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OSCILLATIONS OF GAS FLOW IN A TUBE WITH PERMEABLE WALLS

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Characteristics of linear waves propagating in a gas flow in a long tube with permeable walls are investigated. Linear approximation of governing equations is used, and it is considered that time-averaged flow properties depend on the space co-ordinate along the tube. They are obtained from the steady flow solution. Two examples showing the comparison of theoretical and experimental results are presented.

1. INTRODUCTION

In power and cement plants, bag filters with porous walls are often used to separate solid particles from the gas. The particles which get stuck in the wall are removed by back flow (from the inside to the outside of the bag) which is switched periodically. Particle shedding becomes more effective when the steady back flow which persists for about 0·5 s during one switching is superimposed with forced oscillations of moderate amplitude. This technical problem motivated the present study of linear waves transmission in a tube with a permeable wall.

To some extent an analogous problem appears in the case of vehicle mufflers having perforated sections. The analysis of perforated elements of the mufflers was begun by Sullivan and Crocker [1]. They suggested an analytical approach based on one-dimensional linear equations of tube flow to predict the transmission loss of a concentric-tube resonator. According to the linear theory, the perturbation velocity through the perforated section was assumed proportional to the perturbation pressure difference between the inner and outer tubes (cavity). The factor of proportionality was defined by the acoustic impedance of the perforated section.

Various formulas for the acoustic impedance based on experimental data have been proposed for both the grazing and the cross pipe flows [2]. The formulas show the acoustic impedance to be dependent on the frequency of the transmitted wave, and on the pipe flow Mach number as well as the geometrical properties of the perforation. The imaginary part of the impedance can be attributed to inertia of the gas flow in the holes in the pipe wall. This effect, however, can be neglected for holes of small length for which the resonance frequency is of some orders of magnitude larger than the frequency of the wave transmitted along the pipe. In this case, only the resistance of the permeable wall to the cross flow seems to be of practical significance.

In previous papers, the perforated section was considered as a discrete element of the acoustic waveguide [1–6]. In the case of multi-chamber mufflers having a few perforated sections a segmentation procedure was developed [3] which allows one to consider the muffler as a lumped system. A kind of segmentation method was also used by Cummings and Kirby [7] to analyze the acoustic field in a long permeable tube; they divided the tube into several cells of assumed constant impedance. Their model accounts for the coupling of internal and external sound fields via acoustic motion through the wall but ignores the effect of mean tube flow.

Contrary to the papers mentioned above, in the present paper, the wave strength variation is regarded as a continuous function of the space co-ordinate along the tube with a permeable wall. A rigid-walled tube of constant cross-section open at one end and closed at the other end is considered. The non-uniform distribution of the mean flow Mach number along the tube is taken into account.

2. GOVERNING EQUATIONS

The main assumptions used in the formulation of governing equations are as follows: (1) variations of flow properties over any cross-section of the tube are small in comparison with longitudinal variations; (2) amplitudes of acoustic pressure and acoustic flow velocity normalized with mean time values are small; (3) only losses induced due to the cross flow are considered.

2.1. CONTINUITY EQUATION

Under the above assumptions the conservation of mass for the tube flow is given by

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial x}+\rho\frac{\partial u}{\partial x}+\frac{\gamma}{F}=0, \tag{1}$$

where γ denotes the flow rate of the gas evacuated from the tube per unit length (a list of notation is given in Appendix B). For a tube of circular cross-section the following relationship for γ/F can be used:

$$\frac{\gamma}{F}=\frac{2}{R}\alpha\eta\rho_a w. \tag{2}$$

Here w is the cross flow velocity and ρ_a the density of the gas expanded isentropically to the ambient pressure p_a, and α is the permeability (porosity or perforation) coefficient of the wall, it represents the total cross-sections of pores or orifices in a wall surface unit: η denotes the discharge coefficient of the flow in pores or orifices; it can be obtained from experiment for an assumed model of the cross flow. For the perforated wall the discharge coefficient can be easily determined since both the diameters and the number of orifices in the wall surface unit are known. For the porous wall the product $\alpha\eta$ can only be measured. This product is defined as the effective permeability coefficient. In the present paper an isentropic model of the cross flow is assumed. On this basis, the reference flow rate $\rho_a w$ of the flow through the wall surface unit is calculated as a function of local tube flow properties.

The cross-flow velocity can be obtained from the energy equation

$$\frac{w^2}{2}+\frac{a_a^2}{\kappa-1}=\frac{a^2}{\kappa-1}=\frac{a_0^2}{\kappa-1}-\frac{u^2}{2}. \tag{3}$$

where a, a_0, and a_a are the speed of sound in the gas inside the tube, in the stagnated gas at the pressure p_0, and in the gas expanded to the ambient pressure p_a respectively.

Equations (3) yield

$$\left(\frac{w}{a_0}\right)^2 = \frac{2}{\kappa - 1}\left[1 - \left(\frac{a_a}{a_0}\right)^2\right] - \left(\frac{u}{a_0}\right)^2. \tag{4}$$

Introducing into the above equation the isentropic relationships

$$\left(\frac{a_a}{a_0}\right)^2 = \left(\frac{p_a}{p_0}\right)^{(\kappa-1)/\kappa} \quad \text{and} \quad a_0^2 = \kappa \frac{p_0}{\rho_0}$$

one obtains

$$w = \left\{\frac{2\kappa}{\kappa - 1} p_a^{(\kappa-1)/\kappa}\left(\frac{p_0^{1/k}}{\rho_0}\right)\left[\left(\frac{p}{p_a}\right)^{(\kappa-1)/\kappa} - 1\right]\right\}^{1/2}. \tag{5}$$

During the pipe flow both the pressure and the stagnation density oscillate. However, for isentropic flow the following ratios are constant:

$$\frac{p_0^{1/\kappa}}{\rho_0} = \frac{p^{1/\kappa}}{\rho} = \frac{p_a^{1/\kappa}}{\rho_a} = const. \tag{6}$$

This yields

$$\rho_a = \rho_0 \left(\frac{p_a}{p_0}\right)^{1/\kappa}. \tag{7}$$

Combination of equations (5) and (7) gives

$$\rho_a w = \left\{\frac{2\kappa}{\kappa - 1} p_a^{(\kappa+1)/\kappa}\left(\frac{\rho_0}{p_0^{1/\kappa}}\right)\left[\left(\frac{p}{p_a}\right)^{(\kappa-1)/\kappa} - 1\right]\right\}^{1/2}. \tag{8}$$

2.2. MOMENTUM EQUATION

In the case considered the Euler equation represents conservation of momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0. \tag{9}$$

3. RESULTS

3.1. ANALYSIS

Linear approximation of equation (1) in conjunction with equations (2) and (8) yields

$$\frac{\partial \rho'}{\partial t} + \bar{u}\frac{\partial \rho'}{\partial x} + \bar{\rho}\frac{\partial u'}{\partial x} = -\frac{1}{\bar{U}}\frac{1}{R}p', \tag{10}$$

where the primes denote fluctuations and the bar the time-averaged value,

$$\frac{1}{\bar{U}} = \frac{1}{\bar{a}_0}\alpha\eta\left[2(\kappa - 1)\left(\frac{\bar{p}_0}{p_a}\right)^{(\kappa-1)/\kappa}\right]^{1/2} \frac{(\bar{p}/p_a)^{-1/\kappa}}{[(\bar{p}/p_0)^{(\kappa-1)/\kappa} - 1]^{1/2}}. \tag{11}$$

On the right-hand side of equation (10) one has only the linear term of the power expansion of γ/F; the higher order terms of the expansion are neglected under the assumption that

$$\frac{p'}{p_a} \ll \left(\frac{\bar{p}}{p_a}\right)^{(\kappa-1)/\kappa} - 1.$$

It must be noted that in the flow under consideration $\bar{p} > p_a$. This is because the air supplied to the tube is wholly evacuated through its permeable wall.

The parameters u' and ρ' in equation (10) can be eliminated by making use of the linearized momentum equation

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + \frac{1}{\bar{\rho}}\frac{\partial p'}{\partial t} = 0, \tag{12}$$

and the definition of the speed of sound $a^2 = \mathrm{d}p/\mathrm{d}\rho$.

The result is

$$\frac{1}{\bar{a}^2}\left[\frac{\partial^2 p'}{\partial t^2} + 2\bar{u}\frac{\partial^2 p'}{\partial x\,\partial t} + (\bar{u}^2 - \bar{a}^2)\frac{\partial^2 p'}{\partial x^2}\right] + \left[\frac{1}{\bar{U}R} - \frac{1}{\bar{a}^2\bar{\rho}}\left(\bar{u}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x} - \bar{\rho}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\right)\right]\left(\frac{\partial p'}{\partial t} + \bar{u}\frac{\partial p'}{\partial x}\right)$$
$$+ \left[\frac{\bar{u}}{R}\frac{\mathrm{d}(1/\bar{U})}{\mathrm{d}x} - \frac{1}{\bar{\rho}}\left(\bar{u}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x} - \bar{\rho}\frac{\mathrm{d}\bar{u}}{\mathrm{d}x}\right)\frac{1}{\bar{U}R}\right]p' = 0. \tag{13}$$

In the derivation of equation (13) account was taken of the fact that the parameters marked with the bar are dependent on the space co-ordinate. Replacing the co-ordinates x and t and the pressure p' by dimensionless values x/R, $t\bar{a}_0/R$, $p'/\bar{p}_0$, respectively, one obtains

$$N^2\frac{\partial^2 p'}{\partial t^2} + 2MN\frac{\partial^2 p'}{\partial x\,\partial t} + (M^2 - 1)\frac{\partial^2 p'}{\partial x^2} + \left(\frac{\bar{a}_0}{\bar{U}} + \beta\right)\left(\frac{\partial p'}{\partial t} + \frac{M}{N}\frac{\partial p'}{\partial x}\right) + \vartheta p' = 0, \tag{14}$$

where

$$M = \frac{\bar{u}}{\bar{a}}, \qquad N = \left(1 + \frac{\kappa - 1}{2}M^2\right)^{1/2}, \quad \beta = (M^2 + 1)N^2\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{M}{N}\right), \tag{14a}$$

$$\vartheta = \frac{M}{N}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\bar{a}_0}{\bar{U}}\right) + (M^2 + 1)\frac{\bar{a}_0}{\bar{U}}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{M}{N}\right) \tag{14b}$$

(see Appendix A).

The solution of equation (14) can be expressed as the harmonic function

$$p' = \mathrm{Re}[(A + \mathrm{i}B)\exp(\mathrm{i}\omega t)] \tag{15}$$

or

$$p' = A\cos\omega t - B\sin\omega t, \tag{16}$$

where A and B depend on the x co-ordinate. Introducing equation (16) into equation (14) one has

$$\left[(M^2 - 1)\frac{\mathrm{d}^2 A}{\mathrm{d}x^2} + \frac{M}{N}\left(\frac{\bar{a}_0}{\bar{U}} + \beta\right)\frac{\mathrm{d}A}{\mathrm{d}x} + (\vartheta - N\omega^2)A - 2MN\omega\frac{\mathrm{d}B}{\mathrm{d}x} - \omega\left(\frac{\bar{a}_0}{\bar{U}} + \beta\right)B\right]\cos\omega t$$
$$+ \left[(M^2 - 1)\frac{\mathrm{d}^2 B}{\mathrm{d}x^2} + \frac{M}{N}\left(\frac{\bar{a}_0}{\bar{U}} + \beta\right)\frac{\mathrm{d}B}{\mathrm{d}x} + (\vartheta - N\omega^2)B + 2MN\omega\frac{\mathrm{d}A}{\mathrm{d}x} + \omega\left(\frac{\bar{a}_0}{\bar{U}} + \beta\right)A\right]\sin\omega t = 0 \tag{17}$$

where ω is the Helmholtz number, $\omega = 2\pi f R/a_0$. Equation (17) is valid for every instant of time. This requires that the coefficients in front of $\cos \omega t$ and $\sin \omega t$ are equal to zero. In this way, two ordinary second order differential equations for amplitudes of transmitted and reflected waves A and B are obtained:

$$\frac{d^2A}{dx^2} = \frac{1}{1-M^2}\left[\left(\frac{\bar{a}_0}{\bar{U}}+\beta\right)\left(\frac{M}{N}\frac{dA}{dx}-\omega B\right) - 2MN\omega\frac{dB}{dx} + (\vartheta - N\omega^2)A\right], \quad (18)$$

$$\frac{d^2B}{dx^2} = \frac{1}{1-M^2}\left[\left(\frac{\bar{a}_0}{\bar{U}}+\beta\right)\left(\frac{M}{N}\frac{dB}{dx}+\omega A\right) + 2MN\omega\frac{dA}{dx} + (\vartheta - N\omega^2)B\right]. \quad (19)$$

The boundary conditions for equations (18) and (19) are formulated at the closed end of the tube ($x = l/R$). There, reflective boundary conditions can be assumed, i.e.,

$$A_{l/R} = B_{l/R} \quad \text{and} \quad \left(\frac{dA}{dx}\right)_{l/R} = \left(\frac{dB}{dx}\right)_{l/R} = 0.$$

Equations (18) and (19) were solved numerically together with equation (A9) for time mean flow (see Appendix A) by using the Runge–Kutta–Gill scheme. Calculations were conducted upstream starting at $x = l/R$ where $M = 0$. They were finished when $x = 0$ was reached. The amplitudes $A = B = 1$ were assumed at the starting point.

3.2. EXPERIMENT

Experiments were conducted to verify the theoretical results presented in the previous section. A rigid tube with a narrow slit of constant width along its length (see Figure 1) was

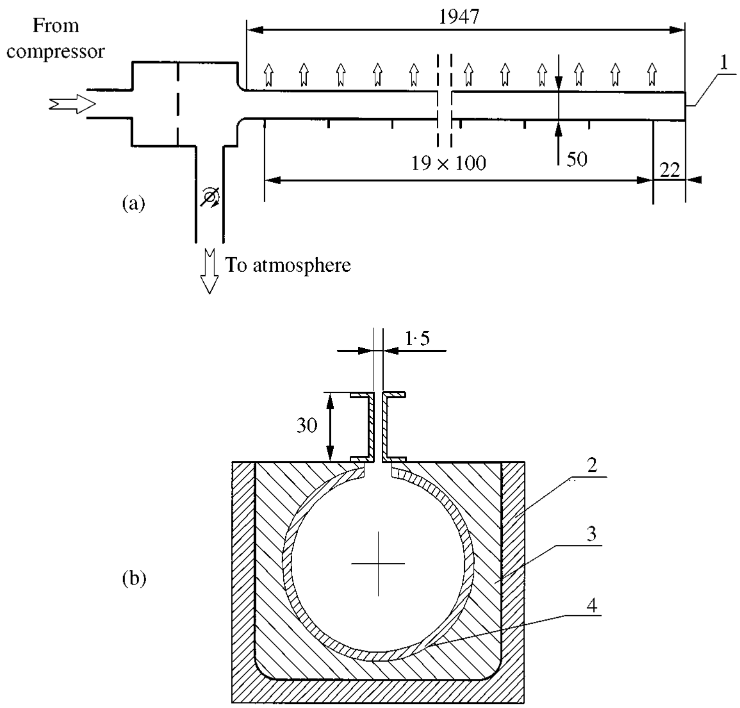

Figure 1. Experimental set-up (a) and cross-section of the tube (b), 1, pressure gage; 2, steel frame; 3, polyester resin; 4, PVC tube. Dimensions in mm.

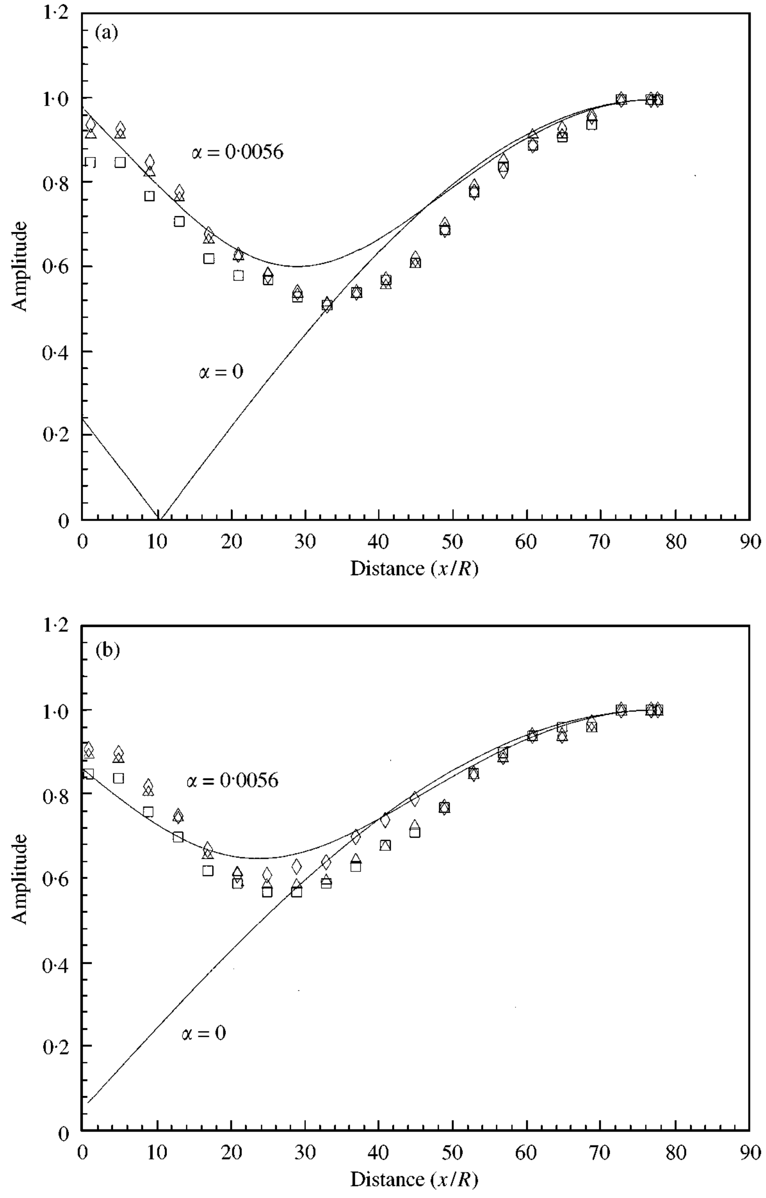

Figure 2. Distributions of normalized pressure amplitude along the tube: lines, theory; points, experiment for silted wall $\omega = 0{\cdot}0233$ (a) and 0·0279 (b).

used. Due to this the permeability of the tube was kept constant. Oscillations were generated by means of a rotating valve. It was located on the side pipe through which compressed air delivered to the system was partly evacuated to the surroundings.

Pressure signals were measured by means of the Kistler 601A pressure transducers. Twenty pressure gages were uniformly distributed along the tube. An additional gage was placed in the plug closing the tube. The pressure signals were filtered to separate the fundamental harmonic corresponding to the rotation number of the valve. In Figures 2(a) and 2(b) the distributions of oscillation amplitude for two frequencies 60·5 and 50·5 Hz are

TABLE 1

Plug pressure amplitudes

$f(s^{-1})$	ω	Pressure amplitudes, peak to peak (bar)		
60·5	0·0279	0·0166	0·0166	0·0167
50·5	0·0233	0·0254	0·0249	0·0245

displayed respectively. The permeability coefficient corresponding to 1·5 mm slit was 0·00955. The discharge coefficient η was measured in the separate experiment for steady flow through the slit $1{\cdot}5 \times 100$ mm. It was 0·586. For these α and η the theoretical lines $\sqrt{0{\cdot}5(A^2 + B^2)}/A_{x=1}$ were obtained by solving equations (18), (19) and (A9). The lines for a tight tube ($\alpha = 0$) are also presented. The experimental data marked by squares, triangles and diamonds were obtained for three separate experiments. In each case, the measured amplitudes along the tube were normalized with corresponding amplitudes at the plug. The plug pressure amplitudes for each experiment are given in Table 1.

The theoretical and experimental data presented above were obtained for the stagnation to an ambient pressure ratio of 1·1 which corresponds to maximal flow Mach number $M_{max} = 0{\cdot}37$.

4. CONCLUDING REMARKS

For the frequencies considered in Figure 2 the oscillation amplitudes in the case of impermeable walls changes drastically along the tube. These changes smoothen considerably when a wall of small permeability is used. This is, first of all, due to the time average tube flow exists in the case of permeable walls.

The differences of the amplitudes for impermeable and permeable walls are large in the inlet section of the tube, where the time average flow velocity, in the case of permeable walls, is relatively high. Contrary to this, the amplitudes for the impermeable and permeable walls are very close to each other in the remaining section of the tube containing gas at nearly stagnation conditions.

To make use of the present analysis the effective permeability coefficient of tube wall for steady flow should be measured. This depends on the fabric wall used in bag filters.

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APPENDIX A: STEADY FLOW IN A TUBE WITH A POROUS WALL

For steady one-dimensional flow the continuity and momentum equations are

$$\frac{\mathrm{d}\rho}{\rho}+\frac{\mathrm{d}u}{u}=-\frac{\gamma}{F}\frac{1}{\rho u}\mathrm{d}x=-\,2\alpha\eta\frac{\rho_a}{\rho}\frac{w}{u}\mathrm{d}x, \tag{A1}$$

$$\frac{\mathrm{d}p}{p}+\kappa M^2\frac{\mathrm{d}u}{u}=0, \tag{A2}$$

where x is the dimensionless co-ordinate (see section 3.1).

From the definition of flow Mach number one obtains

$$\frac{\mathrm{d}M}{M}=2\frac{\mathrm{d}u}{u}-\frac{\mathrm{d}p}{p}+\frac{\mathrm{d}\rho}{\rho}. \tag{A3}$$

For isentropic flow one has

$$\frac{\mathrm{d}p}{p}=\kappa\frac{\mathrm{d}\rho}{\rho}. \tag{A4}$$

Equations (A1–A4) yield

$$\left(1-\frac{\kappa+1}{2}\frac{M^2}{1+(\kappa-1)/2M^2}\right)\frac{\mathrm{d}M}{M}=-\,2\alpha\eta\frac{\rho_a}{\rho}\frac{w}{u}\mathrm{d}x. \tag{A5}$$

The right-hand side of equation (A5) can be expressed as a function of M by considering the energy equation for the cross flow (equation (3) in the main text) and the relationship for the density ratio,

$$\frac{\rho_a}{\rho}=\left[\left(1+\frac{\kappa-1}{2}M^2\right)\Big/\left(1+\frac{\kappa-1}{2}M_{max}^2\right)\right]^{1/(\kappa-1)}, \tag{A6}$$

where M_{max} is the maximal flow Mach number which can be reached when the gas expands from p_0 to p_a. Equation (3) yields

$$\left(\frac{w}{u}\right)^2=\frac{2}{\kappa-1}\left(\frac{a_0}{u}\right)^2\left[1-\left(\frac{a_w}{a_0}\right)^2\right]-1. \tag{A7}$$

For isentropic flow

$$\left(\frac{a_a}{a_0}\right)^2=\left(\frac{p_a}{p_0}\right)^{(\kappa-1)/\kappa}=\left(1+\frac{\kappa-1}{2}M_{max}\right)^{-1}. \tag{A8}$$

Taking into account that

$$\left(\frac{a_0}{u}\right)^2=\left(1+\frac{\kappa-1}{2}M^2\right)\frac{1}{M^2}$$

one obtains from equations (A5), (A7) and (A8) that

$$\frac{dM}{dx} = -2\alpha\eta M\left[\left(\frac{M_{max}}{N_{max}}\frac{N}{M}\right)^2 - 1\right]^{1/2}\left(\frac{N}{N_{max}}\right)^{2/(k-1)}\left[1 - \frac{\kappa+1}{2}\left(\frac{M}{N}\right)^2\right]^{-1}, \qquad \text{(A9)}$$

where $N_{max} = a_0/a_a$ (see equation (A8)).

By using this derivative, the remaining derivatives in equations (14a) and (14b) can be obtained from equation (11) and appropriate isentropic relationships:

$$\frac{d}{dx}\left(\frac{\bar{a}_0}{\bar{U}}\right) = \alpha\eta M\sqrt{2(\kappa-1)}\left(\frac{p_0}{p_a}\right)^{(\kappa-3)/2\kappa}\left[\frac{\kappa+1}{2}\left(\frac{\bar{p}}{p_0}\right)^{(\kappa-1)/\kappa}N_{max}^2 - 1\right]$$
$$\times\left(\frac{\bar{p}}{p_0}\right)^{-(\kappa+1)/\kappa}\left[\left(\frac{\bar{p}}{p_0}\right)^{(\kappa-1)/\kappa}N_{max}^2 - 1\right]^{-1\cdot 5}\left(1+\frac{\kappa-1}{2}M^2\right)^{-(2\kappa-1)/(\kappa-1)}\frac{dM}{dx}, \qquad \text{(A10)}$$

$$\frac{d}{dx}\left(\frac{M}{N}\right) = \left(1+\frac{\kappa-1}{2}M^2\right)^{-1\cdot 5}\frac{dM}{dx}. \qquad \text{(A11)}$$

APPENDIX B: NOMENCLATURE

a	speed of sound
f	frequency
F	cross-sectional area of the tube
κ	ratio of specific heats
l	pipe length
M	flow Mach number
p	pressure
R	radius of the tube
t	time
u	tube flow velocity
w	cross-flow velocity
x	co-ordinate along the tube
α	permeability coefficient
η	discharge coefficient
ρ	density
ω	dimensionless angular velocity