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FREE VIBRATION OF UNIAXIAL COMPOSITE CYLINDRICAL HELICAL SPRINGS WITH CIRCULAR SECTION

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The free vibration problem of unidirectional composite cylindrical helical springs is modelled theoretically as a continuous system considering the rotary inertia, shear and axial deformation effects. The first order shear deformation theory is employed in the mathematical model. The 12 scalar ordinary differential equations governing the free vibration behavior of cylindrical helical springs made of an anisotropic material are solved simultaneously by the transfer matrix method. The overall transfer matrix of the helix is computed up to any desired accuracy by using the effective numerical algorithm available in the literature. The theoretical results are verified with the reported values, which were obtained theoretically and experimentally for straight beams and helical springs. A parametric study is performed to investigate the effects of the number of active coils, the helix pitch angle and material types on the first six natural frequencies of helical springs with circular section and fixed–fixed ends.

1. INTRODUCTION

Helical springs are used in various mechanisms. The primary functions of springs are to absorb energy and mitigate shock, to apply a definite force or torque, to support moving masses or isolate vibration, to indicate control load or torque, etc. In practical applications, helical springs are in the form of cylindrical and non-cylindrical (conical, barrel, and hyperboloidal) types. Having constant curvatures along the axis makes analysis of cylindrical helical springs simpler than non-cylindrical helical springs.

The analytical solution to the static equations of cylindrical helical springs made of isotropic materials was achieved by Cinemre [1], who neglected the axial and shear deformations. In the dynamic analysis, finding the solution in closed form is very difficult. This is because the equations which govern the dynamic behavior of helices become 12 simultaneously partial differential equations containing inertial terms.

Philips and Costello [2], Costello [3], Mottershead [4], and Berdichevsky and Sutryin [5] worked out the non-linear behavior of helical springs. An extensive geometrical non-linear theory was presented in Reference [5].

Experimental determination of the natural frequencies of helical springs is also somewhat difficult because helical springs have free vibration frequencies very close in value.

The analytical formulas for the natural frequencies associated with the axial and torsional modes of cylindrical helical springs made of an isotropic material, which neglect the rotary inertia, axial and shear deformation effects, were presented by Wahl [6]. These formulas are valid for small pitch angle and large cylinder to wire diameter ratios. Yıldırım [7] presented analytical expressions for the first six natural frequencies of cylindrical helical springs made of isotropic material with circular and rectangular sections with a maximum relative error of 5%. This study includes the intervals of the number of active *turns* $n = 3$–16, the helix pitch angle $\alpha = 5, 10, 15, 20$ and 25°, and the ratio of cylinder diameter to the wire diameter $D/d = 4$–16. The rotary inertia, the axial and shear deformation effects were considered in Yıldırım's [7] study.

To the author's knowledge, there are only three papers [8–10] on the vibration analysis of the cylindrical helical springs made of composite materials. The increasing usage of fibre composite materials encompasses applications in helical springs. In order to analyze the dynamic behavior of composite helical springs, an accurate formulation of the mathematical model is required. The natural frequencies depend on the cross-sectional rigidities closely [11]. To achieve the true cross-sectional rigidities, the generalized Hooke's law must be used when considering the classical beam theory instead of adapting the plate cross-sectional rigidities for the beam rigidities [12].

Yıldırım [12] presented governing equations of initially twisted elastic space rods made of laminated composite materials. These equations are easily applicable to the static and dynamic analysis of general space rods with variable cross-sections and curvatures by the classical methods available in the literature. Borri *et al.* [13] proposed a quite general theory for the three-dimensional cross-section analysis accounting for initial twist and curvature of anisotropic and non-homogeneous beams. Cesnik *et al.* [14] offered an asymptotically exact methodology based on geometrical non-linear, three-dimensional elasticity for cross-sectional analysis of initially curved and twisted, non-homogeneous anisotropic beams.

In this study, which is a continuation of references [8–10], the governing equations for composite cylindrical helical springs are presented in a vectorial form based on the first order shear deformation theory. It is assumed that the centroid of the cross-section and the shear center coincide, the material is anisotropic, homogeneous, and linear. Furthermore, warping and pre-twisting of the cross-section are neglected; the inertia moments about normal and binormal axes attached at the mass center of the cross-section are the principal inertia moments.

The cross-sectional rigidities utilized in this study are derived from the generalized Hooke's law in consideration of the classical beam theory [12]. The rotary inertia, shear and axial deformation effects are included in the formulation. The scalar-free vibration equations, which are 12 simultaneous first order differential equations, are obtained by assuming harmonic motion. These equations are solved with the help of the transfer matrix method.

As is well known, the transfer matrix method is one of the well-established methods. However, it has not been used widely for composite analysis [15–17]. This is because the exact overall transfer matrix must be computed to obtain an accurate solution. The accuracy of the overall transfer matrix obtained by using any numerical procedure is an important issue in the transfer matrix method. In order to obtain an accurate solution of the problem, an effective numerical algorithm, which was previously verified for isotropic helices [7, 18, 19], is employed for the computation of the overall transfer matrix for composite helical springs. After verifying the results with the reported values, a through parametric study is performed. It may be noted that in references [9–11] the transfer matrix method was used for the discrete parameters model instead of the distributed parameters model as studied in the present work.

Continuation of this work is expected to lead ultimately to the development of a methodology for relating the spring's natural frequency to the design parameters for the spring. These include geometrical as well as material parameters. Consequently, the work is significant not only from a scientific point of view due to development of the related mathematical and numerical methodology, but also the engineering point of view. This is because the use of composite springs presents a good opportunity for design engineers in vibration isolation as resilient members, energy storage, and power transmission under conditions where weight and chemical resistance are primary concerns. Furthermore, composite springs which involve an organic matrix as the bonding agent and long fibers as the reinforcement will possess enhanced damping characteristics due to viscous damping and possibly due to Coulomb friction.

2. FORMULATION OF THE FREE VIBRATION PROBLEM OF COMPOSITE CYLINDRICAL HELICES

The position vector of a cylindrical helical spring, **r**, is given in terms of Cartesian unit vectors (**i**, **j**, **k**) as follows (see Figure 1(a)):

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} = (R\cos\theta)\mathbf{i} + (R\sin\theta)\mathbf{j} + (h\theta)\mathbf{k}. \tag{1}$$

Here θ is the horizontal angular displacement and R ($= D/2$) is the centerline radius of the helix. With the helix pitch angle denoted by α, the step for unit angle of the helix, h, in equation (1) can be written as

$$h = R\tan\alpha. \tag{2}$$

For a space bar, the Frenet unit vectors associated with the bar axis [see Figure 1(b)] are given as [20]

$$\begin{aligned} \mathbf{t} &= \mathrm{d}\mathbf{r}/\mathrm{d}s, \\ \mathbf{n} &= (\mathrm{d}\mathbf{t}/\mathrm{d}s)/(\mathrm{d}t/\mathrm{d}s), \\ \mathbf{b} &= \mathbf{t}\times\mathbf{n}. \end{aligned} \tag{3}$$

Here **t**, **n**, and **b** denote the tangential, normal and binormal unit vectors respectively. For the helical springs, the infinitesimal length of the bar, ds, is defined as

$$\mathrm{d}s = (\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2)^{(1/2)} = \sqrt{R^2 + h^2}\,\mathrm{d}\theta = c\,\mathrm{d}\theta. \tag{4}$$

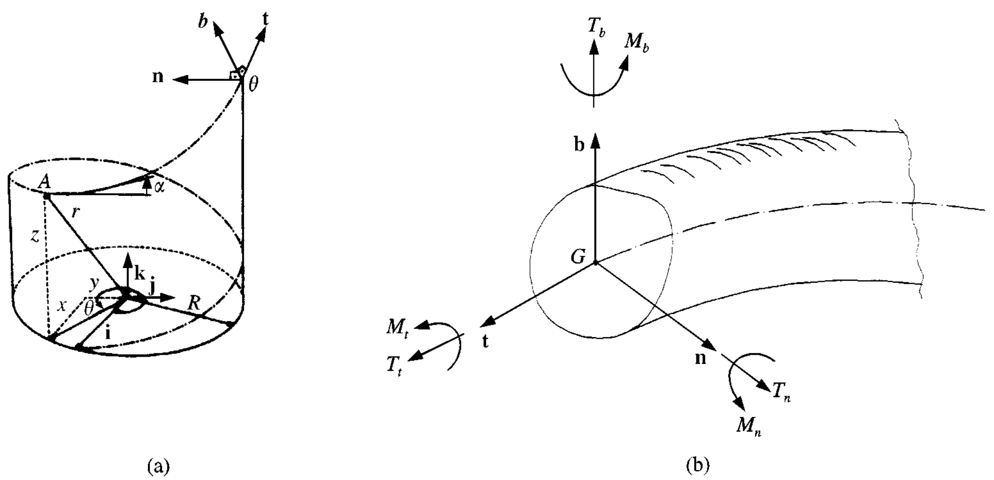

Figure 1. (a) Geometry of cylindrical helical spring; (b) stress resultants and Frenet co-ordinates.

By using equations (1), (3) and (4), the relationship between Frenet co-ordinates (**t**, **n**, **b**) and Cartesian unit vectors (**i**, **j**, **k**) is obtained in the form

$$\mathbf{t} = (-\cos\alpha\sin\theta)\mathbf{i} + (\cos\alpha\cos\theta)\mathbf{j} + \sin\alpha\mathbf{k},$$
$$\mathbf{n} = -\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j},$$
$$\mathbf{b} = (\sin\alpha\sin\theta)\mathbf{i} + (-\sin\alpha\cos\theta)\mathbf{j} + \cos\alpha\mathbf{k}. \tag{5}$$

Frenet unit vectors are related to each other by the following relations [20]:

$$\mathrm{d}\mathbf{t}/\mathrm{d}s = \chi\mathbf{n},$$
$$\mathrm{d}\mathbf{n}/\mathrm{d}s = \tau\mathbf{b} - \chi\mathbf{t},$$
$$\mathrm{d}\mathbf{b}/\mathrm{d}s = -\tau\mathbf{n}. \tag{6}$$

Here χ and τ represent the curvature and tortuosity of a curve in space. These curvatures reduce to the following for a cylindrical helical spring:

$$\chi = \cos^2\alpha/R = R/c^2 = constant, \quad \tau = \sin\alpha\cos\alpha/R = h/c^2 = constant. \tag{7}$$

By cancelling the terms for initial twist of cross-section, the free vibration equations of space rods made of an anisotropic material are obtained in a vectorial form as [12]

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = -\rho\omega^2\mathbf{U},$$
$$\frac{\mathrm{d}M}{\mathrm{d}s} + \mathbf{t}\times\mathbf{T} = -\rho\mathbf{I}_{\mathrm{i}},\ \omega^2\mathbf{\Omega} \quad (i = t, n, b), \tag{8a, b}$$

$$\frac{\mathrm{d}\mathbf{U}}{\mathrm{d}s} = \mathbf{A}'\mathbf{T} + \mathbf{B}'\mathbf{M} + \mathbf{\Omega}\times\mathbf{t}, \quad \frac{\mathrm{d}\mathbf{\Omega}}{\mathrm{d}s} = \mathbf{D}'\mathbf{M} + \mathbf{F}'\mathbf{T} \tag{8c, d}$$

where **T** and **M** are the internal force and internal moment vectors, **U** and **Ω** are the displacement and rotation vectors for a point on the bar axis respectively. The density of the material and the circular frequency (rad/s) are denoted by ρ and ω respectively. $\mathbf{A}'$, $\mathbf{F}'$, $\mathbf{B}'$ and $\mathbf{D}'$ matrices comprise the cross-sectional rigidities for composite bars. In Yıldırım's formulation [12], these matrices contain just material and cross-sectional properties. The effects of χ and τ on the 1-D constitutive matrix that may be important in the analysis are neglected in this study. These effects will be treated in later work.

For symmetric section and 0° fibers, $\mathbf{F}' = \mathbf{B}'^{\mathrm{T}} = 0$ and

$$\mathbf{A}' = \begin{bmatrix} 1/A_{11} & 0 & 0 \\ 0 & 1/A_{22} & 0 \\ 0 & 0 & 1/A_{33} \end{bmatrix}, \quad \mathbf{D}' = \begin{bmatrix} 1/D_{11} & 0 & 0 \\ 0 & 1/D_{22} & 0 \\ 0 & 0 & 1/D_{33} \end{bmatrix}, \tag{9}$$

For unidirectional layers, the elements of extensional and bending stiffness matrices, **A** and **D**, are obtained as

$$A_{11} = \bar{Q}_{11}A, \quad A_{22} = \bar{Q}_{22}A, \quad A_{33} = \bar{Q}_{33}A,$$
$$D_{11} = \bar{Q}_{33}I_b + \bar{Q}_{22}I_n, \quad D_{22} = \bar{Q}_{11}I_n, \quad D_{33} = \bar{Q}_{11}I_b, \tag{10}$$

where A is the undeformed cross-sectional area of the cross-section, I_n and I_b are the inertia moments about the normal and binormal axes. $\bar{Q}_{ij}$ are the elements of the reduced stiffness matrix which can be obtained in terms of the elements of general three-dimensional stiffness and compliance matrices, **C** and **S**, respectively, as [12]

$$\bar{Q}_{11} = C_{11} + (C_{12}S_{12} + C_{13}S_{13})/S_{11}, \quad \bar{Q}_{22} = C_{66}, \quad \bar{Q}_{33} = C_{55}, \tag{11}$$

and

$$\{\bar{\sigma}_1, \bar{\tau}_{12}, \bar{\tau}_{31}\}^{\mathrm{T}} = \bar{\mathbf{Q}}\{\bar{\varepsilon}_1, \bar{\gamma}_{12}, \bar{\gamma}_{31}\}^{\mathrm{T}}, \tag{12}$$

where (1, 2, 3) axes coincide with the Frenet co-ordinates.

For a three-dimensional body the stress–strain relationship, σ–ε, is assumed to be in the form

$$\{\sigma_1, \sigma_2, \tau_3, \tau_{23}, \tau_{31}, \tau_{12}\}^{\mathrm{T}} = \mathbf{C}\{\varepsilon_1, \varepsilon_2, \varepsilon_3, \gamma_{23}, \gamma_{31}, \gamma_{12}\}^{\mathrm{T}}. \tag{13}$$

The scalar components of the vector quantities in equations (8) are [see Figure 1(b)].

$$\mathbf{T} = T_t\mathbf{t} + T_n\mathbf{n} + T_b\mathbf{b}, \quad \mathbf{M} = M_t\mathbf{t} + M_n\mathbf{n} + M_b\mathbf{b}, \quad \mathbf{T} = U_t\mathbf{t} + U_n\mathbf{n} + U_b\mathbf{b},$$
$$\mathbf{\Omega} = \Omega_t\mathbf{t} + \Omega_n\mathbf{n} + \Omega_b\mathbf{b}, \tag{14}$$

By substituting equations (6), (9) and (14) into equations (8), the scalar-free vibration equations of a cylindrical helical spring made of unidirectional composite layers are obtained in (**t**, **n**, **b**) reference frame as follows:

$$\frac{\mathrm{d}U_t}{\mathrm{d}s} = \chi U_n + A'_{11}T_t,$$

$$\frac{\mathrm{d}U_n}{\mathrm{d}s} = -\chi U_t + \tau U_b + \Omega_b + k'A'_{22}T_n,$$

$$\frac{\mathrm{d}U_b}{\mathrm{d}s} = -\tau U_n - \Omega_n + k'A'_{33}T_b,$$

$$\frac{\mathrm{d}\Omega_t}{\mathrm{d}s} = \chi\Omega_n + D'_{11}M_t,$$

$$\frac{\mathrm{d}\Omega_n}{\mathrm{d}s} = -\chi\Omega_t + \tau\Omega_b + D'_{22}M_n,$$

$$\frac{\mathrm{d}\Omega_b}{\mathrm{d}s} = -\tau\Omega_n + D'_{33}M_b,$$

$$\frac{\mathrm{d}T_t}{\mathrm{d}s} = \chi T_n - \bar{A}\omega^2 U_t$$

$$\frac{dT_n}{\mathrm{d}s} = \chi T_b - \chi T_t - \bar{A}\omega^2 U_n,$$

$$\frac{\mathrm{d}T_b}{\mathrm{d}s} = -\tau T_n - \bar{A}\omega^2 U_b$$

$$\frac{dM_t}{\mathrm{d}s} = \chi M_n - \bar{I}_1 n\omega^2 \Omega_t$$

$$\frac{\mathrm{d}M_n}{\mathrm{d}s} = \tau M_b - \chi M_t + T_b - \bar{I}_2\omega^2\Omega_n,$$

$$\frac{\mathrm{d}M_b}{\mathrm{d}s} = -\tau M_n - T_n - \bar{I}_3\omega^2\Omega_b. \tag{15}$$

Here k' represents the shear correction factor. Although equations (15) can be used for any doubly symmetric cross-section, a solid circular section is considered in this study and $k'' = 1{\cdot}1$ is assumed [21]. For the circular section, the other quantities in equations (15) are

$$\bar{A} = \rho A, \quad \bar{I}_2 = \bar{I}_3 = \rho\pi d^4/64, \quad \bar{I}_1 = 2\bar{I}_2. \tag{16}$$

3. SOLUTION BY THE TRANSFER MATRIX METHOD

By referring to equations (15), the state vector can be defined as follows

$$\mathbf{Z}(s) = \begin{bmatrix} U_t \\ U_n \\ U_b \\ \Omega_t \\ \Omega_n \\ \Omega_b \\ T_t \\ T_n \\ T_b \\ M_t \\ M_n \\ M_b \end{bmatrix}. \tag{17}$$

By using this definition, equations (15) can be expressed in a matrix notation as

$$\mathrm{d}\mathbf{Z}(s)/\mathrm{d}s = \mathbf{D}^o\mathbf{Z}(s), \tag{18}$$

where $\mathbf{D}^o$ is the dynamic differential matrix. In the transfer matrix method, the solution to equation (18) is given as [22]

$$\mathbf{Z}(s) = \mathbf{F}\mathbf{Z}(0) \tag{19}$$

where $\mathbf{Z}(0)$ is the state vector at $s = 0$. In this study, a series expression of the overall transfer matrix obtained by the Cayley-Hamilton theorem is utilized [7, 18, 19] as follows

$$\mathbf{F}(s) = \sum_{k=0}^{11} \Phi_k(s)\mathbf{D}^{o^k}. \tag{20}$$

Numerical computation of the overall transfer matrix in an accurate manner is a crucial step in the transfer matrix method. The number of terms from the infinite series $\Phi_k(s)$, which can be considered in the computation determines the accuracy of the solution. The numerical algorithm developed by Yıldırım [7, 18, 19] allows utilization of equation (20) with a variable number of terms depending on the degree of required accuracy for large helix angles, D/d ratios and the number of active turns. This algorithm, which is restricted to unidirectional (0 or 90°) and cross-ply laminates, is also examined for the free vibration of circular composite bars and straight composite beams [23, 24]. For angle-ply laminates, the overall transfer matrix must be obtained by differentiating the free vibration equation set [12].

After accurate computation of all $\Phi_k(s)$ functions, the frequency equation can be obtained from the boundary conditions given at both ends ($s = 0$ and $s = 2\pi nc = 2\pi nR/\cos\alpha$ *with* $n =$ *being the number of active turns*) by using equation (19). The boundary conditions for clamped end are: $U_t = U_n = U_b = 0$ *and* $\Omega_t = \Omega_n = \Omega_b = 0$.

If the transfer matrix consists of sixteen sub-matrices, Equation (19) is rewritten as follows

$$\begin{bmatrix} \mathbf{U} \\ \Omega \\ \mathbf{T} \\ \mathbf{M} \end{bmatrix}_{s = 2\pi nR/\cos\alpha} = \begin{bmatrix} \mathbf{F}_1 & \mathbf{F}_2 & \mathbf{F}_3 & \mathbf{F}_4 \\ \mathbf{F}_5 & \mathbf{F}_6 & \mathbf{F}_7 & \mathbf{F}_8 \\ \mathbf{F}_9 & \mathbf{F}_{10} & \mathbf{F}_{11} & \mathbf{F}_{12} \\ \mathbf{F}_{13} & \mathbf{F}_{14} & \mathbf{F}_{15} & \mathbf{F}_{16} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \Omega \\ \mathbf{T} \\ \mathbf{M} \end{bmatrix}_{s = 0}. \tag{21}$$

Thus, the eigenvalue equation for fixed–fixed ends reduces to the following

$$\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}_{s = 2\pi nR/\cos\alpha} = \begin{bmatrix} \mathbf{F}_3 & \mathbf{F}_4 \\ \mathbf{F}_7 & \mathbf{F}_8 \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{M} \end{bmatrix}_{s=0}. \tag{22}$$

In this study, the free vibration frequencies are obtained by the method of searching determinant. After attributing numerical values to the natural frequency, the overall transfer matrix is computed. The values making the determinant zero are the natural frequencies of the helix. All numerical computations were performed by using the double-precision arithmetic. Mode shapes are not considered in this study. The procedure for determining mode shapes is available in reference [16].

4. NUMERICAL EXAMPLES

First of all, a cantilevered straight beam made of a unidirectional composite material is examined to check the accuracy of the present frequencies. The material and cross-sectional properties of the beam are given as follows: $G_{23} = 2{\cdot}54$ GPa, $E_1 = 129$ GPa, $E_2 = E_3 = 9{\cdot}39$ GPa, $G_{12} = G_{13} = 4{\cdot}3$ GPa, $\nu_{12} = 0{\cdot}3$, $k'' = 1{\cdot}2$, $\rho = 1551{\cdot}47$ kg/m^3, width $= 1{\cdot}27$ cm, thickness $= 0{\cdot}317$ cm, length/thickness $= 60$. The fundamental natural frequencies for the out-of-plane bending oscillations are given in Table 1. A good

TABLE 1

Fundamental frequencies (in Hz) of cantilevered straight beam with rectangular cross-section

	Fiber directions	
	0°	90°
Abarcar and Cunniff [15] (experimental)	126·5	35·5
Hodges *et al.* [11] (finite element)	128·94	34·858
Present	128·62	34·768

Table 2

The first six natural frequencies (Hz) of a helical spring made of an isotropic material. ($\alpha = 8{\cdot}5744°$, $D/d = 10$, $\nu = 0{\cdot}3$, $d = 1$ mm, $R = 5$ mm, $n = 7{\cdot}6$, $\rho = 7900\ \mathrm{kg/m^3}$, $E = 2{\cdot}06 \times 10^{11}\ \mathrm{N/m^2}$)

	Mottershead (experiment) [25]	Mottershead (finite element) [25]	Pearson (transfer matrix) [26]	Xiong and Tabarrok (finite element) [27]	Present (transfer matrix)
ω_1	391	396	395	395	394
ω_2	391	397	398	398	396
ω_3	459	469	456	464	463
ω_4	528	532	518	528	526
ω_5	878	887	860	868	864
ω_6	878	900	875	881	877

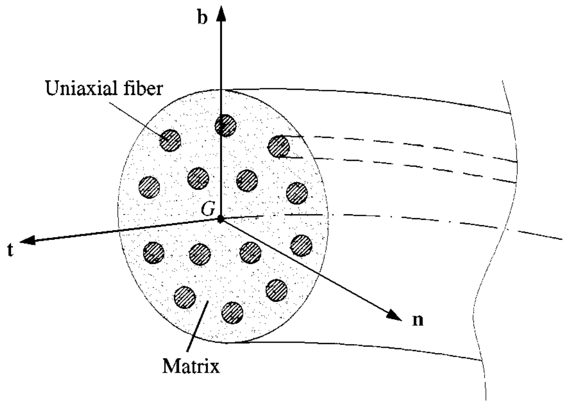

Figure 2. Section of uniaxial composite helical springs.

agreement is observed with the theoretical and experimental results reported in the literature.

As a second example, a helical spring made of an isotropic material is examined. The first six natural frequencies are presented in Table 2. The agreement with previously published results is very good.

After verifying the validity of the present results, a dimensionless parametric study of a 0° unidirectional composite cylindrical helical spring with fixed–fixed ends is performed (see Figure 2). The effects of the number of active turns, the helix pitch angle and the material types on the first six natural frequencies for $D/d = 10$ are investigated. The material properties used for comparison are given in Table 3. The dimensionless frequency is defined as follows:

$$\bar{\omega} = \omega\sqrt{\rho\, AR^4/(E_1\bar{I}_2)}. \tag{23}$$

The variations of the frequencies with the chosen vibrational parameters are shown in Figures 3 and 4. As observed from Figures 3 and 4, the natural frequencies decrease with increasing helix pitch angles for all types of materials. This is due to an increase in the total

TABLE 3

The transversely isotropic material properties used in this study.

	Carbon-epoxy[1] (AS4/3501-6)	Carbon-epoxy[2] (T300/N5208)
E_1 (GPa)	144·8	181·0
E_2 (GPa)	9·65	10·3
G_{12} (GPa)	4·14	7·17
G_{23} (GPa)	3·45	3·433
ρ (kg/m^3)	1389·23	1600·0
ν_{12}	0·3	0·28

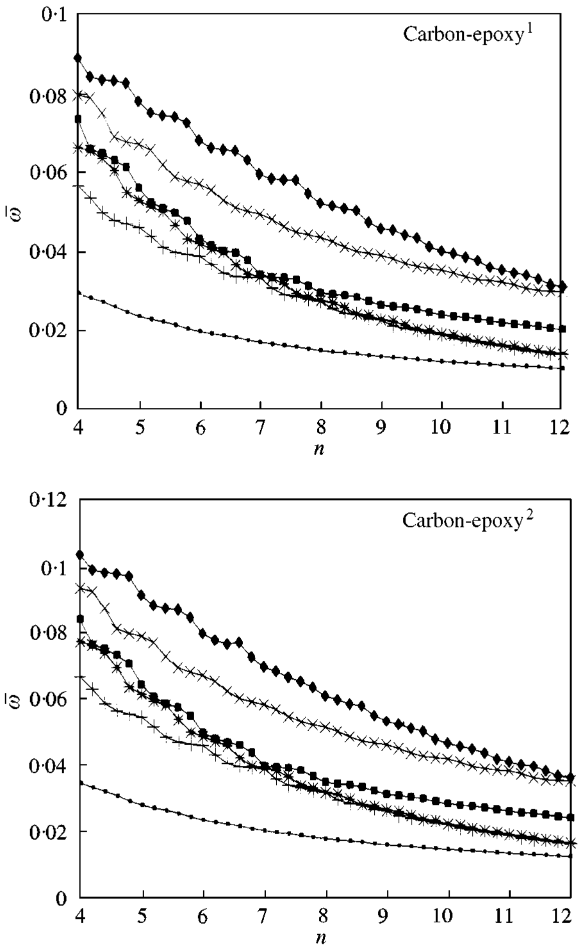

Figure 3. Variation of the first six natural dimensionless frequencies with the number of active turns, and material types (α = helix pitch angle = 5°).

length of the helix ($L = 2\pi nR/\cos\alpha$). If the number of active turns increases, the same result is encountered. Although two materials have similar free vibration characteristics, the carbon-epoxy[2] (graphite-epoxy) material (T300/N5208) gives the highest frequencies.

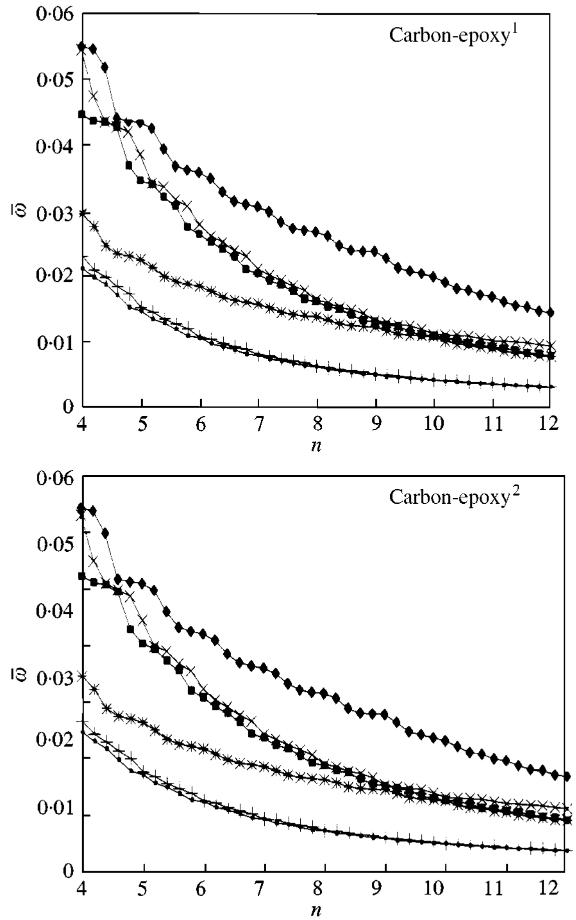

Figure 4. Variation of the first six natural dimensionless frequencies with the number of active turns, and material types (α = helix pitch angle = 25°).

The formulation presented in this work has taken the effects of axial and shear deformations, and rotatory inertia into account. These effects are presented in Tables 4 and 5 for carbon-epoxy[2] material. As can be observed from the tables, neglecting the rotatory inertia, axial and shear deformation effects causes an increase in natural frequencies. The effect of the shear deformation is more significant than others. If the D/d ratios and the number of active turns decrease the effects of the rotatory inertia, axial and shear deformation become significant especially for the higher frequencies.

5. DISCUSSION AND CONCLUSION

As is well known, manufacturing of unidirectional composite helices is less complicated than that of laminated ones. Moreover, helical springs used in practice are mostly manufactured with circular sections without initial twist. In contrast to the helical springs, initial twist is considered for the statical or dynamical analysis of rods like rotor blades. Thus, the linear free vibration of 0° unidirectional composite helices with circular sections is considered in the present work to support the theory presented here by an experimental

TABLE 4

Effects of the axial and shear deformation, and the rotatory inertia on the natural frequencies (in Hz) of carbon-epoxy[2] material ($d = 1$ mm, $\alpha = 5°$, $D/d = 5$) (AD = axial deformation, SD = shear deformation, RI = rotatory inertia)

	No. of active turns (n)	Including AD, SD, and RI effects	Excluding AD effect	Excluding SD effect	Excluding AD and SD effects
ω_1	1	13344·82	13356·80	13932·29	13950·85
ω_2		31418·87	31490·81	38012·82	38204·64
ω_3		36954·71	36961·88	39333·22	39345·25
ω_4		68439·02	69024·64	89818·55	89842·87
ω_5		78187·39	78199·79	106144·8	108222·7
ω_6		127977·0	129470·9	163991·1	164090·5
ω_1	4	2321·33	2321·34	2345·50	2345·51
ω_2		4470·94	4471·06	4515·95	4516·07
ω_3		5158·32	5159·75	5220·88	5222·26
ω_4		5594·30	5598·47	5712·45	5716·58
ω_5		6277·12	6277·42	6324·61	6324·89
ω_6		6916·74	6918·50	7004·90	7007·01
ω_1	8	1167·39	1167·39	1179·73	1179·74
ω_2		2097·16	2098·08	2125·68	2126·62
ω_3		2104·31	2105·33	2133·83	2134·88
ω_4		2320·17	2320·18	2344·64	2344·65
ω_5		3431·79	3431·84	3466·82	3466·87
ω_6		4044·55	4045·72	4092·95	4094·08
ω_1	12	777·67	777·67	785·82	785·83
ω_2		1043·70	1044·01	1052·59	1052·91
ω_3		1049·48	1049·77	1058·69	1058·99
ω_4		1554·69	1554·69	1571·11	1571·12
ω_5		2316·11	2316·14	2340·04	2340·07
ω_6		2375·42	2376·26	2402·11	2402·96

study which will be performed in the near future. The eccentricity (shift in the neutral axis) due to the initial curvature may also be neglected [25–28] for circular or square sections. This effect commonly becomes significant for helical bars with wide rectangular sections, e.g., helical staircases [28]. In this study, the free vibration analysis of composite helical springs having 0° unidirectional fibers along the helix axis is studied based on the transfer matrix method. The rotary inertia, and the axial and shearing deformation terms are considered in the formulation. The effects of the number of active turns ($n = 4$–12), the helix pitch angle ($\alpha = 5°$ and $\alpha = 25°$) and the types of Carbon-epoxy materials are investigated. The first six free vibration frequencies of composite cylindrical helical springs with circular section and clamped–clamped ends are considered and the results are presented in non-dimensional graphical forms for $D/d = 10$.

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TABLE 5

Effects of the axial and shear deformation, and the rotatory inertia on the natural frequencies (in Hz) of carbon-epoxy[2] *material* ($d = 1$ mm, $\alpha = 5°$, $D/d = 10$) (*AD* = *Axial deformation, SD* = *shear deformation, RI* = *rotatory inertia*)

	No. of active turns (n)	Including AD, SD, and RI effects	Excluding AD effect	Excluding SD effect	Excluding AD and SD effects
ω_1	1	3456·36	3457·37	3497·59	3498·92
ω_2		8986·25	8995·26	9555·98	9568·01
ω_3		9883·83	9884·50	10057·58	10058·39
ω_4		22561·67	22563·02	23564·75	23566·48
ω_5		23154·79	23241·29	27004·00	27136·79
ω_6		41444·57	41449·11	44879·97	44887·24
ω_1	4	584·97	584·97	586·50	586·50
ω_2		1126·87	1126·88	1129·71	1129·72
ω_3		1305·87	1305·96	1309·82	1309·91
ω_4		1425·17	1425·43	1432·61	1432·87
ω_5		1581·20	1581·22	1584·20	1584·22
ω_6		1750·18	1750·31	1756·04	1756·18
ω_1	8	294·18	294·18	294·96	294·96
ω_2		531·22	531·28	533·05	533·11
ω_3		532·54	532·61	534·42	534·48
ω_4		584·72	584·72	586·27	586·27
ω_5		864·85	864·85	867·06	867·07
ω_6		1024·57	1024·65	1027·62	1027·68

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