 1999 *Journal of Sound and Vibration* **227**, 239–257. Vibrations of an axially moving beam with time dependent velocity.
17. H. R. ÖZ, M. PAKDEMİRLİ and E. ÖZKAYA *ASME Mechanics and Materials Conference*, *Symposium on Nonlinear Vibrations and Perturbation Methods*, *Final program and Book of Abstracts June* 27–30, *Blacksburg*, *Virginia*, *USA*, 394–395 Linear and non-linear vibration of axially moving beams.
18. H. R. ÖZ, M. PAKDEMİRLİ and H. BOYACI *International Journal of Non-linear Mechanics*. Nonlinear vibrations and stability of an axially moving beam with time dependent velocity (in press).
19. T. B. BENJAMIN 1961 *Proceedings of the Royal Society of London* **261A**, 457–499. Dynamics of a system of articulated pipes conveying fluids: **1** Theory and II. Experiment.
20. S. NEMAT-NASSER, S. S. PRASAD and G. HERRMANN 1966 *American Institute of Aeronautics and Astronautics Journal* **4**, 1276–1280. Destabilizing effect of velocity-independent forces in non-conservative continuous systems.

21. R. W. GREGORY and M. P. PAIDOUSSIS 1996 *Proceeding of the Royal Society of London* **293A**, 512–544. Unstable oscillation of tabular cantilevers conveying fluid, 1. Theory and II. Experiment.
22. M. P. PAIDOUSSIS and G. X. LI 1993 *Journal of Fluids and Structures* **7**, 137–204. Pipes conveying fluid: a model dynamical problem.
23. S. Y. LEE and C. D. MOTE JR 1997 *Journal of Sound and Vibration* **204**, 717–734. A generalized treatment of the energetics of translating continua, Part I: strings and second order tensioned pipes.
24. S. Y. LEE and C. D. MOTE JR 1997 *Journal of Sound and Vibration* **204**, 735–753. A generalized treatment of the energetics of translating continua, Part II: beams and fluid conveying pipes.
25. A. H. NAYFEH 1981 *Introduction to Perturbation Techniques*. New York: John Wiley.
26. A. H. NAYFEH and D. T. MOOK 1979 *Nonlinear Oscillations*. New York: John Wiley.