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DYNAMIC AND ENERGETIC ANALYSES OF A STRING/SLIDER NON-LINEAR COUPLING SYSTEM BY VARIABLE GRID FINITE DIFFERENCE

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In this paper, a string/slider non-linear coupling system with time-dependent boundary condition is considered. One partial differential equation (PDE), describing the transverse small-amplitude vibration of the string, non-linearly coupled with one ordinary differential equation (ODE), describing the horizontal displacement of the slider, are derived by Hamilton's principle. This is a moving boundary problem since the unknown position of the slider has to be determined as a part of the solutions. A transformation of the variable that converts the original non-stationary boundary conditions to a set of fixed boundary conditions is proposed to avoid the increased complication and loss of accuracy associated with unequal space intervals near the moving boundary. The finite difference method with variable grid is employed to show the numerical results of the coupling effect between the string and slider. Finally, some periodic motions of the moving boundary are assigned to show the divergence of the string vibrations.

1. INTRODUCTION

String-like systems, including tapes, belts, chains, band, threads wires fibers, and other materials with negligible bending rigidity and a straight, unsagged equilibrium configuration, have broad applications in the areas of chemical, textile, computer, and tape-recorder industries as well as in many other processes. Many workers [1–3] on the vibration behavior of string-like problems have been studied for the systems with a fixed length and no axial motion. Some researches [4–6] have recently appeared in literature concerning vibration and dynamic stability of axially moving materials such as travelling string, tapes, cables, beams and plates. Although string-like systems exhibit movement, the interest of such studies is still in the fixed length consideration.

Kotera and Kawai [7] analyzed the free vibrations of a string with time-varying lengths by Laplace transformation. Ram and Caldwell [8] introduced a new method of distorted image to resolve the free vibration of a string with moving boundary conditions. Fung *et al.* [9, 10] used the Galerkin approximation with time-dependent basis functions to analyze dynamic response and stability of a string/slider system. The Galerkin approach is too

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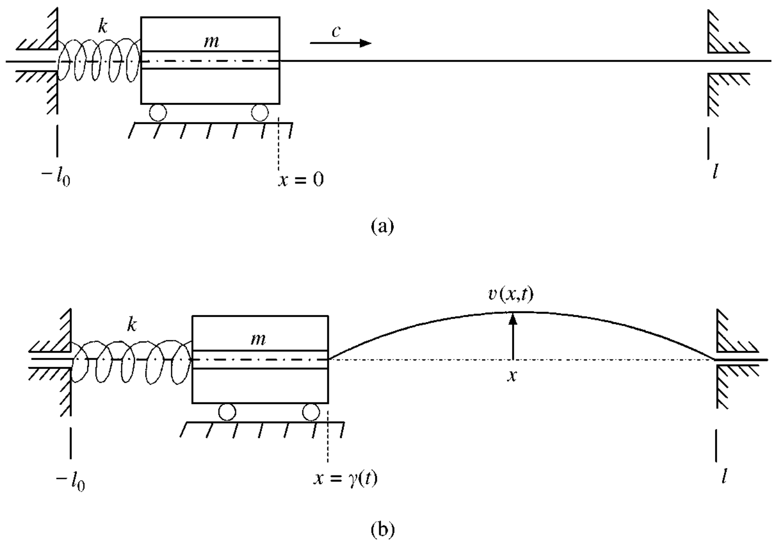

Figure 1. The string/slider coupling system: (a) undeformed configuration, (b) deformed configuration.

computationally intensive due to the time-dependent boundary and its complex mode shape. Wang *et al.* [11] studied a three-dimensional underwater cable with time-dependent length by variable-size finite element method (FEM). The FEM is also time consuming and less amenable to vectorization. However, the FDM [12] is the most popular choice for numerical solutions of the moving boundary problems. By using the FDM, the idea of a co-ordinate transformation to fix a moving boundary [13] is extended to the string/slider coupling system. The FDM simplifies the process of the formulation and programming and conserves the accuracy of the solutions.

The physical model studied in this paper is shown in Figure 1, in which the lumped mass and spring could be regarded as one subsystem to affect the main string system on the vibration amplitudes. In the present work, the transverse small-amplitude vibration of string is considered in the string/slider coupling system, and Hamilton's principle is employed to develop the non-linearly coupled equations of motion. A special FDM with variable-grid scheme is proposed to approximate both the moving boundary and the PDE at the neighboring grid points. The variable-grid method based on co-ordinate transformation, which transforms the time-varying domain into an invariant one, is employed for solving the moving boundary problems. In the numerical simulations, some periodic motions of the moving boundary are given to investigate the energy growth and divergent vibration amplitudes.

2. FORMULATION OF EQUILIBRIUM EQUATIONS

The string/slider coupling system is shown in Figure 1. The mass-spring appearing in the left end is one-degree-of-freedom (d.o.f.) system, which coupled with the continuous string system. It is assumed that the string moving through the slider is frictionless. Position $x = 0$ is the static equilibrium one of the slider, and it is also the left boundary of the string in the undeformed configuration. Position $x = \gamma(t)$ is the current one of the slider and is the left-hand side moving boundary of the string in the deformed configuration. Neglecting the longitudinal elastic motion of the string [1, 2] is an appropriate assumption at lower frequencies and small amplitude of lateral motion. To obtain the coupled equations of motion. Hamilton's principle which states that the variation of the time integral of the kinetic energy minus the potential energy of the string/slider coupling system is zero will be used. However, the application of the principle is not straightforward, since there is a moving boundary involved.

2.1. EQUATION OF MOTION

Hamilton's principle is applicable to a set of particles and when this set tends towards infinity the aggregate of particles are considered to form the continuum. In the process of variation, the aggregate of particles is considered fixed, i.e., between any two times t_1 and t_2 the same particles are considered throughout the variation. To this end, we first consider the entire length of the string and write Hamilton's principle as

$$\int_{t_1}^{t_2} \delta\left(\int_{-l_0}^{\gamma(t)} L'\,\mathrm{d}x + \int_{\gamma(t)}^{l} L\,\mathrm{d}x + \tfrac{1}{2} m\dot{\gamma}^2(t) - \tfrac{1}{2} k\gamma^2(t)\right)\mathrm{d}t = 0, \tag{1}$$

where

$$L = \tfrac{1}{2}\rho(v_t + cv_x)^2 - \tfrac{1}{2} T v_x^2 \tag{2}$$

is the Lagrangian density of string for $\gamma(t) \leqslant x \leqslant l$, in which the transverse small-amplitude vibration $v(x, t)$ of the string is considered; m is the mass of the slider, k is the spring stiffness, c is the travelling speed of the string, ρ is mass per length of the string and T is the initial tension. In the interval $-l_0 \leqslant x \leqslant \gamma(t)$, the integral in equation (1) vanishes since over this interval the transverse deflections and slopes of the string are zero and, consequently, the Lagrangian density function, L', is equal to zero. Hence, equation (1) may finally be expressed in the form

$$\int_{t_1}^{t_2} \delta\left(\int_{\gamma(t)}^{l} L\,\mathrm{d}x + \tfrac{1}{2} m\dot{\gamma}^2(t) - \tfrac{1}{2} k\gamma^2(t)\right)\mathrm{d}t = 0. \tag{3}$$

Performing the variation on equation (3) and collecting the like terms, one obtains

$$0 = \int_{t_1}^{t_2}\left\{\int_{\gamma(t)}^{l}\left(-\frac{\partial}{\partial t}\frac{\partial L}{\partial v_t} - \frac{\partial}{\partial x}\frac{\partial L}{\partial v_x}\right)\delta v\,\mathrm{d}x + \left[\frac{\partial L}{\partial v_x}\delta v\right]_l - \left[\frac{\partial L}{\partial v_x} - \dot{x}\,\frac{\partial L}{\partial v_t}\right]_{\gamma(t)}\delta v(\gamma(t), t)\right.$$
$$\left. + \left[-L - m\ddot{\gamma}(t) - k\gamma(t)\right]_{\gamma(t)}\delta\gamma\right\}\mathrm{d}t + \left[\int_{\gamma(t)}^{l}\frac{\partial L}{\partial v_t}\delta v\,\mathrm{d}x + m\dot{\gamma}(t)\delta\gamma\right]_{t_1}^{t_2}. \tag{4}$$

In Figure 1(b), it is apparent that $v(\gamma(t), t) = v(l, t) = 0$ and $\gamma(t)$ is not defined. In the following variation process, $\gamma(t)$ can be expressed in one relation from the natural boundary condition derived from the process of calculus of variations [14]. In Figure 2, $v^*(x, t)$ is the true path of the transverse displacement. It is apparent that

$$\delta v(x, t) = v(x, t) - v^*(x, t) \tag{5}$$

has no meaning in the interval $[\gamma(t), \gamma(t) + \delta\gamma(t)]$, since $v(x, t)$ is not defined for $x \in (\gamma(t), \gamma(t) + \delta\gamma(t))$. By inspection of Figure 2, we define

$$\delta\bar{v} = v(\gamma(t) + \delta\gamma(t), t) - v^*(\gamma(t), t). \tag{6}$$

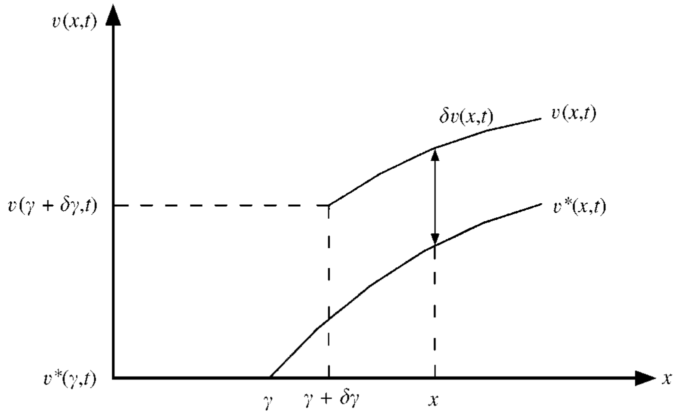

Figure 2. An external, $v^*(x, t)$, and a neighboring comparison curve, $v(x, t)$.

Since the boundary point $x = \gamma(t)$ moves in the horizontal direction, we have the true deflection $v^*(\gamma(t), t) = 0$ and the comparison deflection $v(\gamma(t) + \delta\gamma(t), t) = 0$. From equation (6), we have

$$\delta\bar{v} = 0. \tag{7}$$

Expanding the power series of $v(\gamma(t) + \delta\gamma(t), t)$ about $v(\gamma(t), t)$, substituting into equation (6) and using equation (7), we obtain

$$\delta v(\gamma(t), t) \doteq -v_x(\gamma(t), t)\delta\gamma + \delta\bar{v}, \tag{8}$$

where $\doteq$ means "equal to the first order".
Introducing relation (8) into equation (4), we obtain

$$0 = \int_{t_1}^{t_2}\left\{\int_{\gamma_t}^{l}\left(-\frac{\partial}{\partial t}\frac{\partial L}{\partial v_t} - \frac{\partial}{\partial x}\frac{\partial L}{\partial v_x}\right)\delta v\,\mathrm{d}x + \left[\frac{\partial L}{\partial v_x}\,\delta v\right]_l - \left[\frac{\partial L}{\partial v_x} - \dot{x}\,\frac{\partial L}{\partial v_t}\right]_{\gamma(t)}\delta\bar{v}\right.$$
$$\left. + \left\{\left[-L + v_x\left(\frac{\partial L}{\partial v_x} - \dot{x}\,\frac{\partial L}{\partial v_t}\right)\right]_{\gamma(t)} - m\ddot{\gamma}(t) - k\gamma(t)\right\}\delta\gamma\right\}\mathrm{d}t. \tag{9}$$

Finally, assuming that δv and $\delta\gamma$ vanish at t_1 and t_2, and noting the arbitrariness of $\delta v(x, t)$ for $\gamma(t) < x < l$, the following Euler–Lagrange equation is obtained:

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial v_t} + \frac{\partial}{\partial x}\frac{\partial L}{\partial v_x} = 0 \quad \text{for } \gamma(t) < x < l. \tag{10}$$

The associated boundary conditions $v(\gamma(t), t) = 0$ and $v(l, t) = 0$ are specified physically. One additional condition from the last term of equation (9) is

$$\left\{\left[-L + v_x\left(\frac{\partial L}{\partial v_x} - \dot{x}\,\frac{\partial}{\partial v_t}\right)\right]_{\gamma(t)} - m\ddot{\gamma}(t) - k\gamma(t)\right\}\delta\gamma = 0 \quad \text{at } x = \gamma(t) \tag{11}$$

By substituting into equations (10) and (11), the expression for L, as given in equation (2), the equation of motion and the boundary conditions can be obtained as

$$\rho v_{tt} + 2\rho c v_{xt} + (\rho c^2 - T) v_{xx} = 0, \quad \gamma(t) < x < l, \tag{12}$$

$$m\ddot{\gamma}(t) + k\gamma(t) + (\tfrac{1}{2}\rho v_t^2 - \tfrac{1}{2}\rho c^2 v_x^2 + \tfrac{1}{2} T v_x^2 + \dot{x}\rho v_t v_x)_{\gamma(t)} = 0, \tag{13}$$

$$v(\gamma(t), t) = 0, \quad v(l, t) = 0. \tag{14a, b}$$

It is noted that equation (12) is a linear governing equation of the string and equation (13) is called the transversality condition [14], which can be treated as the governing equation of one-d.o.f. mass–spring system, non-linearity coupled with the motion of the string.

For convenience in studying the effects of system parameters, the dimensionless variables and parameters are defined as

$$V = \frac{v}{l}, \quad \beta = \frac{c}{c_2}, \quad \Gamma = \frac{\gamma}{l}, \quad \xi = \frac{x}{l}, \quad \tau = \frac{c_2 t}{l}.$$

$$M = \frac{\rho l}{m}, \quad \Omega = \frac{\omega_s l}{c_2}, \quad c_2 = \sqrt{\frac{T}{\rho}}, \quad \omega_s = \sqrt{\frac{k}{m}}.$$

By substituting the above variables and parameters into equations (12)–(14), one obtains the dimensionless governing equations and boundary conditions:

$$V_{\tau\tau} + 2\beta V_{\xi\tau} + (\beta^2 - 1) V_{\xi\xi} = 0, \quad \Gamma < \xi < 1, \tag{15}$$

$$\Gamma_{\tau\tau} + \Omega^2 \Gamma + \{\tfrac{1}{2} M [1 - (\Gamma_\tau - \beta)^2] V_\xi^2\}_{\xi = \Gamma} = 0, \tag{16}$$

$$V(\Gamma, \tau) = 0, \tag{17}$$

$$V(1, \tau) = 0. \tag{18}$$

2.2. VARIATION OF THE TOTAL ENERGY

The dimensionless form of the total mechanical energy is

$$E(\tau) = \tfrac{1}{2} \int_\Gamma^1 [(V_\tau + \beta V_\xi)^2 + V_\xi^2] \, \mathrm{d}\xi + \tfrac{1}{2} m_r \Gamma_\tau^2 + \tfrac{1}{2} k_r \Gamma^2. \tag{19}$$

where $m_r = 1/M$, $k_r = kl/\rho c_2^2$. Owing to the translating speed β being not zero, the total derivative operator with respect to dimensionless time is defined as

$$(\dot{\ }) = \frac{\mathrm{d}}{\mathrm{d}\tau} = \frac{\partial}{\partial \tau} + \beta \frac{\partial}{\partial \xi}. \tag{20}$$

Taking the total derivative of equation (19) with respect to dimensionless time, we have

$$\dot{E}(\tau) = \int_\Gamma^1 [(V_z + \beta V_\xi)(V_{zz} + 2\beta V_{\xi\tau} + \beta^2 V_{\xi\xi}) + V_\xi (V_{\xi z} + \beta V_{\xi\xi})] \, \mathrm{d}\xi$$

$$- \Gamma_\tau [\tfrac{1}{2}(V_\tau + \beta V_\xi)^2 + \tfrac{1}{2} V_\xi^2]_{\xi = \Gamma} + m_r \Gamma_\tau \Gamma_{\tau\tau} + k_r \Gamma \Gamma_z. \tag{21}$$

Substituting equations (15)–(18) into equation (21), using integration by parts

$$\int_{\Gamma}^{1}(V_\tau+\beta V_\xi)V_{\xi\xi}\,\mathrm{d}\xi = V_\xi(V_\tau+\beta V_\xi)|_{\Gamma}^{1} - \int_{\Gamma}^{1} V_\xi(V_{\xi\tau}+\beta V_{\xi\xi})\,\mathrm{d}\xi,$$

and the relationship

$$V_\tau(\Gamma,\tau)=\dot{\Gamma}\,V_\xi(\Gamma,\tau),$$

equation (21) is reduced to

$$\dot{E}(\tau)=\beta[V_\xi^2(1,\tau)-V_\xi^2(\Gamma,\tau)]. \tag{27}$$

This expression shows that the total mechanical energy of the coupled string/slider system is time varying. In the case $\beta = 0$, the total mechanical energy does not change as the time increases.

3. THE FDM WITH VARIABLE GRID

The approximate solutions for transverse motion of string with the moving boundary at $\xi = \Gamma(\tau)$ will be solved via a special FDM with variable grid. The way of varying space grid is proposed. The aim is to avoid the increased complication and loss of accuracy associated with unequal space intervals near the moving boundary. The variable-grid method based on co-ordinate transformation [13, 14], which transforms the time-varying domain into a constant one, is found to be the most suitable for the string/slider system with moving boundary.

We keep the number of space intervals between $\xi = \Gamma(\tau)$ and 1, i.e., between a fixed and a moving boundary, constant and equal to N. Thus, for equal space intervals, $\Delta\xi = [1-\Gamma(\tau)]/N$ is different in each time step. The moving boundary is always on the first grid line. The transformation

$$\eta(\tau)=\frac{\xi-\Gamma(\tau)}{1-\Gamma(\tau)} \tag{28}$$

fixes the moving position of the slider at $\eta = 0$ for all τ, and the end-point $\xi = 1$ is also fixed as $\eta = 1$. By using the standard relationships

$$V_\xi=\frac{\hat{V}_\eta}{1-\Gamma},\quad V_{\xi\xi}=\frac{\hat{V}_{\eta\eta}}{(1-\Gamma)^2},\quad V_\tau=\frac{\Gamma_\tau(\eta-1)}{1-\Gamma}\,\hat{V}_\xi+\hat{V}_\tau,$$

$$V_{\xi\tau}=\frac{\Gamma_z(\eta-1)}{(1-\Gamma)^2}\,\hat{V}_{\eta\eta}+\frac{1}{1-\Gamma}\,\hat{V}_{\eta\tau}+\frac{\Gamma_\tau}{(1-\Gamma)^2}\,\hat{V}_\eta,$$

$$V_{\tau\tau}=\frac{(\eta-1)[\Gamma_{\tau\tau}(1-\Gamma)+2\Gamma_z^2]}{(1-\Gamma)^2}\,\hat{V}_\eta+\frac{2\Gamma_z(\eta-1)}{1-\Gamma}\,\hat{V}_{\eta\tau}\,\frac{\Gamma_\tau^2(\eta-1)^2}{(1-\Gamma)^2}\,\hat{V}_{\eta\eta}+\hat{V}_{\tau\tau}, \tag{29a–e}$$

to transform from $V(\xi, \tau)$ to $\hat{V}(\eta, \tau)$, the governing equations and boundary conditions (15)–(18) can be written as

$$\hat{V}_{\tau\tau} + \frac{2\Gamma_\tau(\eta - 1) + 2\beta}{1 - \Gamma}\hat{V}_{\eta\tau} + \frac{\beta^2 - 1 + \Gamma^2(\eta - 1)^2 + 2\beta\Gamma_\tau(\eta - 1)}{(1 - \Gamma)^2}\hat{V}_{\eta\eta}$$

$$+ \frac{(\eta - 1)[\Gamma_{\tau\tau}(1 - \Gamma) + \Gamma_\tau^2] + 2\beta_\Gamma}{(1 - \Gamma)^2}\hat{V}_\eta = 0, \quad 0 < \eta < 1, \tag{30}$$

$$\Gamma_{\tau\tau} + \Omega^2\Gamma + \left\{\frac{M[1 - (\Gamma_\tau - \beta)^2]}{2(1 - \Gamma)^2}\hat{V}_\eta^2\right\}_{\eta = 0} = 0, \tag{31}$$

$$\hat{V}(0, \tau) = 0, \quad \hat{V}(1, \tau) = 0. \tag{32a, b}$$

In the process of co-ordinate transformation, $\xi = \Gamma(\tau)$ becomes $\eta = 0$ and the equal space intervals $\Delta\eta = 1/m$ is equal in each time step. The moving boundary is always on the first grid line. It is seen that the position $\Gamma(\tau)$, speed $\Gamma_\tau(\tau)$ and acceleration $\Gamma_{\tau\tau}(\tau)$ of the moving boundary appear as coefficients in the transformed equation (30). The co-ordinate transformation reduces the problem to one in a fixed domain and then the standard central and forward differences for both the time and space derivatives are used to discrete equations (30) and (31).

4. OBSERVATION

It is seen that Hamilton's principle and calculus of variation are used to derive the governing equations and boundary conditions of the string/slider system. The special FDM with the variable-grid scheme is proposed to approximate the numerical solutions. From the dynamic formulation and approximation analysis of the FDM with variable grid, several important observations can be made:

1. The geometrical non-linearity of the string and non-linear stiffness of the spring are not introduced, but the non-linearities arise in the boundary condition (16).
2. If the geometrical non-linearity of string and non-linear stiffness of spring were introduced, the coupling system will be more complicated and there will be much more possibilities for internal resonance.
3. Undergoing the variable transform (28), (i) the governing equation (30) and the boundary condition (31) and non-linear and time varying, and (ii) The original moving boundary problem is transferred to a fixed boundary one.
4. From equation (27), when the transport speed of the string is zero, the total mechanical energy of the string/slider system is unchanged.

5. NUMERICAL RESULTS

A continuous string and a lumped slider similar to that of Fung and Cheng [9] are used in the numerical simulation. The parameter values are as follows: $T = 200\,\text{N}$, $\rho = 2\,\text{kg/m}$, $l = 1\,\text{m}$, $m = 0{\cdot}5\,\text{kg}$, and the spring stiffness k is adjusted to satisfy the two frequency relationships $\omega_s = \omega_0$ or $2\omega_0$, where $\omega_0 = (\pi/l)\sqrt{c_2^2 - c^2}$ is a constant value of the first

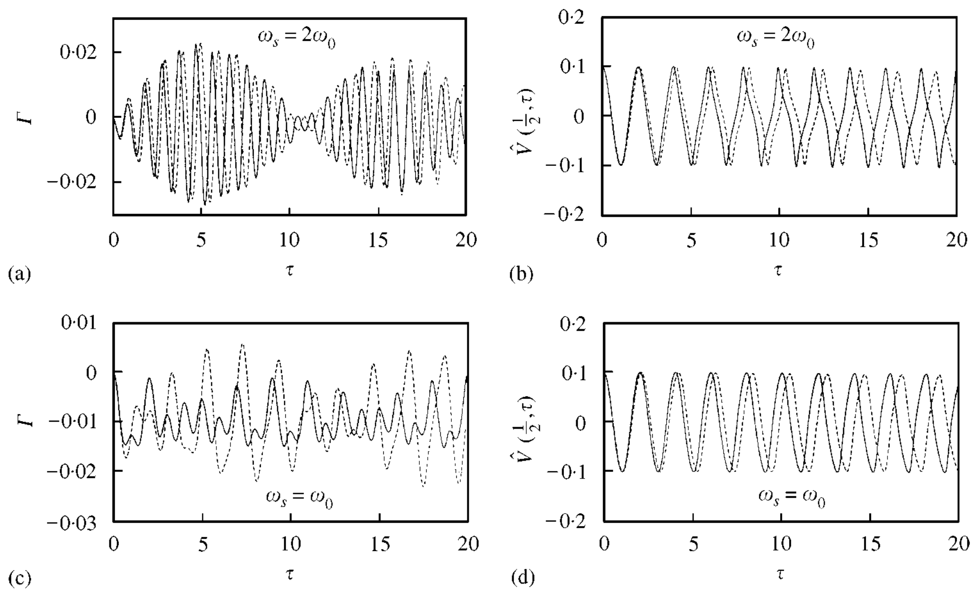

Figure 3. The transient responses of the slider and string: (a) amplitude of the slider with $\omega_s = 2\omega_0$, (b) transverse displacement in the middle of the string with $\omega_s = 2\omega_0$, (c) amplitude of the slider with $\omega_s = \omega_0$, (d) transverse displacement in the middle of the string with $\omega_s = \omega_0$. (—, $\beta = 0$; - - -, $\beta = 0{\cdot}2$).

mode frequency of the string with a fixed length l. The parameters given here are chosen to study the coupling effect on the free vibration of the string/slider system.

5.1. STRING/SLIDER COUPLING SYSTEM

The grid number $N = 1000$ in different translating speeds $\beta = 0$ and 0·2, and the numerical stability criterion with time step 5×10^{-4} are taken in the FDM. The transient amplitude of the slider and the transverse displacement in the middle of the string with $\omega_s = 2\omega_0$ are shown in Figures 3(a) and 3(b) respectively. The non-linear terms are coupled between $\hat{V}(\eta, \tau)$ and $\Gamma(\tau)$. The term $\hat{V}_\eta\ (0, \tau)$ can be treated as the excitations of $\Gamma(\tau)$ in equation (31) and the excited $\Gamma(\tau)$ feedbacks to $\hat{V}\ (\eta, \tau)$ of equation (30). From the coupling effects between $\hat{V}(\eta, \tau)$ and $\Gamma(\tau)$, it is seen that the beating amplitudes of the slider build up and then diminish in a regular pattern. In physical meaning, the energy can transfer between the string and slider through the boundary condition. In addition, the frequencies of vibrations are time-dependent because of the varying length of the string. Figures 3(c) and 3(d) show the transient amplitude of the slider and the transverse displacement in the middle of the string with $\omega_s = \omega_0$. It should be noted that the internal resonance phenomenon does not occur as markedly in this case. Thus, energy rarely transfers between the slider and string. The amplitude of the slider in Figure 3(c) is almost always negative and the values are much smaller than that with $\omega_s = 2\omega_0$, where the internal resonance occurs. The amplitudes (dash lines) with translating speed $\beta = 0{\cdot}2$ have the lower frequencies than those (solid lines) with $\beta = 0$.

5.2. ASSIGN THE BOUNDARY MOTION

Cooper [15] have found that with some periodic motion of the boundary, the energy of the travelling waves may grow without bound and the amplitude of vibration becomes

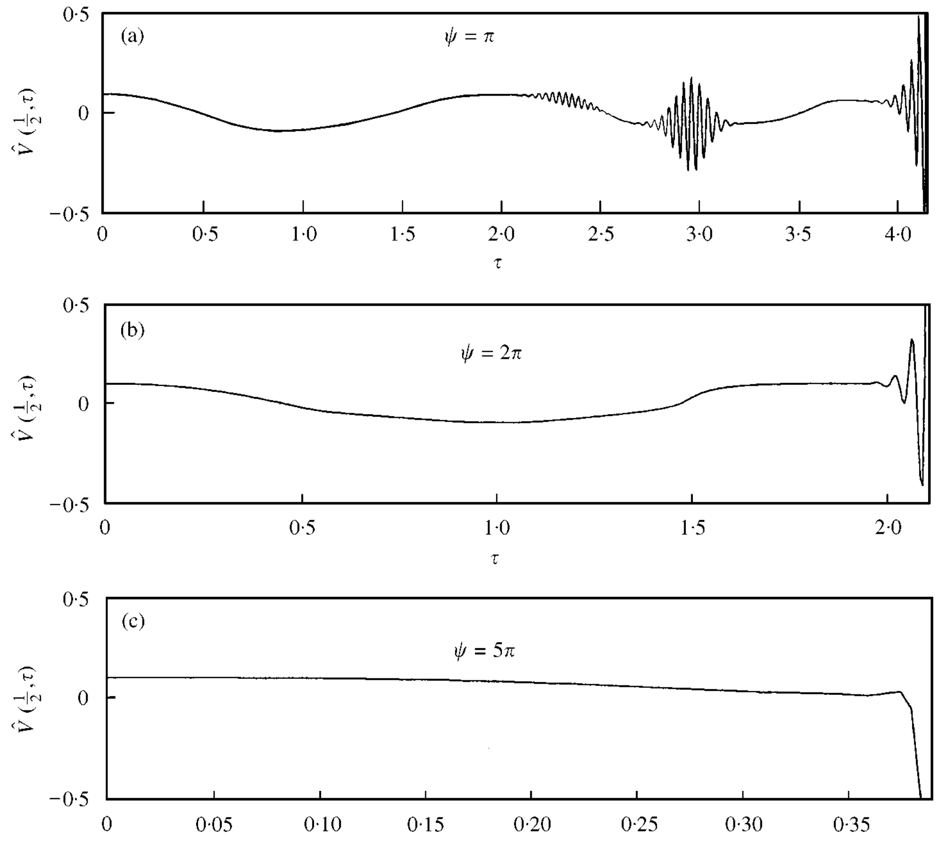

Figure 4. The transient responses of transverse amplitude in the middle of the string with the periodic moving boundaries: (a) $\psi = \pi$, (b) $\psi = 2\pi$, (c) $\psi = 5\pi$.

infinite. In order to show the divergence of the vibration amplitude of the string, the periodic function $\Gamma(\tau)$ is chosen as

$$\Gamma(\tau) = 0{\cdot}1 \sin(\psi\tau), \tag{33}$$

where ψ is the frequency of the moving boundary position. The vibration of the string is assumed to have the initial condition $\hat{V}(\eta, 0) = 0{\cdot}1 \sin(\pi\eta)$. Since $\Gamma(\tau)$ is given in equation (33), only equation (30) needed to be solved for the transient vibrations of the string.

Figure 4 shows the transverse amplitudes in the middle of the string with three different frequencies $\psi = \pi$, 2π and 5π of the periodic moving boundary position. It is shown that the transient responses diverge with a decreasing time as the frequency ψ is π, 2π and 5π. The vibration amplitude explodes finally at $\tau = 4{\cdot}15$ for $\psi = \pi$ and $\tau = 2{\cdot}11$ for $\psi = 2\pi$. The higher frequency of the moving boundary position $\psi = 5\pi$ has the shortest exploding time $\tau = 0{\cdot}39$. Moreover, the vibration energy clearly grows without bound from these figures. Due to the influence of the periodic moving boundary position, the vibration string becomes unstable.

6. CONCLUSIONS

The method for studying the moving boundary effect on the vibration of a string/slider-coupling system has been investigated in this paper. The preliminary results presented here indicate the following conclusions:

1. It is found that the internal resonance occurs when the frequency of the slider is two times of that of the string. The string vibrations and the slider displacement are all the beating phenomena under the internal resonance.
2. From the numerical results, it is seen that when the transport speed of the string is introduced, the natural frequency of the string will increase.
3. The transient responses of the string diverge when an appropriate periodic motion of the moving boundary is given.
4. The vibration amplitude explodes when the frequency of the moving boundary has an integer time of the string and the higher frequency has the shorter exploding time.

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