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ON CALCULATION OF ACOUSTIC POWER

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The previous work on sound power calculation in the presence of a mean flow has been focused on the casting of the basic acoustic equations in the form of an acoustic energy conservation law from which a definition of the acoustic intensity is extracted. The present paper shows that such definitions of sound intensity are deducible from the physical definition of sound power. Also presented is an expression or the sound power in a narrow pipe with a uniform mean flow.

1. INTRODUCTION

A quantity that is often of interest in acoustic analysis is the sound power transmitted through a given surface area. The mathematical definition of sound power, W, crossing surface S is

$$W = \int_S \langle \mathbf{N} \rangle \cdot \mathbf{n} \, dS, \tag{1}$$

where $\mathbf{N}$ denotes the acoustic intensity vector, $\mathbf{n}$ is the unit normal vector to surface S and $\langle \ \rangle$ denotes time averaging. For an ideal and quiescent medium, it is well established that the sound intensity vector is given by

$$\mathbf{N} = p\mathbf{v}, \tag{2}$$

where p is the acoustic pressure and $\mathbf{v}$ is the acoustic particle velocity. Thus, acoustic power has a physical definition as *time-averaged acoustic energy crossing a surface per unit time*. Although this definition is valid generally, no one appears to have used it *ab initio* for sound power or intensity calculation in the presence of a mean flow, the previous work having been focused rather on the casting of the basic acoustic equations in the form of an acoustic energy conservation law from which the definition of the acoustic intensity vector is extracted. A discussion of this approach, which makes the definition of sound intensity to some extent arbitrary, has been given by Morfey [1], who has also derived an acoustic energy conservation law in the presence of a rotational mean flow.

The physical definition of sound power applies regardless of acoustic energy conservation considerations. The present paper will show that Morfey's definition can be deduced from the physical definition of sound power and that a similar definition of sound intensity is also deducible from Doak's momentum potential theory [2]. The paper also presents an expression for the sound power in a circular or rectangular narrow pipe with a uniform mean flow.

2. GENERAL CONSIDERATIONS

From the physical definition of sound power, the sound intensity vector can be defined formally as

$$\mathbf{N} = \left[(\rho^* \mathbf{v}^*) \left(\frac{\mathbf{v}^* \cdot \mathbf{v}^*}{2} \right) + p^* \mathbf{v}^* \right]'_A, \tag{3}$$

where ρ^* is the fluid density and the subscript A denotes an acoustic part. Throughout the analysis, an asterisk (*) denotes a quantity that consists of a sum of a mean part and a fluctuating part with zero mean; a prime ($'$) or, when applicable, dropping the asterisk, denotes a fluctuating part, and the subscript 'o' without the asterisk denotes a mean part.

In equation (3), the first and second terms represent the kinetic energy flux and the compression strain energy flux in the fluid, respectively, due to acoustic motion of the particles. The normal component of this total energy flux is available for transmission over a unit surface area to the neighboring fluid particles as sound waves. In an ideal fluid, this transmission occurs without any energy loss, but in a real fluid, some dissipation is always present due to viscosity and thermal conductivity. In either case, the local acoustic energy flux is given by equation (3).

It is convenient to write equation (3) briefly as the first of the equations

$$\mathbf{N} = [\mathbf{m}^* \Pi^*]'_A \quad \mathbf{m}^* = \rho^* \mathbf{v}^*, \quad \Pi^* = \frac{\mathbf{v}^* \cdot \mathbf{v}^*}{2} + \frac{p^*}{\rho^*}, \tag{4}$$

Here, $\mathbf{m}^*$ is called the momentum density and Π^* is known in thermodynamics as exergy.

Implementation of equation (4) depends on the information available about the fluctuating quantities. Two cases that are of interest here are (1) acoustic components of the fluctuations are known, and (2) the total fluctuations are known. These are considered separately in the following. The analysis is based on small perturbation formulation of acoustic equations with non-linear effects neglected. Insofar as the calculation of the sound power is concerned, the first order terms in the energy flux need not be considered because they will vanish upon time averaging. For this reason, throughout the analysis, the first order terms in $\mathbf{N}$ are tacitly omitted and only the second order terms are given.

2.1. COMPUTATION OF SOUND INTENSITY WHEN ACOUSTIC FLUCTUATIONS ARE KNOWN

Typically, this case arises when one solves the basic acoustic equations with or without mean flow. In this case, equation (4) can be expressed as

$$\mathbf{N} = \mathbf{m}\Pi, \quad \mathbf{m} = \rho_o \mathbf{v} + \rho \mathbf{v}_o, \quad \Pi = \mathbf{v}_o \cdot \mathbf{v} + \frac{p}{\rho_o}, \tag{5}$$

where the mean and fluctuating quantities come from the problem solution. Clearly, the results of equation (5) can be accurate to the extent to which the problem solution is accurate. An application of equation (5) to sound propagation in a narrow pipe with a mean flow is presented in section 3.

2.2. COMPUTATION OF SOUND INTENSITY WHEN THE FLUCTUATIONS ARE KNOWN

This case arises when the fluctuations are measured, or computed by solving the basic fluid dynamic equations. Equation (5) will be valid in this case, too, if the mean flow field is known to be irrotational so that the fluctuations are due only to acoustic motion. Otherwise, it is necessary to separate the acoustic parts of the fluctuations. Two approaches that can be used for this purpose are considered in the following.

2.2.1. *Analysis based on partitioning of velocity fluctuations* [1]

First, assume that the acoustic part of the density fluctuations comes only from the first term of the thermodynamic state equation

$$\mathrm{d}\rho^* = \frac{\mathrm{d}p^*}{c^{*2}} + \left(\frac{\partial \rho^*}{\partial s^*}\right)\mathrm{d}s^*, \tag{6}$$

where s^* is the specific entropy and c^* is the speed of sound. This is tantamount to assuming isentropic sound waves for which the following relationship holds:

$$\rho = \frac{p}{c_o^2}. \tag{7}$$

Thus, the effects of viscosity and thermal conductivity on density fluctuations are neglected in this approach. These effects are generally confined to acoustic boundary layers, or become discernible only after long distances.

The velocity fluctuations are then partitioned as

$$\mathbf{v} = \mathbf{u} + \mathbf{w}, \quad \nabla\cdot\mathbf{w} = 0, \tag{8}$$

where the irrotational part, $\mathbf{u}$, is assumed to be the acoustic particle velocity. This separates the turbulent component of the fluctuating velocity induced by vorticity, but its acoustic component induced by viscosity is lost as well in this process. The effect of this loss is assumed to be ignorable.

Hence, equations (4) become

$$\mathbf{N} = \mathbf{m}\Pi, \quad \mathbf{m} = \rho_o\mathbf{u} + \frac{\mathbf{v}_o p}{c_o^2}, \quad \Pi = \mathbf{v}_o\cdot\mathbf{u} + \frac{p}{\rho_o}. \tag{9}$$

This result coincides with Morfey's definition of acoustic energy flux [1].

2.2.2. *Analysis based on partitioning of momentum density and exergy fluctuations* [2]

The fluctuating part of the momentum density can be partitioned as

$$[\mathbf{m}^*]' = \mathbf{B} - \nabla(\psi + \varphi), \quad \nabla\cdot\mathbf{B} = 0, \tag{10}$$

where the irrotational component, $-\nabla(\psi + \varphi)$, is assumed to consist of acoustic and thermal components $-\nabla\psi$ and $-\nabla\varphi$ respectively. This is justified by the fact that, in terms of the fluctuating part of the momentum potential, $\psi + \varphi$, the mass transport equation can be expressed as [2]

$$\nabla^2(\psi + \varphi) = -\mathrm{i}\omega\rho, \quad \nabla^2\psi = \frac{-\mathrm{i}\omega p}{c_o^2}, \tag{11}$$

where the second equation follows from equation (6), by the assumption that the acoustic part of the fluctuating momentum potential is due to only the pressure fluctuations. Upon assuming further that the solenoidal component of the fluctuating momentum density has no acoustical part, which is tantamount to neglecting the visco-thermal effects, the acoustic component of momentum fluctuations can be expressed as

$$\mathbf{m} = -\nabla\psi, \tag{12}$$

where ψ is determined by the second of equations (11).

To partition the fluctuating exergy, it is first noted that

$$\left[\frac{\mathbf{v}^* \cdot \mathbf{v}^*}{2}\right]' = \frac{\mathbf{v}_o \cdot [\mathbf{m}^*]'}{\rho_o} - \frac{v_o^2 [\rho^*]'}{\rho_o}. \tag{13}$$

Then, by using equation (12) for the acoustic component of the momentum fluctuations, and invoking equation (7) approximately as the acoustic component of the density fluctuations on the right-hand side, the acoustic component of exergy fluctuations can be expressed as

$$\Pi = -\frac{\mathbf{v}_o \cdot \nabla\psi}{\rho_o} + (\mathbf{1} - M_o^2)\frac{p}{\rho_o}, \tag{14}$$

where M_o is the magnitude of the local mean flow velocity Mach number. Thus, equations (4) become

$$\mathbf{N} = \mathbf{m}\Pi, \quad \mathbf{m} = -\nabla\psi, \quad \Pi = -\frac{\mathbf{v}_o \cdot \nabla\psi}{\rho_o} + (1 - M_o^2)\frac{p}{\rho_o}. \tag{15}$$

This result is contained in reference [2], although it is not given specifically as a definition of sound intensity.

3. SOUND POWER IN A NARROW PIPE

This section presents an application of equation (5) for the calculation of sound power in the fundamental mode propagation in a uniform narrow pipe carrying a uniform mean flow. The solution of this problem and the underlying theory is described in reference [3]. In this case, the sound field is obtained by superimposing two waves traveling in opposite directions, the axial component of acoustic particle velocity and the acoustic density of which are given by

$$\rho_o c_o v_x^{\pm}(x, r) = h^{\pm}(r) p^{\pm}(x), \quad c_o^2 \rho^{\pm}(x, r) = g^{\pm}(r) p^{\pm}(x), \tag{16}$$

where x denotes the pipe axis, the superscripts $\pm$ refer to the acoustic pressure wave components traveling in $\pm x$ directions, r denotes the radial co-ordinate, the functions $h^{\pm}(r)$ and $g^{\pm}(r)$ are given in Appendix A and the usual $\exp(-\mathrm{i}\omega t)$ time dependence is assumed for all fluctuating quantities. By substituting the foregoing equations into equation (5), the normal component of the intensity vector for the superimposed field is obtained as

$$\rho_o c_o N_x = A^+ p^+ p^+ + B p^+ p^- + A^- p^- p^-. \tag{17}$$

Here,

$$A^\pm = h^\pm + M_o(g^\pm + h^\pm h^\pm) + M_o^2 g^\pm g^\pm, \tag{18}$$

$$B = h^+ + h^- + M_o(g^+ + g^- + 2g^+ g^-) + M_o^2(g^+ h^- + g^- h^+), \tag{19}$$

where M_o denotes the mean flow velocity Mach number. Hence, from equation (1), the sound power crossing a cross-section of the pipe in the $+x$ direction is obtained as

$$2\rho_o c_o W = \bar{A}_R^+ |p^+|^2 + \bar{B}_R |p^+ p^-| + \bar{A}_R^- |p^-|^2, \tag{20}$$

where an overbar denotes a cross-sectional average and the subscript R denotes the real part of a complex quantity. Clearly, the first and last terms on the right-hand side of equation (20) correspond to sound power of the waves traveling in forward and backward directions respectively. For the inviscid and quiescent case, the factors reduce to $\bar{A}^\pm = \pm 1$ and $B = 0$. For the inviscid case with a uniform mean flow, they reduce to the well-known result that follows from Morfey's definition of sound intensity:

$$\bar{A}^\pm = \pm(1 \pm M_o)^2, \quad \bar{B} = 0. \tag{21}$$

This result can also be deduced by applying equation (15). Thus, in the inviscid case, the sound field in the pipe is 'canonical'; that is, $\bar{B} = 0$, which means that the sound power of a reflective field is equal to the algebraic sum of the powers of the forward and backward waves. Another case of a canonical sound field occurs when the mean flow velocity is zero. For this case, it can be shown that, for example, for a circular pipe,

$$\bar{A}_R^\pm = \left[\frac{-K^\pm \mathrm{J}_2(s\sqrt{i})}{\mathrm{J}_o(s\sqrt{i})}\right]_R, \quad \bar{B} = 0, \tag{22}$$

where $K^\pm$ denote the propagation constants, s is the shear wavenumber and J_n denotes a Bessel function of order n. For large s, say $s > 40$, equation (22) can be expressed approximately as

$$\bar{A}_R^\pm = \pm\left[1 - \frac{1}{s\sqrt{2}}\left(1 - \frac{\gamma - 1}{\sigma}\right)\right], \tag{23}$$

where γ is the ratio of specific heat coefficients and σ is the square root of the Prandtl number (see Appendix A for the definition of the parameters in the foregoing equations). For example, for $\gamma = 1{\cdot}4$ and $\sigma^2 = 0{\cdot}7$, the factors are larger than 99% in absolute value for $s > 40$. Thus, in this case, the visco-thermal effect on sound power is negligibly small, as expected.

With mean flow and visco-thermal effects present, however, the sound field is no longer canonical and the factors $\bar{A}_R^\pm$ may be reduced considerably. This is shown in Figure 1, where the factors $\bar{A}_R^\pm$ and $\bar{B}_R$ of a circular pipe are shown as functions of the Stokes number and the mean flow Mach number. As can be seen from these characteristics, the absolute

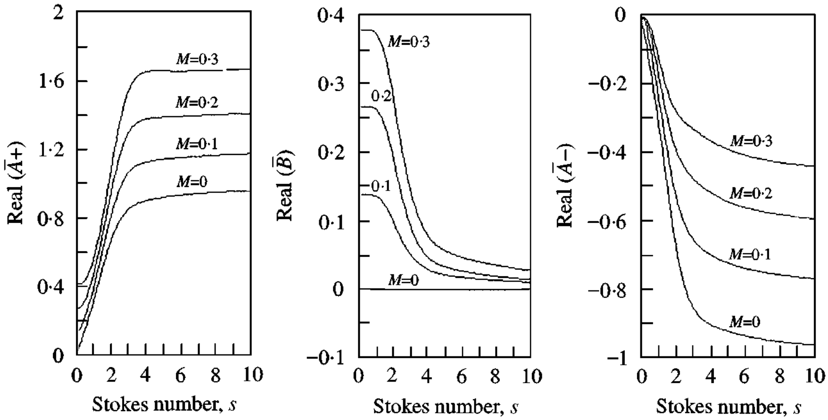

Figure 1. Sound power factors for a circular pipe with $\sigma^2 = 0{\cdot}7$ and $\gamma = 1{\cdot}4$.

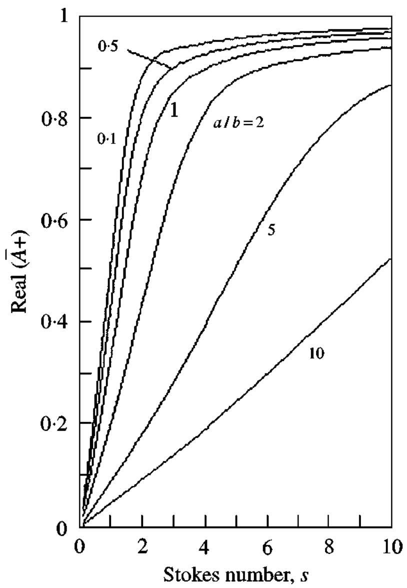

Figure 2. Effect of the aspect ratio on the sound power factor for a rectangular pipe with zero mean flow, $\sigma^2 = 0{\cdot}7$ and $\gamma = 1{\cdot}4$.

value of these factors vary approximately linearly for $s < 3$, and gradually tend to their inviscid values given by equation (21) as s increases. The shape of the pipe cross-section may have a slight effect on these characteristics. To give an idea of the amount of deviations that may occur due to the shape of the pipe cross-section, the factor $\bar{A}_R^+$ is shown in Figure 2 for rectangular pipes with different aspect ratios for the case of zero mean flow, and the factors $\bar{A}_R^\pm$ and $\bar{B}_R$ are shown in Figure 3 for $M_o = 0{\cdot}1$. It should be noted that, the aspect ratios $a/b = 0{\cdot}1$ and $0{\cdot}5$ correspond to the same cross-sections as the aspect ratios $a/b = 10$ and 2, respectively, the characteristics look different only because the Stokes number is based on side a. Thus, it is seen that the effect of a 10 fold change in the aspect ratio amounts to approximately less than about 20% change in the magnitude of the factors $\bar{A}_R^\pm$ and $\bar{B}_R$.

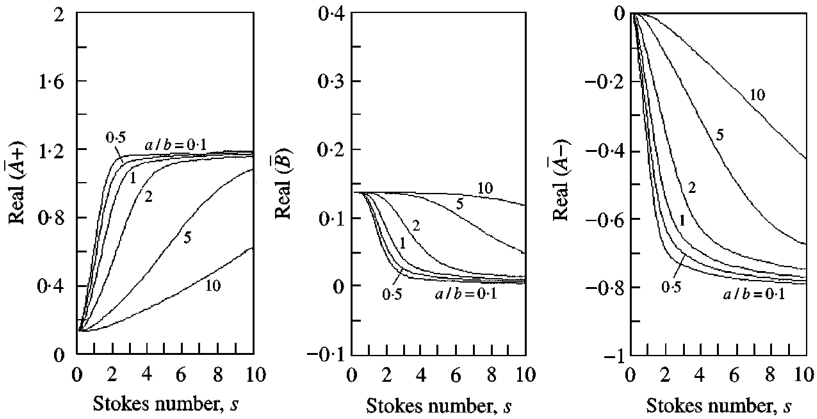

Figure 3. Effect of the aspect ratio on the sound power factors for a rectangular pipe with mean flow, $M_o = 0{\cdot}1$, $\sigma^2 = 0{\cdot}7$ and $\gamma = 1{\cdot}4$.

4. CONCLUSION

Morfey's definition of sound intensity is shown to be deducible from the physical definition of sound power. The partitioning proposed by Doak yields an alternative definition of sound intensity. When the acoustic field is known, equation (5) can be used directly to compute the sound power. The sound power in a narrow pipe with a uniform mean flow is thus computed for the first time.

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APPENDIX A: SOUND WAVE TRANSMISSION IN A NARROW PIPE

Summarized in this appendix are the expressions that are relevant to computation of sound power in a uniform narrow pipe with a uniform mean flow. The underlying theory is described in reference [3].

The propagation constants $K^{\pm}$ are computed from the dispersion equation

$$\gamma + (\gamma - 1)I(\sigma\beta a) + \left(\frac{K}{1 - M_o K}\right)^2 I(\beta a) = 0, \quad \beta a = s\sqrt{\mathrm{i}(1 - KM_o)}. \tag{A.1}$$

Here a denotes a characteristic dimension of the pipe cross-section, I is a function that depends on the pipe cross-section, γ is the ratio of specific heat coefficients, σ^2 denotes the Prandtl number, $\sigma^2 = \mu c_p/\kappa$, where μ is the shear viscosity coefficient, c_p is the specific heat coefficient at constant pressure and κ is the thermal conductivity, s is the Stokes number, which is defined as $s = a\sqrt{(\rho_o\omega/\mu)}$, where ω is the radian frequency.

Circular pipes:

$$h^{\pm}(r) = \left[\frac{K^{\pm}}{1 - M_o K^{\pm}}\right]\left[1 - \frac{J_o(\beta^{\pm} r)}{J_o(\beta^{\pm} a)}\right], \tag{A.2}$$

$$g^{\pm}(r) = 1 + (\gamma - 1)\left[\frac{J_o(\sigma\beta^{\pm} r)}{J_o(\sigma\beta^{\pm} a)}\right], \tag{A.3}$$

$$I(\xi) = \frac{J_2(\xi)}{J_o(\xi)}, \tag{A.4}$$

where a is the pipe radius, r is the radial coordinate and J_n denotes a Bessel function of order n.

Rectangular pipes:

$$h^{\pm}(y, z) = \left[\frac{16}{\pi^2}\frac{K^{\pm}}{1 - M_o K^{\pm}}\right]\sum_{m,n=1,3,\ldots}\frac{\sin(m\pi y)\sin(n\pi z)}{mn\alpha_{mn}(\beta^{\pm} a)}, \tag{A.5}$$

$$g^{\pm}(y, z) = \left[\gamma - (\gamma - 1)\frac{16}{\pi^2}\right]\sum_{m,n=1,3,\ldots}\frac{\sin(m\pi y)\sin(n\pi z)}{mn\alpha_{mn}(\sigma\beta^{\pm} a)}, \tag{A.6}$$

$$\alpha_{mn}(\xi) = 1 - \pi^2(m^2 + n^2a^2/b^2)/4\xi^2, \tag{A.7}$$

$$I(\xi) = -\frac{64}{\pi^4}\sum_{m,n=1,3,\ldots}\frac{1}{m^2n^2\alpha_{mn}(\xi)}, \tag{A.8}$$

where the pipe cross-section occupies the region $2a \geqslant y \geqslant 0$ and $2b \geqslant z \geqslant 0$.