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A STUDY ON THE INTERACTION BETWEEN THE PRIMARY AND SECONDARY STRUCTURES OF COMBINED STRUCTURES

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A study on the transfer functions of primary structures and the relative power flow between primary and secondary structures under evolutionary earthquake input gives a physical insight into the interaction between the primary and secondary structures of combined structures. The interaction between primary and secondary structures depends mainly on the mass ratio, stiffness ratio, coupling stiffness and frequency tuning between the primary and secondary structures. The interaction also depends on the location at which the secondary structures are attached to the primary system. The analysis shows that the secondary systems, instead of being vibration absorbers, have a significant driving effect on the primary structure when the mass ratio of the secondary to the primary system is very large. An asynchronous driving principle is proposed to reduce the driving effect.

1. INTRODUCTION

The study on the resonance effect (tuning) is quite active in the research area of primary–secondary (PS) systems. In the theory of tuned mass damper (TMD), it is pointed out that some secondary systems attached to a primary structure can absorb the vibration energy of the primary structure. It is generally agreed that TMD is quite effective as far as wind-resistant is concerned, but as to whether it is effective in earthquake resistance, there are some contradictory conclusions. Some studies show that TMD is effective in earthquake resistance [1, 2], while some others show that it does not work at all [3, 4]. These contradictory results show that the effectiveness of the TMD in earthquake resistance depends on the parameters of the primary and secondary structures.

This paper studies another aspect of the interaction between the primary and secondary structure, i.e., the driving effect of the secondary systems on the primary structure. When secondary structures are appended to a primary structure, their motion inevitably exerts driving forces on the primary structure. However, the driving effect of the secondary structures on the primary structure was often ignored or less discussed [5–7]. In the study of TMD, it is suggested that a bigger mass ratio of the secondary structures to the primary structure will have a better effect of vibration absorption. When the TMD theory is applied to wind vibration control of a TV tower [5], it is reasonable that a bigger mass ratio of the

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TMD to the TV tower is more effective. On this occasion, the driving effect can be neglected because the primary structure is quite massive and thus a big mass ratio is impossible. However, when TMD is applied to a microwave communication tower, a bigger mass ratio of TMD or a bigger number of TMDs will not only lead to the failure of vibration absorption, but also increase the response of the primary structure [6]. Unfortunately, the explanation of the foregoing result was not given. Recently, the mega subconfiguration is put forward as an innovative way of vibration control in tall buildings [7]. It is proposed that the secondary structures in tall buildings can act as a kind of energy absorber, but the driving effect of the secondary structures with a large mass ratio was not discussed. Therefore, the foregoing result should be considered carefully. In some recent studies [8, 9], it is reported that the megaframe with suspension systems (MFSS) is a kind of new structure system in vibration control. These studies further confirm the feasibility of using mega subconfiguration as an effective way of vibration control in tall buildings, but it is also suggested that the secondary structures (suspension systems) will be unlikely to be a vibration absorber, and that on the contrary, the combined system will be mainly driven by the secondary structures and vibrates in the fundamental periods of the secondary structures. These studies indicate that it is the most essential to consider the driving effect of the secondary structure in the study of a combined structure with mega subconfiguration. Some principles in the design of TMD and MTMD [10–12] is no longer valid in vibration control of a combined structure with a large mass ratio of the secondary structures to the primary structure. The vibration control strategy should be based on how to control the driving effect.

In this paper, a useful physical insight of the interaction between the primary and secondary structures is given through the study of the transfer functions and the relative energy flow between the primary and secondary structures under an evolutionary seismic input. An asynchronous driving principle is also presented in this paper for the control of the driving effect. It is essential to understand the interaction between the primary and secondary structures in designing combined structures.

2. EQUATION OF MOTION AND MODAL REDUCTION

The dynamical behavior of PS systems is inherently complex. One of the difficulties in the analysis of the PS system arises when both the primary and secondary systems are multi-degree-of-freedom systems. In the component mode analysis of such a combined system, if only the modes of lower orders are retained while those of higher orders are neglected, some absurd results may occur [13]. An admissible transformation in modal reduction of combined structures presented by Muscolino [13] is adopted in this paper.

Consider a N-degree-of-freedom primary megaframe with N secondary suspension systems having s_i ($i = 1, 2, \dots, N$) suspended floors respectively. The equation of motion of the combined systems can be written as follows:

$$\mathbf{M}_t \ddot{\mathbf{u}}_t + \mathbf{C}_t \dot{\mathbf{u}}_t + \mathbf{K}_t \mathbf{u}_t = -\mathbf{M}_t \tau_t \eta_g(t) \tag{1}$$

in which $\eta_g(t)$ is the ground acceleration input, τ_t is the influence coefficient vector, the dot over a variable denotes its time derivatives and $\mathbf{u}_t$ is the total displacement vector (of order $n = N + \sum_{i=1}^{N} s_i$) defined as

$$\mathbf{u}_t = [\mathbf{y}_1^{\mathrm{T}}, \mathbf{y}_2^{\mathrm{T}}, \dots, \mathbf{y}_N^{\mathrm{T}}, \mathbf{y}_p^{\mathrm{T}}]^{\mathrm{T}}, \tag{2}$$

where $\mathbf{y}_p^{\mathrm{T}}$ and $\mathbf{y}_i^{\mathrm{T}}$ $(i = 1, \ldots, N)$ are the displacement vectors, measured with respect to the ground of the megaframe and suspension systems. In equation (1) the matrices $\mathbf{M}_t$ and $\mathbf{K}_t$ are given by

$$\mathbf{M}_t = \begin{bmatrix} \mathbf{M}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_2 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{M}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{M}_p \end{bmatrix}, \quad \mathbf{K}_t = \begin{bmatrix} \mathbf{K}_1 & \mathbf{0} & \cdots & \mathbf{0} & K_{1p} \\ \mathbf{0} & \mathbf{K}_2 & \cdots & \mathbf{0} & \mathbf{K}_{2p} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{K}_N & \mathbf{K}_{Np} \\ \mathbf{K}_{1p}^{\mathrm{T}} & \mathbf{K}_{2p}^{\mathrm{T}} & \cdots & \mathbf{K}_{Np}^{\mathrm{T}} & \mathbf{K}_p + \sum_{i=1}^{N} \mathbf{K}_{0i} \end{bmatrix}, \tag{3}$$

where $\mathbf{M}_i$, $\mathbf{K}_i$ $(i = 1, \ldots, N)$, $\mathbf{M}_p$ and $\mathbf{K}_p$ are the mass and stiffness matrices of the suspension systems and the megaframe fixed at their own basis (i.e., the megaframe fixed at the ground and the suspension systems fixed at the megaframe). $\mathbf{K}_{ip}$ $(i = 1, \ldots, N)$ is the coupling matrix between the megaframe and the ith suspension system. $\mathbf{K}_{0i}$ $(i = 1, \ldots, N)$ represents the increment to the stiffness matrix of the megaframe due to the ith suspension system. $\mathbf{C}_t$ in equation (1) is the structural damping matrix of the combined structure.

To apply the admissible co-ordinate transformation [13] to equation (1), the so-called relative displacement vector is introduced as follows:

$$\mathbf{u}_r = [\mathbf{x}_1^{\mathrm{T}}, \mathbf{x}_2^{\mathrm{T}}, \ldots, \mathbf{x}_N^{\mathrm{T}}, \mathbf{y}_p^{\mathrm{T}}]^{\mathrm{T}}. \tag{4}$$

in which $\mathbf{x}_i = (i = 1, \ldots, N)$ is the nodal points displacement vector of the ith suspension system with respect to the connecting point. The relationship between the vectors $\mathbf{u}_t$ and $\mathbf{u}_r$ is

$$\mathbf{u}_t = \begin{bmatrix} \mathbf{I}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{N}_{1p} \\ \mathbf{0} & \mathbf{I}_2 & \cdots & \mathbf{0} & \mathbf{N}_{2p} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_N & \mathbf{N}_{Np} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I}_p \end{bmatrix} \mathbf{u}_r, \tag{5}$$

where $\mathbf{I}_i$ $(i = 1, \ldots, N)$ and $\mathbf{I}_p$ are the identity matrix of orders $(s_i \times s_i)$ and $(N \times N)$ respectively. $\mathbf{N}_{ip}$ $(i = 1, \ldots, N)$ is the so-called pseudostatic influence matrix, given as

$$\mathbf{N}_{ip} = -\mathbf{K}_i^{-1} \mathbf{K}_{ip}. \tag{6}$$

In this paper, the suspension systems and the megaframe are assumed to be mono-connected. Thus, we have [13]

$$\mathbf{K}_{0i} - \mathbf{K}_{ip}^{\mathrm{T}} \mathbf{K}_i^{-1} \mathbf{K}_{ip} = 0. \tag{7}$$

By means of the co-ordinate transformation defined in equation (5), we obtain the equation of motion of the combined system in terms of the relative displacement as

$$\mathbf{M}_r \ddot{\mathbf{u}}_r + \mathbf{C}_r \dot{\mathbf{u}}_r + \mathbf{K}_r \mathbf{u}_r = -\mathbf{M}_r \tau_r \eta_g(t), \tag{8}$$

in which $\mathbf{M}_r$, $\mathbf{C}_r$, $\mathbf{K}_r$ and $\boldsymbol{\tau}_r$ are the mass, damping, stiffness and influence coefficient matrices in the relative displacement coordination defined in equation (5).

Equation (8) can be further reduced to a system of equations expressed in modal co-ordinates. According to the component-mode synthesis, we have the co-ordinate transformation as

$$\mathbf{u}_r = \Gamma_r \mathbf{q}, \qquad \Gamma_r = \begin{bmatrix} \boldsymbol{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_2 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Phi}_N & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Phi}_p \end{bmatrix}, \tag{9}$$

where $\boldsymbol{\Phi}_i$ $(i = 1, \ldots, N)$ and $\boldsymbol{\Phi}_p$ are the modal matrices (of orders $n_{si} \leqslant s_i$ and $n_p \leqslant N$) normalized with respect to $\mathbf{M}_i$ $(i = 1, \ldots, N)$ and $\mathbf{M}_p$ respectively. They can be obtained by solving the following eigenproblems:

$$\mathbf{M}_i \boldsymbol{\Phi}_i \boldsymbol{\Omega}_i^2 = \mathbf{K}_i \boldsymbol{\Phi}_i \ (i = 1, \ldots, N), \qquad \mathbf{M}_p \boldsymbol{\Phi}_p \boldsymbol{\Omega}_p^2 = \mathbf{K}_p \boldsymbol{\Phi}_p, \tag{10}$$

where $\boldsymbol{\Omega}_i$ $(i = 1, \ldots, N)$ and $\boldsymbol{\Omega}_p$ are diagonal matrices consisting of the natural radian frequencies of the suspension systems and the megaframe respectively.

By using the co-ordinate transformation defined in equation (10), equation (8) becomes a set of equations of order $n = n_p + \sum_{i=1}^{N} n_{s_i}$, which is generally smaller than the order of original set of equations. It can be written as

$$\mathbf{M}_q \ddot{\mathbf{q}} + \mathbf{C}_q \dot{\mathbf{q}} + \mathbf{K}_q \mathbf{q} = -\mathbf{V} \eta_g(t), \tag{11}$$

where

$$\mathbf{M}_q = \begin{bmatrix} \mathbf{I}_1 & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Phi}_1^{\mathrm{T}} \mathbf{M}_1 \mathbf{N}_{1p} \boldsymbol{\Phi}_p \\ \mathbf{0} & \mathbf{I}_2 & \cdots & \mathbf{0} & \boldsymbol{\Phi}_2^{\mathrm{T}} \mathbf{M}_2 \mathbf{N}_{2p} \boldsymbol{\Phi}_p \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_N & \boldsymbol{\Phi}_N^{\mathrm{T}} \mathbf{M}_N \mathbf{N}_{Np} \boldsymbol{\Phi}_N \\ \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{N}_{1p}^{\mathrm{T}} \mathbf{M}_1 \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{N}_{2p}^{\mathrm{T}} \mathbf{M}_2 \boldsymbol{\Phi}_2 & \cdots & \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{N}_{Np}^{\mathrm{T}} \mathbf{M}_N \boldsymbol{\Phi}_N & \mathbf{I}_p + \sum_{i=1}^{N} \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{N}_{ip}^{\mathrm{T}} \mathbf{M}_i \mathbf{N}_{ip} \boldsymbol{\Phi}_p \end{bmatrix}.$$

$$\mathbf{K}_q = \begin{bmatrix} \boldsymbol{\Omega}_{s_1}^2 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Omega}_{s_2}^2 & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \boldsymbol{\Omega}_{s_N}^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{\Omega}_p^2 \end{bmatrix}, \qquad \mathbf{C}_q = \begin{bmatrix} 2\xi \boldsymbol{\Omega}_{s_1} & 0 & \cdots & 0 & 0 \\ 0 & 2\xi \boldsymbol{\Omega}_{s_2} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 2\xi \boldsymbol{\Omega}_{s_N} & 0 \\ 0 & 0 & \cdots & 0 & 2\xi \boldsymbol{\Omega}_p \end{bmatrix},$$

$$\mathbf{V} = [\boldsymbol{\Phi}_1^{\mathrm{T}} \mathbf{M}_1 \ \ \boldsymbol{\Phi}_2^{\mathrm{T}} \mathbf{M}_2 \ \ \cdots \ \ \boldsymbol{\Phi}_N^{\mathrm{T}} \mathbf{M}_N \ \ \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{M}_p + \textstyle\sum_{i=1}^{N} \boldsymbol{\Phi}_p^{\mathrm{T}} \mathbf{N}_{ip}^{\mathrm{T}} \mathbf{M}_i]^{\mathrm{T}}. \tag{12}$$

3. THE ANALYSIS OF THE TRANSFER FUNCTIONS

The study of the transfer functions of combined systems can clearly show the effect of the secondary structures on the primary structure in the frequency domain, which is shown in some study in the TMD theory [10–12]. In this section, the transfer functions of PS systems will be deduced to observe the interaction between the secondary and the primary structures.

The excitation and the response in equation (11) can be expressed as

$$\eta_g(t) = F\mathrm{e}^{-\mathrm{j}\omega t}, \qquad \mathbf{q}(t) = F\mathbf{H}(\omega)\mathrm{e}^{-\mathrm{j}\omega t}. \tag{13}$$

The transfer functions in modal co-ordinates can be obtained as

$$\mathbf{H}_i(\omega) = [v_i + \omega^2 \mu_i \mathbf{H}_p(\omega)][\Omega_i^2 - \omega^2 \mathbf{I}_i - 2\xi\omega\Omega_i \mathrm{j}]^{-1} \quad (i = 1, \ldots, N), \tag{14}$$

$$\mathbf{H}_p(\omega) = \left[v_p + \sum_{i=1}^{N} \mathbf{Z}_i(\omega) v_i \right] \left[\mathbf{\Omega}_p^2 - \omega^2 \mu_p - 2\xi\omega\mathbf{\Omega}_p \mathrm{j} - \omega^2 \sum_{i=1}^{N} \mathbf{Z}_i(\omega)\mu_i \right]^{-1}, \tag{15}$$

where

$$\mathbf{Z}_i(\omega) = \omega^2 \mu_i^{\mathrm{T}} [\Omega_i^2 - \omega^2 \mathbf{I}_i - 2\xi\omega\Omega_i j]^{-1}, \tag{16a}$$

$$\mu_i = \mathbf{\Omega}_i^{\mathrm{T}} \mathbf{M}_i \mathbf{N}_{ip}^{\mathrm{T}} \mathbf{\Phi}_p, \qquad v_i = \mathbf{\Phi}_i^{\mathrm{T}} \mathbf{M}_i, \tag{16b, c}$$

$$\mu_p = \mathbf{I}_p + \sum_{i=1}^{N} \mathbf{\Phi}_p^{\mathrm{T}} \mathbf{N}_{ip}^{\mathrm{T}} \mathbf{M}_i \mathbf{N}_{ip} \mathbf{\Phi}_p, \qquad v_p = \mathbf{\Phi}_p^{\mathrm{T}} \mathbf{M}_p + \sum_{i=1}^{N} \mathbf{\Phi}_p^{\mathrm{T}} \mathbf{N}_{ip}^{\mathrm{T}} \mathbf{M}_i \tag{16d, e}$$

in which j is the imaginary unit, and $\mathbf{H}_i(\omega)$ $(i = 1, \ldots, N)$ and $\mathbf{H}_p(\omega)$ are the transfer functions in the component mode co-ordinate respectively. The transfer functions in the total displacement co-ordinate u_t can then be obtained from the result of equations (13) and (14) according to the following co-ordinate transformation:

$$\mathbf{u}_t = \Gamma_t \mathbf{q}, \qquad \Gamma_t = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{N}_{1p}\mathbf{\Phi}_p \\ \mathbf{0} & \mathbf{\Phi}_2 & \cdots & \mathbf{0} & \mathbf{N}_{2p}\mathbf{\Phi}_p \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{\Phi}_N & \mathbf{N}_{Np}\mathbf{\Phi}_p \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Phi}_p \end{bmatrix}. \tag{17}$$

However, it is in modal co-ordinates that the interaction between the suspension systems and the megaframe can be shown more clearly. The term $\omega^2 \mu_i \mathbf{H}_p(\omega)$ in equation (14) indicates that the suspension systems have additionally input a base acceleration amplified by the megaframe dynamically. The second term in equation (16d) describes the increase in the mass of the megaframe contributed by the suspension systems statically. The second term in equation (16e) is the seismic force contributed by the suspension systems statically. The term $\sum_{i=1}^{N} \mathbf{Z}_i(\omega) v_i$ in equation (15) is the driving force transferred from the suspension systems dynamically, and the term $-\omega^2 \sum_{i=1}^{N} \mathbf{Z}_i(\omega)\mu_i$ in equation (15) is the dynamic stiffness contributed by the suspension systems.

The foregoing discussions indicate that the dynamic effect of the suspension systems in the megaframe is twofold. When the frequency of the suspension systems is tuned to the frequency of the seismic force, the suspension systems provide a large driving force and dynamic stiffness. The enlargement of the dynamic stiffness can reduce the vibration of the megaframe, which is the principle of TMD or MTMD. The principle and characteristics of TMD and MTMD have been widely discussed [1–7, 10–12]. Nevertheless, the driving effect of the secondary structure on the primary structure is less considered or even neglected in the previous studies of TMD or MTMD.

When the secondary systems have different natural frequencies, as is shown in equations (15) and (16a), the tuned forces of the secondary systems have different phases, therefore, the secondary structures asynchronously drive the primary system. The sum of the driving forces is less than the sum of the synchronous driving forces, and the peak values of frequency response are distributed in a relatively wider range of frequency. This is called the asynchronous driving principle. The following discussion will show its feature and effectiveness in vibration control of the driving effect.

4. RELATIVE POWER FLOW

In the study of the interaction between the secondary and the primary structures, the concept of power flow is valuable because it combines the forces and velocities in a very simple concept with the relative phase angle considered. Some studies present the way to study the power flow between the primary and secondary structures under harmonic [14] and white noise [15] input. Under the seismic input, because the energy comes from the ground, passes through the primary structure, and lastly, arrives at the secondary structures, it is certainly true that the secondary structures are always energy absorbers under the seismic input. In this paper, in contrast to the power flow, relative power flow is proposed to appraise the dynamic effect of the secondary structure on the primary structure.

Figure 1 shows a modal of a single-degree PS structure under seismic input. In the figure, m_s, m_p, m_g are the mass of the secondary, primary structures and the ground respectively. k_s, k_p, k_g are the stiffness of the secondary, primary structures and the ground, which are also the absolute value of the coupling stiffness between the nearby structures. c_s, c_p, c_g are the damping of the secondary, primary structures and the ground, which are also the coupling damping between the nearby structures. u_e is the absolute displacement of the epicentre. u_g is the relative displacement to the epicentre. y_s, y_p are the relative displacement of the secondary, primary structures to the ground. The excitation of the system is an acceleration at the epicenter, and its value is $\ddot{u}_e$. The power flow from the secondary structure to the

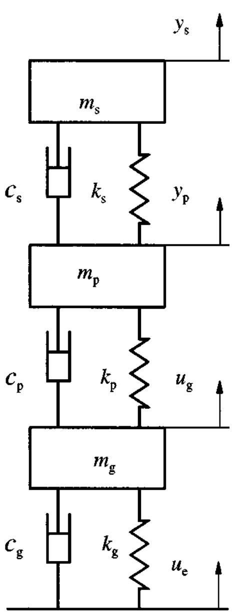

Figure 1. Schematic sketch of a PS system over the epicenter.

primary structure is

$$P_a = F_s(\dot{u}_e + \dot{u}_g + \dot{y}_p) = [k_s(y_s - y_p) + c_s(\dot{y}_s - \dot{y}_p)](\dot{u}_e + \dot{u}_g + \dot{y}_p), \tag{18}$$

in which F_s is the force acting on the primary structure by the secondary structure, and can be written as

$$F_s = k_s(y_s - y_p) + c_s(\dot{y}_s - \dot{y}_p). \tag{19}$$

Because the power flow passes from the primary structure to the secondary structure, we can always have

$$P_a < 0 \tag{20}$$

which indicates that the secondary structure is always an energy absorber. However, we are more interested in the effect of the secondary structure on the relative displacement of the primary structure to the ground, which cannot be shown in equation (18). Thus, we introduce relative power flow from the secondary structures to the primary structures as

$$P_r = F_s\dot{y}_p = [k_s(y_s - y_p) + c_s(\dot{y}_s - \dot{y}_p)]\dot{y}_p \tag{21}$$

and the mean relative power flow is

$$E[P_r] = E[F_s\dot{y}_p] = E[F_s]E[\dot{y}_p], \tag{22}$$

where $E[\cdot]$ means stochastic average. From equation (22) we have

$$E[F_s] = \frac{E[P_r]}{E^2[\dot{y}_p]}E[\dot{y}_p]. \tag{23}$$

The equation of motion of the primary structure can be written as

$$m_p\ddot{y}_p + c_p\dot{y}_p + k_py_p = -m_p\ddot{u}_g + F_s. \tag{24}$$

Taking $E[\cdot]$ operation on both sides of equation (24) yields

$$m_pE[\ddot{y}_p] + c_pE[\dot{y}_p] + k_pE[y_p] = -m_pE[\ddot{u}_g] + E[F_s]. \tag{25}$$

Substituting equation (23) into equation (25), we have

$$m_pE[\ddot{y}_p] + \left(c_p - \frac{E[P_r]}{E^2[\dot{y}_p]}\right)E[\dot{y}_p] + k_pE[y_p] = -m_pE[\ddot{u}_g]. \tag{26}$$

As is shown in equation (26), when the mean relative power flow is negative, the effect of the secondary structure on the primary one is equivalent to a damper. When the mean relative power flow is positive, the secondary structure can be taken as a driver.

A previous study [16] proposed a technique for non-stationary stochastic analysis of combined primary and secondary sybsystems subjected to a non-stationary zero mean Gaussian base excitation. The technique is employed here for the study of the mean relative power flow between the primary and secondary structure under the non-stationary zero mean Gaussian seismic input. In the following analysis, the deduction from equations (27) to equations (39) is adapted from a study by Falsone *et al.* [16].

By introducing the $2n$-state vector approach, equations (11) can be written as a set of $2n$ first order differential equations as

$$\dot{\mathbf{z}} = \mathbf{D}\mathbf{z} + \mathbf{v}\eta_g(t), \tag{27}$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{M}_q^{-1}\mathbf{K}_q & -\mathbf{M}_q^{-1}\mathbf{C}_q \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \end{bmatrix}, \qquad \mathbf{v} = -\begin{bmatrix} \mathbf{0} \\ \mathbf{M}_q^{-1} \end{bmatrix}\mathbf{V}. \tag{28}$$

For seismic input, the filtered input is obtained as the solution of a first order differential equation

$$\eta_g(t) = \mathbf{n}^{\mathrm{T}}\mathbf{z}_g(t), \qquad \dot{\mathbf{z}}_g = \mathbf{D}_g\mathbf{z}_g + \mathbf{v}_g\varphi(t)\xi(t). \tag{29}$$

$\mathbf{z}_g$, $\mathbf{v}_g$ and $\mathbf{n}$ are vectors of order n_g, $\mathbf{D}_g$ is a matrix of order $n_g \times n_g$, $\varphi(t)$ is a deterministic shaping function, and $\xi(t)$ is a zero mean Gaussian white noise stationary process having one-side power spectral density S_0. Associating equation (27) with equation (29), we have

$$\dot{\bar{\mathbf{z}}} = \bar{\mathbf{D}}\bar{\mathbf{z}} + \bar{\mathbf{v}}\varphi(t)\xi(t), \tag{30}$$

$$\bar{\mathbf{z}} = \begin{bmatrix} \mathbf{z} \\ \mathbf{z}_g \end{bmatrix}, \qquad \bar{\mathbf{D}} = \begin{bmatrix} \mathbf{D} & \mathbf{D}_{cg} \\ \mathbf{0} & \mathbf{D}_g \end{bmatrix}, \qquad \bar{\mathbf{v}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{v}_g \end{bmatrix}, \qquad \mathbf{D}_{cg} = \mathbf{v}\mathbf{n}^{\mathrm{T}}. \tag{31}$$

To obtain the stochastic response of the linear structure subjected to a non-stationary zero mean Gaussian input process, only the second order moments of the response have to be evaluated. By using Kronecker algebra, these quantities can be written as

$$m_2[\bar{\mathbf{z}}] = E[\bar{\mathbf{z}} \otimes \bar{\mathbf{z}}] = E[\bar{\mathbf{z}}^{[2]}]. \tag{32}$$

The exponent is square brackets means Kronecker power, and the symbol $\otimes$ means Kronecker product. Differentiating equation (32) with respect to time, we have

$$\dot{m}_2[\mathbf{z}] = E[\dot{\bar{\mathbf{z}}} \otimes \bar{\mathbf{z}}] + E[\bar{\mathbf{z}} \otimes \dot{\bar{\mathbf{z}}}]. \tag{33}$$

Substituting equation (30) into equation (33), and then taking into account the peculiar property of the white noise processes, equation (33) can be reduced to [17].

$$\dot{m}_2[\bar{\mathbf{z}}] = \bar{\mathbf{D}}_2 m_2[\bar{\mathbf{z}}] + \bar{\mathbf{v}}^{[2]}F(t), \tag{34}$$

where equation (34) is a set of r^2 ($r = 2n + n_g$) first order differential equations, in which

$$\bar{\mathbf{D}}_2 = \mathbf{I}_r \otimes \bar{\mathbf{D}} + \bar{\mathbf{D}} \otimes \mathbf{I}_r, \qquad F(t) = \pi S_0 \varphi^2(t). \tag{35, 36}$$

Let the time space be divided into small intervals of equal length Δt, and let $t_0 = 0$, $t_1, \dots, t_{k-1}$, $t_k, \dots, t_n$ be the division times. The numerical solution of equation (34) is

$$m_2[\bar{\mathbf{z}}(t_{k+1})] = \bar{\Theta}_2(\Delta t) m_2[\bar{\mathbf{z}}(t_k)] + \bar{\mathbf{L}}_2(\Delta t)\bar{\mathbf{v}}^{[2]}F(t_k), \tag{37}$$

where

$$\bar{\Theta}_2(\Delta t) = \sum_{j=0}^{\infty} \frac{1}{j!} (\Delta t \bar{\mathbf{D}}_2)^j \cong \sum_{j=0}^{J} \frac{1}{j!} (\Delta t \bar{\mathbf{D}}_2)^j, \tag{38}$$

$$\bar{\mathbf{L}}_2(\Delta t) = \Delta t \left[\sum_{j=1}^{\infty} \frac{1}{j!} (\Delta t \bar{\mathbf{D}}_2)^{(j-1)} \right] \cong \Delta t \left[\sum_{j=0}^{J-1} \frac{1}{(j+1)!} (\Delta t \bar{\mathbf{D}}_2)^j \right]. \tag{39}$$

Equations (37) is a numerical procedure for the integration of equation (34). The stability conditions of equation (37) has been studied by Falsone *et al.* [17]. Once $m_2[\bar{\mathbf{z}}]$ is obtained, the nodal cross covariance of the combined structure can be obtained as

$$E[\mathbf{U}_t \otimes \mathbf{U}_t] = (\mathbf{Q}_t \otimes \mathbf{Q}_t) m_2[\bar{\mathbf{z}}], \tag{40}$$

where

$$\mathbf{U}_t = \{\mathbf{u}_t, \dot{\mathbf{u}}_t, \mathbf{z}_g\}^{\mathrm{T}}, \qquad \mathbf{Q}_t = \begin{bmatrix} \Gamma_t & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Gamma_t & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{(2n_g)} \end{bmatrix} \tag{41, 42}$$

in which $I_{(2n_g)}$ is the indentity matrix of order $(2n_g \times 2n_g)$.

5. NUMERICAL EXAMPLE

Consider an MFSS with three megafloors, each of which is suspended with six subfloors. The diameters d_0 of the four booms are designed to carry the weight of the suspension floors with a safety coefficient of 1·2. The cross-section areas of the four columns of the megaframe are designed to carry the weight above it with a safety coefficient of 1·5. The height of the suspended floors is 3 m. The equivalent distributing load acting on the suspended floor is supposed to be 15 kN/m^2. The plane and the elevation of the MFSS are shown in Figures 2 and 3 respectively. The way to find out the lateral stiffness of the suspension systems is presented in a previous study [8].

In the analysis of the relative power flow in the numerical example, the seismic excitation in equation (29) is defined as a Tajimi–Kanai-like filter. Some parameters of the seismic

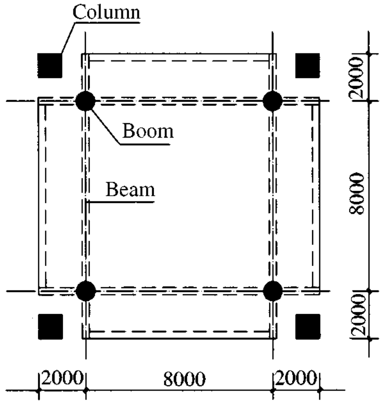

Figure 2. Plane of MFSS.

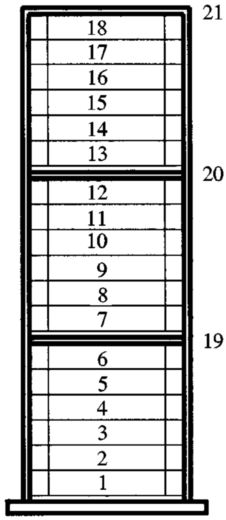

Figure 3. Elevation of MFSS.

excitation modal shown in Figure 1 are taken as follows:

$$c_g = \varsigma_g, \qquad k_g = \omega_g^2, \qquad m_g = 1, \qquad \ddot{u}_e = \varphi(t)\xi(t) \tag{43}$$

in which ς_g and ω_g are two parameters which define the characteristics of the filter, $\varphi(t)$ is a deterministic shape function and $\xi(t)$ is a zero-mean Gaussian white noise process with a one-sided power spectral density S_0.

Then seismic excitation process can be written as

$$\eta_g(t) = -\omega_g^2 u_g(t) - 2\varsigma_g\omega_g\dot{u}_g(t), \tag{44}$$

where u_g and $\dot{u}_g$ are the solutions of a differential equation of a single oscillator subjected to a non-stationary white noise input, i.e.,

$$\ddot{u}_g + 2\varsigma_g\omega_g\dot{u}_g + \omega_g^2 u_g = -\varphi(t)\xi(t). \tag{45}$$

The parameters are taken as $\varsigma_g = 0{\cdot}6$, $\omega_g = 5\pi$ rad/s, $S_0 = 1$ cm^2/s^3. $\varphi(t)$ is chosen as a Geto modal:

$$\varphi(t) = b\frac{t}{t_g}\exp\left(1 - \frac{t}{t_g}\right) \tag{46}$$

in which $b = 1$ and $t_g = 5$. t_g is the time when the function reaches its peak value.

According to equations (21) and (22), in the model of Figure 3, the mean relative power flow from the ith suspension system to the megaframe is

$$P_i = E[k_i(u_{t(3\cdot i)} - u_{t(18+i)})\dot{u}_{t(18+i)} + c_i(\dot{u}_{t(3i)} - \dot{u}_{t(18+i)})\dot{u}_{t(18+i)}] \quad (i = 1, \dots, 3), \tag{47}$$

where k_i and c_i are the coupling stiffness and coupling damping between the ith suspension system and ith megafloor respectively. Moreover, the sum of the mean relative power flow from the secondary structure to primary structure is

$$P_s = \sum_{i=1}^{3} P_i \quad (i = 1, \ldots, 3). \tag{48}$$

It should be noted that because the action of the secondary structure on primary structure is also determined by the position of the secondary structure, P_s is only an approximate index of the interaction between the secondary and the primary structures.

In order to apply the asynchronous driving principle in the system, the diameters of the suspension systems in different megafloors are variegated to obtain different frequencies of the suspension systems. Suppose the diameter of the booms of the suspension system at the ith megafloor is arranged in the following sequence:

$$d_i = d_0[a + c(-0{\cdot}5(N + 1) + i)] \quad (i = 1, 2, 3) \tag{49}$$

in which ad_0 is the average value of all the diameters, and cd_0 is the difference between two neighboring suspension systems. Here μ is defined as the mass ratio of one sub-floor to one megafloor. The frequencies of the megaframe are

$$\omega_1 = 7{\cdot}6 \text{ rad/s}, \qquad \omega_2 = 21{\cdot}3 \text{ rad/s}, \qquad \omega_3 = 30{\cdot}6 \text{ rad/s}. \tag{50}$$

In order to study the interaction between the primary and secondary structures in different cases, the megaframe with different suspension systems attached are studied. The different suspension systems attached are listed in Table 1 and the corresponding transfer functions of the megaframe in Figure 4.

In case 1, the suspension systems are tuned to the megaframe with a small mass ratio, while in other cases, the suspension systems are tuned to the megaframe with a large mass ratio. In cases 5 and 6, the suspension systems drive the megaframe synchronously and asynchronously respectively. As can be seen from these figures, when the mass ratios of the

TABLE 1

Different suspension systems and their dynamic property

Case	a	c	μ (%)	ξ_p	ξ_s	Megafloor number	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
1	5·4	0	5	0·02	0·03	All	7·15	20·9	33·5	44·0	51·9	56·8
2	1	0	85	0·02	0·02	All	1·43	3·92	6·22	8·16	9·73	11·3
3	1	0	85	0·02	0·06	All	The same with case 2					
4	1	0	85	0·02	0·12	All	The same with case 2					
5	1·6	0	85	0·02	0·02	All	2·59	7·46	11·9	15·6	18·3	20·2
6	1·6	0·3	85	0·02	0·02	1	2·34	6·70	10·7	14·0	16·6	18·4
						2	The same with case 5					
						3	3·63	10·5	16·9	22·2	26·3	28·9
7	2·5	0	85	0·02	0·02	All	4·49	12·9	20·3	26·5	31·3	34·3
8	2·5	1·0	85	0·02	0·02	1	2·37	6·81	10·8	14·2	16·8	18·6
						2	The same with case 7					
						3	6·48	17·9	27·9	36·5	43·5	50·5
9	4·05	0	85	0·02	0·02	All	7·60	20·6	32·1	41·8	50·0	61·4
10	4·05	2·5	85	0·02	0·02	1	2·48	7·13	11·3	14·9	17·6	19·4
						2	The same with case 9					
						3	13·7	35·8	55·3	71·3	88·7	129

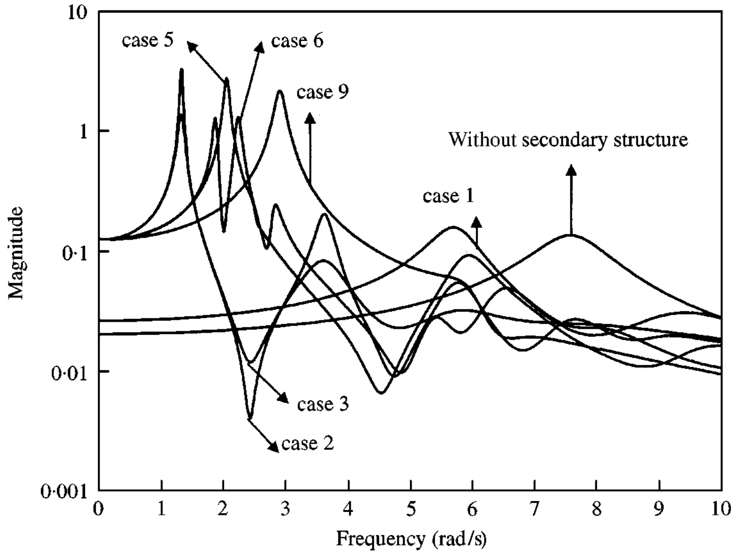

Figure 4. Comparison of transfer functions.

suspension systems are large, they can provide the megaframe with large dynamic stiffness in the fundamental frequency domain of the megaframe. However, it can also be seen that in the frequency domain from zero to the fundamental frequency of the megaframe, the driving effect of the suspension systems in cases 2, 3, 5, 6, 9 is much larger than that in case 1, too. When the frequency of the excitation force is within the driving effect domain. the driving force from the suspension systems can be very large. As it is shown, in the frequency domain from zero to the first frequency of the suspension system, the suspension systems have a great driving effect, while in the frequency domain greater than the fundamental frequency of the suspension system, the driving effect drops sharply. It is also noticed that the transfer function of the megaframe with more flexible suspension systems will have higher peak value within a narrower band, while the one with more rigid suspension systems has a lower peak value within a wider band. The effect of the secondary structures on the primary structure is a comprehensive effect of these factors. But we can approximately reach the conclusion that a more flexible secondary structure with a larger mass ratio will have less driving effect on the primary structure. The comparison between cases 2 and 3 indicates that when the secondary structure has a larger damping ratio, the driving effect is smaller. The comparison between the transfer functions in cases 5 and 6 shows that when the suspension systems drive the megaframe asynchronously, the peak value of the transfer function can be cut off successfully.

Figures 5–14 depict the relative power flow in case 1–10 respectively. Figure 15 is the comparison among the MS displacements at point 21 in different cases. As can be seen in Figure 5, when the mass ratio is very small, all the secondary structures act as vibration absorbers. When a secondary structure has a large mass ratio (Figures 6–14), a more flexible secondary structure provides less driving effect on the primary structure. However, even in case 2, in which the most flexible parameters are available for this kind of structure, the secondary structures also provide a driving effect on the primary structure. Figures 7 and 8 show that when the secondary structures have different damping ratios, they also act as drivers, too. Therefore, we can reach the conclusion that the secondary structure can be considered theoretically as a kind of absorbers, but is invalid in practice for some combined

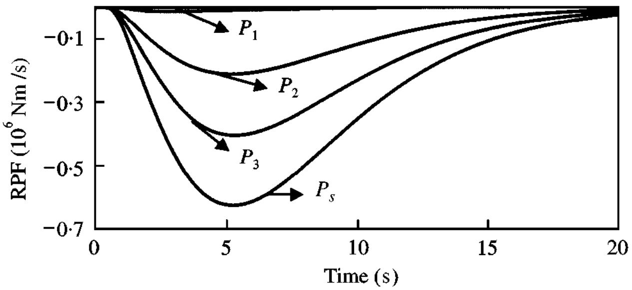

Figure 5. Mean RPF in case 1.

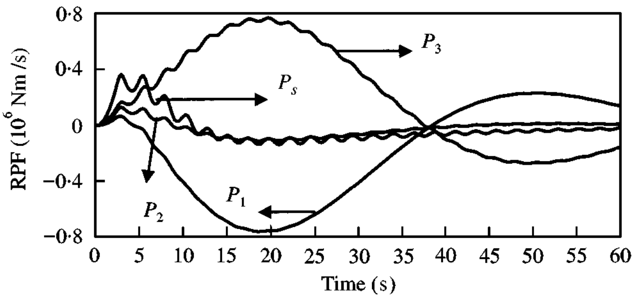

Figure 6. Mean RPF in case 2.

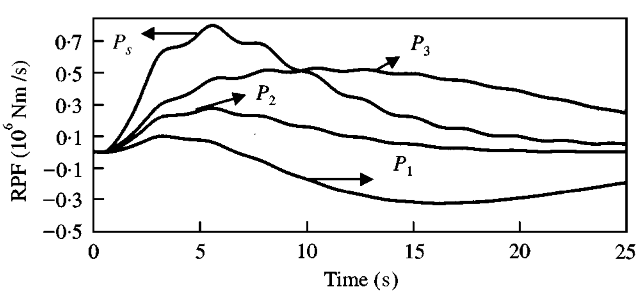

Figure 7. Mean RPF in case 3.

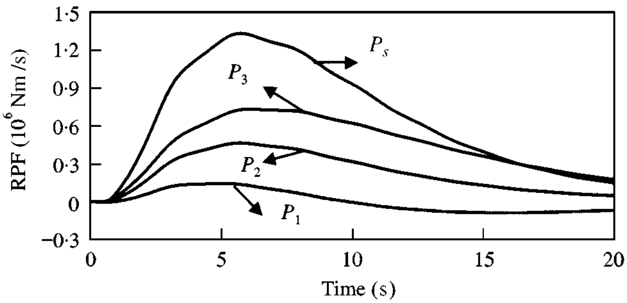

Figure 8. Mean RPF in case 4.

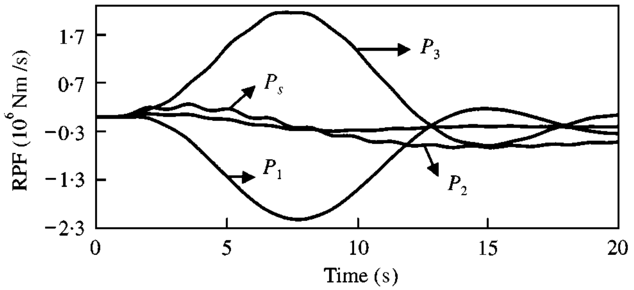

Figure 9. Mean RPF in case 5.

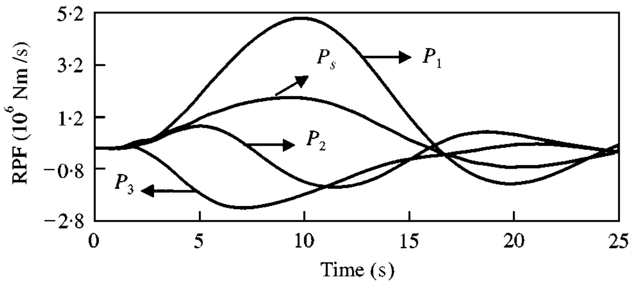

Figure 10. Mean RPF in case 6.

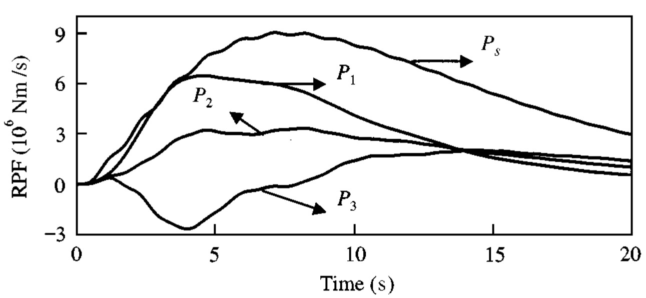

Figure 11. Mean RPF in case 7.

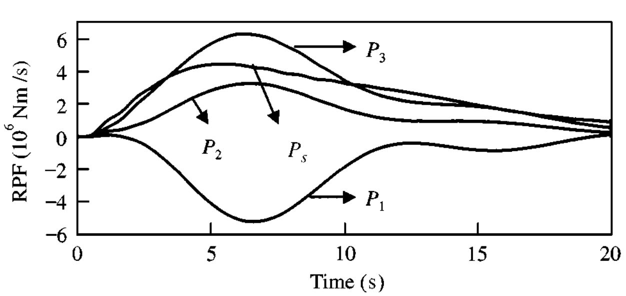

Figure 12. Mean RPF in case 8.

structures. To reduce the driving effect, the stiffness of the secondary structures should be reduced. This tendency is also shown in Figure 15. In a traditional structure system, the secondary structures have large stiffness, such as in case 9. It provides a large driving force to the primary structure, and the response of the latter is very large. When the secondary

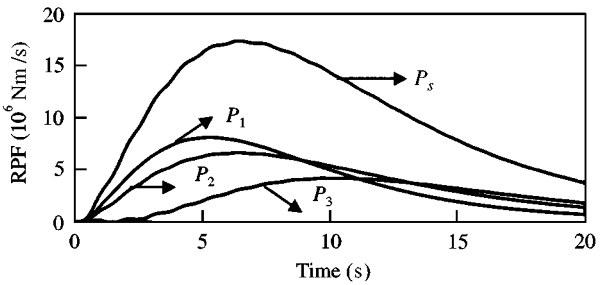

Figure 13. Mean RPF in case 9.

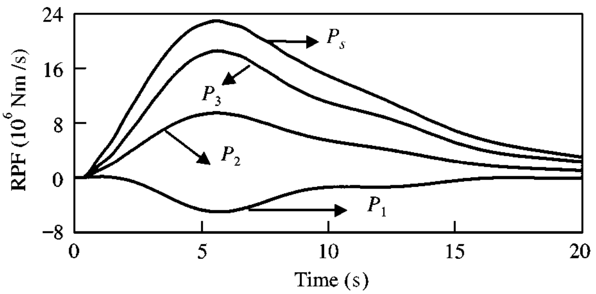

Figure 14. Mean RPF in case 10.

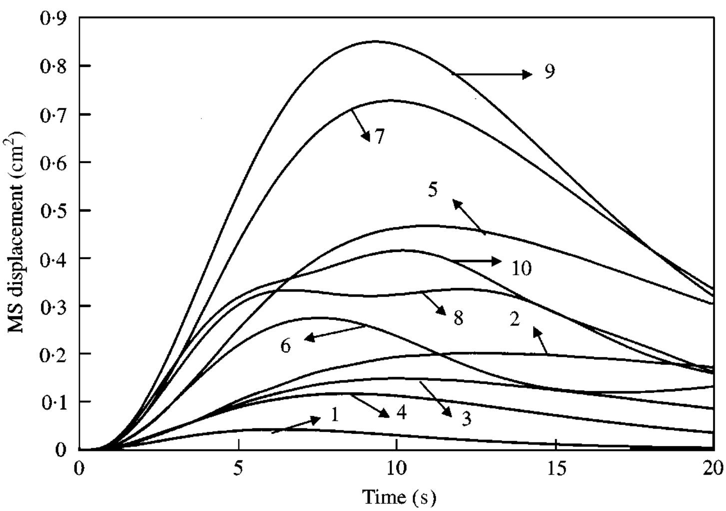

Figure 15. Comparison of MS displacement of point 21.

structures are designed to be flexible, such as in cases 2–6, the driving force can be much smaller and the response of the primary structure is much weaker. This effect can be easily confused with that of TMD. In fact, their mechanisms and the effects are not the same.

Figures 6–14 also indicate that even if the secondary structures drive the primary structure as a whole, the secondary structures may act as absorbers in different periods of time and in different positions of the primary structure. This kind of phenomena is quite common when the secondary structures are very flexible, as shown in Figures 5–10. When the asynchronous driving principle is not used, the secondary structures in the higher megafloors act as drivers, while those in the lower megafloors act as absorbers. When the asynchronous driving principle is used, the secondary structures in the higher megafloors act as absorbers, while those in the lower megafloors act as drivers. Consequently, although the secondary structures act as drivers as a whole, but the driving effect can be reduced. In case 9, as shown in Figure 13, when the secondary structures are also very stiff, all the secondary structures act as drivers. As shown in Figure 14, when the asynchronous driving principle is used, the flexible secondary structure in the lowest megafloor acts as an absorber. These are the mechanisms of the asynchronous driving principle. Certain quantitative measures and performance indexes relative to the use of the asynchronous driving principle are presented in some other studies [18, 19].

6. CONCLUSIONS

The dynamic property of a combined structure depends on the interaction of the primary and secondary structures. When the mass ratio of the secondary structures to the primary structure is small, the secondary structures can be simply taken as a kind of vibration absorber. When the mass ratio is very large, the driving effect of the secondary structures on the primary structure plays a dominant role in the interaction between the primary and secondary structures. In most cases, the secondary structures can be taken as vibration drivers. Using the asynchronous driving principle has been shown an effective way to reduce the driving effect of the secondary structures on the primary structure.

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