Journal of Sound and Vibration (2000) **236**(5), 781–800
doi:10.1006/jsvi.1999.3032, available online at http://www.idealibrary.com on IDEAL®

PARTICLE IRREGULARITY AND AGGREGATION EFFECTS IN AIRBORNE SUSPENSIONS AT AUDIO- AND LOW ULTRASONIC FREQUENCIES

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(*Received* 22 *April* 1999, *and in final form* 15 *November* 1999)

In the inertial regime of frequency-radius space, irregularity and aggregation of particles can result in values of acoustic attenuation that are significantly different from those predicted by assuming separated smooth spherical particles. Data obtained previously from suspensions of alumina particles and olivine sand in air at audio-frequencies, together with new data obtained at low ultrasonic-frequencies in suspensions of glass beads and silica flour, are compared with predictions. It is shown that neither a coupled phase theory modified to allow for non-spherical shapes nor effective radius theories are able to account for these data. Qian [21] has suggested that suspensions may be treated as fractal media and used the acoustic Reynolds number as the fractal dimension in modifying scattering theory. A new fractal modification of multiple-scattering theory for acoustic attenuation is derived. The theory uses $\omega\tau_v$ (ω is the angular acoustic frequency, τ_v is the dynamic relaxation time of the particles) as a fractal scale. It requires an empirical determination of the difference between the fractal dimension of the measured suspension and that of a hypothetical suspension of spheres with the same particle size distribution. However, values obtained at a single frequency also enable fits with data at other frequencies. The new fractal modification of scattering theory is found to enable better agreement with measured attenuation as a function of concentration for irregular particles than an effective radius model. Also, the fractal modification is able to predict the observed frequency dependence at a given concentration rather better than effective radius approaches. Moreover, the fractal approach is found to enable discrimination between the effects of particle irregularity and aggregation.
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1. INTRODUCTION

Acoustic propagation in airborne suspensions of solid particles is of interest in many industrial contexts. The presence of suspended airborne solid particles, smaller than the

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incident sound wavelength, results in acoustic attenuation due to viscous and thermal effects. When the particles are significantly smaller than the incident sound wavelength, scattering theories [1–4] and coupled phase theories [5–8] have provided identically good agreement with experimental results [9–12]. Data and predictions show a clear relationship between the acoustic attenuation and particle concentration at a given sound frequency and for a given particle size distribution. Most of these data relate to particles small compared to the viscous or thermal boundary layer thickness [13]. With this restriction on particle size, the effects of particle shape and surface roughness on acoustic attenuation are insignificant and, therefore, may be neglected.

On the other hand, there are data [9, 10, 14–17] that reveal significant departures from predictions that are based on the assumption of smooth spherical particles. Such deviations occur when the particles are large compared to the viscous or thermal boundary layer thickness. The disagreements could be due to particles having irregular shapes and rough surfaces, or a tendency to form aggregates, or a combination of both. The influence of particle irregularity *and* aggregation on acoustic attenuation in airborne particulate suspensions has received very little attention in the literature. Evans [7] has extended a coupled phase theory [5, 6] to include the influence of particle shape on the acoustic attenuation in airborne suspensions. Two different effective radii are used in the drag and heat transfer terms [18] of the coupled phase theory. He predicted that the influence of particle shapes on the acoustic attenuation in airborne suspensions would be fairly small in the frequency-particle size regime of interest.

Schaafsma and Hay [15] have studied acoustic attenuation in non-aggregating water-borne suspensions of irregularly shaped sediment particles for $ka > 1$, where k is the wavenumber and a is the mean radius of the particles. They modified scattering theory by using two "equivalent" sphere dimensions. The two parameters are determined empirically but may be given geometrical interpretations. Later, Schaafsma *et al.* [16] used finite and boundary element methods (FEM-BEM) to study the attenuation cross-section of irregularly shaped sediment particles. Essentially, the approach used by Schaafsma and Hay [15] corresponds to a form of effective radius model. It is restricted to high values of dimensionless acoustic radius (ka). Indeed, the authors remark that their approach is not particularly successful for small values of ka.

An alternative way of taking account of particle irregularity in predictions of sound attenuation is based on fractal theory [19, 20]. This approach has been suggested by Qian [21] using the acoustic Reynolds number, Re (i.e. the viscous wave number) as the fractal dimension. The resulting predictions of attenuation agree closely with the results of the Wu *et al.* experiments in water-saturated riverbed coarse sands [14]. Wu *et al.* [14] found that the non-sphericity and surface roughness of particles have strong effects on the acoustic attenuation when Re is much larger than 1 and ka is smaller than 1.

Moss [9] and Moss *et al.* [10] have reported significant discrepancies between predictions assuming smooth spherical particles and measurements on flowing suspensions of rough olivine sand. In these measurements, the particles were quite large compared to the viscous or thermal boundary layer thickness. Consequently, these sand particles might not be regarded as smooth and spherical even through the Reynolds number was not as large as in the Wu *et al.* experiments [14].

The present work investigates the influence of particle irregularity and aggregation on acoustic attenuation in airborne suspensions at low ultrasonic- and audio-frequencies with particle radii that are comparable with the viscous or thermal boundary layer thickness. First theoretical predictions of attenuation in suspensions are reviewed and a new fractal modification of multiple-scattering theory [1] is proposed to account for the effects of particle irregularity and aggregation. In section 3, new experiments designed to test these effects are described. In Section 4, the data from these experiments and other data obtained by Moss and co-workers [9, 10, 17] are compared with the predictions of the fractal modification. Finally, some conclusions are given.

2. THEORIES OF ACOUSTIC ATTENUATION IN SUSPENSIONS

According to the multiple-scattering theory of Lloyd and Berry [1], the attenuation of sound in suspensions may be calculated from

$$\alpha_0 = \mathrm{Im}\left[k_f\sqrt{\left(1 + A_0'\frac{\phi_m\rho_f}{\rho_s}\right)\left(1 - 3A_1'\frac{\phi_m\rho_f}{\rho_s}\right) + 6A_1'\left(\frac{\phi_m\rho_f}{\rho_s}\right)^2}\right], \tag{1}$$

where

$$A_0' = -3\mathrm{i}\sum_n\left[\Phi_n\frac{A_{0n}}{(k_f a_n)^3}\right], \tag{2}$$

$$A_1' = 3\mathrm{i}\sum_n\left[\Phi_n\frac{A_{1n}}{(k_f a_n)^3}\right], \tag{3}$$

$$A_{0n} = \frac{\mathrm{i}(k_f a_n)^3}{3}\left(\frac{\rho_f c_f^2}{\rho_s c_s^2} - 1\right) - k_f^2 a_n c_f T_f \rho_f \kappa_f H\left(\frac{\Theta_f}{\rho_f C_{pf}} - \frac{\Theta_s}{\rho_s C_{ps}}\right)^2, \tag{4}$$

$$A_{1n} = \frac{-\mathrm{i}(k_f a_n)}{3}\left\{\frac{3\rho_f}{(\rho_s - \rho_f)} + 2\left[1 + 3(1+\mathrm{i})\frac{\delta_v}{2a_n} + 3\mathrm{i}\frac{\delta_v^2}{2a_n^2}\right]^{-1}\right\}^{-1}, \tag{5}$$

$$H = \left\{\frac{1}{(1 - iz_f)} - \frac{\kappa_f}{\kappa_s}\frac{\tan(z_s)}{\tan(z_s) - z_s}\right\}^{-1}, \tag{6}$$

$$z = \frac{(1+\mathrm{i})a_n}{\delta_t}, \tag{7}$$

$$\delta_v = \sqrt{\frac{2\mu_f}{\omega\rho_f}}, \tag{8}$$

$$\delta_t = \sqrt{\frac{2\kappa}{\omega\rho C_p}}. \tag{9}$$

Here ω is the angular frequency, ρ is the density, μ is the dynamic viscosity, Φ_n is the volume fraction of particles with radii a_n, A_{0n} and A_{1n} are the scattering coefficients of single particle for a_n. $\phi_m = \phi_v\rho_s/\rho_f$ is the particle mass concentration, where ϕ_v is the volume concentration. c is the speed of sound, T is the absolute temperature, C_p is the specific heat capacity at constant pressure, Θ is the coefficient of thermal volume expansion, κ is the thermal conductivity, δ_v is the viscous boundary layer thickness and δ_t is the thermal boundary layer thickness. Subscripts f and s denote the continuous phase (i.e., fluid) and particulate phase (i.e., solid particle), respectively. The thermal boundary layer thickness is the product of the viscous boundary layer thickness and the Prandtl number. In air, this means that the viscous and thermal boundary layers are of comparable thickness but the thickness of the thermal boundary layer is slightly smaller.

Evans [7], Moss [9] and Moss and Attenborough [17] have found that, for dilute airborne suspensions where $\phi_v < (1 + \delta_t/a)^{-3}$, scattering theories [1–4] agree with coupled phase theories [5–8]. Evans [7] has extended coupled phase theories [5, 6] to include heat transfer and a compressible particulate phase. According to Evans' work, the attenuation of sound in suspensions may be calculated from

$$\alpha_0 = \mathrm{Im}\left[\omega\sqrt{(1-\phi_v)\kappa_f\rho(\omega)\frac{1+\sum_n F_{hn}}{1+\gamma_f^{-1}\sum_n F_{hn}}}\right], \tag{10}$$

where

$$F_{hn} = \frac{\Phi_n \rho_s \gamma C_{ps} S_{hn}}{(1-\phi_v)C_{pf}(\rho_s C_{ps} + \rho_f S_{hn})}, \tag{11}$$

$$S_{hn} = -\frac{3\kappa_f}{i\omega a_n^2 \rho_f}\left[\frac{1}{1 - iz_f} - \frac{\kappa_f}{\kappa_s}\frac{\tan(z_s) + \dfrac{3}{z_s} - \dfrac{3\tan(z_s)}{z_s^2}}{\tan(z_s) - z_s}\right]^{-1}, \tag{12}$$

$$\rho(\omega) = \frac{\rho_{va} - \rho_s^2 \sum_n \dfrac{\Phi_n}{\rho_s + \rho_f S_n}}{1 + (\rho_{va} - 2\rho_s)\sum_n \dfrac{\Phi_n}{\rho_s + \rho_f S_n}}, \tag{13}$$

$$\rho_{va} = \rho_f(1-\phi_v) + \rho_s\phi_v, \tag{14}$$

$$S_n = \frac{9}{4}\mathrm{i}\frac{\delta_v^2}{a_n^2} + \frac{9}{4}(1+\mathrm{i})\frac{\delta_v}{a_n} + \frac{1}{2}. \tag{15}$$

Here γ is the ratio of specific heats.

Figure 1 shows that there is negligible difference between predictions of sound attenuation in airborne suspensions of smooth spheres with volume concentrations much less than 0·01 m^3 of particles per m^3 of air, in the long wavelength regime, using the multiple-scattering theory [equation (1)] and the coupled phase theory [equation (10)]. It should be remarked that neither of the predictions compares well with the data for attenuation in airborne suspensions of olivine sand [9, 10] also shown in this figure. Given the close agreement between the predictions for smooth spherical particles with relevant properties, only the multiple-scattering theory of Lloyd and Berry will be used in subsequent comparisons of predictions of attenuation in suspensions of smooth spherical particles. Possible explanations for the large difference between such predictions and data will be explored.

Evans [7] has investigated the influence of the particle shape on acoustic attenuation in suspensions by modifying couple phase theory using an effective radius a_v in the drag term and another effective radius a_h in the heat transfer term according to expressions given by Clift *et al.* [18] for steady state conditions. Expressions for the effective radii a_v and a_h for particles of spheres, spheroids and cubes of the same volume are given in Table 1 [7]. In Table 1, a_{ob} is the semi-major axis of spheroids, $h = b/a_{ob}$, where b is the semi-minor axis, $q = \sqrt{1-h^2}$ and $l = (4\pi a_n^3/3)^{1/3}$. Figure 1 shows that use of effective radii for spheroids and cubes does not result in predictions of acoustic attenuation that differ significantly from those for spherical particles. If anything, this method for including deviations from spherical particles results in a decrease rather than an increase in the predicted attenuation thus

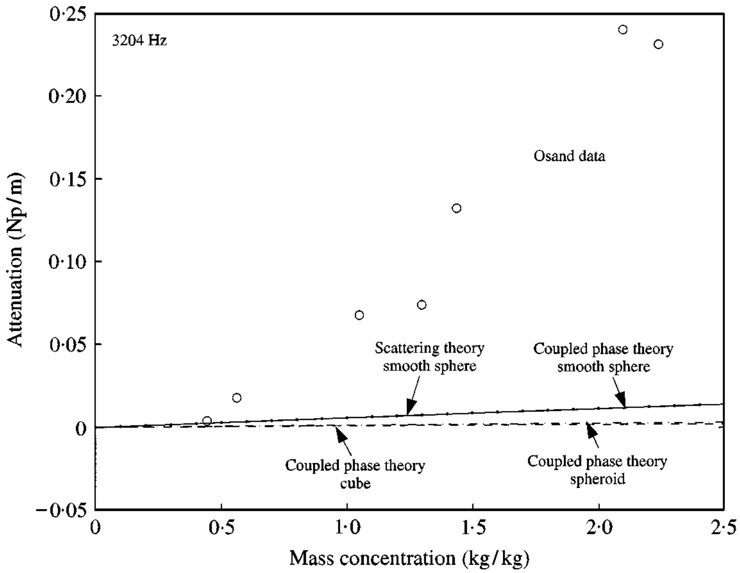

Figure 1. Attenuation versus mass concentration for a suspension of olivine sand in air. ○, data [9, 10]; ——, prediction of the multiple scattering theory using equation (1) with spheres; ...., prediction of the coupled phase theory using equation (10) with spheres; · – · –, - - -, predictions of the coupled phase theory [7] with spheroids and cubes respectively.

TABLE 1

Effective radii of spheres, spheroids and cubes of the same volume [7]

Particle shape	a_v	a_h	a_v/a_n	a_h/a_n
Sphere	a_n	a_n	1	1
Spheroid	$\frac{a_{ob}q}{\arccos(h)}$	$\frac{a_{ob}q}{\arccos(q)}$	1·4576 ($h = 0{\cdot}1$)	1·4576 ($h = 0{\cdot}1$)
Cube	$(2/3)l$	$0{\cdot}656l$	1·0747	1·0575

increasing the discrepancy between prediction and data for rough irregular-shaped olivine sand suspended in air [9].

Schaafsma and Hay [15] have shown good agreement between their model predictions and data obtained in suspensions of irregular particles for $ka > 1$. They introduce two effective radii, intended to allow for the effect of irregular particle shapes, corresponding to the radius of a circle of area equal to the projected area of the particle and to the radius of an equal volume sphere respectively. However, the long wavelength regime where $ka \ll 1$ is of interest. In this regime, dilatational wave scattering effects on acoustic attenuation scale as $(ka)^4$ and are negligible compared to viscous and thermal loss mechanisms. The restriction of the Schaafsma and Hay [15] approach to $ka > 1$, means that it is not possible to apply their approach to explain the data reported here which correspond to $ka \ll 1$. It is not clear how effective radius models would enable discrimination between the measured effects of particle irregularity and aggregation or how the effective radii of irregularly shaped and aggregated particles would be determined.

Previous data [9, 10, 17] and the new data considered here are consistent with a power law relationship between the specific acoustic attenuation and the particle concentration.

According to Mandelbrot [19, 20], this suggests that the suspensions of interest may be considered to be fractal systems. It is assumed that each suspension has its own fractal structure and can be characterized by particular fractal scales and dimensions. If this is true, then the effects of the irregularity and aggregations of particles in any suspension should be described by fractal models. The discrepancies between predictions of acoustic attenuation based on the assumption of smooth spherical particles and data obtained from suspensions of irregular particles may be considered to result from their different fractal structures. Qian [21] has modified scattering theory to predict sound attenuation in water-borne sediments by using the acoustic Reynolds number (Re) as a fractal scale. The acoustic Reynolds number [13] is defined by

$$Re = \frac{a}{\delta_v} = a\sqrt{\frac{\rho_f \omega}{2\mu_f}}. \tag{16}$$

Wu *et al.* [14] found that the irregularity of water-borne coarse sand particles cause the acoustic attenuation, for $60 < Re < 240$, to increase significantly over the values predicted for smooth spherical particles. It should be noted that, given sand particles with an average radius of 170 μm, suspended in water, then, for frequencies where $60 < Re < 240$, the particles are large compared with the viscous or thermal boundary layer thickness. Kytömaa [13] has identified viscous, inertial and scattering regimes for sound propagation in suspensions of solids in a liquid distinguished by ranges of Re and ka. These are shown in terms of frequency and particle radius in Figure 2. The results of Wu *et al.*'s measurement [14] suggest that particle irregularity has a significant influence on acoustic attenuation in the inertial regime.

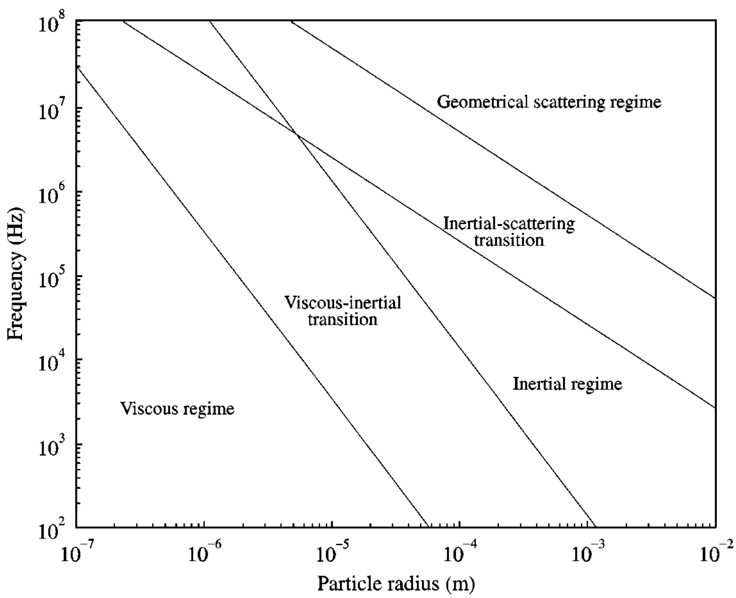

Figure 2. Frequency-radius map of regimes of sound propagation through a suspension of solid particles in water according to Kytömaa [13]. $Re = 1$ and 20 define the transition between the viscous regime and the inertial regime while $ka = 0{\cdot}1$ and 2 identify the transition between the inertial regime and the geometrical scattering regime.

The large discrepancies, shown in Figure 1, between predictions and data for attenuation in airborne suspensions of olivine sand imply that particle irregularity may have an important influence on attenuation of sound waves even when the acoustic Reynolds number is not as large as in the Wu *et al.* experiments [14]. For sound propagation through airborne suspensions, Temkin and Dobbins [5] have distinguished the viscous and inertial regimes and the transition between them by using the dimensionless quantity $\omega\tau_v$, where τ_v is the viscous relaxation time denoted by

$$\tau_v = \frac{2a_{32}^2\rho_s}{9\mu_f}, \tag{17}$$

Here a_{32} is the Sauter mean particle radius [10], $a_{32} = \sum_n N_n a_n^3 / \sum_n N_n a_n^2$, and N_n is the number of particles of radius a_n. The Sauter mean particle radius has been identified to be the characteristic particle size in the long-wavelength regime [9, 11]. $\omega\tau_v$ may be considered as the dominant scale in the long-wavelength regime and, therefore, applicable to suspensions with low acoustic Reynolds number [5] which is the case for the suspensions of solid particles in air considered here.

Figure 4 shows a frequency-radius map of regimes for describing sound propagation through a suspension of spherical particles in air. When $0 \leqslant \omega\tau_v \leqslant 1$, corresponding to sound propagation in the viscous regime [13, 22], the attenuation has been shown to vary as $a_{32}^2\omega^2/\mu_f$. The viscous or thermal boundary layer thickness is large compared to the particle radius, the viscous relaxation time is shorter than the period of excitation, and Stokes' law can be used for the drag force. In this regime, particle irregularity has not been observed to influence the acoustic attenuation in suspensions and models that assume smooth and spherical particles are reasonably accurate.

When a certain value of $\omega\tau_v$, of the order of 10^2, is exceeded, the acoustic attenuation varies as $\sqrt{\omega\mu_f}/a_{32}$ and sound propagation is said to be in the inertial regime [13, 22]. There is a transition between the viscous regime and the inertial regime. In the inertial regime, the viscous or thermal boundary layer is small compared to the particle radius. Flow outside the thin boundary layer is governed by inertial effects, and the history term [11] begins to influence the particle drag. The nature of the particle shape and surface begins to be important. The different frequency dependencies of attenuation in the viscous and inertial regimes and the transition between them are shown in Figure 3.

The effects of particle irregularity become more visible as $\omega\tau_v$ is increased. In the inertial regime, the presence of irregularity and aggregation should mean that particles may not be regarded as smooth and spherical, and it is expected that data will deviate significantly from predictions that assume smooth spherical particles.

The ranges $0{\cdot}1\pi \leqslant ka_{32} \leqslant \pi$ and $ka_{32} > \pi$, identify the Rayleigh scattering and resonant scattering regimes respectively. In this paper, Qian's approach [21] is extended to sound propagation in airborne particulate suspensions at long wavelengths including the inertial propagation regime and the transition between the viscous and inertial regimes. Instead of the acoustic Reynolds number, $\omega\tau_v$ is used as the scale, since it is known as the self-similar scale in these regimes. It is postulated that the acoustic attenuation in a suspension consisting of irregular particles and aggregates, α, may be related to that in a suspension of smooth spheres with the same particle size distribution, α_0, by

$$\alpha = \alpha_0(\omega\tau_v)^{D-D_0}, \quad \omega\tau_v \geqslant 1. \tag{18}$$

Here D and D_0 are the fractal dimensions of suspensions containing rough non-spherical and smooth spherical particles respectively. In contrast to the effective radius concept,

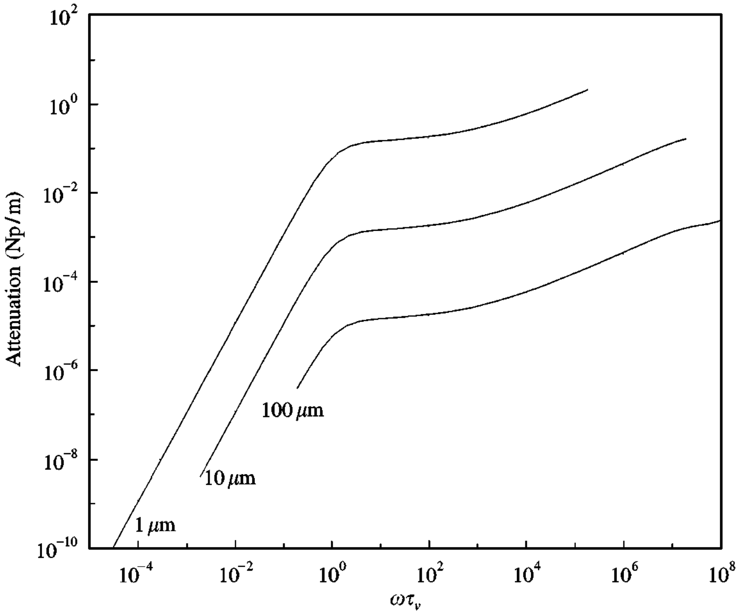

Figure 3. Attenuation in a suspension of glass beads in air predicted by multiple-scattering theory [1] as a function of $\omega\tau_v$ for three Sauter mean radii (1, 10 and 100 μm).

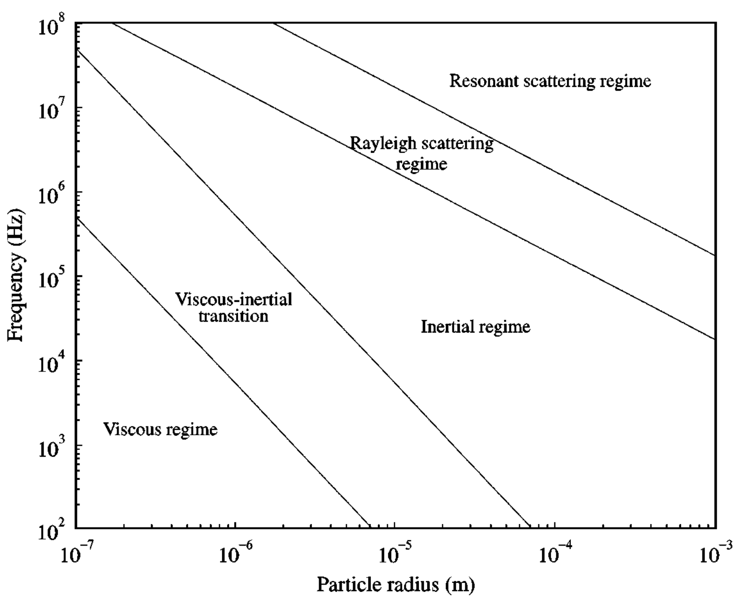

Figure 4. Frequency-radius map of regimes of sound propagation through a suspension of fine glass beads in air. $\omega\tau_v = 1$ and 100 define the transition between the viscous regime ($\omega\tau_v < 1$) and the inertial regime ($\omega\tau_v > 100$) while $ka_{32} = 0{\cdot}1\pi$ and π identify the lower limits of the Rayleigh scattering regime and the resonant scattering regime respectively.

equation (18) predicts changes in the frequency dependence as well as changes due to particle radius as a result of changes in fractal structure. The value of $D - D_0$ indicates the influence of particle irregularity and aggregation on the fractal dimension of the suspension.

It should be noted that equation (18) is valid only for wavelengths significantly larger than the typical particle dimensions. In the scattering regime $ka_{32} \geqslant 0{\cdot}1\pi$, corresponding to that studied by Schaafsma and Hay [15], a different fractal scale, i.e., ka_{32}, should be used instead of $\omega\tau_v$.

The value of $D - D_0$ is unknown, and is peculiar to each suspension. However, it can be estimated empirically by using equation (18) to fit the data. If $D - D_0 = 0$, then $\alpha = \alpha_0$, and propagation may be modelled as though it takes place through non-aggregating smooth spheres. If $D - D_0 \neq 0$, then $\alpha \neq \alpha_0$ and the particles may not be treated as if they are smooth and spherical or non-aggregating. As the value of $\omega\tau_v$ increases, the measured acoustic attenuation is expected to deviate more from predictions assuming smooth spherical particles. This suggests that the influence of particle irregularity plays a much more important role in the inertial regime than in the viscous regime.

3. EXPERIMENTS AND COMPARISONS WITH THEORIES

As discussed earlier, audio and low ultrasonic frequency sound propagation through small particles suspended in air occurs in the viscous regime and the transition between the viscous and inertial regimes. It is expected that the influence of particle and irregularity increases with increasing values of $\omega\tau_v$. Some experimental results obtained in suspensions of rough non-spherical particles, with $\omega\tau_v < 1$, show good agreement with theoretical predictions assuming smooth spherical particles [11, 12, 17]. Therefore, cases for which $\omega\tau_v < 1$ will not be discussed any further.

It is proposed that equation (18) will be useful for predicting acoustic attenuation in suspensions that consist of arbitrarily shaped particles and aggregates where $\omega\tau_v > 1$ corresponding to the inertial regime and the transition between the viscous and inertial regimes. It is assumed that the fractal dimensions of these suspensions can be determined empirically. To test these predictions, existing data [9] have been supplemented with new measurements on quasi-static suspensions where turbulence can be ignored. To suspend high concentrations of various particles without flow, a vertical rig with a mechanical particle feeder has been built to produce a uniform falling column of particles, moving through still air (Figure 5).

First, predictions of the modified theory are tested against data for the acoustic attenuation in airborne suspensions of glass beads where the measurement frequencies and particle sizes were chosen such that sound propagation was in the inertial regime. For fine glass beads, with mean radius $a_{32} = 11{\cdot}2\,\mu$m (radii between 1·6 and 31 μm) and low ultrasonic frequencies between 40 and 60 kHz, $\omega\tau_v > 100$ and $ka_{32} \ll 0{\cdot}1\pi$. Figure 6 shows the results of these measurements. Also shown are theoretical predictions for the attenuation, α_0, that assume smooth spherical particles (i.e., $D - D_0 = 0$ and $\alpha = \alpha_0$) but allow for the distribution of particle sizes. The attenuation is plotted in Np/m (1 Np/m = 8·7 dB/m) as a function of mass concentration in kg/kg (i.e., kg solid particles per kg air). The maximum concentration of glass beads in suspension (4·5 kg/kg) corresponds to a volume concentration of 0·0022 m^3/m^3. Scanning electron microscope pictures (see Figure 7) confirm that these fine glass beads are approximately smooth spheres with a well-defined distribution of radii. The measurements were made at eight different mass concentrations and with ultrasonic pure tones of 40 and 60 kHz respectively. It can be seen from these figures that there is good agreement between predictions assuming smooth spherical particles and these data for suspensions of glass beads in air.

Next the fractal formulation of scattering theory was tested against data, which was introduced partly in Section 2, for sound propagation through airborne suspensions of

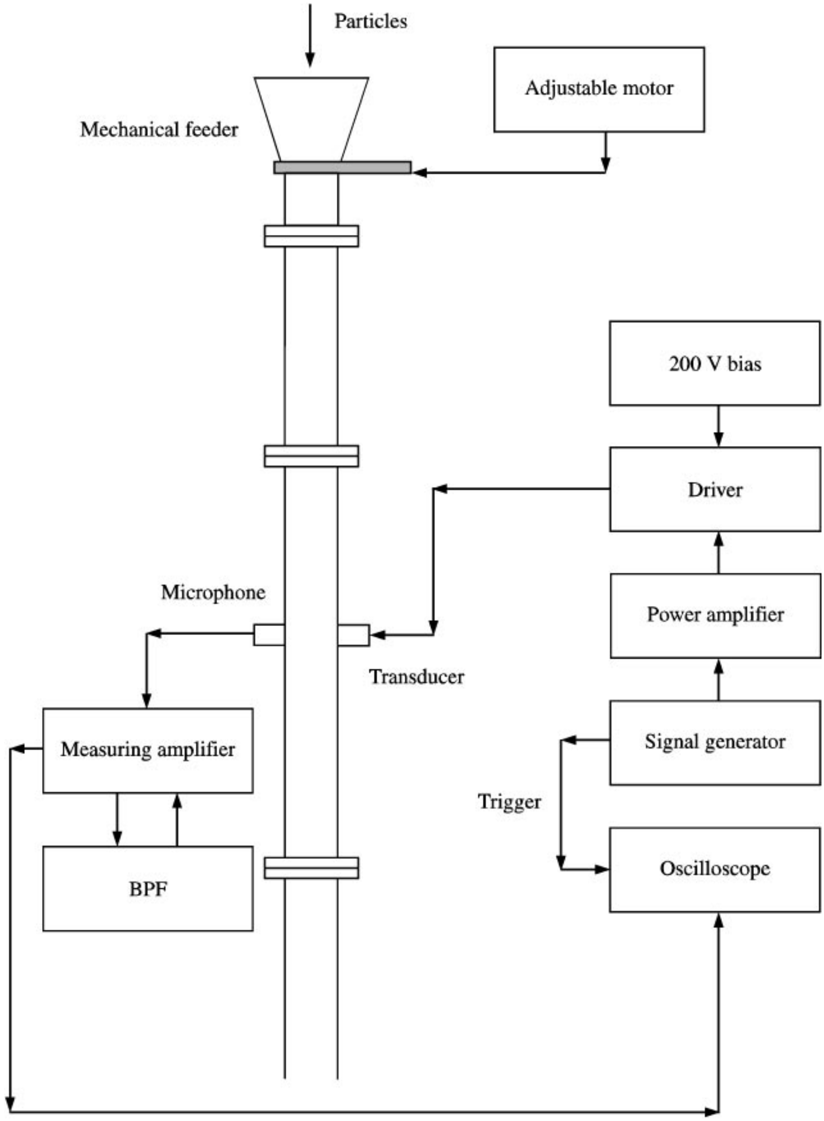

Figure 5. Apparatus to measure the low-ultrasonic attenuation in a quasi-static airborne suspension.

rough irregular particles in the inertial regime where $\omega\tau_v > 100$ and $ka_{32} < 0{\cdot}1\pi$. These data were obtained on suspensions of flowing olivine sand with Sauter mean "radius" $a_{32} = 263\,\mu$m ("radii" between 86·3 and 422 μm determined by a laser diffraction particle sizer) and at six frequencies between 600 and 3200 Hz [9, 10]. The maximum mass concentration of sand, 2·3 kg/kg, corresponds to a volume concentration of $8{\cdot}6\times10^{-4}$ m^3/m^3. Figure 8 shows that the olivine sand particles used in the experiments have irregular crystalline structures but do not aggregate. Figure 9 shows that the measured attenuation is much larger than that predicted by assuming that the particles are smooth spheres. The scatter in these data in this case may be due to flow turbulence [23] since the measurements were made on flowing suspensions. In general, these data are in much better agreement with the fractal formulation of scattering theory [equation (18)] using the value

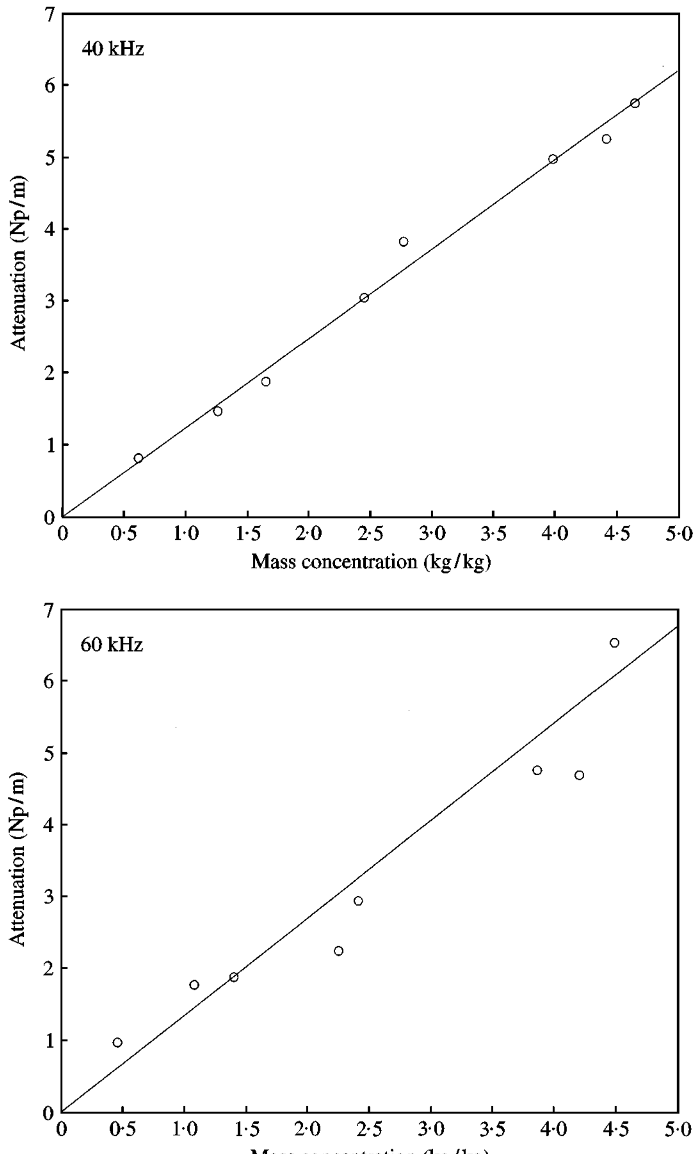

Figure 6. Attenuation versus mass concentration for a suspension of glass beads in air: ○, measurement data; ——, predictions according to equation (18) where $D = D_0$ thus equation (18) gives the same prediction as equation (1), i.e., the smooth sphere prediction.

of $D - D_0$ ($= 0{\cdot}25$) that gives best fit with data at one of these six frequencies (3204 Hz). This value for $D - D_0$ enables good agreement with the data at the other five frequencies (e.g., at 644, 1156 Hz, 1868 Hz, 2180 Hz and 2692 Hz) also shown in Figure 9. The fractal dimension of the suspension of olivine sand is larger than that for a suspension of smooth spherical particles. The roughness and irregularity of the sand particles cause the acoustic attenuation to increase greatly over values predicted for the same concentrations of smooth spherical particles. It was remarked earlier that the proposed fractal modification of scattering theory results in a change in frequency dependence for a given concentration. The

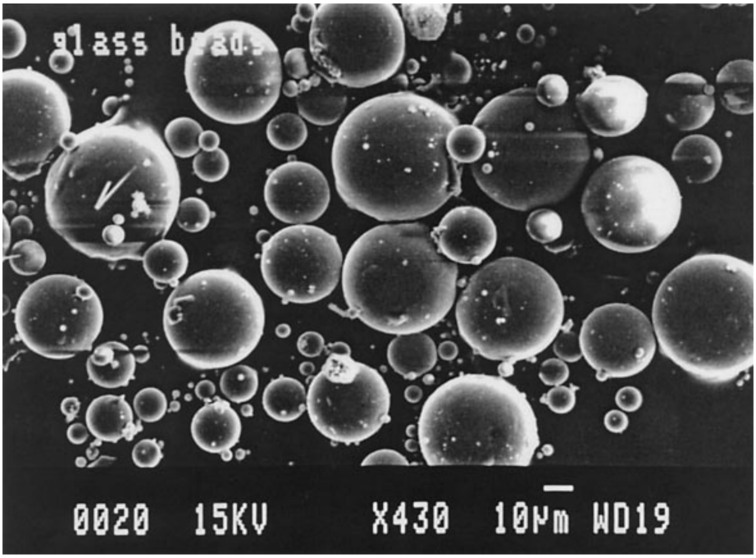

Figure 7. Scanning electron microscope photograph of fine glass beads.

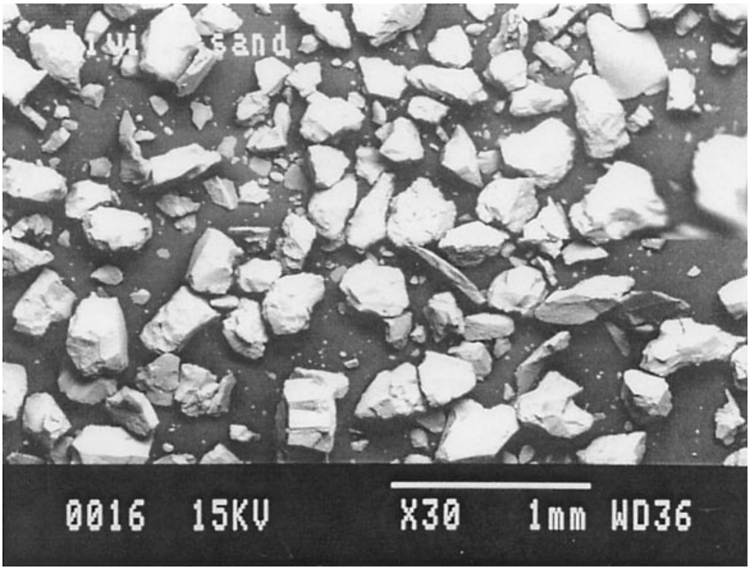

Figure 8. Scanning electron microscope photograph of olivine sand.

improvement in the resulting agreement between predicition and data is demonstrated in Figure 10. The data for the change in attenuation in an olivine sand suspension with frequency is clearly different from the predicted by scattering and coupled phase theories. On the other hand, the fractal-based theory, with $D - D_0 = 0{\cdot}25$, gives quite reasonable agreement.

To investigate acoustic attenuation in airborne suspensions that consist of irregular *and* aggregated particles, new data have been obtained at 30, 40, 55 and 60 kHz for attenuation

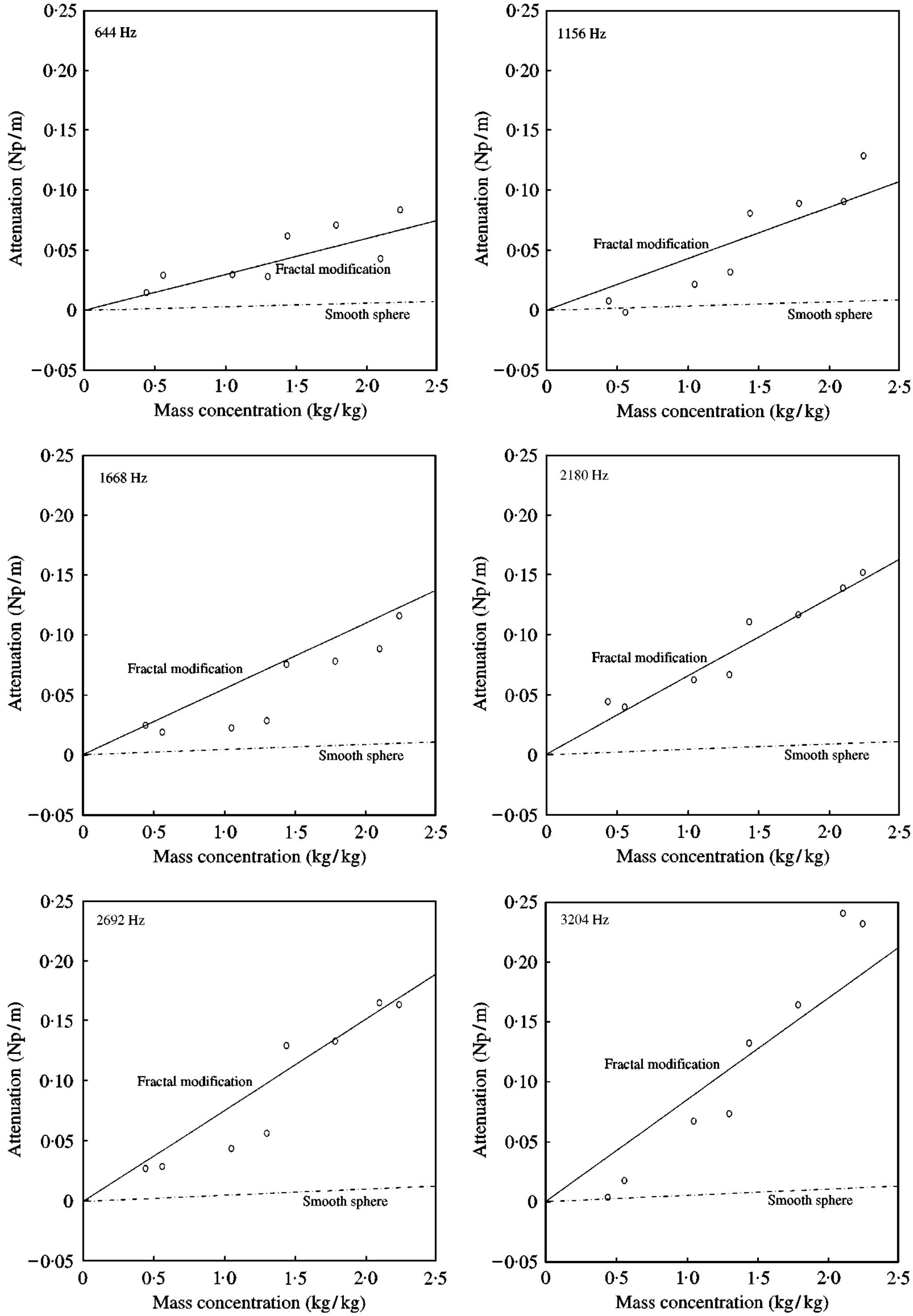

Figure 9. Attenuation versus mass concentration for a suspension of olivine sand in air: ○, measurement data of moss; ——, predictions according to equation (18); –·–, predictions of the multiple-scattering theory with smooth spherical particles according to equation (1).

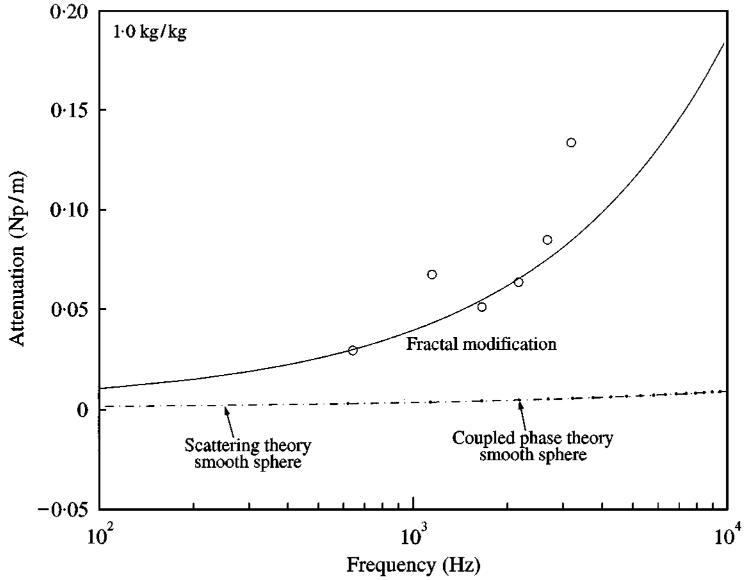

Figure 10. Attenuation versus frequency for a suspension of olivine sand in air at a mass concentration of 1·0 kg/kg: ○, measurement data of moss; ——, prediction according to equation (18); –·–, prediction of the multiple-scattering theory with smooth spherical particles according to equation (1); ...., prediction of the coupled phase theory with smooth spherical particles according to equation (10).

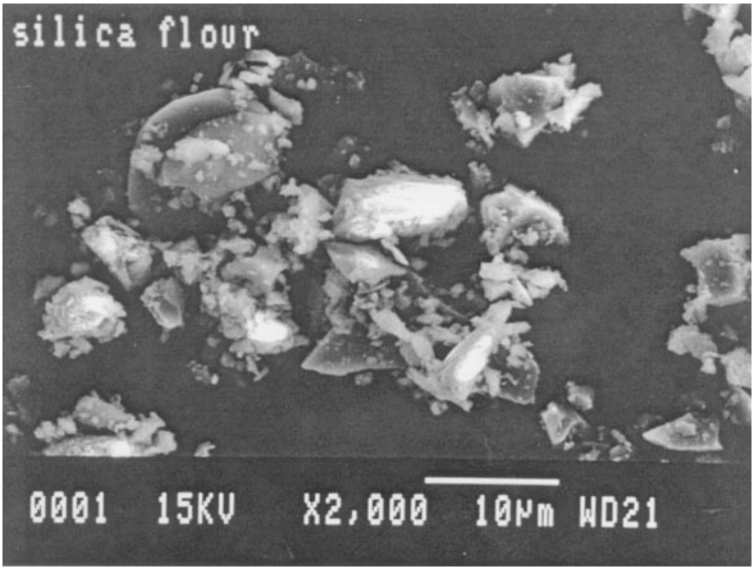

Figure 11. Scanning electron microscope photograph of silica flour.

in quasi-static suspensions of silica flour with mean particle 'radius' $a_{32} = 8{\cdot}7\ \mu$m ('radii' between 2·5 and 20 μm determined from air-sieve analysis). Figure 11 is a scanning electron microscope photograph confirming that these silica flour particles are irregular *and* form aggregates. Measurements at the cited frequencies in these silica flour particles should correspond to sound propagation in the inertial regime since $\omega\tau_v > 100$ and $ka_{32} \ll 0{\cdot}1\pi$. The radii of the aggregations depend on growth conditions and are larger than those of the separated individual particles. The acoustic attenuation is expected to decrease with increasing particle radius in the inertial regime [13] and in the transition close to the inertial regimes (see also Figure 4). Consequently, acoustic attenuation in a suspension with aggregates is expected to be smaller than that predicted in a similar suspension without aggregates but with the original particle size distribution. On the other hand, increasing particle roughness should increase the attenuation above that predicted for smooth particles in the same propagation regime taking into account the original particle size distribution. Figure 12 shows the results of measured acoustic attenuation, indicated by circles, in falling suspensions of silica flour through air. The solid and dash-dotted lines represent the predicted attenuation with and without the fractal modification of the smooth spherical particle scattering model. The measurements encompass different mass concentrations. From the data at 60 kHz the value of $D - D_0$ is estimated to be $-0{\cdot}11$ and

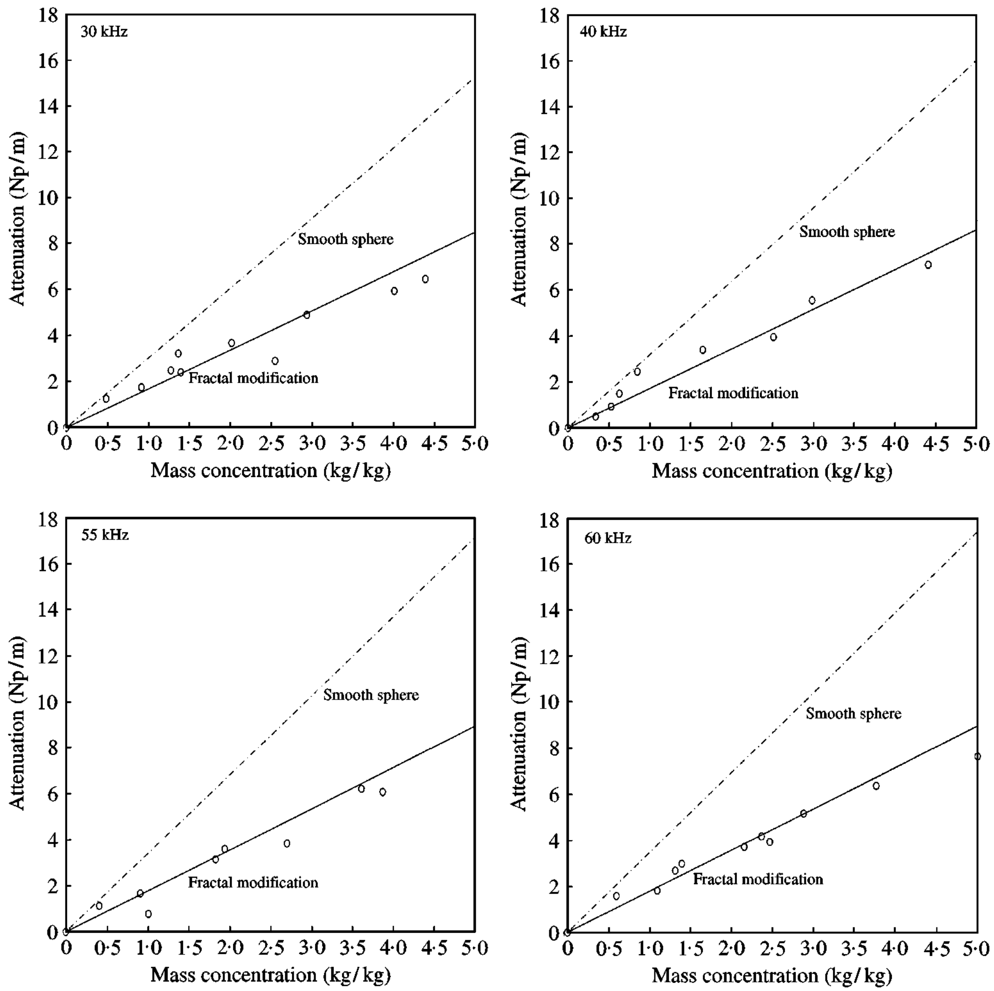

Figure 12. Attenuation versus mass concentration for a suspension of silica flour in air: ○, measurement data; ——, predictions according to equation (18); –·–, predictions of the multiple-scattering theory with smooth spherical particles according to equation (1).

therefore $\alpha < \alpha_0$. The same value for $D - D_0$ enables the good fits with data at 30, 40 and 50 kHz also shown in Figure 12. The irregularity of the particles should result in $D - D_0 > 0$. It is likely that the negative best-fit value for $D - D_0$ is a consequence of the fact that the aggregates are larger than the original particle sizes and suggests that, for propagation through these silica flour particles, aggregation effects dominate over irregularity effects on acoustic attenuation. The effective Sauter mean 'radius' deduced from the measured acoustic attenuation, assuming particles are smooth and spherical, is 12·8 μm, i.e., significantly larger than the 'original' non-aggregated value of 8·7 μm.

Finally, predictions are compared with data obtained in quasi-static particulate suspensions of F800 alumina powder with Sauter mean 'radius' $a_{32} = 2{\cdot}8\ \mu$m ('radii' determined in a WASP between 0·7 and 5·7 μm) [9, 17]. These data correspond to $1 \leqslant \omega\tau_v < 100$ and hence to sound propagation in the transition between the viscous and inertial regimes. Scanning electron microscope images (see Figure 13) reveal that these alumina particles are irregular *and* form aggregates. Figure 14 shows the comparisons between measured acoustic attenuation data for F800 alumina particles and the scattering theory predictions with and without the fractal modification. The data show close agreement with both theoretical predictions at 444 Hz where $\omega\tau_v \approx 1$ and propagation is close to that in the viscous regime. The fractal modification of the scattering theory has a small effect at frequencies sufficiently low that the sound propagates close to the viscous regime. As frequency increases and hence the value of $\omega\tau_v$ increases, the acoustic conditions depart from the viscous regime and the differences between the two predictions become larger. The acoustic attenuation predicted by assuming separated smooth spherical particles is consistently higher than the data. On the other hand, predictions of the fractal-modified scattering theory give much better agreement with these data than those based on the smooth spherical particle assumption. The influence of particle irregularity and aggregation on acoustic attenuation appears to become more important as acoustic propagation conditions approach the inertial regime. The value of $D - D_0$, estimated from the data at 1733 Hz, is $-0{\cdot}18$. The same value enables the good fits with the data at 444, 666, 933 Hz, 1200 Hz and 1466 Hz also shown in Figure 14. Again, the negative value of $D - D_0$ indicates that aggregation has the dominant effect on acoustic attenuation in these alumina particles. The effective Sauter mean 'radius' deduced from the measured acoustic attenuation, assuming particles are smooth and spherical, is 3·5 μm, i.e., significantly larger

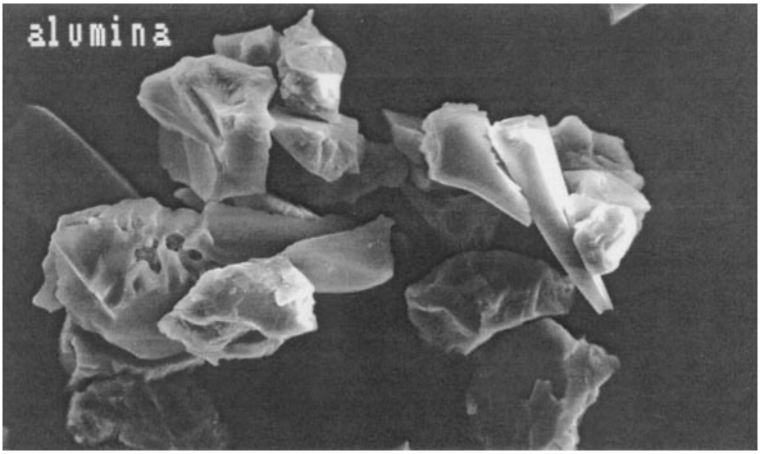

Figure 13. Scanning electron microscope photograph of F800 alumina particles.

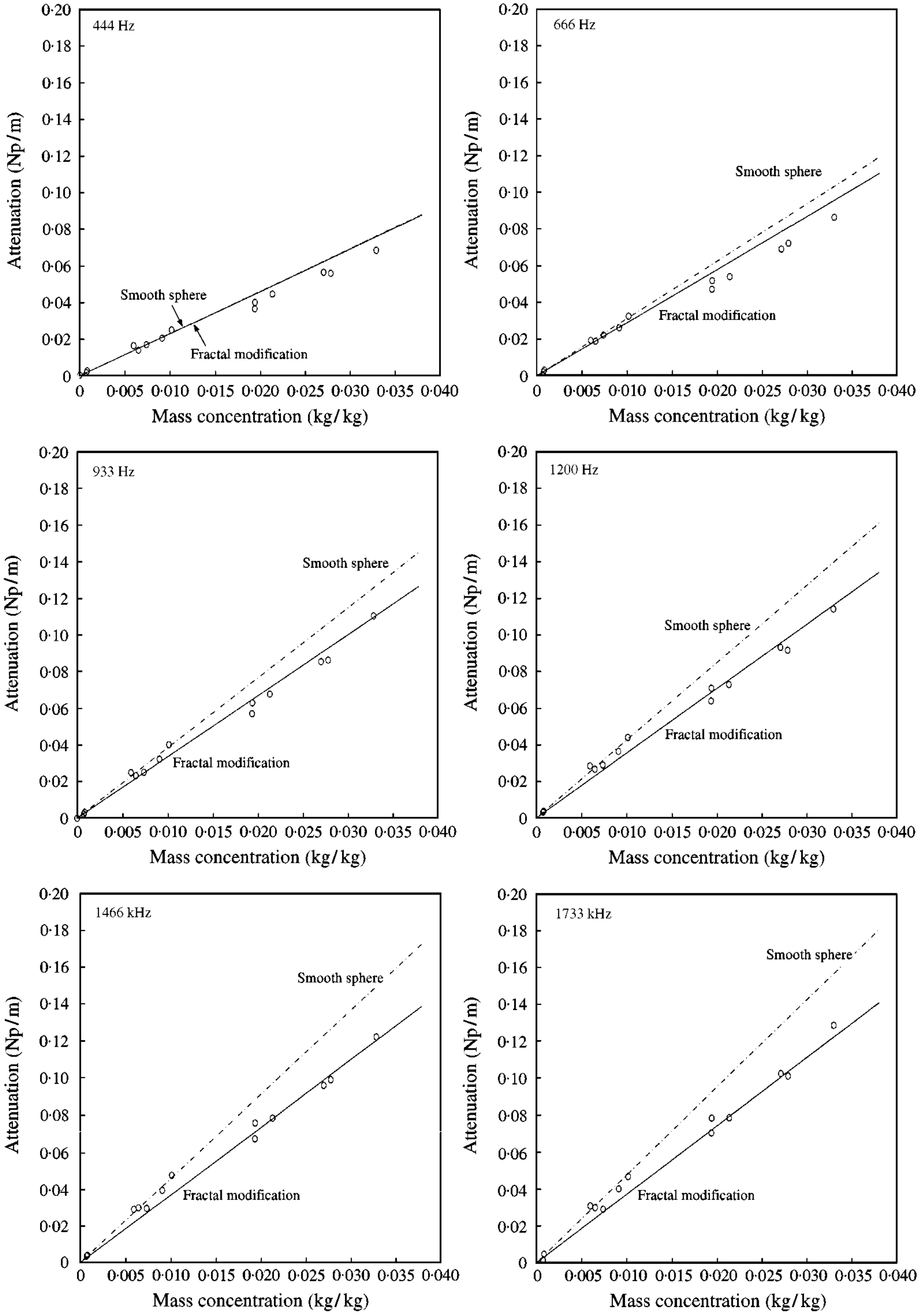

Figure 14. Attenuation versus mass concentration for suspensions of F800 alumina particles in air: ○, measurement data of moss; ——, predictions according to equation (18); – · –, predictions of the multiple-scattering theory assuming smooth spherical particles according to equation (1).

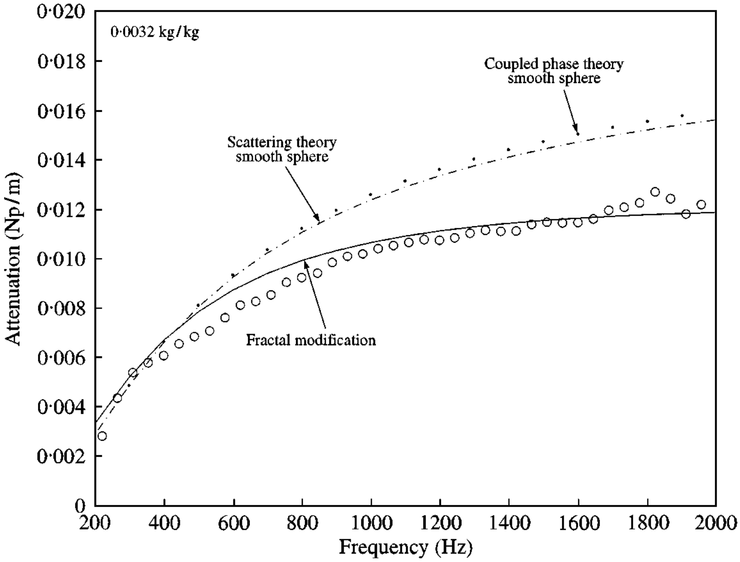

Figure 15. Attenuation versus frequency for a suspension of F800 alumina particles in air at a mass concentration of 0·0032 kg/kg: ○, measurement data of moss; ——, prediction according to equation (18); – · –, prediction of the multiple-scattering theory with smooth spherical particles according to equation (1); ...., prediction of the coupled phase theory with smooth spherical particles according to equation (10).

than the 'original' non-aggregated value of 2·8 μm. Figure 15 shows comparisons between theoretical predictions and data for attenuation as a function of frequency with a mass concentration of 0·0032 kg/kg. Again the fractal formulation, with $D - D_0 = -0{\cdot}18$, enables good agreement with the measured frequency dependence.

4. CONCLUSIONS

Effects of irregularity and aggregation of particles on acoustic attenuation in airborne particulate suspensions have been investigated. It has been found that when the particles have irregular shapes, rough surfaces and aggregate the measured acoustic attenuation differs greatly from that predicted by assuming separated smooth spherical particles. When the particles and aggregates are small compared to the incident sound wavelength, the most significant influence will occur in the inertial regime such that $\omega\tau_v > 100$. Following its success in explaining data for sound attenuation in water-saturated sediments [21], the fractal dimension approach has been extended to predict attenuation in airborne particulate suspensions including influences due to particle irregularity and aggregations. The product of angular frequency and viscous relaxation time, $\omega\tau_v$, has been used as a fractal scale. The new fractal formulation of scattering theory enables good agreement with attenuation data obtained at audio- and ultrasonic frequencies in a variety of suspensions. The data used in the comparisons include frequencies between 440 Hz and 60 kHz and concentrations between 0·5 and 4·5 kg/kg. The particles used to produce the airborne suspensions included glass beads (with radii between 1·6 and 31 μm), flowing olivine sand (with radii between 8·6 and 422 μm), silica flour (with radii between 2·5 and 20 μm) and alumina particles (with radii between 0·7 and 5·7 μm). The values of fractal dimension (or rather the difference between the fractal dimension of the suspension and that of an equivalent array of smooth spheres) have been determined empirically from the variation of attenuation with concentration at a single frequency. However, the same value enables good agreement with data at other frequencies and good agreement with the frequency dependence at a given concentration. Fractal dimensions of steady state suspensions have been determined empirically by measuring PH and thermal diffusion [24, 25]. It would be extremely difficult to employ these techniques on the falling and flowing suspensions considered here because of their transitory nature. On the other hand, the good fits between the fractal modification of scattering theory and data suggest that acoustical determination of fractal dimensions in suspensions is a possibility. This will be the subject of further work.

ACKNOWLEDGMENTS

This work was supported by EPSRC Grants GR/L36512 and GR/L39797.

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