Journal of Sound and Vibration (2000) **237**(5), 793–803
doi:10.1006/jsvi.2000.3052, available online at http://www.idealibrary.com on IDEAL®

A METHOD TO PREDICT SOUND RADIATION FROM A PLATE-ENDED CYLINDRICAL SHELL EXCITED BY AN EXTERNAL FORCE

J. H. Wu, H. L. Chen and W. B. An

Institute of Vibration and Noise Control, School of Mechanical Engineering, Xi'an Jiaotong University, Xi'an Shaanxi 710049, *People's Republic of China. E-mail*: *wwwjiuhui@263.net*

(*Received* 18 *January* 2000, *and in final form* 24 *April* 2000)

The vibroacoustic coupling between a finite circular cylindrical shell closed at each end by a piece of circular plate and its enclosed cavity is analyzed in this paper by using a new method, called the covering-domain method. The covering-domain method, which transforms the calculation of the scattering sound field of a complicated-shaped close cavity to that of a series of simply regular-shaped close shells such as close spherical shell and infinite cylindrical shell, is applied to calculating the scattering sound field of the plate-ended cylindrical shell structure. So we can predict radiated sound pressure of the composite elastic structure excited by an external force by using the acoustical reciprocity theorem. The new method expands practical application of the reciprocity theorem. It is verified to be valid by a corresponding test.

1. INTRODUCTION

The plate-ended cylindrical shell structure is of particular interest for many industrial applications. The vibroacoustic coupling analysis of the composite elastic structure and the quantitative prediction of its sound radiation are very important topics in the study on structural vibration and noise control. Representative examples for this are the designs of aeronautical or space structures and industrial vessels.

Up to now, the many studies of vibroacoustic characteristics of these shell–plate structures are mostly limited to the free vibration analysis [1, 2]. Bafilios *et al.* had developed an analytical model to predict the structure-borne noise in a double-wall cylindrical composite shell in reference [3], in which the motions of the shell and the end plates are taken to be independent, so the acoustic pressure inside the enclosure can be obtained by the superposition of the corresponding acoustic radiation pressure due to shell and plate motions. Tavakoli and Singh [4] had applied the state-space method (SSM), a transfer-matrix-based sub-structuring technique, to analyze the vibration characteristic of a hermetic cavity which is composed of a circular cylinder with two circular end plates. Since the SSM requires distinct boundaries, the eigenproblem becomes singular at the center of the end plate. A "pinhole" with free edges is introduced at the center of the end plate as the initial boundary, which may produce inaccurate moment and force estimations near the center of the plate. The state-space method, the receptance method [5] and the transfer matrix method [6, 7] can be used only to analyze the free vibration of those composite structures. Once the forced vibration and sound radiation of those combined structures are considered, these methods will be complex.

On the basis of a free vibration model, and using an artificial spring system to consider the boundary conditions and shell–plate joint conditions, Cheng [8] studied the full

0022-460X/00/450793 + 11 $35.00/0

coupling of a plate-ended circular cylindrical shell with its enclosed acoustic medium. However, it is not easy to establish the corresponding free vibration model for a complicated structure. Slepyan and Sorokin [9, 10] presented a two-level boundary integral equation method to analyze forced vibrations of a composite elastic structure immersed in compressible inviscid fluid. Because both the interaction between the acoustic medium and the composite structure and the interactions between the parts of the structure are described by boundary integral equations, it is very complicated to perform calculations.

Therefore, this paper presents a new method, covering-domain method, to predict the internal sound radiation from a plate-ended circular cylindrical shell due to the action of an external force. Reference [11] has given a general expression about Fredholm integration equation which expresses the strain-stress relation of an arbitrary-shaped elastomer due to an external force by using a concept of covering domain. The concept is used for reference in the paper. This method can also be used to deal with more complicated structures.

2. THEORY OF THE COVERING-DOMAIN METHOD

Suppose elastomers A and B are, respectively, fixed in two separate co-ordinate systems. When the two co-ordinate systems are overlapped, it is concluded that B covers A if point $M \in A$, then $M \in B$.

In general case, the boundary curved surface C of an arbitrary-shaped closed shell A can always be fit by n pieces of spherical surfaces $C_1, C_2, \dots, C_n$. To calculate the interior sound field of the closed shell A, a series of close spherical shells $A_k (k = 1, 2, \dots, n)$ can be used to cover A. The spherical shell A_k has only a piece of its boundary L_k to coincide with C_k and has the same thickness as the original spherical surface C_k. It is obvious that the common domain of all of A_k is the domain occupied by the closed shell.

Although it is difficult to calculate the interior sound field of a closed shell with complicated shape directly, it is easy to calculate the interior sound field of these spherical shells. So we can make use of the concept of covering domain in order to change the problem of the interior sound field of a complicated shell into a simple problem of a series of closed spherical shells. Then the interior scattered sound field of the arbitrary-shaped closed shell can be expressed as follows:

$$P_s(\mathbf{r}) = \sum_{k=1}^{n} P_S^{(k)}(\mathbf{r}), \tag{1}$$

where $P_S^{(k)}(\mathbf{r})$ is the scattering sound field of the kth covering spherical shell at an interior point $\mathbf{r}$ of the arbitrary-shaped closed shell.

For a closed composite structure B, which consists of a finite cylindrical shell and two circular end plates, the internal sound field of the elastic structure B excited by an external force is complex. In order to obtain the internal scattered sound field of B, we can use an infinite cylindrical shell and two close spherical shells with radii big enough to fit the end plates to cover B. The radius and wall thickness of the infinite cylindrical shell are the same as those of the finite cylindrical shell and the wall thickness of the close spherical shell is the same as that of the end plate. Thus, the common domain of the infinite cylindrical shell and the two spherical shells is the internal domain of the closed composite structure B. So the internal scattered sound field of the composite structure B can be expressed as

$$P_S(\mathbf{r}) = P_{SC}(\mathbf{r}) + P_{SS1}(\mathbf{r}) + P_{SS2}(\mathbf{r}), \tag{2}$$

where $P_{SC}(\mathbf{r})$ is the internal scattering sound field of the infinite cylindrical shell and $P_{SS1}(\mathbf{r})$ and $P_{SS2}(\mathbf{r})$ are the internal scattering sound field of the two spherical shells respectively.

According to the acoustic reciprocity theorem, to calculate the radiation sound pressure of an elastomer at a space point $\mathbf{r}_0$ due to the action of an external force, it is supposed that there is a point sound source q with unit intensity at $\mathbf{r}_0$. If the scattered sound field $P_S(\mathbf{r}, \mathbf{r}_0)$ by the elastomer at a point $\mathbf{r}$ due to the action of the q is known, the radiation sound pressure of the elastomer at the point $\mathbf{r}_0$ excited by the external force can be calculated by the following equation:

$$P(\mathbf{r}_0) = -\frac{1}{i4\pi\omega\rho}\iint_S \frac{\cdot P_S(\mathbf{r}, \mathbf{r}_0)}{\partial \mathbf{n}} \cdot f(\mathbf{r})\,\mathrm{d}S, \tag{3}$$

where $f(\mathbf{r})$ is the distributive external force acting on the elastomer at $\mathbf{r}$, S the elastic surface, $\mathbf{n}$ the normal of the elastic surface which directs toward outside, $\mathrm{i} = \sqrt{-1}$, ω the circular frequency, ρ the medium density, $P(\mathbf{r}_0)$ the radiation sound pressure at $\mathbf{r}_0$, and $P_S(\mathbf{r}, \mathbf{r}_0)$ the scattering sound field of the elastic surface.

Obviously, if the internal scattering sound fields $P_{SC}(\mathbf{r})$, $P_{SS1}(\mathbf{r})$ and $P_{SS2}(\mathbf{r})$ can be obtained, the internal radiated sound field by the plate-ended cylindrical shell structure excited by an external force can be obtained by equation (3).

3. CALCULATING THE INTERNAL SCATTERING SOUND FIELD OF THE INFINITE CYLINDRICAL SHELL

When there is a point sound source with unit intensity at a point $\mathbf{r}_0$ in an infinite cylindrical shell, the spherical wave radiated by the point sound source can be decomposed into the form of cylindrical waves whose axes cross at the point $\mathbf{r}_0$ according to the following formula [12]:

$$\frac{\mathrm{e}^{\mathrm{i}kR_1}}{R_1} = \frac{\mathrm{i}k}{2}\int_\Gamma \mathrm{H}_0^{(1)}(k\lambda\sin\xi)\sin\xi\,\mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi}\,\mathrm{d}\xi, \tag{4}$$

where $\lambda = \sqrt{r^2 + r_0^2 - 2rr_0\cos(\varphi - \varphi_0)}$, $R_1 = \sqrt{\lambda^2 + z^2}$, k is the wave number, (r_0, φ_0, z_0) are the cylindrical co-ordinates for the point $\mathbf{r}_0$, (r, φ, z) are the cylindrical co-ordinates for an internal point $\mathbf{r}$, $\mathrm{H}_0^{(1)}(\cdot)$ is the first kind of Hankel function with zero order, ξ denotes the angle between the axes of the cylindrical shell and cylindrical wave, and Γ is its integral path shown in Figure 1.

Now we consider an infinite cylindrical shell with inside radius b and outside radius a, longitudinal wave speed c_l and transverse wave speed c_t. The density of air medium inside and outside the shell is ρ and wave speed is c.

An internal cylindrical wave, whose axis forms an angle ξ with the axis of the cylindrical shell, can be expressed as follows:

$$\begin{aligned} P_{0C}(r, \varphi, z) &= \mathrm{H}_0^{(1)}(k\lambda\sin\xi)\,\mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi} \\ &= \mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi}\sum_{m=0}^{\infty}\varepsilon_m \mathrm{J}_m(kr_0\sin\xi)\,\mathrm{H}_m^{(1)}(kr\sin\xi)\cos m(\varphi - \varphi_0), \end{aligned} \tag{5}$$

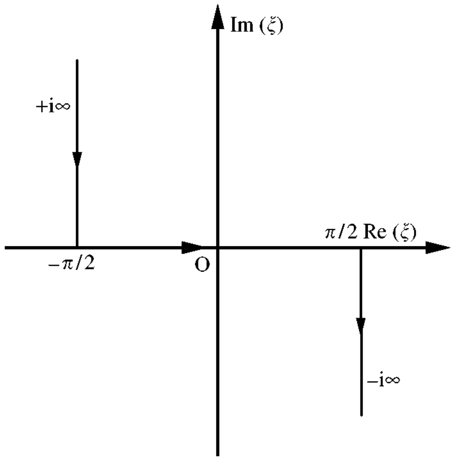

Figure 1. Schematic diagram of integral path Γ.

where $k = \omega/c$, ε_m is the Neumann coefficient, $\mathrm{J}_m(\cdot)$ the Bessel function and $\mathrm{H}_m^{(1)}(\cdot)$ the first kind of Hankel function. In the air medium inside the hollow cylindrical shell, the scattered sound pressure P_{S1} can be expressed

$$P_{S1}(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k|z - z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m B_m \mathrm{J}_m(kr \sin\xi) \cos m(\varphi - \varphi_0), \tag{6}$$

where B_m is the unknown scattering coefficient.

Within the solid shell, the vibrational displacement vector $\mathbf{u}$ is given by

$$\mathbf{u} = \nabla\Psi + \nabla\times\mathbf{A}, \tag{7}$$

where Ψ is a scalar potential representing longitudinal waves and $\mathbf{A}$ is a vector potential representing transverse waves. The displacement equations are satisfied if Ψ and $\mathbf{A}$ verify the differential equations

$$\nabla^2\Psi - \frac{1}{c_l^2}\frac{\partial^2\Psi}{\partial t^2} = 0, \tag{8}$$

$$\nabla^2\mathbf{A} - \frac{1}{c_t^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = 0. \tag{9}$$

Then, Ψ and $\mathbf{A}$ can be expressed as

$$\Psi(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k|z - z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m \left[C_{m1}\mathrm{U}_m(k_l r \sin\theta_l) + C_{m2}\mathrm{V}_m(k_l r \sin\theta_l)\right] \cos m(\varphi - \varphi_0), \tag{10}$$

$$\mathrm{A}_r(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k|z - z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m \left[D_{m1}\mathrm{U}_{m+1}(k_t r \sin\theta_t) + D_{m2}\mathrm{V}_{m+1}(k_t r \sin\theta_t)\right] \sin m(\varphi - \varphi_0), \tag{11}$$

$$\mathrm{A}_{\varphi}(r, \varphi, z) = -\,\mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m \left[D_{m1}\mathrm{U}_{m+1}(k_t r \sin\theta_t) + D_{m2}\mathrm{V}_{m+1}(k_t r \sin\theta_t)\right] \cos m(\varphi - \varphi_0), \tag{12}$$

$$\mathrm{A}_z(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m \left[E_{m1}\mathrm{U}_m(k_t r \sin\theta_t) + E_{m2}\mathrm{V}_m(k_t r \sin\theta_t)\right] \sin m(\varphi - \varphi_0), \tag{13}$$

where $\sin\theta_l = \sqrt{1-(c_l/c)^2\cos^2\xi}$, $\sin\theta_t = \sqrt{1-(c_t/c)^2\cos^2\xi}$, $k_l = \omega/c_l$, $k_t = \omega/c_t$, C_{m1}, C_{m2}, D_{m1}, D_{m2}, E_{m1}, and E_{m2} are unknown coefficients.

Depending on the angle of ξ, $\sin\theta_l$ and $\sin\theta_t$ can be real or imaginary, so functions $\mathrm{U}_m(\cdot)$ and $\mathrm{V}_m(\cdot)$ are the first (J_m) and second kind (N_m) Bessel functions or the modified Bessel functions of the first (I_m) and second kind (K_m) (cf. reference [13]).

In the air medium outside the cylindrical shell, transmission sound pressure P_{1C} can be written as follows:

$$P_{1C}(r, \varphi, z) = \mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi} \sum_{m=0}^{\infty} \varepsilon_m F_m \mathrm{H}_m^{(1)}(kr\sin\xi)\cos m(\varphi - \varphi_0), \tag{14}$$

where F_m is the unknown coefficient.

By applying the boundary conditions of displacements and stresses continuity at the two surfaces (at $r = a$ and b) of the cylinder, the coefficients B_m, C_{m1}, C_{m2}, D_{m1}, D_{m2}, E_{m1}, E_{m2} and F_m can be determined by a set of equations. Then B_m can be obtained by solving the set of equations.

Therefore, when there is a point sound source with unit intensity at an internal point $\mathbf{r}_0$ in the infinite cylindrical shell, the internal scattered sound field by the cylindrical shell can be expressed as

$$P_{SC}(r, \varphi, z) = \frac{\omega\rho k}{8\pi} \sum_{m=0}^{\infty} \int_{\Gamma} \mathrm{e}^{\mathrm{i}k|z-z_0|\cos\xi} B_m \mathrm{J}_m(kr\sin\xi)\cos m(\varphi - \varphi_0)\sin\xi \mathrm{d}\xi. \tag{15}$$

4. CALCULATING THE INTERNAL SCATTERING SOUND FIELD OF CLOSE THIN-WALLED ELASTIC SPHERICAL SHELL

Considering a system with spherical co-ordinates (r, θ, φ), we can make the center of the co-ordinate system coincide with the center of a close thin-walled elastic spherical shell. It is supposed that there is a point sound source q with unit intensity at the interior point $\mathbf{r}_0\,(r_0, \theta_0, \varphi_0)$ of the spherical shell. Due to the action of q, the spherical shell will bring about vibration, generate interior sound field $P_{1S}(\mathbf{r})$, and external sound field $P_{2S}(\mathbf{r})$ respectively. $P_{1S}(\mathbf{r})$ consists of two parts, i.e., free sound field $P_{0S}(\mathbf{r})$ generated by q, and the interior scattered sound field $P_{SS}(\mathbf{r})$.

It is easy to get $P_{0S}(r)$ as follows:

$$\begin{aligned} P_{0S}(r, \theta, \varphi) &= -\frac{\mathrm{i}\omega\rho}{4\pi}\frac{\mathrm{e}^{\mathrm{i}kR_2}}{R_2}\mathrm{e}^{-\mathrm{i}\omega t} \\ &= \sum_{n=0}^{\infty}\sum_{m=-n}^{n} \frac{\mathrm{i}\omega\rho}{4\pi}(2n+1)k\frac{(n-m)!}{(n+m)!}\mathrm{P}_n^m(\cos\theta)\mathrm{P}_n^m(\cos\theta_0)\mathrm{e}^{\mathrm{i}m(\varphi-\varphi_0)}\mathrm{j}_n(kr_0)\mathrm{h}_n^{(1)}(kr)\mathrm{e}^{-\mathrm{i}\omega t}. \end{aligned} \tag{16}$$

The external sound field and the interior scattered sound field, which are related to the vibration of the spherical shell, can be given as follows:

$$P_{2S}(r,\theta,\varphi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}B_n \mathrm{h}_n^{(1)}(kr)\,\mathrm{P}_n^m(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{17}$$

$$P_{SS}(r,\theta,\varphi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}C_n\, \mathrm{j}_n(kr)\,\mathrm{P}_n^m(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{18}$$

where $R_2=\sqrt{r^2+r_0^2-2rr_0\cos\beta}$, β is the angle between vector $\mathbf{r}$ and $\mathbf{r}_0$, B_n and C_n are unknown coefficients, $\mathrm{h}_n^{(1)}(\cdot)$ is the first kind of the spherical Hankel function, $\mathrm{j}_n(\cdot)$ is the spherical Bessel function, and $\mathrm{P}_n^m(\cdot)$ is the first kind of the associated Legendre function.

According to reference [14], the radial displacement w of the spherical shell should meet the following equation:

$$\varepsilon\nabla^6 w+r_1\nabla^4 w+r_2\nabla^2 w+r_3 w+W=0, \tag{19}$$

where $\varepsilon=h^2/12R^2$,

$$r_1=\varepsilon[3-\mu-2(1+\mu)k_s]+\varepsilon[k_t+k_r+k_tK_S](kR)^2,$$

$$\begin{aligned}r_2&=1-\mu^2-k_t(kR)^2+2\varepsilon[1-\mu-(3+2\mu-\mu^2)k_s]\\&\quad+\varepsilon[(1-\mu)k_t+2k_r-2(1+\mu)k_rk_s-4\mu k_tK_S](kR)^2\\&\quad+\varepsilon k_t[k_r+(k_t+k_r)K_S](kR)^2\,\omega^2,\end{aligned}$$

$$\begin{aligned}r_3&=[2(1-\mu^2)+(1+3\mu)k_t(kR)^2-k_t^2(kR)^2\omega^2]-4\varepsilon(1-\mu^2)k_s\\&\quad-2\varepsilon k_s[(1+3\mu)k_t+2(1+\mu)k_r](kR)^2\\&\quad+\varepsilon k_t[2k_tk_s-(1+3\mu)k_rK_S](kR)^2\omega^2+\varepsilon k_t^2k_rK_S(kR)^2\omega^4,\end{aligned}$$

$$W=-[1-\varepsilon K_S(\nabla^2+1-\mu+k_r(kR)^2)]H,$$

$$H=\frac{(1-\mu^2)R^2}{Eh}(\nabla^2+1-\mu+k_t(kR)^2)(P_{1S}-P_{2S}),$$

$$\nabla^2\equiv\frac{\partial^2}{\partial\theta^2}+\operatorname{ctg}\theta\,\frac{\partial}{\partial\theta}+\frac{1}{\sin^2\theta}\,\frac{\partial^2}{\partial\varphi^2},$$

where h is the thickness of the spherical shell, R the nominal radius of the closed spherical shell, E the Young's modulus, μ the Poisson ratio of the shell material, and k_s is an averaging coefficient of the shear.

Here suppose

$$w=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}A_n P_n^m(\cos\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{20}$$

Then the following boundary condition should be met on the interfaces of the shell and acoustic medium:

$$\frac{1}{\mathrm{i}\omega\rho}\frac{\partial P_{1S}}{\partial r}\bigg|_{r=R-h/2}=\frac{1}{\mathrm{i}\omega\rho}\frac{\partial P_{2S}}{\partial r}\bigg|_{r=R+h/2}=\frac{\partial w}{\partial t}. \tag{21a}$$

Because the shell wall considered here is thin ($h \ll \lambda_0$, in which λ_0 is the wavelength of sound wave in the material of the shell), the medium vibration velocity on the interior and external surface of the shell ($r = R \pm h/2$) can be replaced by the vibration velocity on the middle surface of the shell $r = R$. In this case, the boundary conditions may be written as

$$\frac{1}{\mathrm{i}\omega\rho}\frac{\partial P_{1s}}{\partial r}\bigg|_{r=R}=\frac{1}{\mathrm{i}\omega\rho}\frac{\partial P_{2S}}{\partial r}\bigg|_{r=R}=\frac{\partial w}{\partial t}. \tag{21b}$$

Then the following results can be obtained by equations (19)–(21b):

$$C_n=\frac{b_{n1}}{a_{n2}}\frac{\mathrm{i}\omega}{4\pi c}(2n+1)\frac{(n-m)!}{(n+m)!}P_n^m(\cos\theta_0)\,\mathrm{e}^{-\mathrm{i}m\varphi_0}\,\mathrm{j}_n(kr_0), \tag{22}$$

where

$$a_{n2}=-a_{n1}\frac{j_n'(kR)}{\rho c\omega}+b_n\,j_n(kR),$$

$$b_{n1}=\frac{a_{n1}}{c}h_n^{(1)'}(kR)-b_n\omega\rho h_n^{(1)}(kR),$$

$$a_{n1}=a_n+b_n\frac{h_n^{(1)}(kR)}{h_n^{(1)'}(kR)}\omega\rho c,$$

$$a_n=-\varepsilon n^3(n+1)^3+r_1n^2(n+1)^2-r_2n(n+1)+r_3,$$

$$b_n=\frac{(1-\mu^2)R^2}{Eh}[-n(n+1)+(1-\mu)+k_t(kR)^2]$$

$$\times[1-\varepsilon K_S(-n(n+1)+(1-\mu)+k_r(kR)^2)].$$

Therefore, if there is a point sound source with unit intensity in the closed thin spherical shell, its interior scattered sound field can be obtained by equation (18).

Obviously, $P_{SS1}(\mathbf{r})$ and $P_{SS2}(\mathbf{r})$ in equation (2) can be obtained by equation (18), which only need different parameters.

5. EXPERIMENTS

To verify the method presented in this paper, we measured the internal radiated sound pressure by the combined cylindrical shell structure as shown in Figure 2. The combined structure consists of a finite cylindrical shell and two end plates, which are jointed by bolts. In the experiment, the structure is suspended by soft ropes at its ends. There is a pore at the center of each end plate, through which a horizontal regulating stem for measuring can be moved. The origin of the cylindrical co-ordinate system (r, φ, z) is located at the center of the

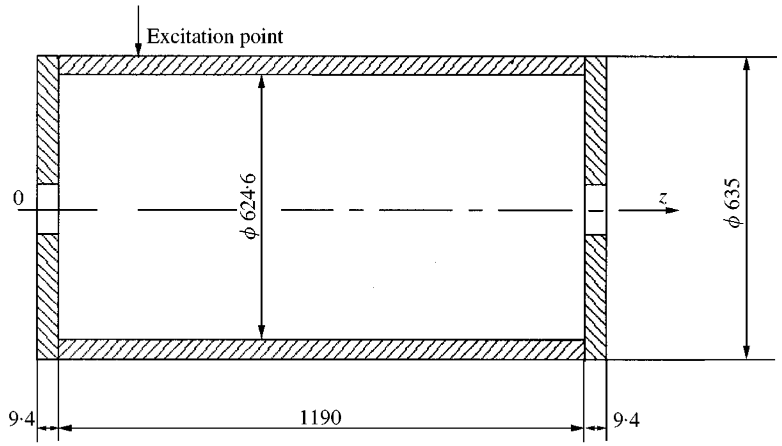

Figure 2. Experimental model of the composite cylindrical shell structure (unit: mm).

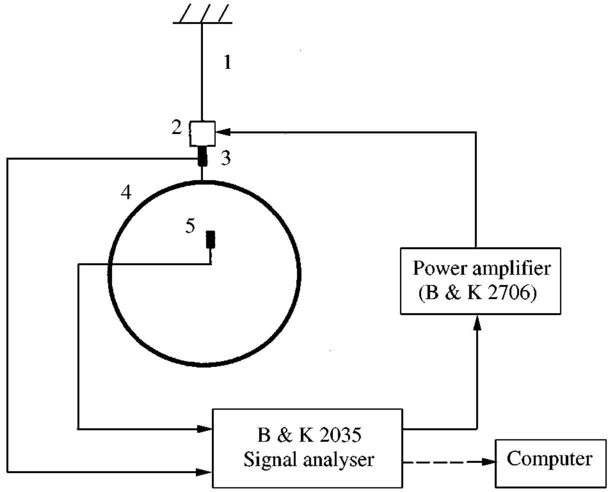

Figure 3. Sketch of essential testing system for the combined structure with cylindrical shell: 1. Rubbing sling; 2. Generator (B&K4809); 3. Force sensor (B&K8001); 4. Combined cylindrical shell structure (cross section); 5. Microphone (B&K4155).

left end plate, and the z-axis is equal to the axis of the finite cylindrical shell in the direction of the right.

In order to reduce the environmental influence, all the experiments are done in an anechoic room. As shown in Figure 3, the essential testing system consists of two parts, i.e., excitation part and corresponding testing part. The signal analyzer B&K2035 is used in all the experiments.

To measure the inside sound pressure of the combined structure, we design an adjustable measuring device, which consists of longitudinal regulating stem, radial measuring rod and microphone. The measuring rod, whose radial size is adjustable, can be moved along the longitudinal regulating stem. The microphone is installed on the radial measuring rod. The position of the microphone can be read by a scale and regulated by the corresponding movement and rotation.

The generator is suspended by rubber slings and the excitation point is located at the point (110, $3\pi/2$, 120) (by the millimeter, the same as the following). When the harmonic exciting forces with corresponding frequencies of 800, 900, 1000 and 1100 Hz are used, respectively, we can measure the sound pressure at different internal points which are taken as $1, 2, \ldots, 10$ whose co-ordinates are shown in Table 1. The amplitudes of the forces are all 3 N. The corresponding theoretical calculations are conducted by using the covering-domain method above. The comparisons between measured results and computed results are shown in Figure 4 in which the horizontal co-ordinate denotes the order of measured points and the vertical co-ordinate denotes the sound pressure amplitude. It can be seen that there is a good agreement between them at 800 and 1000 Hz. But there is a little discrepancy at several points at 900 and 1100 Hz, which may be caused by difference

TABLE 1

The cylindrical co-ordinates of the measured points

Order of measured points	1	2	3	4	5
Co-ordinates of measured points	(110, $3\pi/2$, 700)	(110, $\pi/2$, 700)	(110, $3\pi/2$, 600)	(110, $\pi/2$, 600)	(110, $3\pi/2$, 500)
Order of measured points	6	7	8	9	10
Co-ordinates of measured points	(110, $\pi/2$, 500)	(110, $3\pi/2$, 400)	(110, $\pi/2$, 400)	(110, $3\pi/2$, 300)	(110, $\pi/2$, 300)

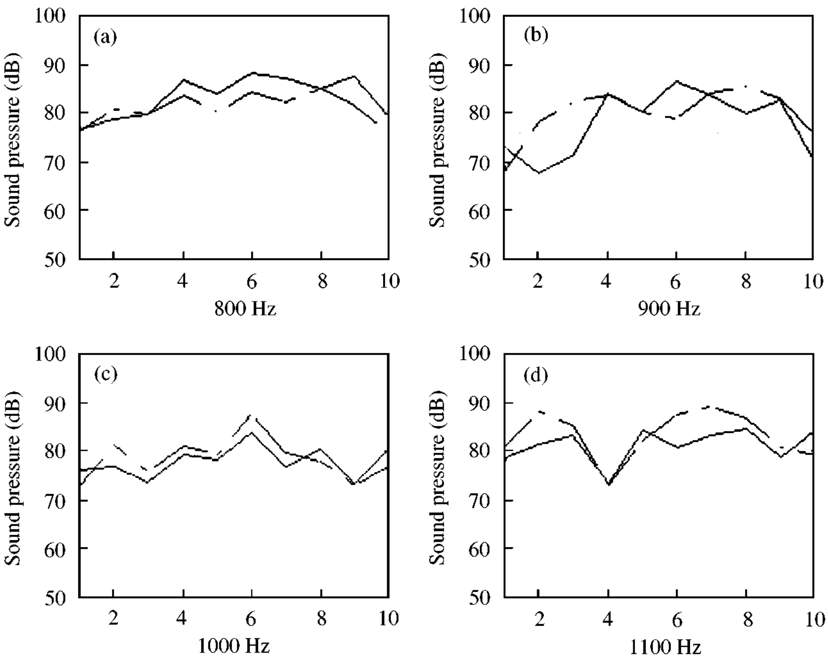

Figure 4. Comparison between measuring results and calculating results: ——, calculating results; – – –, measuring results.

between theoretic model and experimental model, and by random measuring error in practice. As a whole, there is a credible agreement between them. So the method presented in this paper is verified to be valid.

6. CONCLUSIONS

This paper introduces the covering-domain method to predict the internal sound radiation from a plate-ended circular cylindrical shell due to the action of an external force. The formulas for calculating the internal scattering sound fields of an infinite cylindrical shell and a closed thin-walled spherical shell is given in this paper. In addition, although we calculate only the internal sound field of the composite elastic structure, by using the method presented above, the external radiation sound field of the composite elastic structure or other complicated structures excited by arbitrary force can also be calculated. This will largely expand practical applications of the acoustic reciprocity theorem.

ACKNOW LEDGMENTS

The authors wish to express their gratitude to the National Nature Science Foundation of China (No. 59875069) and the Doctorate Foundation of Xi'an Jiaotong University of China.

REFERENCES

1. L. CHENG and J. NICOLAS 1992 *Journal of Sound and Vibration* **155**, 231–247. Free vibration analysis of a cylindrical shell-circular plate system with general coupling and various boundary conditions.
2. G. YAMADA, T. IRIE and T. TAMIYA 1986 *Journal of Sound and Vibration* **108**, 297–304. Free free vibration of a circular cylindrical double-shell system closed by end plates.
3. D. A. BAFILIOS and C. S. LYRINTZIS 1991 *AIAA Journal* **29**, 1193–1201. Structure-borne noise transmission into cylindrical enclosures of finite extent.
4. M. S. TAVAKOLI and R. SINGH 1990 *Journal of Sound and Vibration* **136**, 141–145. Modal analysis of a Hermetic can.
5. S. AZIMI 1988 *Journal of Sound and Vibration* **120**, 19–35. Free vibration of circular plates with elastic edge supports using the receptance method.
6. T. IRIE, G. YAMADA and Y. MURAMOTO 1984 *Journal of Sound and Vibration* **95**, 31–39. Free vibration of joined conical–cylindrical shells.
7. T. IRIE, G. YAMADA and Y. MURAMOTO 1984 *Journal of Sound and Vibration* **92**, 107–115. The axisymmetrical response of a circular cylindrical double-shell system with internal damping.
8. L. CHENG 1994 *Journal of Sound and Vibration* **174**, 641–654. Fluid-structural coupling of a plate-ended cylindrical shell: vibration and internal sound field.
9. L. I. SLEPYAN and S. V. SOROKIN 1995 *Journal of Sound and Vibration* **184**, 195–211. Analysis of structural-acoustic coupling problems by a two level boundary integral method, Part I: a general formulation and test problems.
10. L. I. SLEPYAN and S. V. SOROKIN 1995 *Journal of Sound and Vibration* **184**, 213–228. Analysis of structural-acoustic coupling problems by a two level boundary integral method, Part II: vibrations of a cylindrical shell of finite length is an acoustic medium.
11. WANG LIANG-GUO, LIN XIAO 1987 Acta Mechanica Sinica **19**, 323–331. A general expression and its discussing for solutions of a set of Fredholm integration equation in elastic mechanics.
12. L. M. BREKHOVSKIKH 1980 *Waves in Layered Media*. New York: Academic Press, second edition.
13. F. LÉON, F. LECROQ, D. DÉCULTOT and G. MAZE 1992 *Journal of the Acoustical Society of America* **91**, 1388–1397. Scattering of an obliquely incident acoustic wave by an infinite hollow cylindrical shell.
14. C. PRASAD 1964 *Journal of the Acoustical Society of America* **36**, 489–494. On vibration of spherical shells.

APPENDIX: NOMENCLATURE

ω	circular frequency
ρ	density of the acoustic medium
b, a	inside radius and outside radius of the finite cylindrical shell
c_l	longitudinal wave speed
c_t	transverse wave speed
c	speed of sound in the acoustic medium
k	wave number corresponding to c
P_{SC}	internal scattering sound field of the infinite cylindrical shell
P_{SS1}, P_{SS2}	internal scattering sound field of the two spherical shells
P_S	internal scattering sound field of the combined cylindrical shell structure
$f(\mathbf{r})$	distributive external force acting on an elastomer at $\mathbf{r}$
S	boundary of the acoustic medium
$\mathbf{n}$	outward normal to the fluid domain at the surface of an elastomer
$P(\mathbf{r}_0)$	radiation sound pressure at $\mathbf{r}_0$
i	$=\sqrt{-1}$
P_{0C}	an internal cylindrical wave
P_{S1}	scattered sound pressure of P_{0C} inside the cylindrical shell
ε_m	Neumann coefficient
$\mathbf{u}$	vibrational displacement vector
Ψ	scalar potential representing longitudinal waves
$\mathbf{A}$	vector potential representing transverse waves
A_r, A_φ, A_z	component of $\mathbf{A}$ in cylindrical co-ordinates
k_l, k_t	wave number corresponding to c_l, c_t respectively
P_{1C}	transmission sound pressure of P_{0C} outside the cylindrical shell
q	point sound source
P_{0S}	free sound field generated by q
P_{SS}	internal scattering sound field of the spherical shell generated by q
P_{1S}, P_{2S}	interior sound field and external sound field of the spherical shell generated by q, respectively
h	thickness of the spherical shell
R	nominal radius of the closed spherical shell
E	Young's modulus
μ	the Poisson ratio of the shell material
k_s	an averaging coefficient of the shear
w	radial displacement of the spherical shell