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LINEARITY TESTS FOR IN-DUCT ACOUSTIC ONE-PORT SOURCES

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Acoustic one-port source data are commonly used to predict the plane wave sound generation in duct and pipe systems connected to fluid machines. The source data are usually determined experimentally, which assumes that linear time-invariant system theory can be used. Since some machines such as IC-engines and compressors generate very high sound levels in the connecting ducts or pipes it is of interest to investigate whether the assumption of linearity is justified. Linearity tests for linear system identification when both input and output signals can be measured are common in the literature. In the case when only the output signal can be measured linearity tests are not so readily found. This paper presents two different linearity coefficients for determining whether an acoustic one-port source under test is linear. Their sensitivity to random noise and their ability to detect non-linearities are investigated by simulations and measurements on several types of machines.

1. INTRODUCTION

Acoustic one-port source models can be used to calculate the acoustic plane wave field generated in duct systems by fluid machines, e.g., pumps, fans, IC-engines. They can also be used for the design of mufflers and silencers and to gain a better understanding for the sound generating mechanisms in these machines. A number of experimental methods have been developed for determining the source data for fluid machines. The source has to behave as a linear time-invariant system for the one-port model to be valid. For machines such as IC-engines and compressors, which generate high sound levels in the connected pipes, the condition of linearity might be violated. There is therefore a need for experimental methods to check whether the conditions of linearity and time invariance are fulfilled. In the frequency domain, an acoustic one-port can be described completely by a source strength and a source impedance (or a reflection coefficient) (see Figure 1) and the following equation (1):

$$p_s Z = p Z_s + p Z, \tag{1}$$

where p_s is the source pressure, Z_s is the normalized source impedance, p is the acoustic pressure at the outlet of the source and Z is the normalized acoustic impedance of the rest of the system seen from the source. The impedances are normalized using the density and the speed of sound of the medium at the source duct cross section. A review of different measurement methods for determining the source data can be found in references [1].

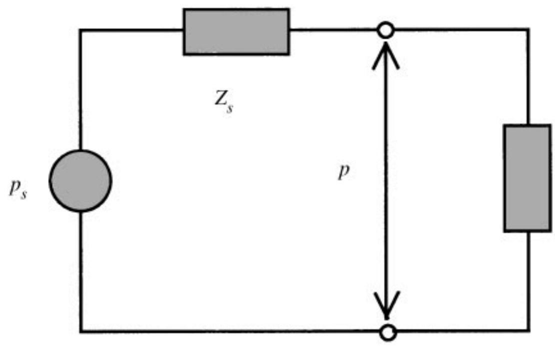

Figure 1. One-port model for the plane wave region frequency domain source data for a fluid machine.

The measurement methods used to determine the source data of acoustic one-ports can be divided into methods "with an external source" or direct methods [2, 3] and "without an external source" or indirect methods [4–8].

The methods "with an external source" are two step methods. First, the source impedance is determined by exciting the source with the sound field from an external source and using, for example, the two-microphone method [2, 3]. In the second step, the external source is turned off and the source strength is determined by making a pressure measurement when a known acoustic load is applied to the source.

When using the methods "without an external source" the two unknowns p_s and Z_s are determined by applying acoustic loads with known impedances (Z) and measuring the acoustic pressure (p). Since there are two unknowns two measurements should be sufficient, which leads to the two-load method [4]. If more than two loads are used an overdetermined problem is obtained, which can be useful for improving the measurement results [3, 6, 9], and for checking if the source behaves as a linear system [3]. The two-load method requires that it is possible to make complex pressure measurements, which means that a reference signal related to the sound generating mechanism of the source is needed. Such a reference signal is not always possible to find, for example, when broadband as well as harmonic components in the spectrum from the source are considered. For this reason and also for purely practical considerations, for example, being able to use a single channel measurement system instead of a two-channel system, methods have been developed where the auto-spectra of the pressures are measured instead of the complex pressures. The first such method was the three-load method [7]. By taking the squared magnitude of equation (1), a real equation with three unknowns is obtained, i.e., $G_{ps} = |p_s|^2$ and the real and imaginary parts of Z_s, ($\mathrm{Re}(Z_s)$, $\mathrm{Im}(Z_s)$). To determine the unknowns, it is therefore necessary to make measurements using three different loads. The resulting system of equations is non-linear and can have more than one real solution. This method is quite impractical to use and has also been reported to give large measurement errors [5]. In the four-load method [8], a fourth measurement is used to eliminate the non-linear term containing $|Z_s|^2$. This method has also been reported to give large measurement errors [5, 10]. An analysis of the reasons for these problems and an alternative method of evaluating the data used in the four-load method, which gives better results, was presented in reference [11].

Linearity tests can be found in the literature for systems where both input and output signals can be measured (e.g., see references [12, 13]). The acoustic one-port source is an example of a system where only output data are available. Linearity tests for this case are

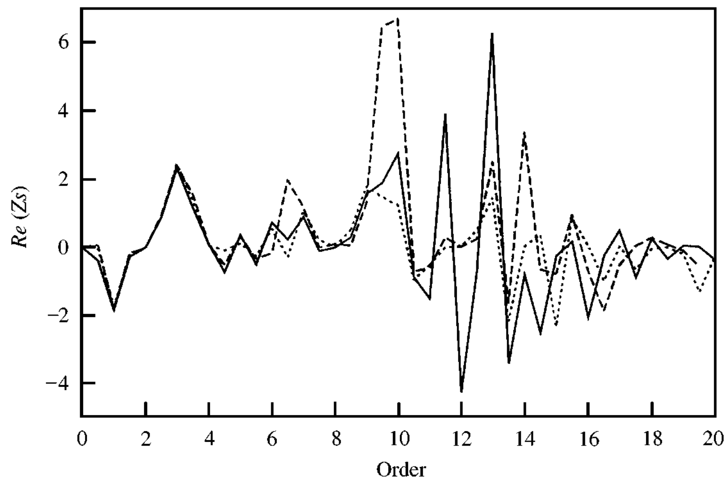

Figure 2. Real part of normalized source impedance as a function of engine orders. Engine sped 1200 r.p.m., three different engine loads.

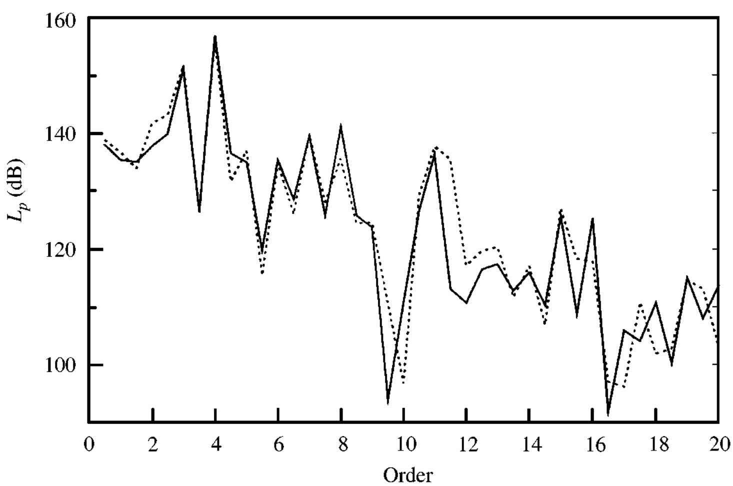

Figure 3. Sound pressure level (re. 2×10^{-5} Pa) in the exhaust system of a diesel engine as a function of engine orders: —— measured; - - - - - calculated.

the main topic of this article even though a short discussion of linearity tests for the case when input and output signals are available is given in Section 2. One would of course prefer to be able to calculate the frequency domain one-port source data from first principles, but this is, in most cases, not possible. Non-linear time domain models are available, (e.g., see reference [14]), but the problem is then to couple this description to the linear frequency domain model normally used for the rest of the system. The resulting so-called hybrid models have so far not been fully successful [15–17]. A number of experimental methods for determining the linear frequency domain source data have been developed [1]. It is therefore of interest to assess whether the experimental data obtained are consistent with a linear source model. Figure 2 shows a typical example of the real part of the measured normalized source impedance for the exhaust side of a Diesel engine

[18, 19]. As can be seen, the real part of the source impedance is negative for many frequencies, which is not physically correct for a linear source model. This could be an indication of non-linear source behaviour. For a linear time-invariant passive system the real part of the impedance must be positive since this shows the direction of energy. Energy can only be lost into the system; it cannot be created since the system is passive. For a non-linear system energy can be transferred from one frequency to another. This could, at certain frequencies, suggest that the system was no longer passive by giving a negative real part for the impedance.

The most relevant test is of course how well the source data can be used to predict the noise from the machine. Figure 3 shows an example of the prediction of pressure in the exhaust system for a Diesel engine, compared to measurements [19]. As can be seen the prediction gives results which are satisfactory if the aim is engineering accuracy.

Many fluid machines such as compressors and IC-engines generate high sound pressure levels or high flow velocities, and it is not certain that linear models can be used to describe them. It is therefore useful in the experimental situation to have methods to determine whether a linear model can be used. Such a linearity test has been suggested for the methods with an external source in reference [3]. In this paper, general linearity tests for all measurement methods are presented.

2. LINEARITY TESTS FOR SYSTEMS WITH INPUT AND OUTPUT SIGNALS AVAILABLE

A number of different methods can be used to check whether the sound propagation in the system is linear if the input and output signals are available. One example of such a system is a duct where the pressure is measured at two different duct cross sections. This is the measurement situation when the source impedance is measured using the two-microphone method which is a "direct" or "external source" method [1]. This linearity test will only indicate whether or not the system in-between the two measurement positions is linear. It will therefore not indicate whether the source under test is linear. An ordinary coherence function measurement can be used to detect non-linearitites in-between the measurement points. The Hilbert transform can also be used for the some purpose (e.g., see reference [12]). If certain assumptions are made regarding the character of the nonlinearities the system can be separated into its linear and non-linear components [13]. If the pressure is measured at three different duct cross sections, two of the pressure measurements can be used to determine the reflection coefficient using the two-microphone methods [18]. The reflection coefficient can then be used to calculate the transfer function between the first and the third measurement position, using linear acoustic theory. The agreement between this linear calculation result and the directly measured transfer function indicates the validity of the linear model. Sometimes the machine is instead used as the acoustic source to determine the acoustic load impedance. Figure 4 shows an example of such a comparison and Figure 5 shows a coherence function measurement from measurements in a Diesel engine exhaust system [19], indicating that a linear propagation model is valid.

3. GENERAL LINEARITY TESTS FOR SYSTEMS WITH ONLY OUTPUT SIGNALS AVAILABLE

If it is assumed that there is a problem with m complex unknowns and that n measurements are made, the over-determined equation system for determining the

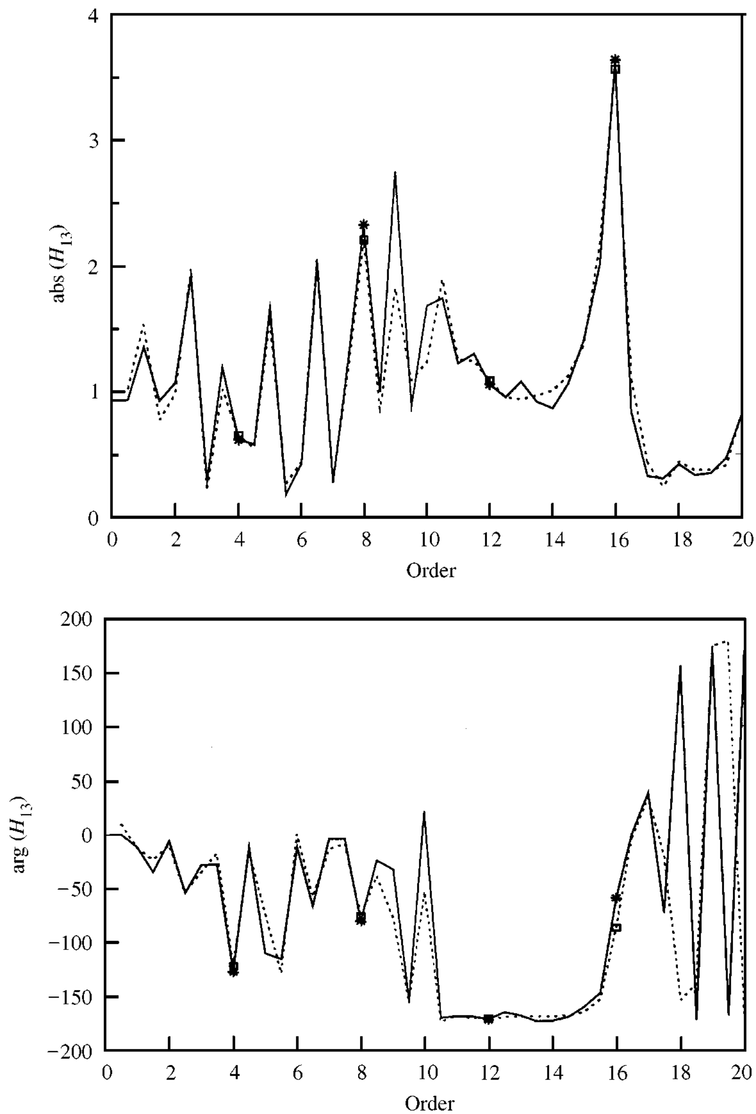

Figure 4. Pressure transfer function between two measurement positions 80 cm apart in a Diesel engine exhaust system as a function of engine orders. ——, measured; - - - - -, calculated, $*$ main engine harmonics.

unknowns (**x**) can be written in the following way:

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b}, \tag{2}$$

where **A** is an $(n \times m)$ matrix, **x** is an $(m \times 1)$ vector and **b** is an $(n \times 1)$ vector. The idea is now to formulate tests, i.e., linearity coefficients, which can indicate whether the measured data

in **A** and **b** is consistent with the linear relationship of equation (2). The number of measurements, n in equation (2), has to be larger than m for the linearity coefficients to be meaningful. If n equals m the linearity coefficients will always indicate a linear relationship. It can be of advantage to scale (2) in such a way that **b** is always the unity vector. This can improve the result if some of the n equations in equation (2) are much larger in magnitude

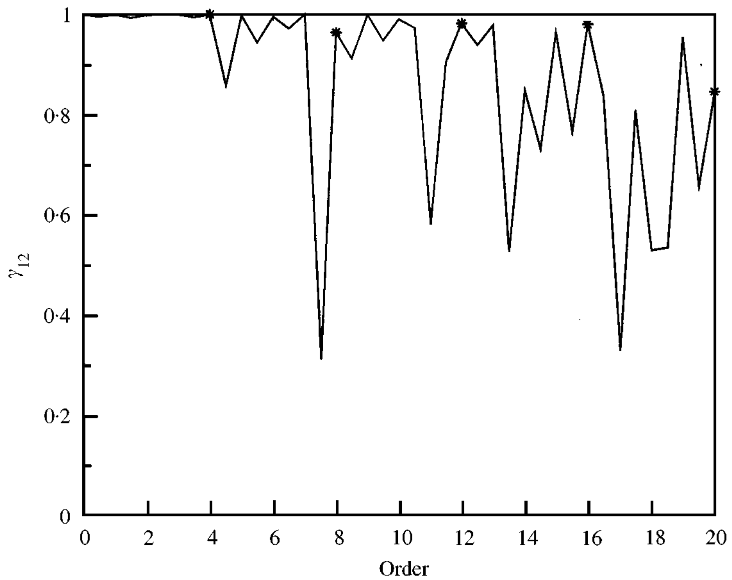

Figure 5. Coherence function measurement between two measurement positions in a diesel engine exhaust system as a function of engine orders, * main engine harmonics.

than the others. A linearity coefficient which is similar to the coherence function can then be defined as

$$\gamma_c^2 = \mathbf{x}^I \cdot \mathbf{x} = \mathbf{b}^I \cdot \mathbf{A} \cdot \mathbf{A}^I \cdot \mathbf{b}, \tag{3}$$

where superscript I denotes the pseudo-inverse of a matrix [20]. This linearity coefficient will have a value in the interval $0 \leqslant \gamma_c^2 \leqslant 1$, where the upper limit represents a perfect linear relationship.

One alternative approach to define a linearity measure is to calculate all solutions for all possible combinations consisting of m of the n equations. The number of solutions is given by

$$\binom{n}{m} = \frac{n(n-1)(n-2)\cdots(n-m-1)}{m!} = \frac{n!}{m!\cdot(n-m)!}. \tag{4}$$

The mean (μ) and standard deviation (σ) of all the solutions can then be calculated for each component of $\mathbf{x}$. To get dimensionless quantities the standard deviation could be divided by the mean. This will give a result which is equal to zero when there is a perfect linear relationship and larger than zero otherwise. A linearity coefficient which has a value between zero and one and is equal to one when there is a perfect linear relationship can be defined as

$$\gamma_s^2 = \frac{\mu^2}{\mu^2 + \sigma^2}. \tag{5}$$

A third possibility is to use the residual

$$e = (\mathbf{A} \cdot \hat{\mathbf{x}} - \mathbf{b})^{\mathrm{H}} (\mathbf{A} \cdot \hat{\mathbf{x}} - \mathbf{b}), \tag{6}$$

where $\hat{\mathbf{x}}$ is the estimate of $\mathbf{x}$ and $(\ )^{\mathrm{H}}$ stands for transposition and complex conjugation. To get a dimensionless quantity the following quantity can be defined:

$$\varepsilon = \frac{(\mathbf{A}\cdot\hat{\mathbf{x}} - \mathbf{b})^{\mathrm{H}}(\mathbf{A}\cdot\hat{\mathbf{x}} - \mathbf{b})}{(\mathbf{A}\cdot\hat{\mathbf{x}})^{\mathrm{H}}(\mathbf{A}\cdot\hat{\mathbf{x}})}. \tag{7}$$

A linearity coefficient which is equal to one when there is a perfect linear fit for the data and which goes to zero when there is a bad fit can be defined in the following way:

$$\gamma_e^2 = \frac{1}{(1+\varepsilon)}. \tag{8}$$

It can in fact be shown that γ_e^2 gives the same result as γ_c^2 if $\hat{\mathbf{x}}$ calculated by taking $\mathbf{b}$ multiplied by the pseudo-inverse of $\mathbf{A}$ gives the same result as $\hat{\mathbf{x}} = (\mathbf{A}^{\mathrm{H}}\cdot\mathbf{A})^{I}(\mathbf{A}^{\mathrm{H}}\cdot\mathbf{b})$, i.e., if the problem is not numerically ill conditioned. A proof of the identity of γ_e^2 and γ_c^2 is given in Appendix A.

The suitability of the different linearity coefficients for detecting non-linearities will, in the following sections, be tested both by computer simulations and by measurements. All three linearity coefficients will also be affected by random noise disturbances. To separate the effects of noise and non-linearities in a measurement the number of averages and, if possible, the level of the excitation should be varied. The effect of random noise should decrease with an increased number of averages and the effect of non-linearities should increase if the level of excitation is increased.

4. LINEARITY TESTS FOR THE TWO-LOAD METHOD

For the two-load method, $\mathbf{A}, \mathbf{x}$ and $\mathbf{b}$ from equation (2) can be expressed in the following way:

$$\mathbf{A} = \begin{bmatrix} Z_1 & -P_1 \\ Z_2 & -P_2 \\ \vdots & \vdots \\ Z_n & -P_n \end{bmatrix}, \qquad \mathbf{x} = \begin{pmatrix} P_s \\ Z_s \end{pmatrix}, \qquad \mathbf{b} = \begin{bmatrix} Z_1\cdot P_1 \\ Z_2\cdot P_2 \\ \vdots \\ Z_n\cdot P_n \end{bmatrix}. \tag{9}$$

Equation (3) and (5) then give the linearity coefficients. The number of measurements, n, in equation (9), has to be larger than two for the linearity coefficient to be meaningful. If n equals two the linearity coefficient will always be equal to one. It is recommended that five to six measurements be made, which will also improve the quality of the result [21–23].

5. LINEARITY TEST FOR THE FOUR-LOAD METHOD AND THE DIRECT LEAST-SQUARES METHOD

In the four-load method and the direct least-squares method attempts are made to fit the data to the model given in equation (10)

$$\frac{|Z|^2}{G_p}G_{ps} - 2\cdot\mathrm{Re}(Z)\cdot\mathrm{Re}(Z_s) - 2\cdot\mathrm{Im}(Z)\cdot\mathrm{Im}(Z_s) - |Z_s|^2 = |Z|^2, \tag{10}$$

where Re( ) stands for the real part, Im( ) stands for the imaginary part and G_p is the squared magnitude of P. The following system of equations in vector notation is obtained.

$$\mathbf{A} = \left[\frac{G_p}{Z} \; -2\cdot\mathrm{Re}(\mathbf{Z}) \; -2\cdot\mathrm{Im}(\mathbf{Z}) \; -\mathbf{I} \right], \qquad \mathbf{x} = \begin{bmatrix} G_{ps} \\ \mathrm{Re}(Z_s) \\ \mathrm{Im}(Z_s) \\ |Z_s|^2 \end{bmatrix}, \qquad \mathbf{b} = |\mathbf{Z}|^2, \tag{11}$$

where $\frac{G_p}{Z}$ is an $(n \times 1)$ vector containing the elements $\frac{G_{p1}}{Z_1}, \frac{G_{p2}}{Z_2}, \ldots, \frac{G_{pn}}{Z_n}$, $\mathbf{Z}$ is an $(n \times 1)$ vector containing the elements $Z_1, Z_2, \ldots, Z_n$, and $\mathbf{I}$ is an $(n \times 1)$ vector with all elements equal to 1. The linearity coefficients are then given by equations (3) and (5).

6. LINEARITY TESTS FOR "EXTERNAL SOURCE" OR DIRECT METHOD

When using the two-microphone method the source impedance and the source strength are determined in two separate measurements. For the source impedance determination the following expressions are obtained:

$$\mathbf{A} = \mathbf{q}^e, \qquad x = Z_s, \qquad \mathbf{b} = \mathbf{p}^e, \tag{12}$$

where $\mathbf{p}^e$ and $\mathbf{q}^e$ are $(n \times 1)$ vectors containing the acoustic pressures and volume velocities (due to the external source) at the measurement cross-section. For determining the source strength the following expressions are obtained:

$$\mathbf{A} = \mathbf{I} + \frac{\mathbf{Z}_s}{\mathbf{Z}}, \qquad x = p_s, \qquad \mathbf{b} = \mathbf{p}. \tag{13}$$

The linearity coefficients are as above given by equations (3), (5) and (8). Equation (3) gives the linearity coefficient suggested in reference [3].

An impedance measurement can be regarded as a single input/single output problem, where $q(f)$ is the input, $p(f)$ is the output and $Z(f)$ is the linear system which is to be determined. It is therefore also possible to use the non-linear system identification technique developed by Bendat [13] as mentioned in section 2.1. In this method, there is no need for any over-determination. The measured signals are instead analyzed in a special way, where for instance the input signal is squared and cubed before the auto-spectra and cross-spectra are calculated.

7. NUMERICAL SIMULATIONS

To check how well the linearity coefficients are able to detect different degrees of non-linearities some numerical simulations have been made. It is also important to understand how the proposed linearity coefficients react to noise since there will be some remaining random noise in all measurements. To investigate the effect of noise on the calculation the following model was used:

$$\mathbf{A} = \mathbf{I}, \qquad x = 1, \qquad \mathbf{b} = \mathbf{A} + \kappa \cdot \mathbf{b}_r, \tag{14}$$

where $\mathbf{b}_r$, is a vector of the same size as $\mathbf{A}$ containing random numbers with the Gaussian distribution with zero mean and a variance equal to one and κ is a constant which is varied. All three linearity coefficients gave the same result when κ was varied for a given noise sample and degree of over-determination. Figure 6 shows an example of the result of this simulation. The degree of over-determination can be varied by changing the length of vector

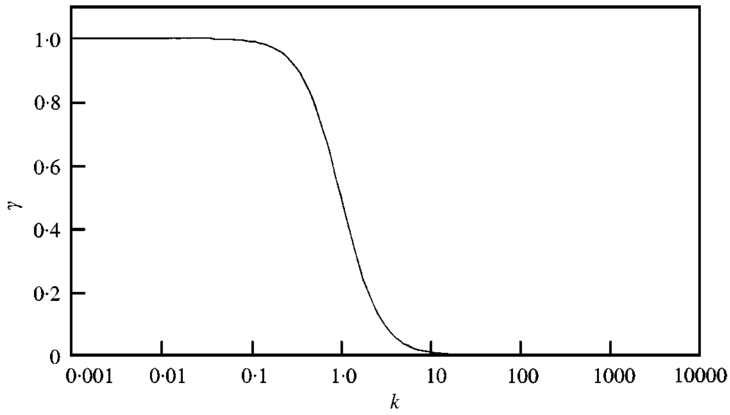

Figure 6. Effect of added noise on the linearity coefficients. For a system ($\mathbf{b} = \mathbf{A}$) where $\mathbf{k}$ is the relative magnitude of the noise compared to the noise free data —— γ_c^2; - - - - - γ_s^2.

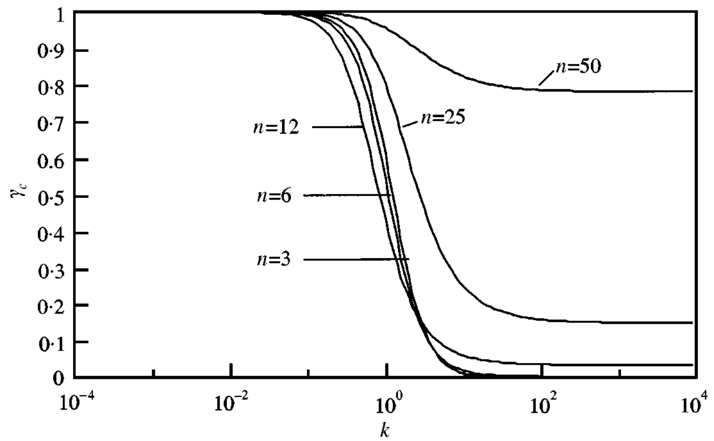

Figure 7. Effect of the degree of overdetermination (n) on γ_c^2 for a system ($\mathbf{b} = \mathbf{A}$) with added noise where $\mathbf{k}$ is the relative magnitude of the noise compared to the noise free data.

A. The result of varying the degree of over-determination is shown in Figure 7, and it can be seen that when it exceeds 12 no further improvement in the noise detection is obtained.

The results presented in Figure 6 show that the three linearity coefficients are equally sensitive to noise. This is important since it means that any large differences in the calculated linearity coefficients is not caused by measurement noise and must therefore be caused by some non-random type of disturbance, i.e., non-linearity.

To investigate how the two linearity coefficients react to a non-linear system some simple tests have been made. The model used was, $\mathbf{b} = \mathbf{A}^{\kappa}$, where κ is a constant which has been varied. $\mathbf{A}$ was a vector with six components with small ($\mathbf{A} = (1\ 1{\cdot}1\ 1{\cdot}2\ 1{\cdot}3\ 1{\cdot}4\ 1{\cdot}5)^{t}$), medium ($\mathbf{A} = (1\ 2\ 3\ 4\ 5\ 6)^{t}$) and large ($\mathbf{A} = (1\ 10\ 10^2\ 10^3\ 10^4\ 10^5)^{t}$) variation in the data. The results are presented in Figure 8. It can be noted that γ_c^2 can give a result close to unity even if the system is strongly non-linear and the variation in the data is large when the non-normalized version of equation (2) was used. When the normalized version of equation

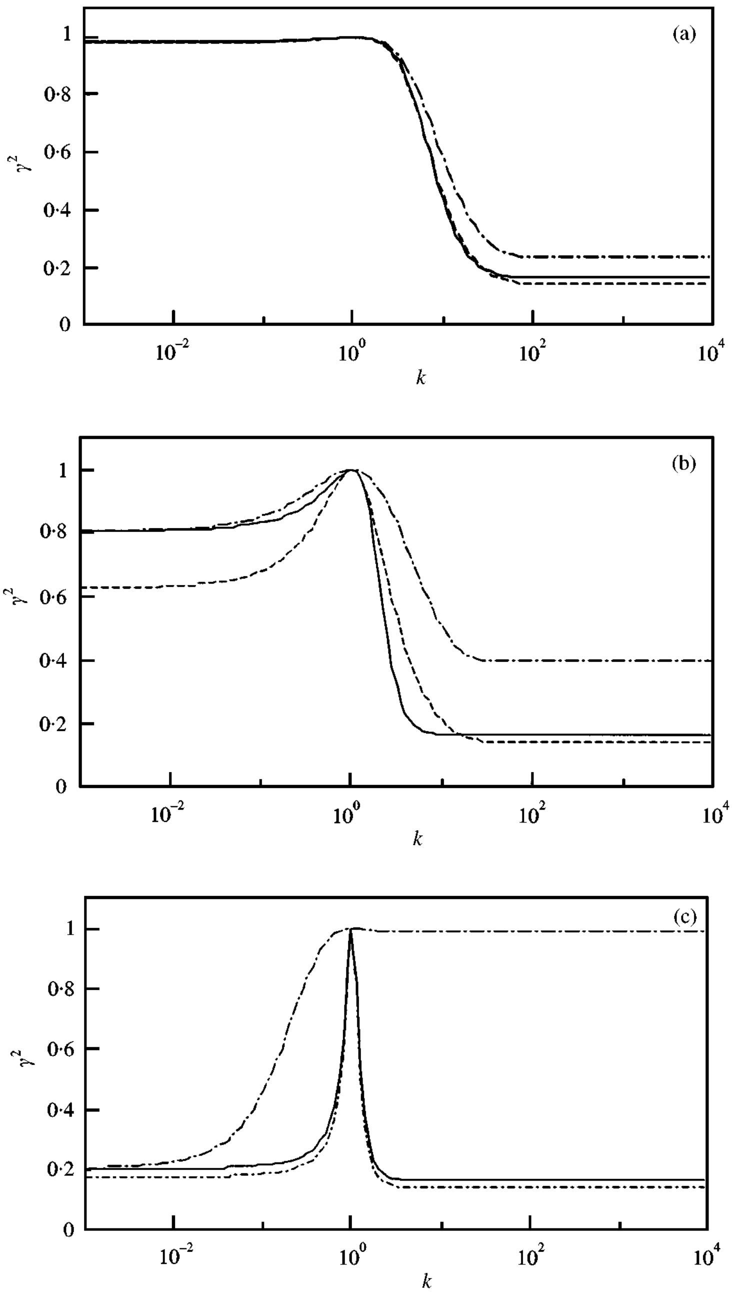

Figure 8. Linearity coefficients for the system $\mathbf{b} = \mathbf{A}^k$: —— γ_c^2 normalized; – – – – – – γ_c^2 non-normalized; - - - - - γ_s^2. (a) Small variation in the data, $\mathbf{A} = [1, 1{\cdot}1, 1{\cdot}2, 1{\cdot}3, 1{\cdot}4, 1{\cdot}5]$. (b) Medium variation in the data, $\mathbf{A} = [1, 2, 3, 4, 5, 6]$. (c) Large variation in the data, $\mathbf{A} = [1, 10, 100, 1000, 10\,000, 100\,000]$.

(2) was used the result was much more similar to the result from γ_s^2. Using the coherence like linearity coefficient γ_c^2 calculated from the normalized version of equation (2) or the standard deviation based linearity test γ_s^2 therefore seems to be the best choice in a measurement situation.

A simulation of non-linear data for the two-load method has also been made. Six different loads were used and the following data were assumed:

$$P_s = 1, \qquad Z_s = 0{\cdot}5 + \mathrm{j}0{\cdot}5, \qquad \mathbf{Z} = \begin{pmatrix} a + \mathrm{j}b \\ b + \mathrm{j}c \\ c + \mathrm{j}c \\ d + \mathrm{j}g \\ e + \mathrm{j}k \\ f + \mathrm{j}m \end{pmatrix},$$

$$P = P_s \frac{Z}{Z + Z_s} + \kappa P_s^2 \frac{Z}{(Z + Z_s)^2} \frac{|Z + Z_s|}{|P_s|},$$

where κ determines the magnitude of the non-linear term compared to the linear term. The result is shown in Figure 9. It can be seen that if the non-linear term is smaller than 10% of the linear term the linearity coefficients are very close to one, when it is approximately 50% of the linear term the beginning of a decrease in the linearity coefficients is seen and when it is equal to the linear term in magnitude non-linearities from the measured coefficients will be certainly suspected.

8. EXPERIMENTAL RESULTS

8.1. IMPEDANCE MEASUREMENT

To test the linearity coefficients in an experiment where the measurement errors should be small and the system should be linear a standard impedance measurement was made. The data were measured using the two-microphone method and the measurement objects were a straight open ended duct and the same duct with absorbing material at the far end. Figure 10 shows the measured impedances and Figure 11 shows the calculated linearity coefficients. As expected both linearity coefficients are very close to unity.

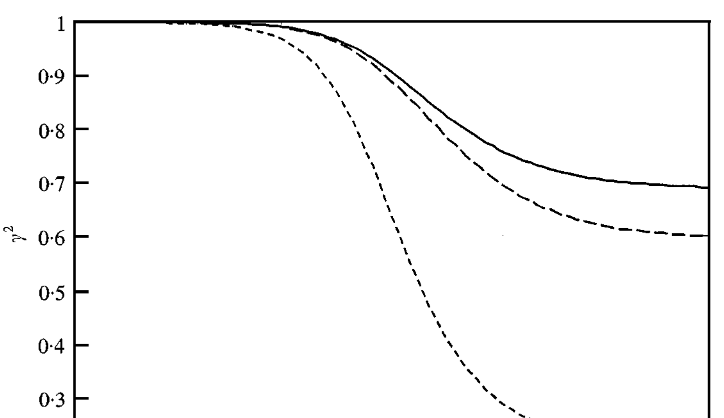

Figure 9. Effect of non-linearity on the linearity coefficients for a simulated two-load measurement, **k** is the relative magnitude of the non-linear term compared to the linear term: —— γ_c^2; - - - - - γ_s^2 for p_s; – – – – – γ_s^2 for Z_s.

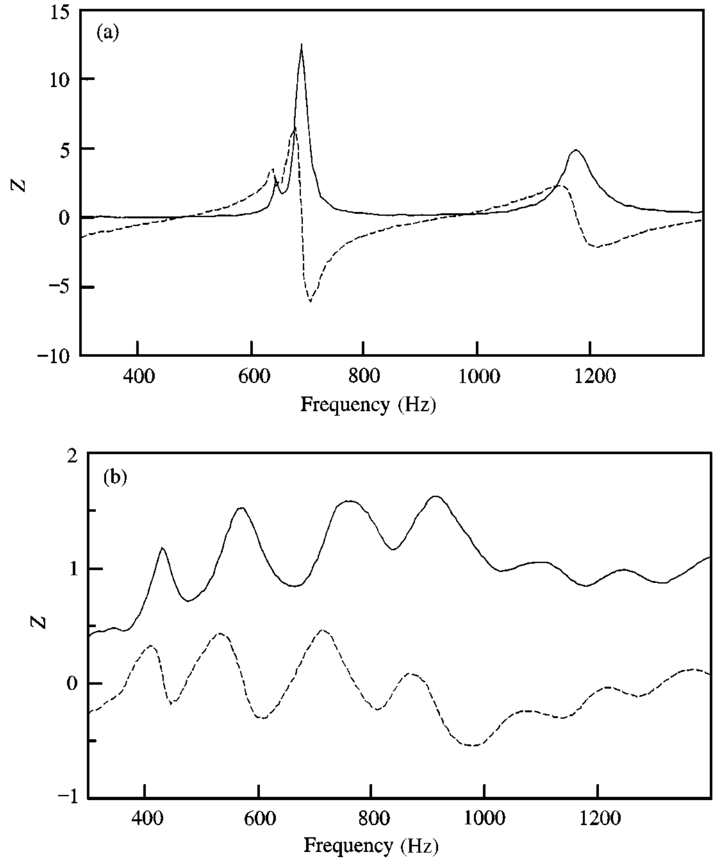

Figure 10. Measured normalized impedances: —— real part; – – – – – imaginary part. (a) Open ended duct. (b) Duct filled with absorbing material at the end.

8.2. AXIAL FLOW FAN

The data presented in this section were originally used for determining the acoustic two-port source data for a fan [11] and have been reanalyzed in order to calculate the one-port data and the linearity coefficients. In reference [11] the source data were determined using an external source and the two-microphone method. To get overdetermination five different loads were used. An example of the results is shown in Figure 12. All previous experience [3, 11] indicates that the axial flow fan is a linear acoustic source. The decrease in the linearity coefficients at certain frequencies is therefore probably caused by insufficient suppression of flow noise.

8.3. INTERNAL COMBUSTION ENGINE

Data from earlier studies on car engines [21–23] using the two-load method have been reanalyzed to calculate the linearity coefficients. Linearity coefficients have also been determined in more recent experiments for truck engines using the two-load-method [19] and the direct least-squares method [18]. Data from both the exhaust and the intake side have been analyzed.

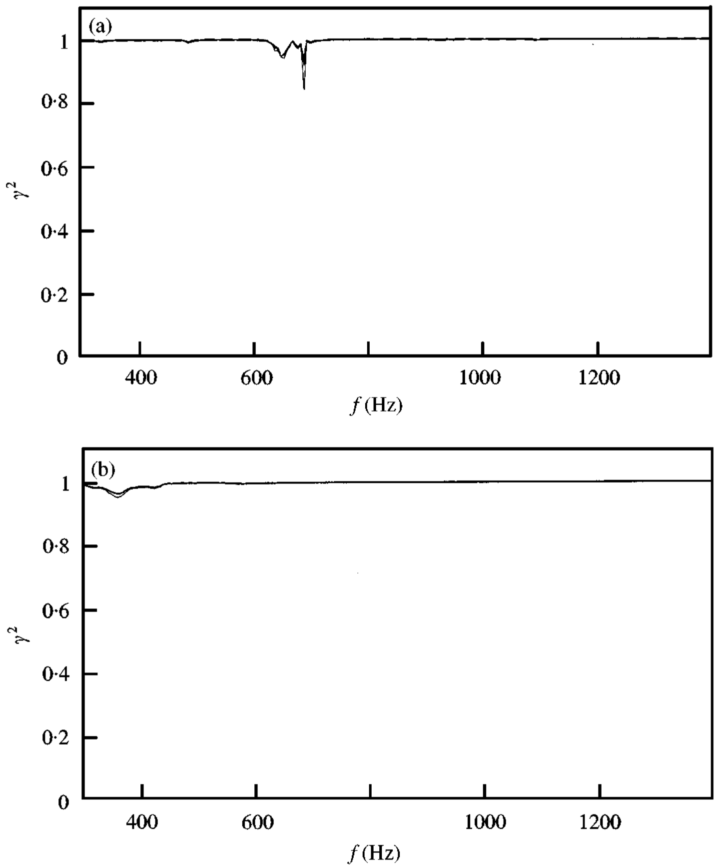

Figure 11. Measured linearity coefficients for impedance measurements; ——γ_c^2; - - - - - γ_s^2. (a) Open ended duct. (b) Duct filled with absorbing material at the end.

For the car engine exhaust side measurements were made for 20 acoustic loads and a number of different speeds and engine loads. Figure 13 shows an example of the measured linearity coefficients, which clearly indicate some non-linearity. This could be expected from earlier analysis of these data [21–23].

For the car engine inlet side measurements were made for eight acoustic loads on five engines and for a number of speeds and engine loads. An example of the results is presented in Figure 14. This figure illustrates how difficult it is to interpret the results of the linearity test when they do not indicate a perfect linear relationship. The normalized coherence function-like linearity coefficient and the standard deviation based linearity are, for most engine harmonics, significantly lower than unity. This shows that the result deviates from a linear relationship, but to what degree and what consequences it will have when using the source data for predictions is difficult to say. An example of a comparison between predictions of the pressure in the intake system made using the measured source data and direct measurements is shown in Figure 15 [22]. It can be seen that an acceptable prediction of the pressure can be obtained even though the linearity tests indicate a non-linear source behaviour.

Figures 16 and 17 show examples of the linearity coefficients from measurements on the exhaust side of a truck Diesel engine [19]. Measurements were made using six acoustic

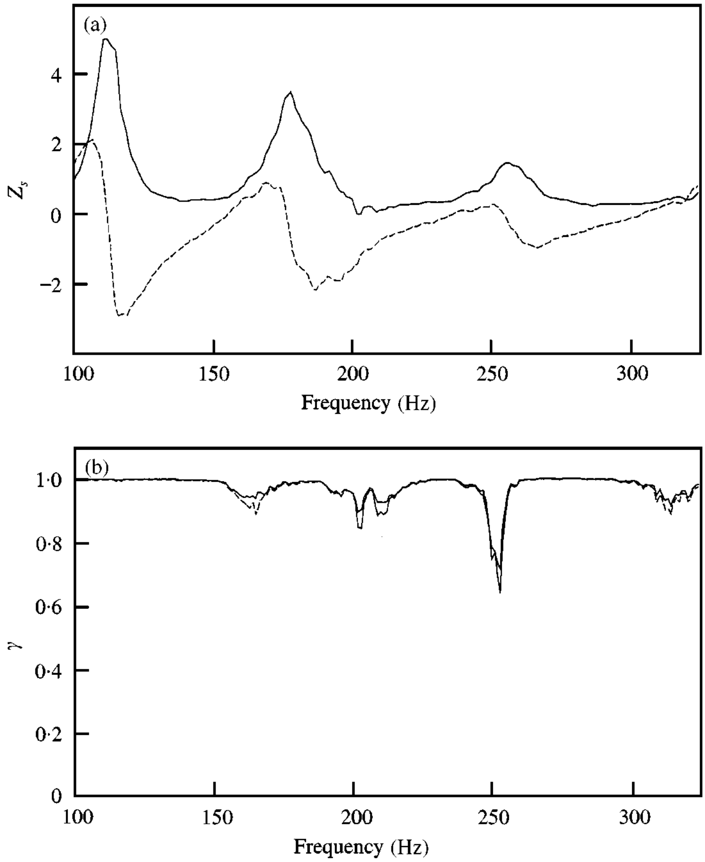

Figure 12. Measurements on an axial flow fan. (a) Measured normalized source impedance; —— real part; – – – – – imaginary part. (b) Measured linearity coefficients; —— γ_c^2; – – – – – γ_s^2.

loads and for a number of different speeds and engine loads. Also in this case the standard linearity coefficients indicates a non-linear source behaviour. As could be seen from Figure 3 an acceptable prediction of the pressure in the exhaust system could be obtained despite the indicated non-linear source characteristics.

Figure 18 shows an example of the linearity coefficients from measurements on the air inlet side of a truck Diesel engine [18]. Measurements were made using six acoustic loads and for a number of different speeds and engine loads. As for the result for the exhaust side, the linearity coefficients indicate non-linear source behaviour but a reasonably good prediction of the pressure in the inlet system could still be obtained as can be seen from Figure 19.

8.4. COMPRESSOR

Data from earlier studies [10] using the two-load method have been reanalyzed to calculate the linearity coefficients. Measurements were made for 24 acoustic loads and a number of different configurations. Some examples of the results are presented in Figure 20. These results show that the compressor behaves as a non-linear source.

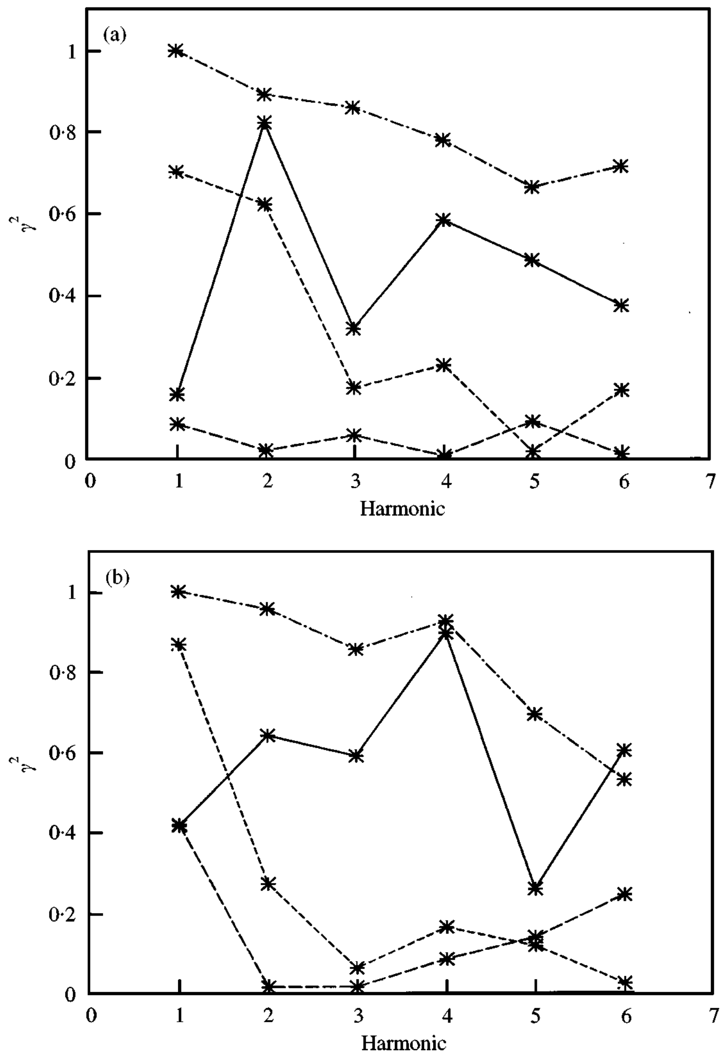

Figure 13. Measured linearity coefficients for source data measurements on the exhaust side of a car engine as a function of engine harmonics: —— γ_c^2; normalized; - - - - - - - γ_c^2, not normalized; – – – – – γ_s^2 for Z_s; - - - - - γ_s^2 for p_s. (a) The catalytic converter included in the source. (b) Catalytic converter not included in the source.

9. CONCLUSIONS

Two different linearity coefficients for determining if an acoustic one-port source under test is linear has been suggested. It has been shown that both linearity coefficients have the same sensitivity to random noise. By simulations and measurements it has been shown that non-linearities can be detected. Measurements for low-level sound sources such as a loudspeaker and an axial fan indicate linear source characteristics using both linearity coefficients. When the linearity coefficients are close to unity, as they are for the loudspeaker and the axial fan, the conclusion can be drawn that a linear source model is valid. Tests on high-level sound sources such as IC-engines and a compressor indicated some non-linear behaviour. Numerical simulations have shown that the normalized version of equation (2) should be used when calculating the coherence function-like linearity coefficient. If the

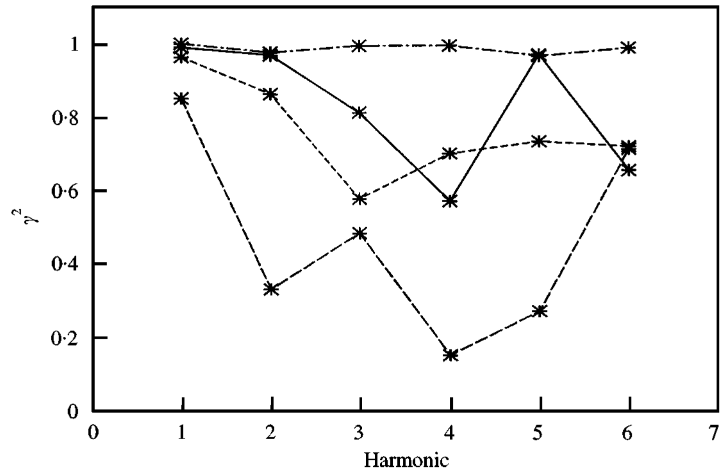

Figure 14. Measured linearity coefficients for source data measurements on the air intake side of a car engine as a function of engine harmonics; —— γ_c^2, normalized; - - - - - - - γ_c^2, not normalized; – – – – – γ_s^2 for Z_s; - - - - - γ_s^2 for p_s.

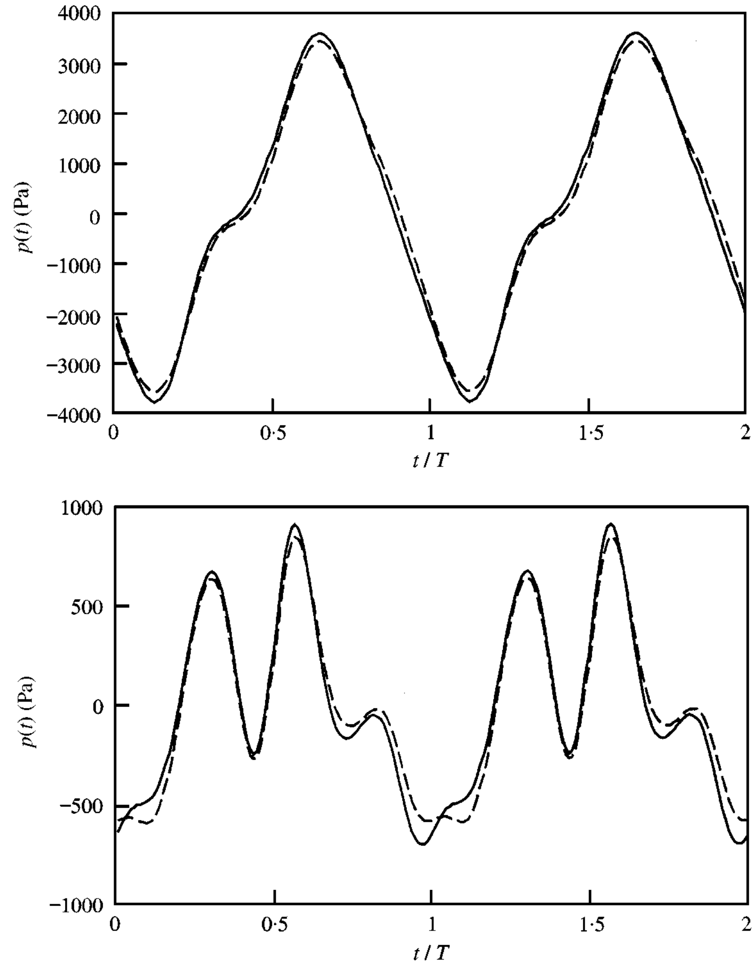

Figure 15. Two examples of sound pressure measurements in the air inlet system of a Otto engine as a function of time: —— measured; - - - - - calculated.

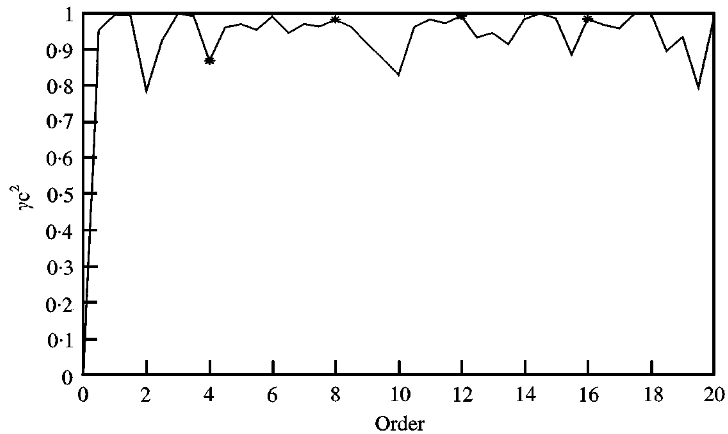

Figure 16. Linearity measured γ_c^2 for measurements on a 8-cylinder diesel engine, $*$ main engine harmonics.

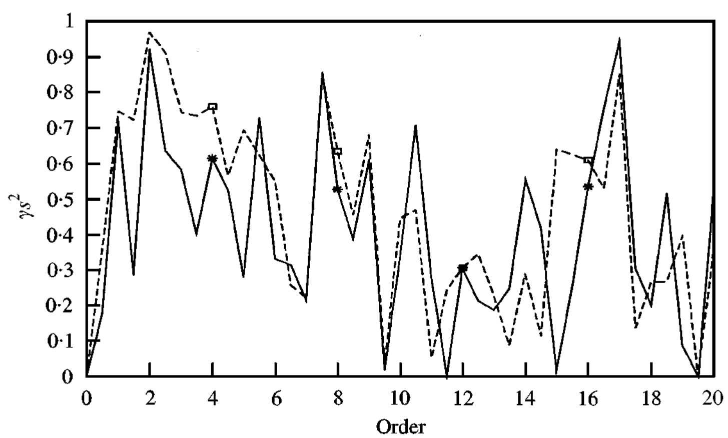

Figure 17. Linearity measure γ_s^2 for measurements on the exhaust side of a 8-cylinder diesel engine; ——, source strength; – – – – –, source impedance; $*$ main engine harmonics.

normalization is performed before the calculation both the standard deviation based linearity coefficient and the coherence function-like linearity coefficient give a clear indication of nonlinearity. One of the problems with these tests is that they cannot distinguish between non-linearity and other types of measurement errors. Even if measurement noise can be reduced by various signal-processing methods the interpretation of the results when the test does not indicate a perfect linear source behaviour is a problem. One way to get an indication of whether a deviation from linearity according to the tests proposed in this paper is caused by measurement noise or non-linear source characteristics is to use both proposed tests and compare the results. Numerical simulation has shown that both tests give the same result for measurement noise but give slightly different results for non-linear source characteristics. The relationship between the value for the linearity coefficients and the degree of non-linearity and what the consequences will be when using the measured source data for predictions still remains to be explained.

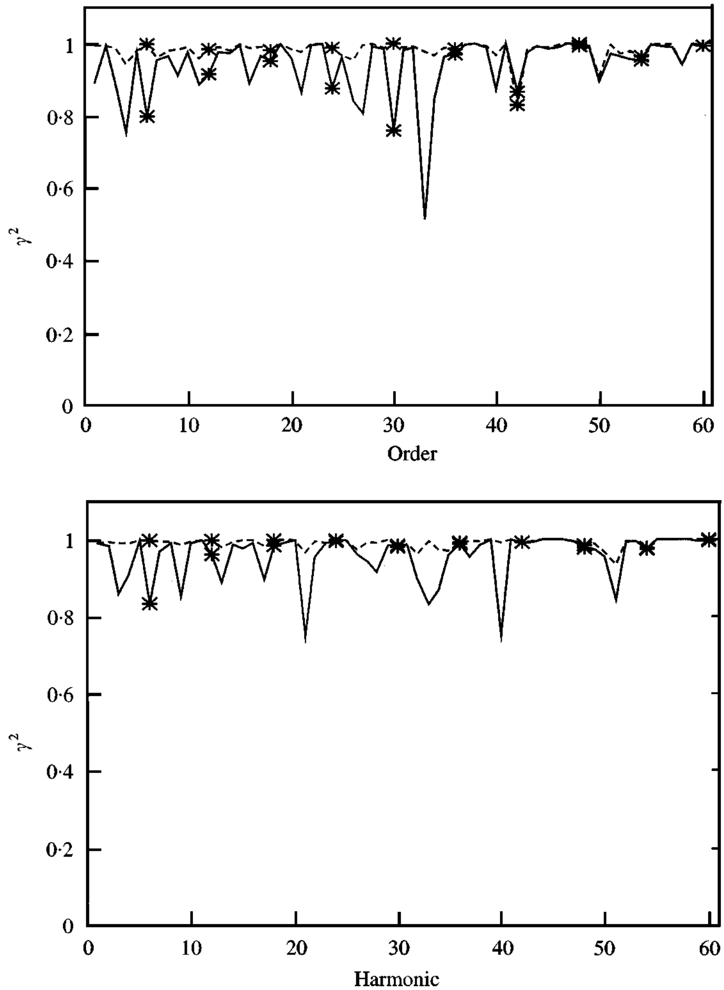

Figure 18. Linearity measure γ_c^2 for measurements on the inlet side of a 6-cylinder diesel engine; ——, normalized; – – – – –, non-normalized; $*$ main engine harmonics.

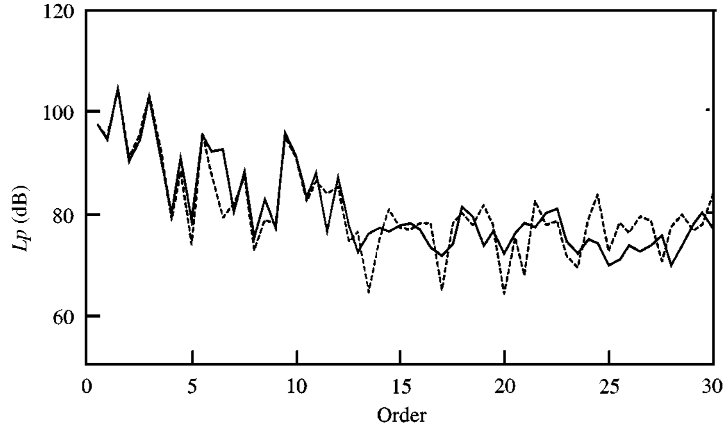

Figure 19. Pressure in the air inlet system of a Diesel engine; —— measured; - - - - calculated.

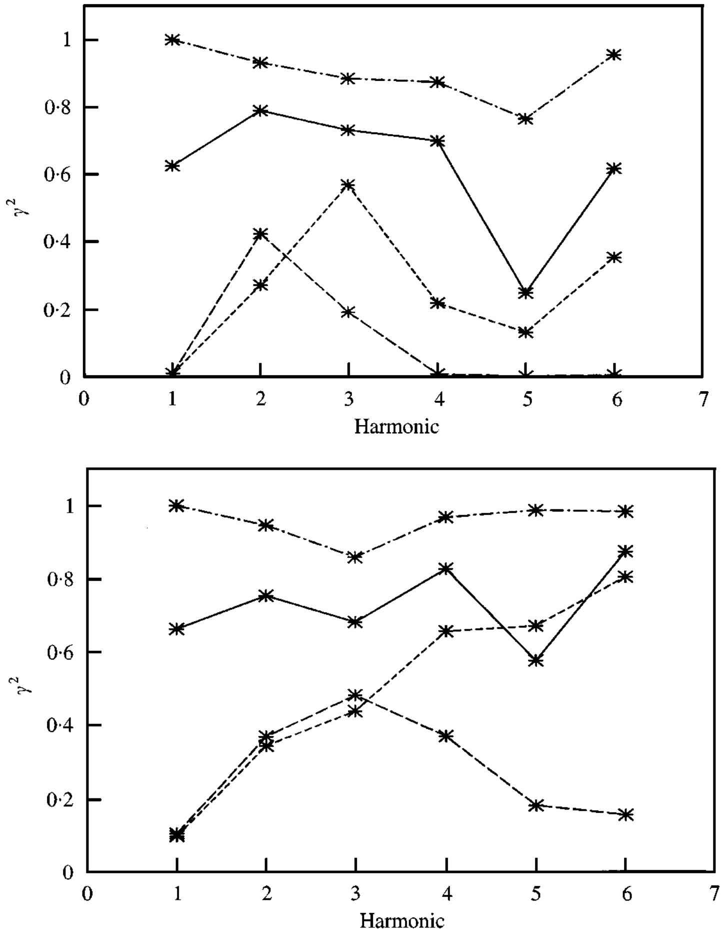

Figure 20. Measured linearity coefficients for source data measurements of a compressor: ——— γ_c^2, normalized; - - - - - - - γ_c^2; not normalized; — — γ_s^2 for Z_s; - - - - - γ_s^2 for p_s.

REFERENCES

1. H. BODÉN and M. ÅBOM 1995 *Acta Acustica* **3**, 549–560. Modelling of fluid machines as sources of sound in duct and pipe systems.
2. M. G. PRASAD and M. J. CROCKER 1983 *Journal of Sound and Vibration* **90**, 479–490. Acoustical source characterization studies on a multi-cylinder engine exhaust system.
3. J. LAVRENTJEV, H. BODÉN and M. ÅBOM 1992 *Journal of Sound and Vibration* **155**, 534–539. A linearity test for acoustic one-port sources.
4. M. L. KATHURIYA and M. L. MUNJAL 1979 *Journal of the Acoustical Society of America* **65**, 240–248. Experimental evaluation of the aeroacoustic characteristics of a source of pulsating gas flow.
5. H. S. ALVES 1986 *M.Sc. Thesis, University of Calgary*, Characterization of noise sources in ducts.
6. H. BODÉN 1991 *Journal of Sound and Vibration* **148**, 437–453. The multiple load method for measuring the source characteristics of time-variant sources.
7. H. S. ALVES and A. G. DOIGE 1987 *NOISE-CON* 87, 329–334. A three-load method for noise source characterization in ducts.
8. M. G. PRASAD 1987 *Journal of Sound and Vibration* **114**, 347–356. A four-load method for evaluation of acoustical source impedance in a duct.

9. H. BODÉN, M. ÅBOM and J. LAVRENTJEV 1993 *InterNoise* 93. A measurement method for determining the source data of acoustic two-port sources.
10. CROCKER and SHRIDHARA 1993 *Journal of the Acoustical Society of America* **65**, 240–248. Error analysis for the four load method.
11. H. BODÉN 1995 *Journal of Sound and Vibration* **180**, 725–743. On multi-load methods for measuring the source data of acoustic one-port sources.
12. G. R. TOMLINSON 1987 *Mechanical Systems and Signal Processing* **1**, 151–171. Development in the use of the Hilbert transform for detecting and quantifying non-linear associated with frequency response functions.
13. J. S. BENDAT 1998 Non-linear system techniques and applications. New York: Wiley-Interscience.
14. A. D. JONES 1984 *Noise Control Engineering* **23**, 12–31. Modelling the exhaust noise radiated from reciprocating internal combustion engines — a literature review.
15. V. H. GUPTA and M. L. MUNJAL 1992 *Journal of the Acoustical Society of America* **92**, 2716–2725. On numerical prediction of the source characteristics of an engine exhaust system.
16. P. O. A. L. DAVIES and M. F. HARRISON 1993 *Proceedings of the Institute of Acoustics* **15**, 369–374. Hybrid systems for IC engine breathing noise synthesis.
17. J. M. DESANTES, A. J. TORREGROSA and A. BROATCH 1995 *SAE Paper*-950545. Hybrid linear/non-linear method for exhaust noise prediction.
18. H. BODÉN 1997 *Report TRITA-FKT*-1997:35, MWL, KTH. Measurement of acoustic source data for truck engines: Part 1.
19. F. ALBERTSON 1998 *Report TRITA-FKT*-1998:14. Measurement of acoustic source data for truck engines: Part 2.
20. S. J. LEON 1995 Linear Algebra with Applications. New York: MacMillan Publishing Company.
21. H. BODÉN 1988 *Report TRITA-TAK*-8803. Characterization of a multi-cylinder internal combustion engine as a source of fluid borne noise.
22. H. BODÉN 1989 *13th International Congress on Acoustics*. Characterization of IC engines as sources of exhaust noise.
23. H. BODÉN 1992 *Proceedings of the 25th ISATA Conference on Mechatronics*. Characterisation of IC-engines as sources of exhaust and intake noise.

APPENDIX A

In this appendix, it is proved mathematically that the two measures γ_c^2 and γ_e^2 are equivalent under some conditions. First, some theorems and definitions are given which form a basis for the proof.

Definition. Let $\mathbf{A}$ be a quadratic matrix of order m. Then if $\det(\mathbf{A}) \neq 0$, $\mathbf{A}$ is said to be *non-singular*.

In numerical calculations this is somewhat similar to a low condition number. There are however cases when the matrix $\mathbf{A}$ still behaves as non-singular even for higher condition numbers. Hence, a low condition number is not equivalent to the matrix being nonsingular.

Definition. Let $\mathbf{A}$ be an $(m \times n)$ matrix. Then the *pseudo-inverse* $\mathbf{A}^I$ is given by $\mathbf{A}^I = \mathbf{V}\mathbf{\Sigma}^I\mathbf{U}^{\mathrm{H}}$. Here, $\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathrm{H}}$,$\mathbf{U}$ is an unitary $(m \times m)$ matrix, $\mathbf{V}$ is an unitary $(n \times n)$ matrix, and

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{D} & 0 \\ 0 & 0 \end{bmatrix},$$

where $\mathbf{D}$ is a $r \times r$ diagonal matrix containing the singular values of $\mathbf{A}$.

It is well known that the Penrose conditions are satisfied for the pseudo-inverse. That is $(\mathbf{A}\mathbf{A}^I)^{\mathrm{H}} = (\mathbf{A}\mathbf{A}^I)$, $(\mathbf{A}^I\mathbf{A})^{\mathrm{H}} = (\mathbf{A}^I\mathbf{A})$, $\mathbf{A}^I\mathbf{A}\mathbf{A}^I = \mathbf{A}^I$, $\mathbf{A}\mathbf{A}^I\mathbf{A} = \mathbf{A}$. Furthermore, if $r = n$, then $\mathbf{A}^I\mathbf{A} = \mathbf{E}$.

Theorem. *The matrix* $\mathbf{A}^t\mathbf{A}$ *is non-singular if and only if the columns of* $\mathbf{A}$ *are linearly independent.*

According to this theorem, the least-squares solution is unique if the columns of $\mathbf{A}$ are linearly independent. It is known that the least-squares solution is

$$\mathbf{Ax} = \mathbf{b} \Leftrightarrow \mathbf{A}^t\mathbf{Ax} = \mathbf{A}^t\mathbf{b} \Leftrightarrow \mathbf{x} = (\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t\mathbf{b}$$

and the pseudo-inverse solution is

$$\mathbf{Ax} = \mathbf{b} \Leftrightarrow \mathbf{x} = \mathbf{A}^I\mathbf{b},$$

where $\mathbf{A}^I$ is the pseudo-inverse of $\mathbf{A}$. Hence, if the matrix $\mathbf{A}^t\mathbf{A}$ is non-singular then

$$\mathbf{x}(\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t\mathbf{b} = \mathbf{A}^I\mathbf{b}.$$

The two measures γ_c^2 and γ_e^2 have been defined and found to be numerically equivalent in our applications. This can be proved as follows. According to equation (3)

$$\gamma_c^2 = \mathbf{b}'\mathbf{AA}^I\mathbf{b}$$

and equations (7) and (8) give

$$\gamma_e^2 = \frac{(\mathbf{A\hat{x}})^{\mathrm{H}}(\mathbf{A\hat{x}})}{(\mathbf{A\hat{x}})^{\mathrm{H}}(\mathbf{A\hat{x}}) + (\mathbf{A\hat{x}} - \mathbf{b})^{\mathrm{H}}(\mathbf{A\hat{x}} - \mathbf{b})},$$

where $\hat{\mathbf{x}}$ is the least-squares solution to the overdetermined equation system $\mathbf{Ax} = \mathbf{b}$. Since $\mathbf{A}^t\mathbf{A}$ is non-singular $\hat{\mathbf{x}} = (\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t\mathbf{b} = \mathbf{A}^I\mathbf{b}$. The expression $\hat{\mathbf{x}} = \mathbf{A}^I\mathbf{b}$ can be substituted into the linearity measure γ_e^2. Some algebra reveals that this measure can be rewritten as

$$\gamma_e^2 = \frac{\mathbf{b}^{\mathrm{H}}\mathbf{AA}^I\mathbf{b}}{\mathbf{b}^{\mathrm{H}}\mathbf{b}}.$$

Now, to study the equality $\gamma_c^2 = \gamma_e^2$, that is

$$\mathbf{b}^I\mathbf{AA}^I\mathbf{b} = \frac{\mathbf{b}^{\mathrm{H}}\mathbf{AA}^I\mathbf{b}}{\mathbf{b}^{\mathrm{H}}\mathbf{b}}.$$

Multiplying both sides with $\mathbf{b}^{\mathrm{H}}\mathbf{b}$ and factorizing the equation gives

$$(\mathbf{b}^{\mathrm{H}}\mathbf{bb}^I - \mathbf{b}^{\mathrm{H}})\mathbf{AA}^I\mathbf{b} = 0.$$

By using the Penrose condition $\mathbf{bb}^I = (\mathbf{bb}^I)^{\mathrm{H}}$ and the above proven condition, $\mathbf{b}^I = (\mathbf{b}^t\mathbf{b})^{-1}\mathbf{b}^t$, it is found that

$$(\mathbf{b}^{\mathrm{H}} - \mathbf{b}^{\mathrm{H}})\mathbf{AA}^I\mathbf{b} = 0$$

and the equation holds.

Thus the following theorem is proven.

Theorem. *Let* $\mathbf{A}$ *be a non-singular* $(m \times n)$ *matrix, i.e., a matrix with linearly independent columns and let* $\mathbf{b}$ *be a* $(m \times 1)$ *matrix. Then*

(i) *The pseudo-inverse* $\mathbf{A}^I = (\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t$ *and* $\mathbf{b}^I = (\mathbf{b}^t\mathbf{b})^{-1}\mathbf{b}^t$.
(ii) *The linearity measures* γ_c^2 *and* γ_e^2 *are equivalent.*