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LETTERS TO THE EDITOR

DYNAMIC INTERACTION AMONG FOOTINGS

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1. INTRODUCTION

Extensive investigations conducted in recent decades have demonstrated that for most structures, the dynamic interaction between soil and structure and the interaction through the soil among adjacent structures play a very important role in seismic analysis [1–3]. There are a number of factors that may affect the dynamic behaviour of the interaction system [2–7]. A thorough literature survey reveals that most of the studies on cross-interaction for surface and shallow or deeply buried foundations are focused on the interaction between members which are next to each other. In practical situations, each individual member of the foundation of a building may be influenced not only by its immediate neighbour but also those separated by some intermediate ones which may be considered to be its distant neighbors. For pile foundations, the group effect was studied in some detail. Usually, results for regularly arranged pile groups, say a 2×2 or 3×3 sub-group, were presented for design purposes. The group effect can be calculated by the direct analysis of the four- or nine-pile group against the results of the simple sum of single piles, which is referred to as the group efficiency ratio. It should be noted that for a pile foundations, piles are normally interconnected with each other by a rigid cap. The group effects are a mixed combination of each member pile. In cases of mat foundations, individual foundations are independent of one another. The group effect can be considered as the sum of the influence coefficients of direct adjacent foundations and that of the other indirect adjacent ones.

Numerical studies for 2 or 4 independent or structurally interconnected rigid surface foundations arranged in direct adjacency were made by Qian and Beskos [3] and compared with ATC-3 provisions. It also provided an up-to-date literature survey on analytic and numerical solutions for the sub-soil coupling problems. Other factors concerning foundation flexibility and edge effect were discussed in later publications by Qian *et al.* [4, 5] and Tham *et al.* [6]. Efforts on the sub-soil coupling effects for system including distant neighbors were made by Triantaffildis and Prange [8] and Mohammadi and Karabalis [9]. In their studies, ties of a railway track were considered to be massive surface foundations with rigid interconnection. In a recent publication of Mulliken and Karabalis [10], a discrete model was proposed for the analysis of dynamic sub-soil coupling effects.

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Frequency-independent springs and dashpots were used to represent either the soil–foundation interaction or the sub-soil coupling among foundations. It has to be noted that the off-diagonal stiffness and damping coefficients adopted in reference [10] were based on numerical results provided by Huang [11] for two identical square foundations thus the group effect among distant foundations had not been included.

Most studies indicate that interaction effects decay with the increase of separation distance [3–8]. It seem that an indirect neighbor might have less influence than the immediate ones because of a greater distance from the one concerned. However, the combined effects of a large number of distant neighbors may have significant contributions. It is the purpose of this paper to study the effect of distant neighbors on a footing.

In this study, the footings are considered to be rigid and in perfect contact with the surface of the elastic half-space. The frequency domain boundary element method proposed by the authors [3] is employed. The group effects of the through the soil coupling on the dynamic stiffness for a multi-foundation system as a function of separation distance and relative position are assessed through an extensive parametric study.

2. FUNDAMENTALS OF BEM FORMULATION

A detailed description of the analytical model and the solution procedure was reported by the writers [3, 4]. Therefore, for brevity, only an outline of the formulation is given here.

In the frequency domain, the integral representation of surface displacements for a rigid, massless, surface footing of arbitrary planform S_f in full contact with an elastic half-space can be written as

$$u_i^s(x) = \iint_s \mathbf{G}_{ij}(x - x', y - y') t_i^s(x') \, \mathrm{d}x' \, \mathrm{d}y', \tag{1}$$

where $\mathbf{G}_{ij}$ is the surface half-space Green's function for the surface displacement in the ith direction at $(x, y, 0)$ due to a unit force acting in the jth direction at $(x', y', 0)$. The explicit form of $\mathbf{G}_{ij}$ and its numerical computation can be found elsewhere in reference [3].

For a numerical solution of equation (1), the soil–footing interface S_f is discretized into a number of eight-nodded quadratic isoparametric boundary elements. Thus, equation (1) is rewritten in the discretized form as

$$\{U\} = \{G\}\{T\}. \tag{2}$$

For a rigid footing, its dynamic response can be described by three displacements Δ_i and rotations $\Phi_i (i = x, y, z)$ at the center of the footings.

Applying the condition of compatibility of displacements at the contact area and equilibrium at the interface of the solid and the rigid footing, one obtains

$$\{P\} = [K(\omega)]\{\Delta\}, \tag{3}$$

where $\{P\}$ is the external force vector and $\{\Delta\} = \{\Delta_x, \Delta_y, \Delta_z, \Phi_x, \Phi_y, \Phi_z\}^T$. The 6×6 matrix $[K(\omega)]$ is called the impedance or dynamic stiffness matrix of the rigid, massless footing.

For a system of N rigid surface footings, $\{P\}$ and $\{\Delta\}$ will be $6N \times 1$ vectors and the matrix $[K]$ can be rewritten in a sub-matrix form as

$$\begin{bmatrix} K^{11} & K^{12} & \bullet & K^{1n} \\ K^{21} & K^{22} & \bullet & K^{2n} \\ \bullet & \bullet & \bullet & \bullet \\ K^{n1} & K^{n2} & \bullet & K^{nn} \end{bmatrix} \begin{Bmatrix} \Delta^1 \\ \Delta^2 \\ \bullet \\ \Delta^n \end{Bmatrix} = \begin{Bmatrix} P^1 \\ P^2 \\ \bullet \\ P^n \end{Bmatrix} \tag{4}$$

with each sub-matrix being 6×6. The sub-matrix $[K^{ij}]$ represents the influence of footing j to footing i while $\{\Delta^i\}$ and $\{P^i\}$ are the displacement and the external force vectors of footing i respectively.

3. NUMERICAL STUDY OF THE COMPLICATE SUB-SOIL COUPLING EFFECTS

A simple interaction system of three-square footing placed in sequence, as shown in Figure 1, is employed to study the group effects. The three footings are identical with a side of $2L$. The separation distances between footings are equal and footings are in perfect contact with the elastic half-space. The half-space has the Poisson ratio $\nu = 1/3$ and the shear module G. The dimensionless distance is defined as $d = D/L$, in which D is the clear distance between footings "1" and "3" as shown in Figure 1. The dimensionless frequency is defined as $\omega_0 = \omega L/V_s$, where V_s is the shear wave velocity in the elastic half-space. A two-footing system, e.g., without footing "2" in the system shown in Figure 1, is chosen for comparison purposes demonstrating the effect of the intermediate footing on the sub-soil coupling between footings "1" and "3". The complete stiffness matrix for the three-footing

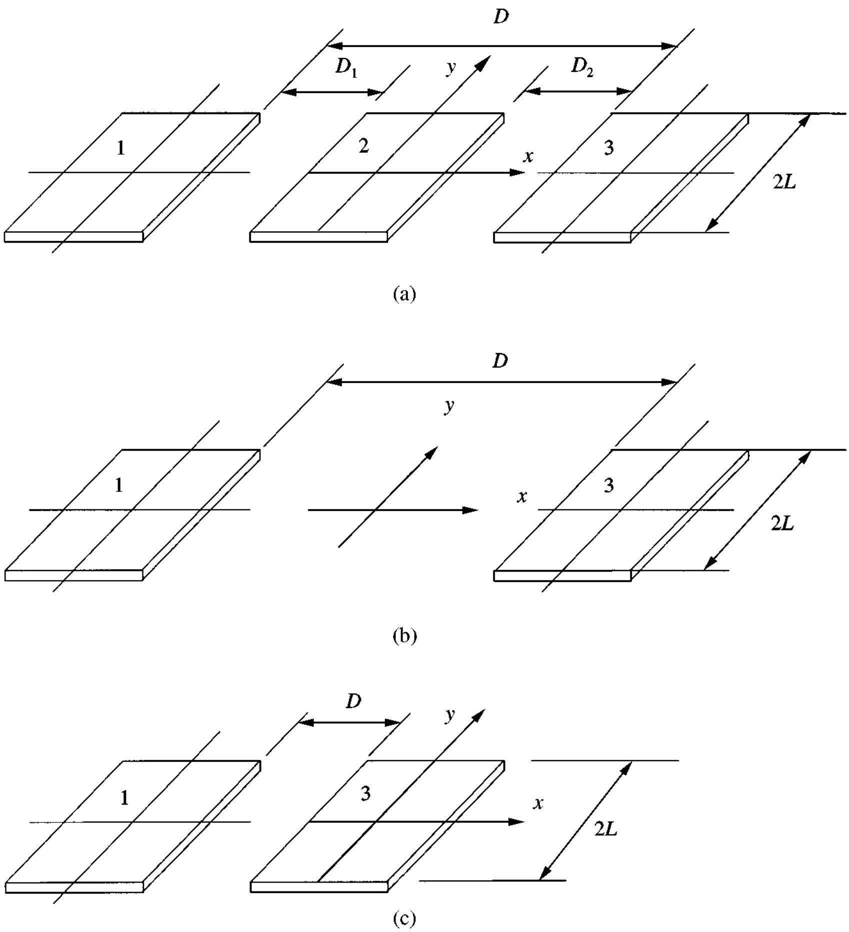
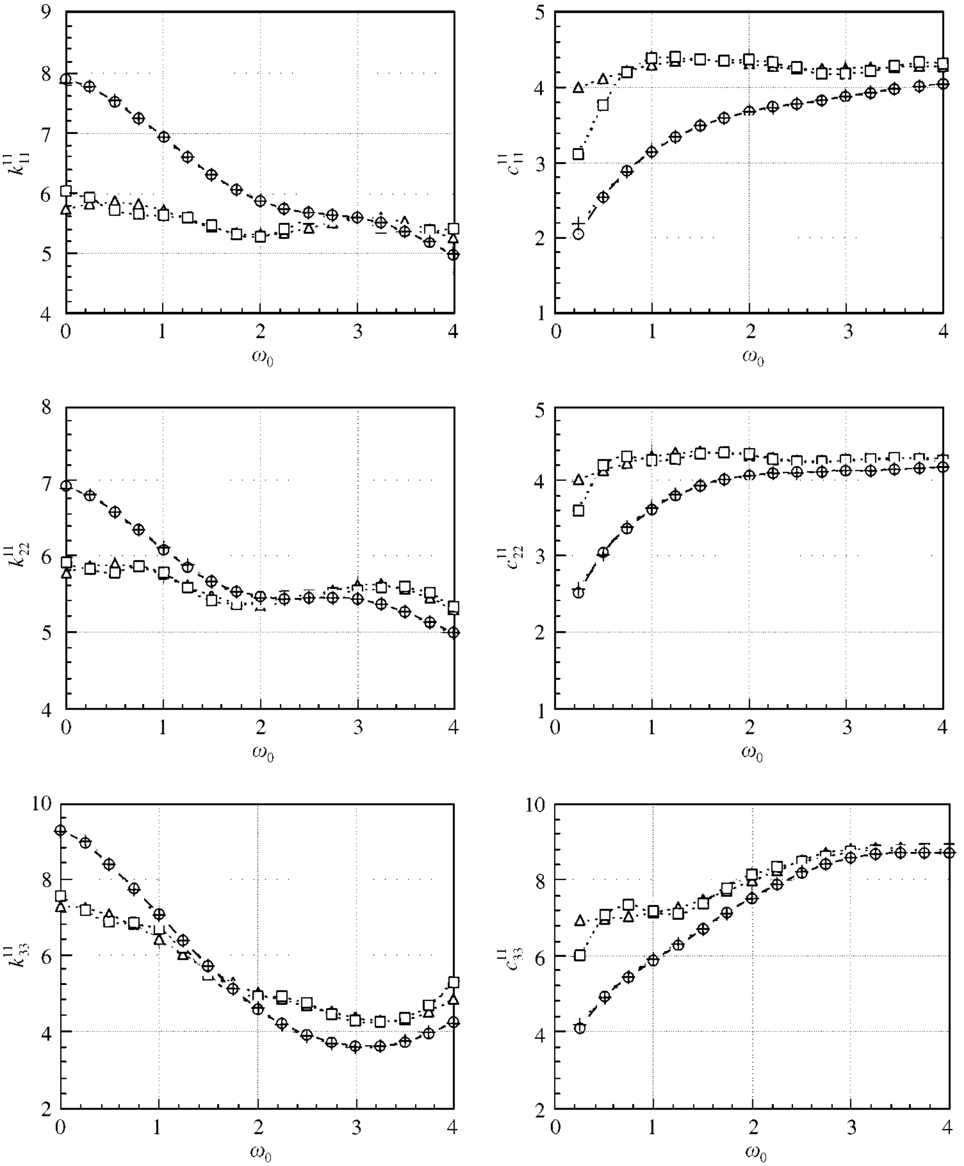


Figure 1. Two- and three-footing system. (a) 3F; (b) 2F; (c) 2Fa.



(a)

Figure 2. Spring and damping coefficients for $[K^{11}]$. ---Δ---1F; ---□---2F; --- + ---2Fa; —○— 3F.

system is composed of 3×3 sub-matrices with the following properties $[K^{ij}] = [K^{ji}]$ and $K^{rs}_{ij} = K^{sr}_{ji}$ ($r, s = 1, 2, 3$ and $i, j = 1, 2, \ldots, 6$). K^{rs}_{ij} can be explained as contact force in the ith direction on footing r caused by a unit displacement in the jth direction taking place at footing j. The spring and damping coefficients are introduced to define the real and imaginary parts of the complex dynamic stiffness K^{rs}_{ij}, respectively, that is,

$$k^{rs}_{ij} = \mathrm{Re}\, K^{rs}_{ij}/G \quad (r, s = 1, 2, 3 \quad \text{and} \quad i, j = 1, 2, \ldots, 6),$$
$$C^{rs}_{ij} = \mathrm{Im}\, K^{rs}_{ij}/G\omega_o. \tag{5}$$

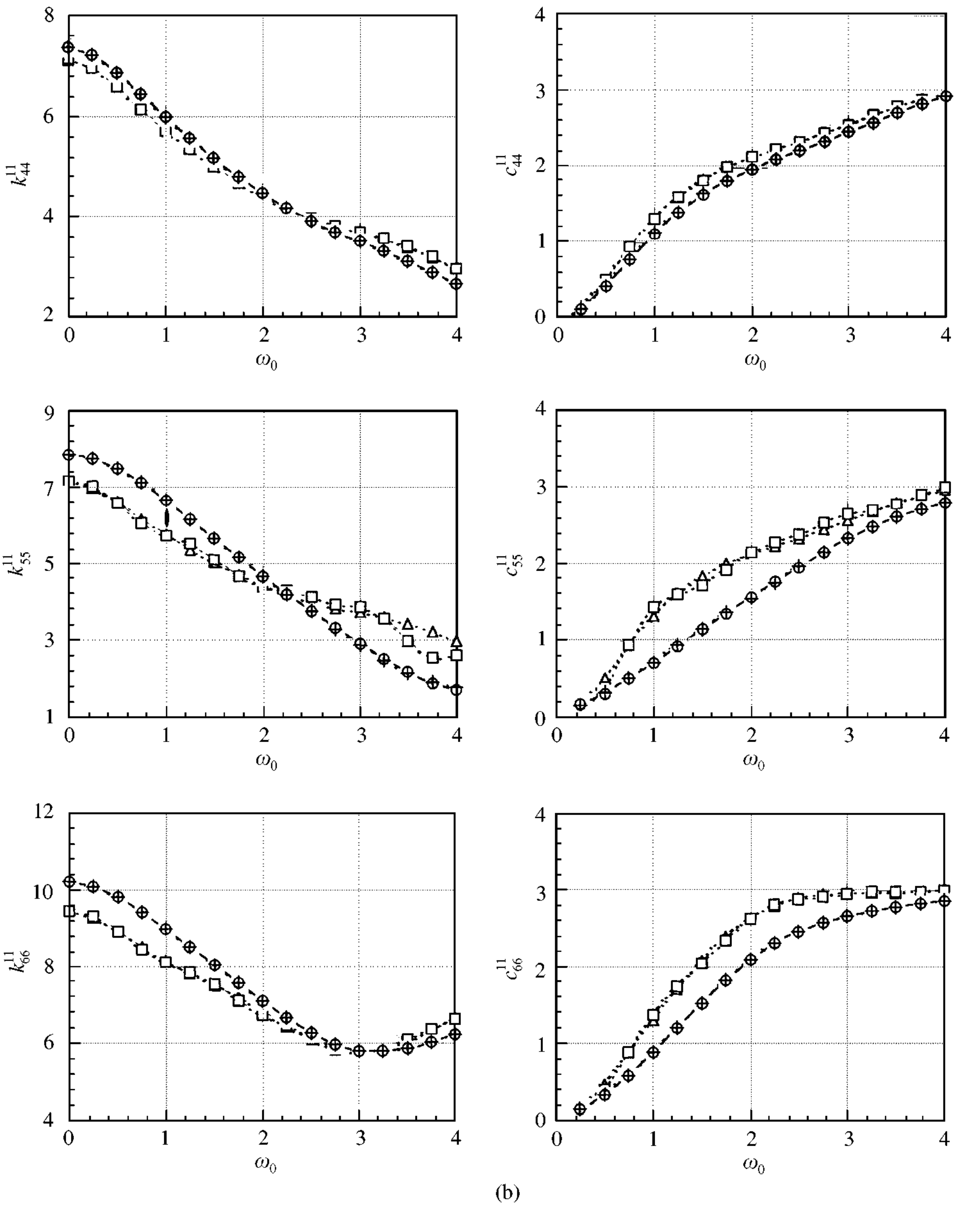
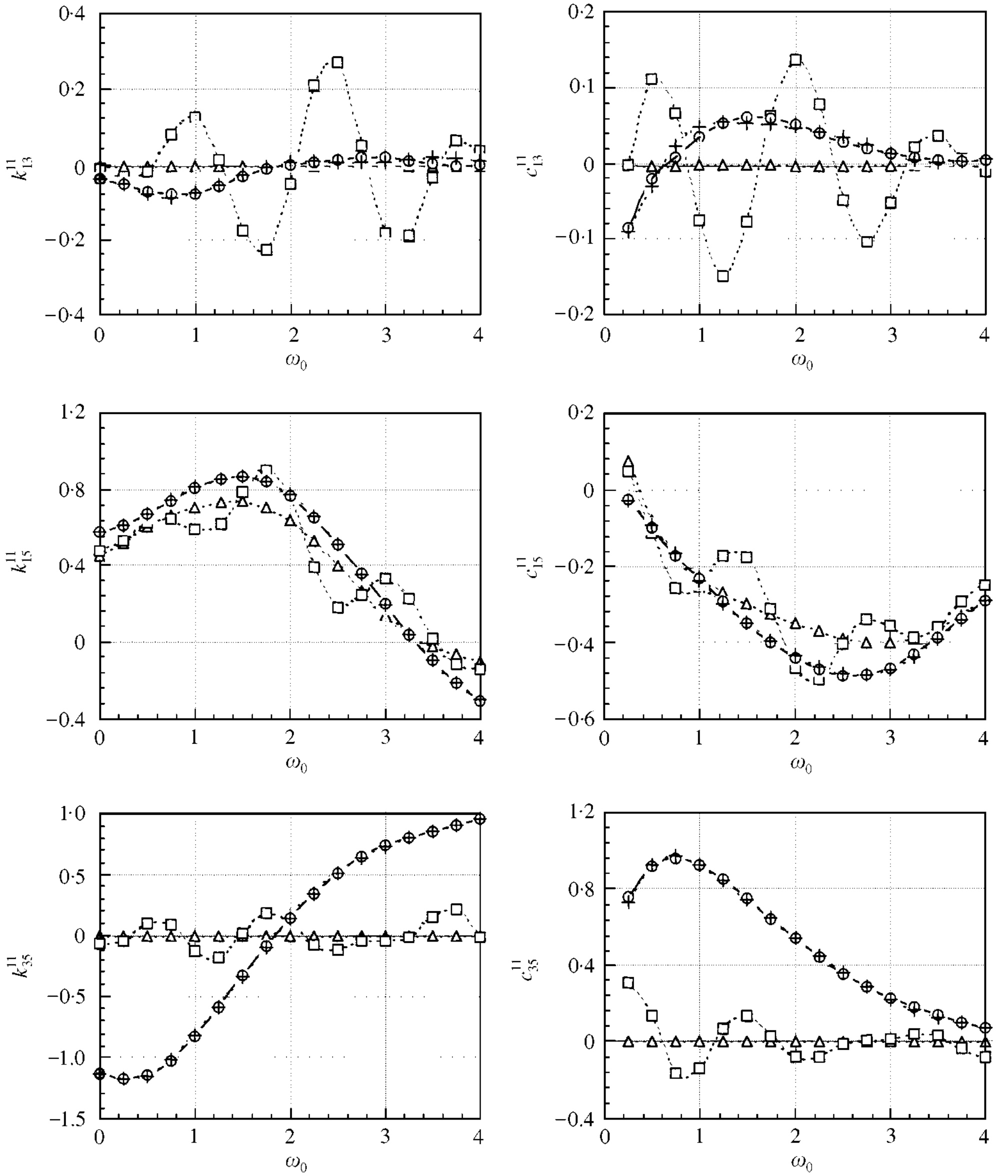


Figure 2. Continued.

In the following discussion, coefficients in the diagonal sub-matrix $[K^{rr}]$ are referred to as the self-correlative dynamic stiffness while that in the off-diagonal sub-matrix $[K^{rs}]$, $r \neq s$, is called the cross-correlative dynamic stiffness.

3.1. EFFECTS OF THE INTERMEDIATE FOOTING ON THE SUB-SOIL COUPLING

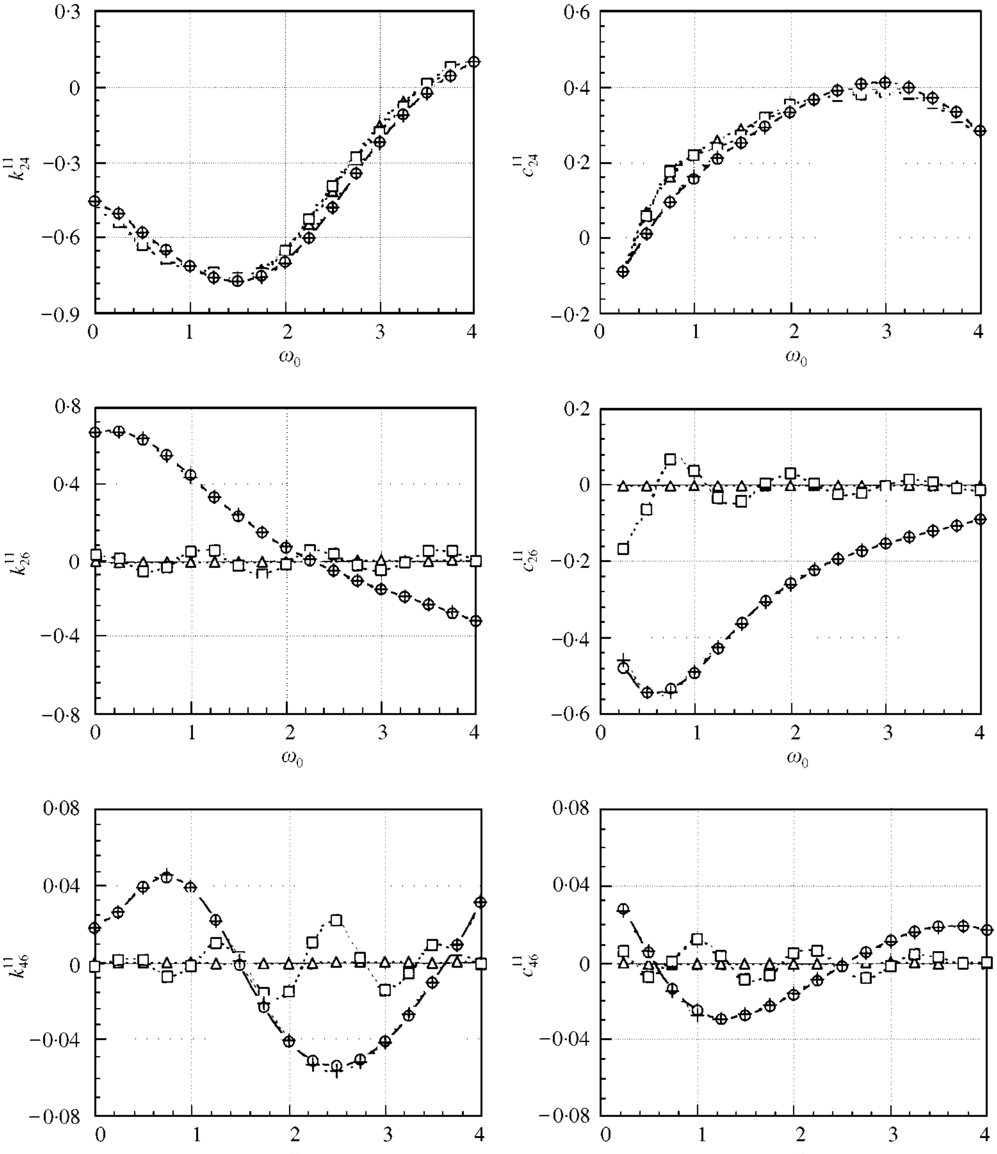
Figure 2 shows all the non-zero components of the self-correlative terms in $[K^{11}]$ versus the excitation frequency ω_0 for closely spaced footings with $d = 2{\cdot}5$. Curves labelled by 1F,



(c)

Figure 2. Continued.

2F and 3F represent dynamic stiffness for the interaction system composed of 1, 2 and 3 footings respectively. It is clear that results for 1F show the interaction effect between footing and soil only while in that for 2F and 3F the sub-soil coupling effects among footings will be included. Results labelled by 2F and 2Fa are both for a two-footing system with different separation distances $d_1 = D_1/L = 2{\cdot}5$ and 0·25 respectively (Figure 1). The latter is equal to the distance between footings "1" and "2" for the three-footing system. It is interesting to see that there is no significant difference in $[K^{11}]$ for cases 3F and 2Fa. The results indicate that footings in a group immediately next to the one concerned contribute



(d)

Figure 2. Continued.

the most to the sub-soil coupling effect, while the others arranged consequently have only a minor effect not only due to their relatively greater distance from the one in consideration but also due to the shielding effect of the footings in between.

All the non-zero components of the off-diagonal sub-matrix $[K^{13}]$ are plotted in Figure 3 against the dimensionless frequency ω_0. $[K^{13}]$ represents the sub-soil coupling effect between footings, e.g., contact forces on footing "1" induced by unit displacements taking place at footing "3" or *vice versa*. As observed in previous studies [3, 4], either the spring or the damping coefficients exhibit wavy variations with the frequency. The presence of the

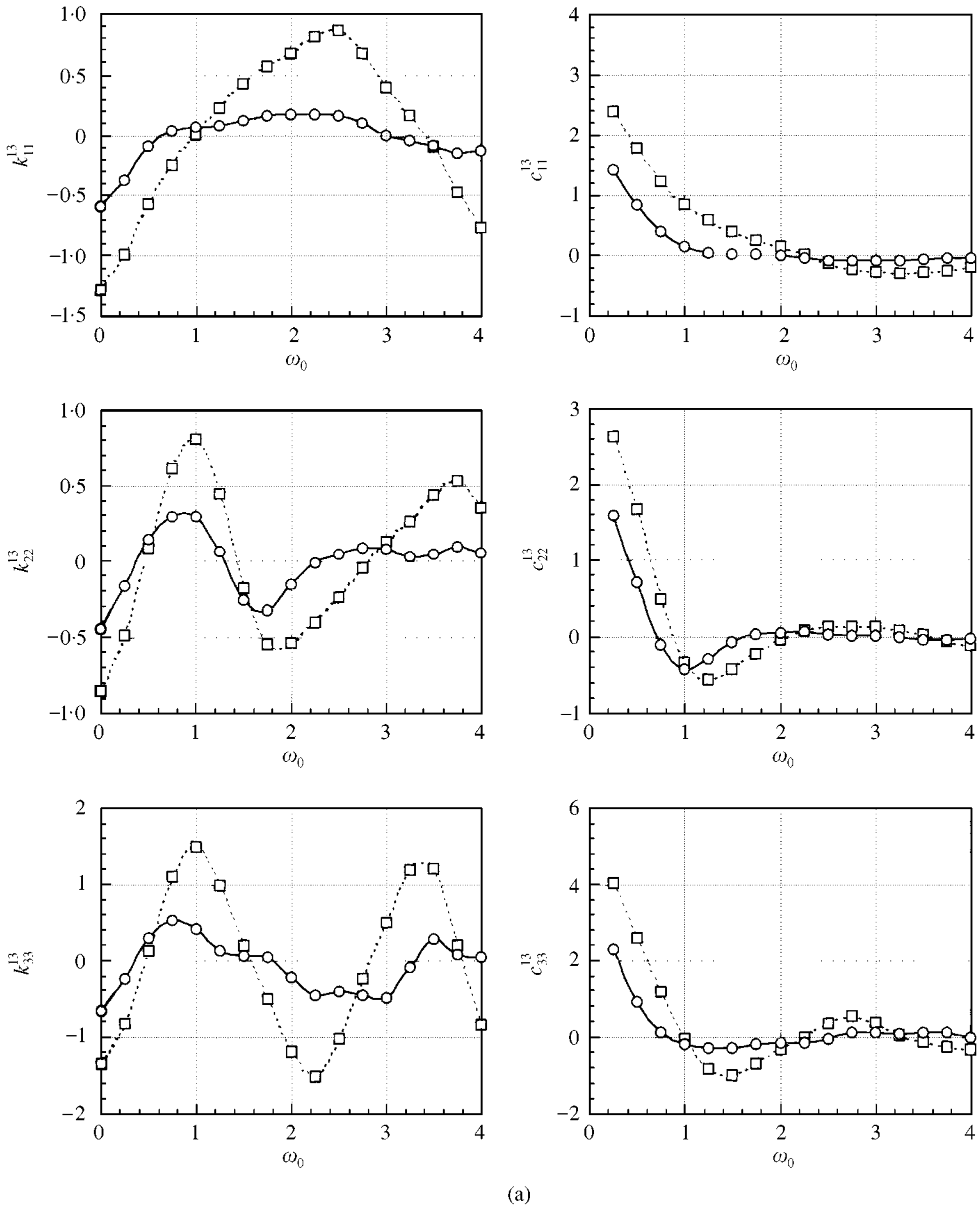


Figure 3. Spring and damping coefficients for $[K^{13}]$. ---Δ---1F; ----□--- + ---2F; —○— 3F.

intermediate footing does not change this wavy variation nature while reducing its amplitude markedly.

For a variety of separation distances d, i.e., from 2·5 to 18·0, a similar trend has been observed. Figure 4 shows dynamic stiffness K_{ij}^{11} at a low frequency $\omega_0 = 0{\cdot}25$. Values labelled by 2Fa have been calculated for a two-footing system with $d = 2(d_1 + 1)$, that is, the right-hand footing is placed at the position where footing "2" is located for case 3F. Again, results of 2Fa and 3F are almost the same for all the diagonal terms K_{ij}^{11} $(i = 1, 2, \ldots, 6)$. The

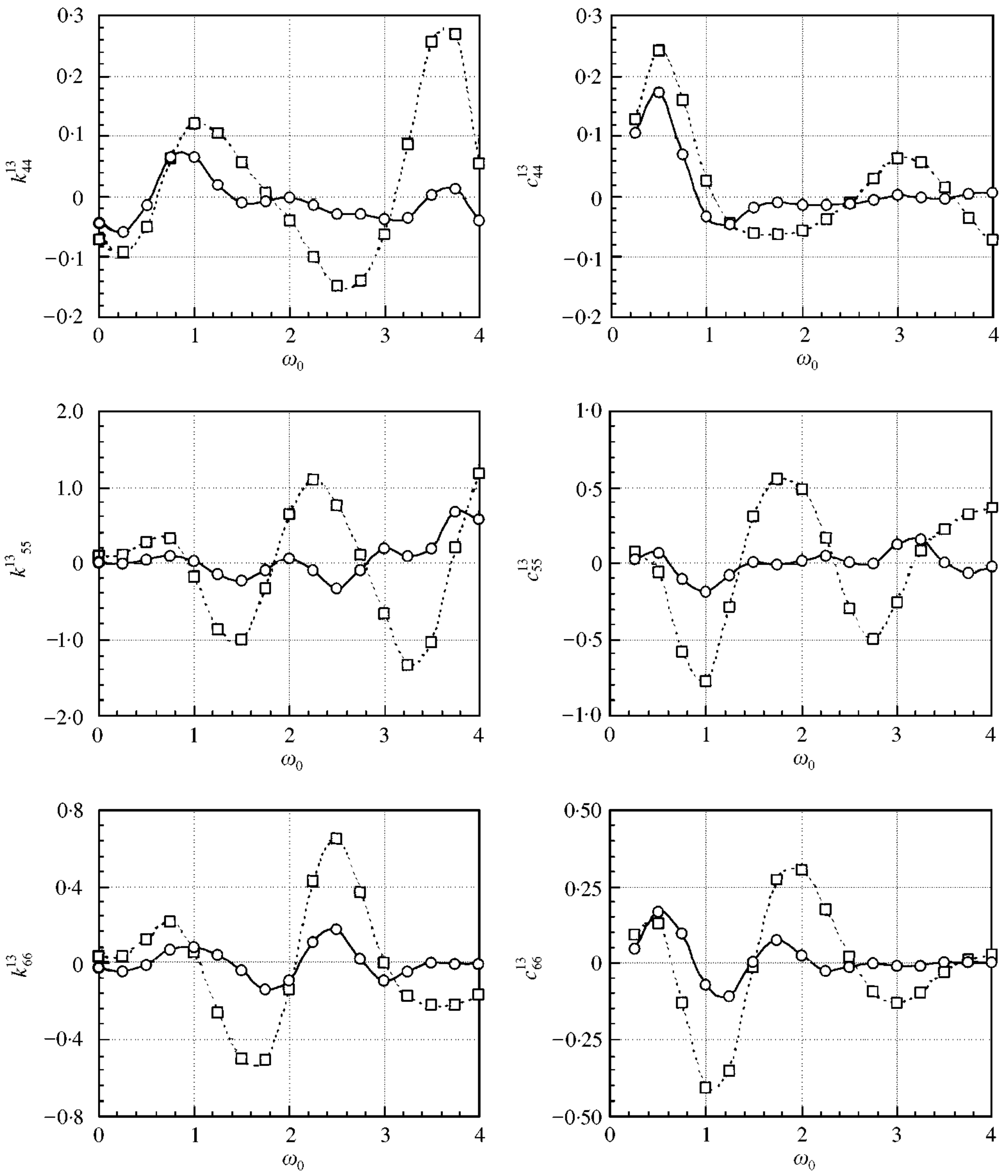
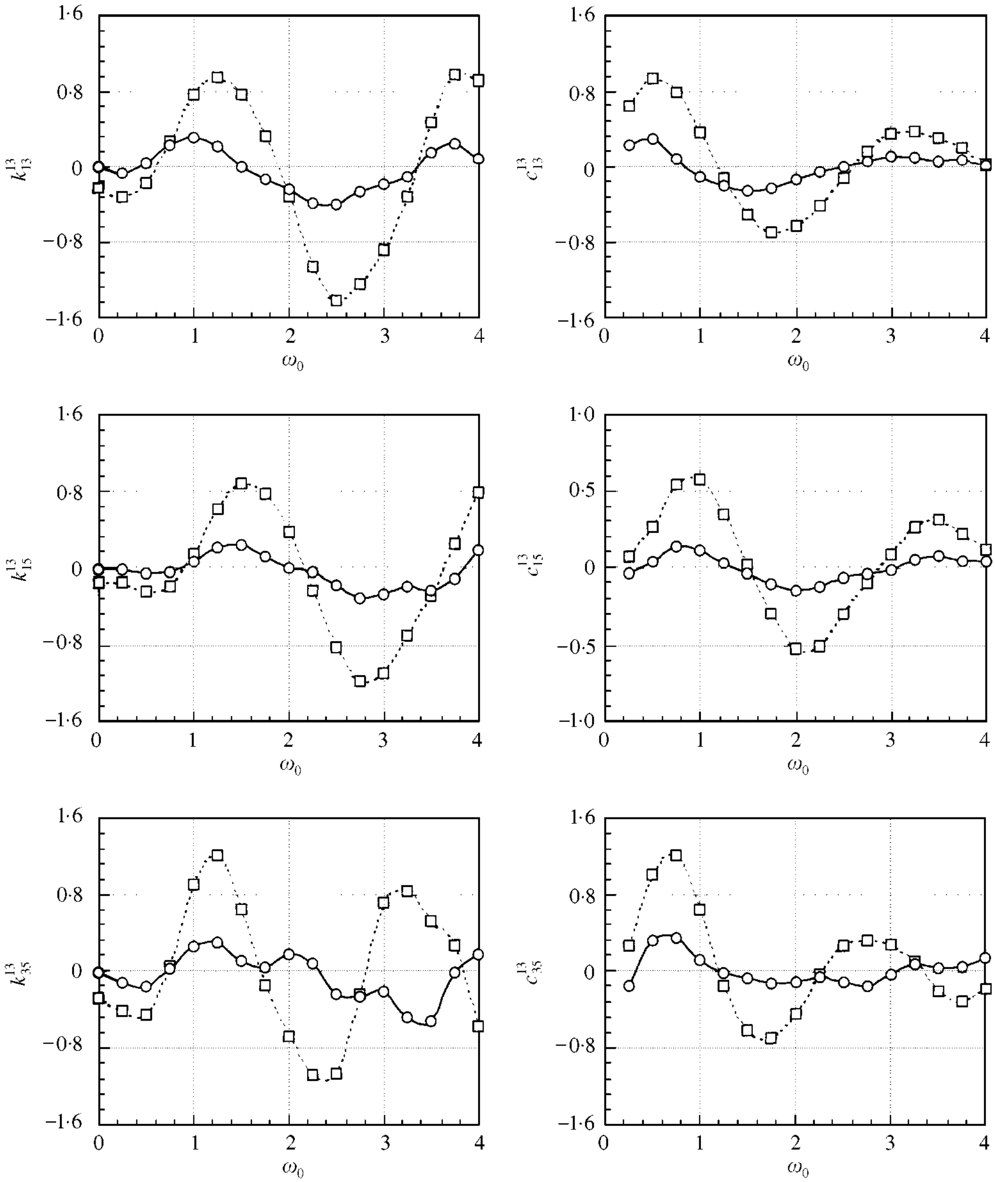


Figure 3. Continued.

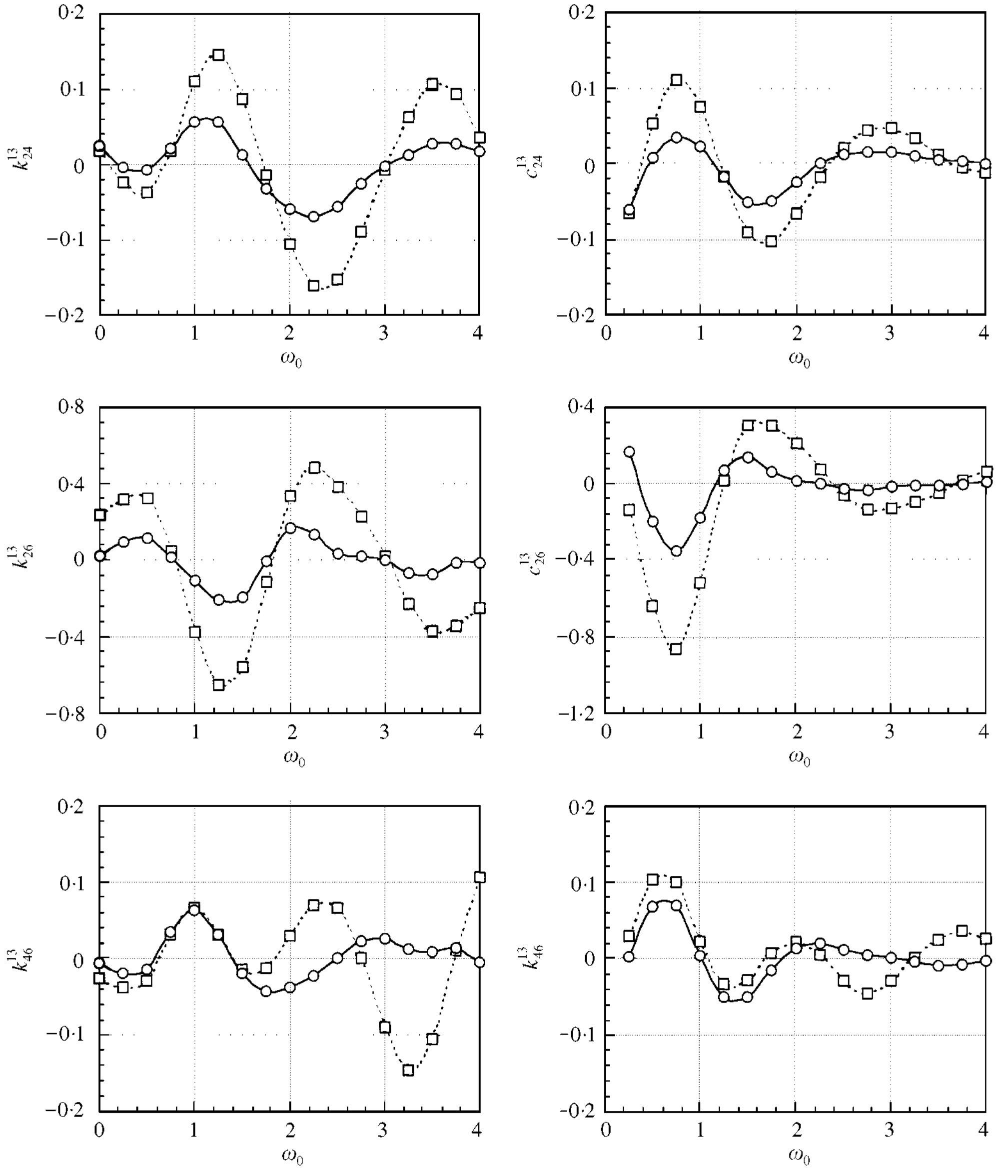
presence of the third footing has a recognizable effect only on those minor coupling terms to a certain degree. This is also true for higher excitation frequencies as can be seen in Figure 5, which is calculated at the frequency $\omega_0 = 2{\cdot}50$. As indicated before, [3, 4], either spring or damping coefficients vary with the separation distance. They fluctuate around the values of 1F and decay rapidly with increasing the spacing. Numerical results show that the dominant factor for a group of footings is the minimum value of spacing. In other words, a group of consequently placed footings may have very little influence on an isolated footing except one, which is immediately next to that to be considered.



(c)

Figure 3. Continued.

For a better understanding of the complicate effects of sub-soil coupling among a group of footings, the total subgrade stress σ_z for a three-footing system under uniform transverse displacements with different parameter combinations are listed in Tables 1 and 2. The values computed by the present method (BEM 3F) are compared with those for a single footing (BEM 1F) as well as those obtained by simply summing the values of each member pair in the same footing system ($\sum$2F). The dimensionless distance is defined as $d_i = D_i/L$ ($i = 1, 2$). The relative differences are presented in parentheses under each number. It can be clearly seen that difference is more significant for small spacing and lower frequency.

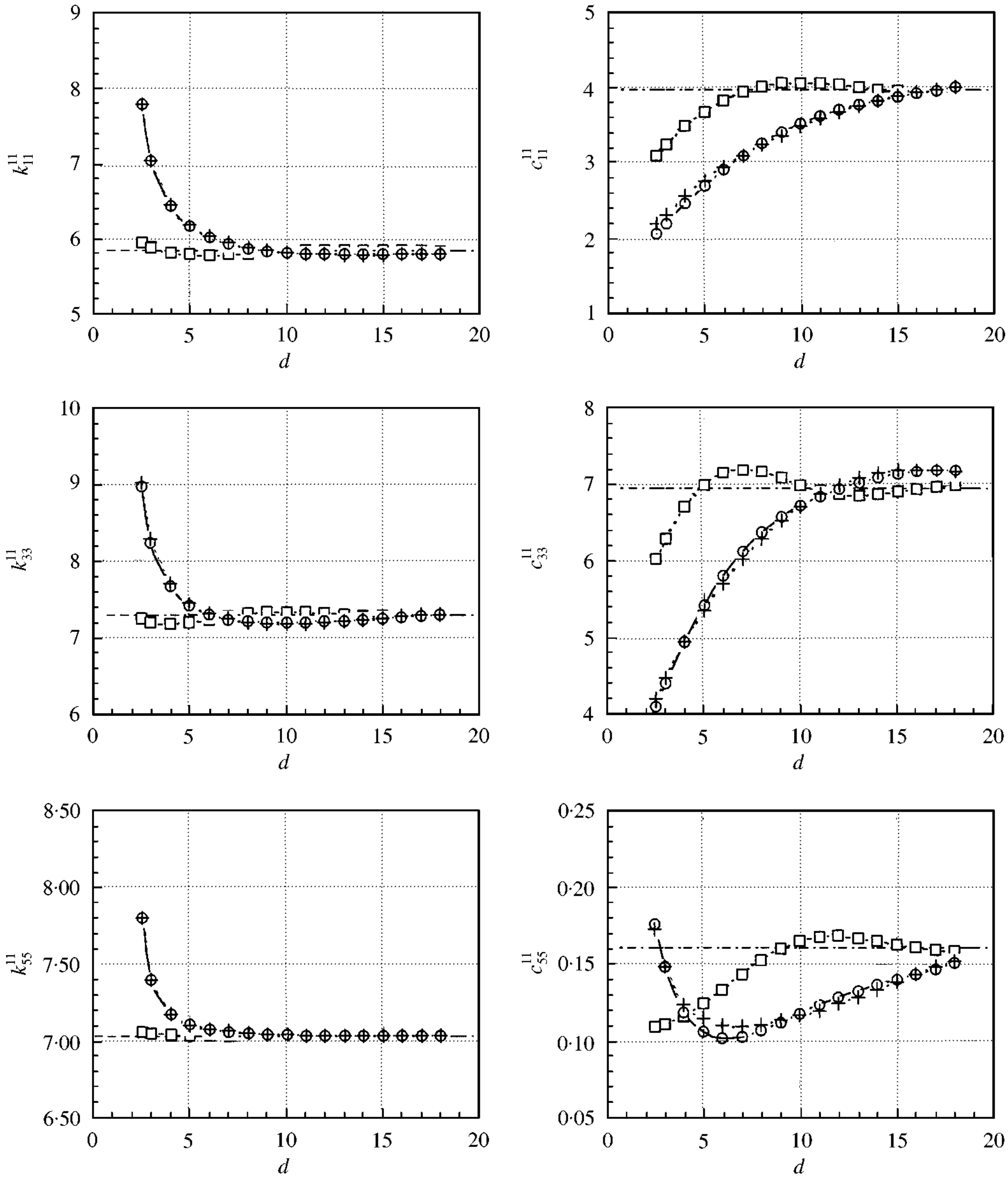


(d)

Figure 3. Continued.

3.2. EFFECTS OF RELATIVE POSITION OF THE INTERMEDIATE FOOTING ON THE SUB-SOIL COUPLING

Effects of relative position of the intermediate footing on dynamic stiffness have been studied by shifting the location of footing "2" from the left end to the right in between footings "1" and "3". The results for two critical situations are given in Figure 6 with frequency $\omega_0 = 0{\cdot}25$, and in Figure 7 with $\omega_0 = 2{\cdot}50$. Values labelled by 3Fa are calculated by placing footing "2" extremely close to footing "1" with the clear distance $d_1 = 0{\cdot}25$ while

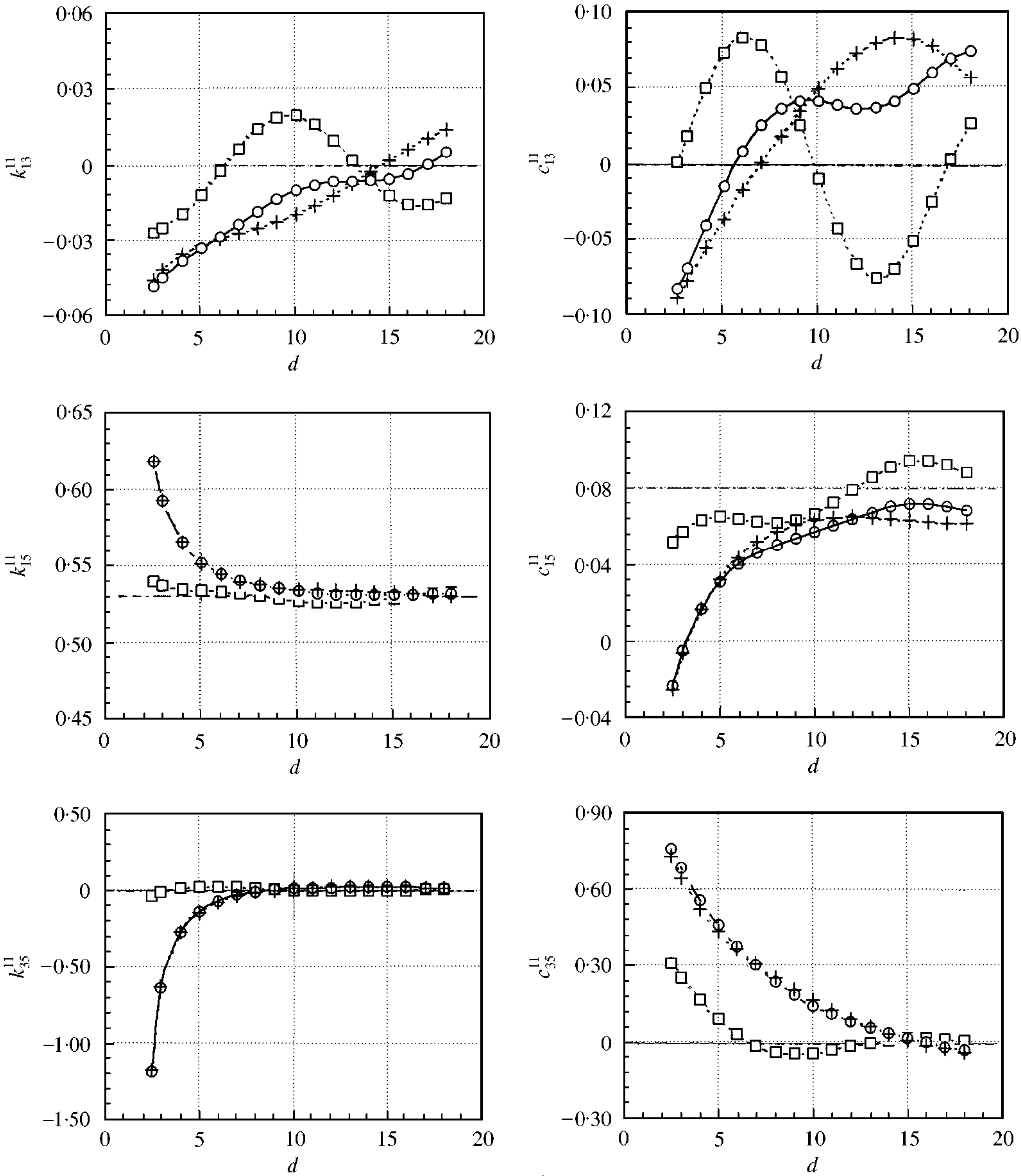


(a)

Figure 4. Spring and damping coefficients for $[K^{11}]$. –·–·– 1F; ----□---2F; --- + ---2Fa; —○— 3F.

3Fb is calculated by placing footing "2" extremely close to footing "3" with distance $d_2 = 0{\cdot}25$.

In general, spring coefficients are not very sensitive to the shifting location of the intermediate footing while the damping coefficients, that is the imaginary part of $[K]$, may change with the shifting location of footing "2", especially for rotational terms at lower frequencies. Dynamic stiffness can be either reduced by sub-soil coupling between footings or increased depending on frequency, spacing and relative position of the intermediate ones. The results seem to indicate that the presence of the intermediate footing may smoothen the variation of the dynamic stiffness with spacing especially at higher frequencies.



(b)

Figure 4. Continued.

4. CONCLUSIONS

A boundary element approach is successfully applied to study the group effect of a group of footings. An extensive parametric study suggests the following conclusions:

1. It is confirmed again that dynamic group effects among a group of footings can be strongly frequency dependent, and are governed by the ratio of spacing to the excitation wavelength.

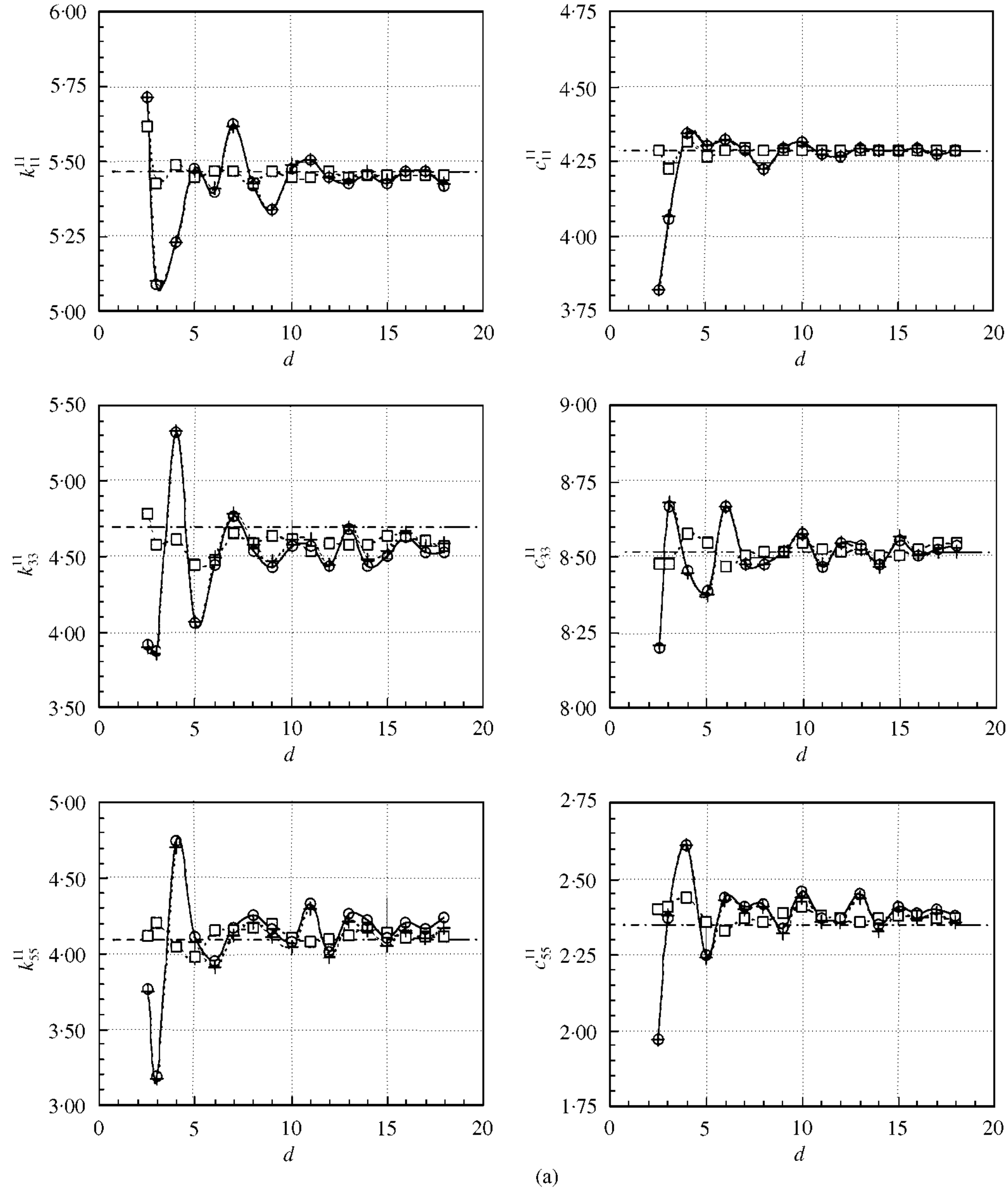
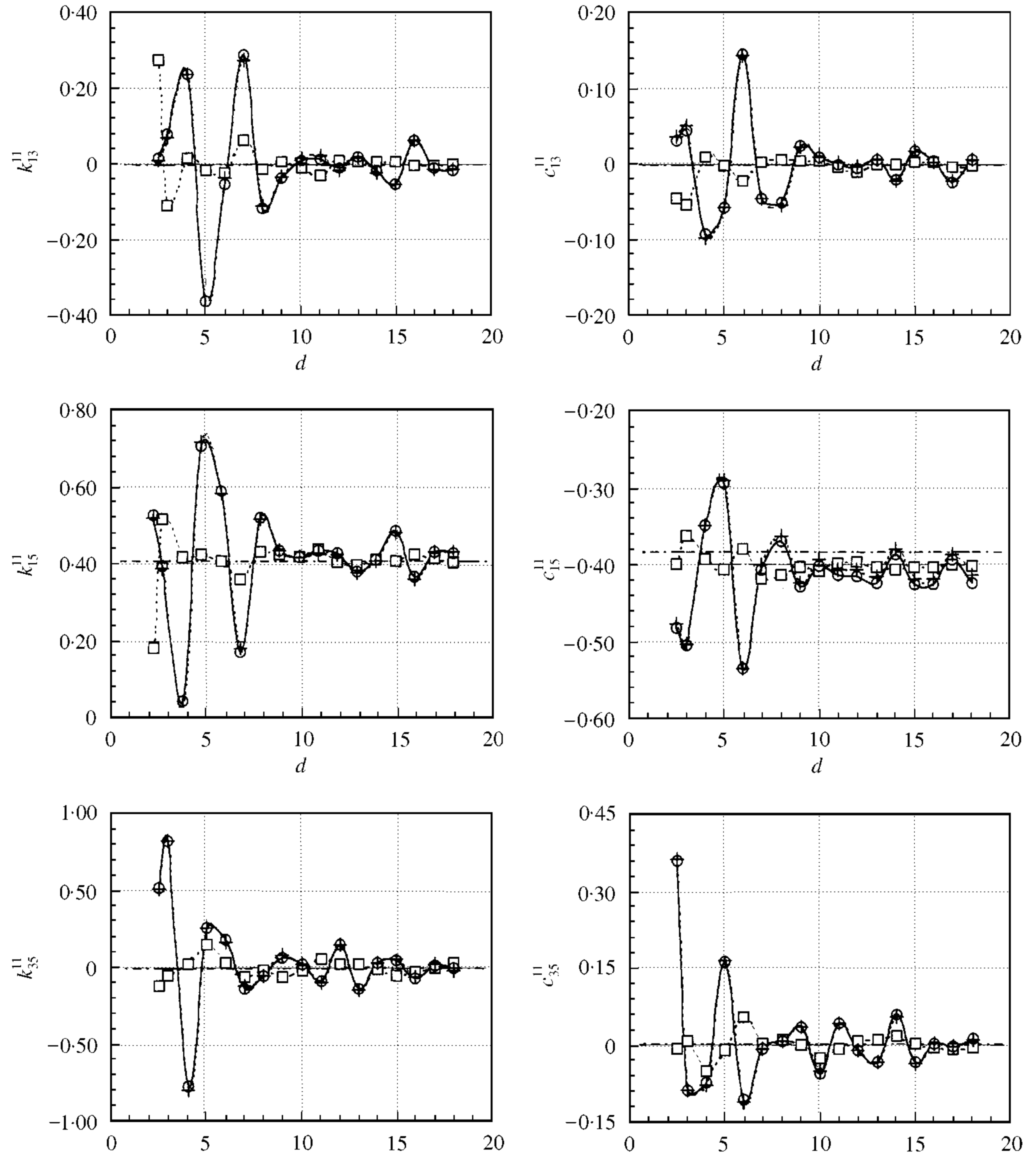


Figure 5. Spring and damping coefficients for $[K^{11}]$. –·–·– 1F; ----□---2F; --- + ---2Fa; —○— 3F.

2. Group effect may cause the dynamic stiffness of a group of footings to be considerably different from the value predicted by a simple sum of the value for each member pair in the cluster.
3. Sub-soil coupling among a group of footings can either reduce or increase the value of dynamic stiffness depending on frequency, spacing and the relative location of the intermediate footings.
4. The presence of the intermediate footing may markedly smoothen the variation of the cross-correlative stiffness terms with spacing, especially at higher frequencies.



(b)

Figure 5. Continued.

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Table 1

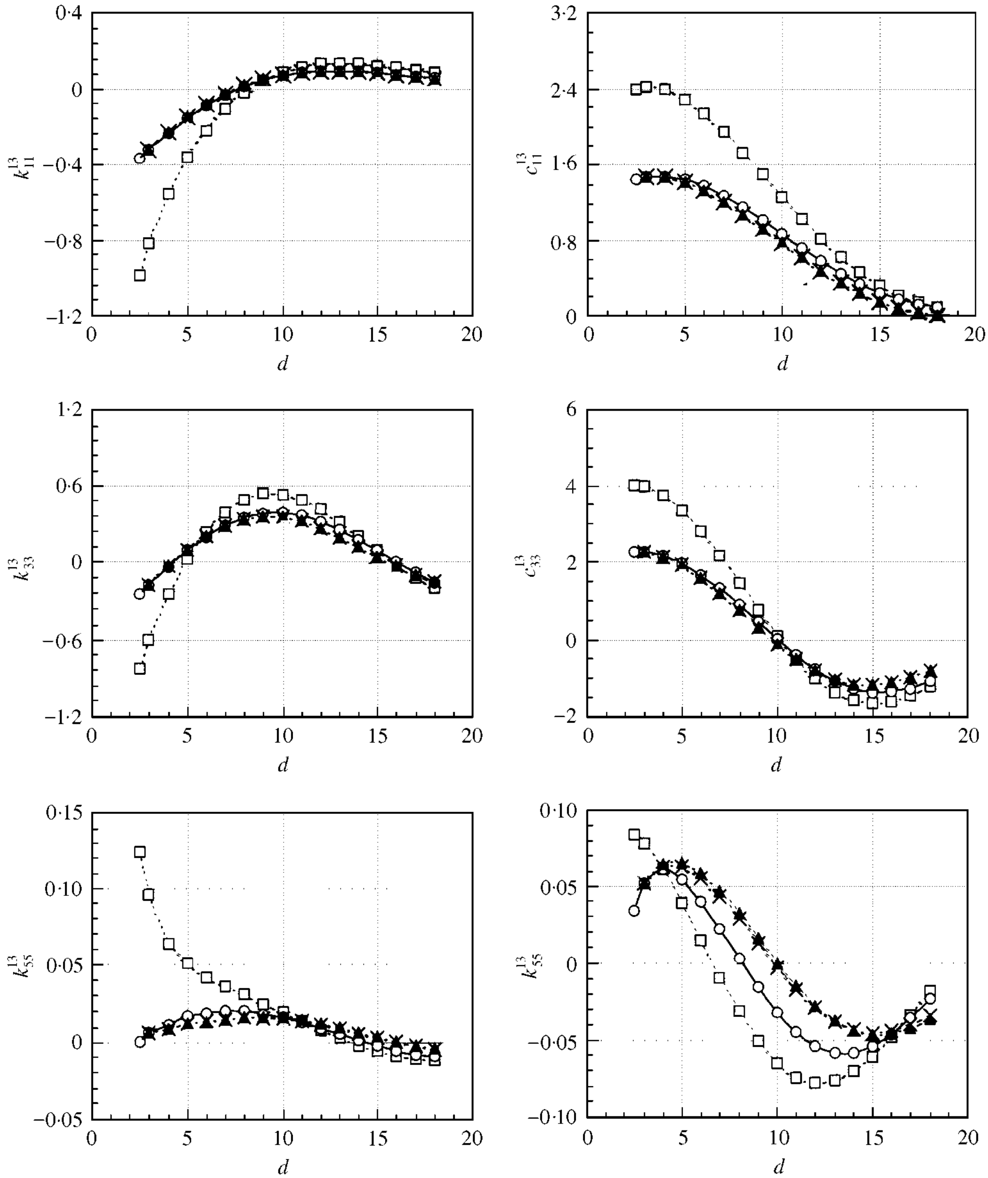
Total normalized subgrade stress σ_z/F_z beneath footing "1"

Case	$\omega_0 = 0{\cdot}25$		$\omega_0 = 2{\cdot}50$	
	$d_1 = d_2 = 0{\cdot}25$	$d_1 = d_2 = 2{\cdot}50$	$d_1 = d_2 = 0{\cdot}25$	$d_1 = d_2 = 2{\cdot}50$
BEM 1F	7·2923, 1·7412 (41·4%, − 22·5%	7·2923, 1·7412 (10·1%, − 39·6%)	4·7058, 21·2955 (40·6%, − 4·8%)	4·7058, 21·2955 (38·4%, − 3·8%)
∑2F	4·5718, 2·9314 (− 11·4%, 30·5%)	6·8156, 3·0706 (2·9%, 6·4%)	2·7474, 23·4275 (− 17·9%, 4·7%)	2·9567, 22·3281 (− 13·0%, 0·9%)
BEM 3F	5·1580, 2·2465	6·6220, 2·8849	3·3460, 22·3668	3·3996, 22·1398

Table 2

Total normalized subgrade stress σ_z/F_z beneath footing "2"

Case	$\omega_0 = 0{\cdot}25$		$\omega_0 = 2{\cdot}50$	
	$d_1 = d_2 = 0{\cdot}25$	$d_1 = d_2 = 2{\cdot}50$	$d_1 = d_2 = 0{\cdot}25$	$d_1 = d_2 = 2{\cdot}50$
BEM 1F	7·2923, 1·7412 (103%, − 3·7%)	7·2923, 1·7412 (35·1%, − 46·8)	4·7058, 21·2955 (61·4%, − 10·5%)	4·7058, 21·2955 (64·7%, − 7·0%)
∑2F	1·7604, 2·7791 (− 50·9%, 53·7%)	5·6034, 3·5356 (3·8%, 8·0%)	3·6382, 24·5647 (24·8%, 3·2%)	2·7386, 22·9689 (− 4·2%, 0·3%)
BEM 3F	3·5879, 1·8079	5·3960, 3·2746	2·9154, 23·7944	2·8577, 22·9030



(a)

Figure 6. Spring and damping coefficients for $[K^{13}]$. ---□---2F; —○—3F; ---▲---3Fa; ---×---3Fb.

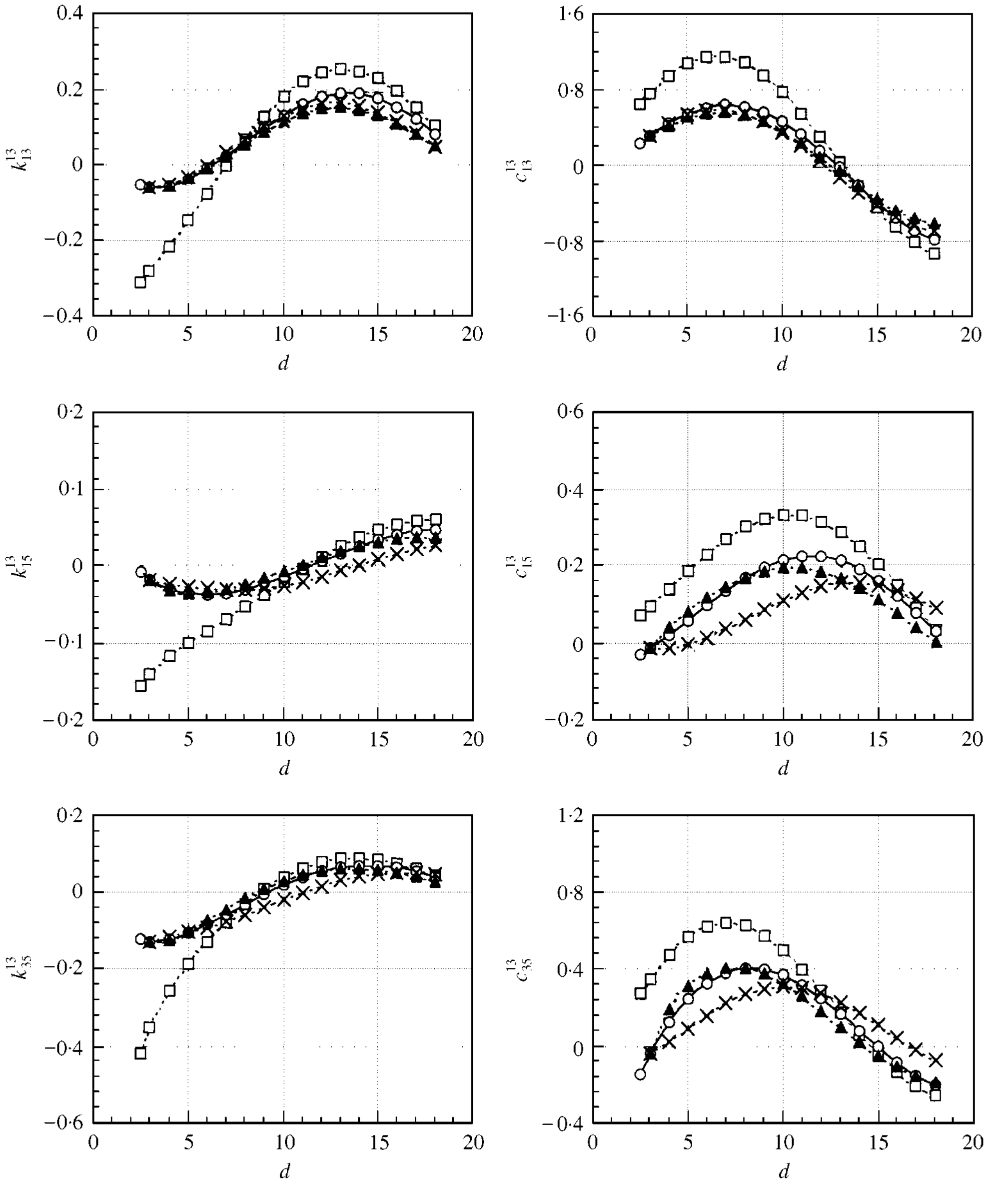


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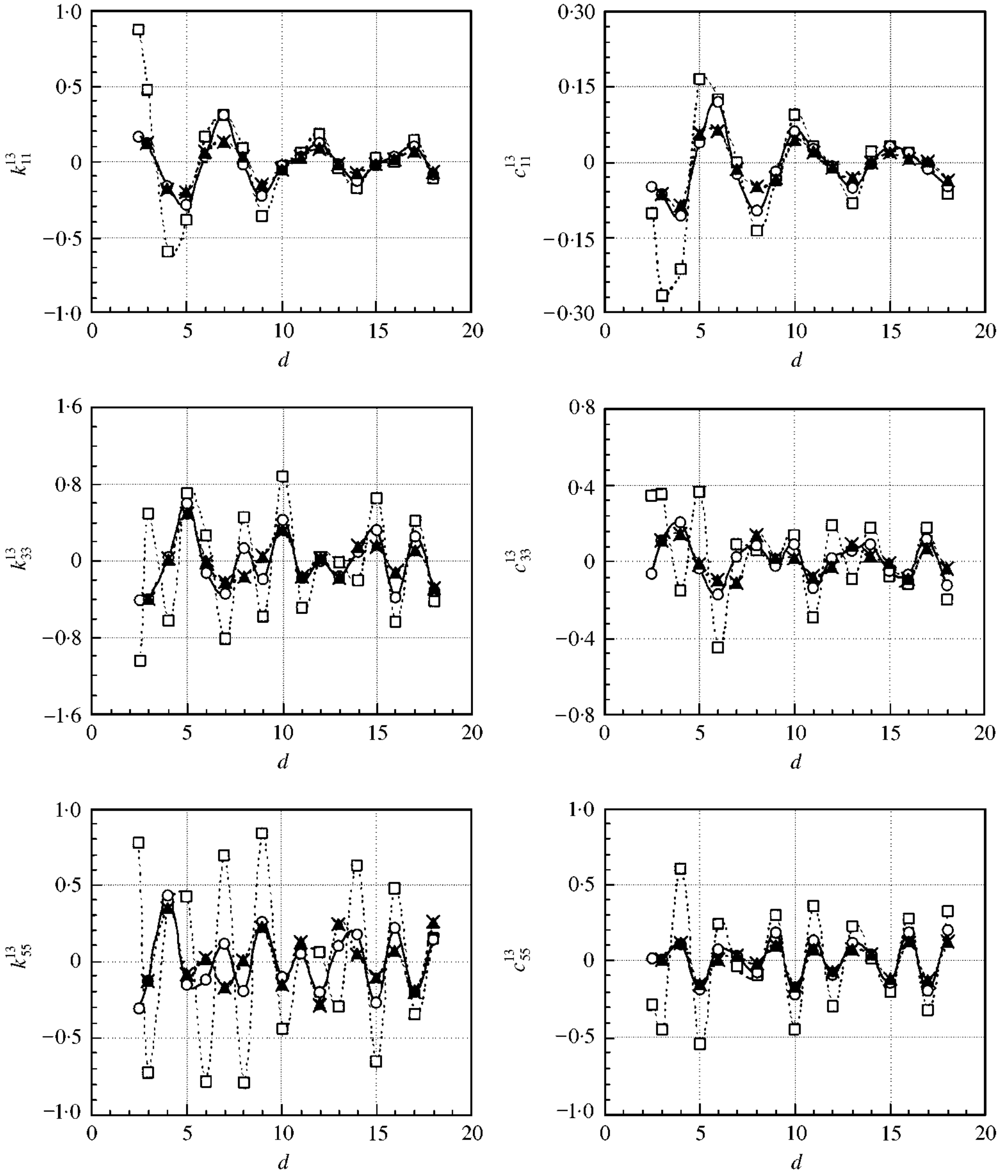
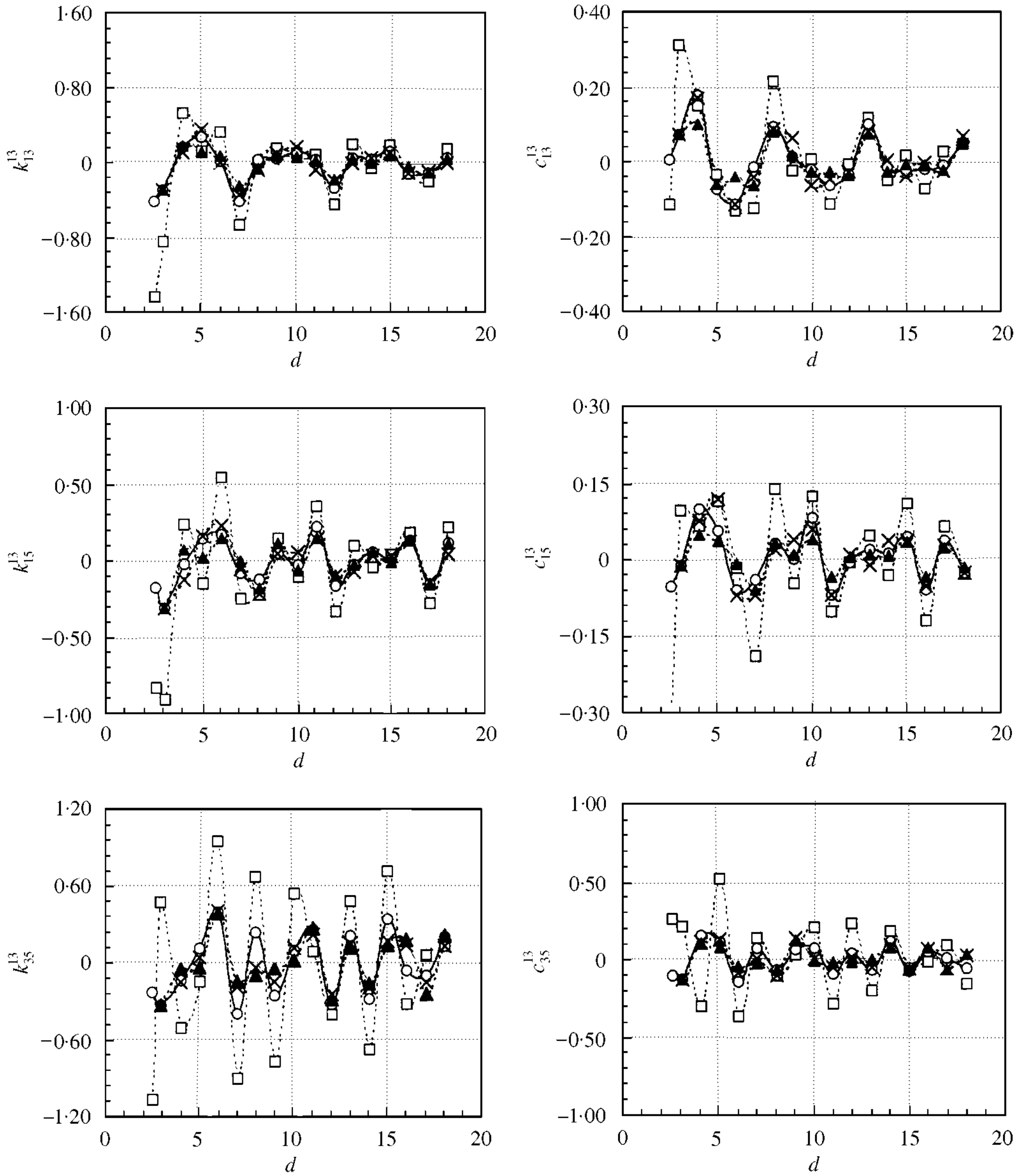
Figure 6. Continued.



(a)

Figure 7. Spring and damping coefficients for $[K^{13}]$. ---□---2F; —○—3F; ---▲---3Fa; ---×---3Fb.



(b)

Figure 7. Continued.

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