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COMMENTS ON "FUNDAMENTAL FREQUENCY OF A WAVY NON-HOMOGENEOUS CIRCULAR MEMBRANE"

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The author is to be congratulated for this very elegant and useful treatment of an important membrane vibrations problem [1]. Professor Wang has determined the fundamental frequency of a circular membrane where the density is assumed to be a sinusoidal function of radius. It is also the purpose of this discussion to point out the existence of another membrane type problem similar to the one treated by the author.

Consider a vibrating simply connected membrane of complicated boundary shape. Using classical vibrations theory, in the case of normal modes the problem is governed by Helmholtz equation

$$\nabla^2 \Phi + \lambda^2 \Phi = 0, \tag{1}$$

where λ is the eigenvalue of the problem.

When the membrane is fixed at the boundary one has

$$\Phi[L(x, y) = 0] = 0, \tag{2}$$

where $L(x, y) = 0$ is the functional relation that defines the boundary of the domain. Let

$$z = x + \mathrm{i}y = f(\xi), \quad \xi = r\mathrm{e}^{\mathrm{i}\theta} \tag{3}$$

be the analytic function which transforms the given domain in the z-plane onto a unit circle in the ξ-plane. Substituting equation (3) into equation (1) one obtains

$$\nabla^2 \Phi + \lambda^2 |f'(\xi)|^2 \Phi = 0, \tag{4}$$

while the boundary condition becomes

$$\Phi(1, \theta) = 0. \tag{5}$$

Consequently, the original problem of a vibrating membrane of complicated boundary shape and constant density has been transformed into a vibrations problem of a circular membrane of unit radius and non-uniform density where the variation is defined by $|f'(\xi)|^2$.

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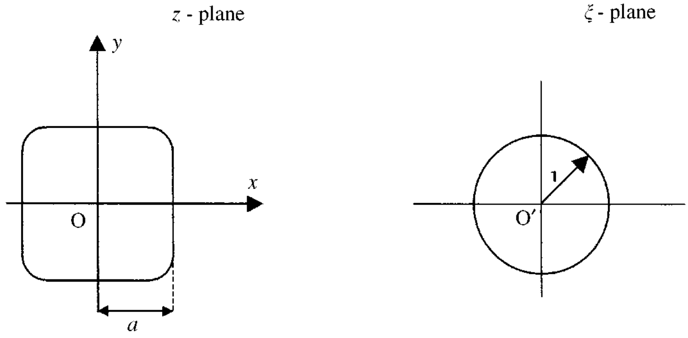

Figure 1. Square ($2a \times 2a$) domain with rounded corners and its image in the ξ-plane.

Consider a square shape ($2a \times 2a$) with rounded corners; the mapping function is [2], see Figure 1:

$$z = aN(\xi + \eta\xi^5), \tag{6}$$

where $N = \frac{25}{24}$, $\eta = -\frac{1}{25}$. For this case one immediately obtains

$$|f'(\xi)|^2 = 1 + 10\eta r^4 \cos 4\theta + 25\eta^2 r^8. \tag{7}$$

It is observed that the "density" of the unit membrane is a function of r and also varies in a sinusoidal fashion with respect to the azimuthal variable.

The transformed differential system (4) and (5) has been solved in previous publications using variational techniques [3]. On the other hand, very close upper and lower bound, have been obtained for certain domains of complicated boundary shape [4–6] using the Kohn–Kato approach.

For instance, in the case of the configuration shown in Figure 1 the optimized Kohn–Kato bounds for the fundamental eigenvalue are [6, 7]

$$2{\cdot}3073 \leqslant (\lambda a)_1 \leqslant 2{\cdot}3075,$$

whereas the result available in the literature is [2]

$$(\lambda a)_1 = 2{\cdot}308.$$

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