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TUNING A WINE GLASS VIA MATERIAL TAILORING — AN APPLICATION OF A METHOD FOR OPTIMAL ACOUSTIC DESIGN

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This paper describes a method for optimally designing a structure to "best fit" a specified set of acoustic characteristics, e.g., sound spectrum or radiated power. The method links the disciplines of structural dynamics, acoustics and optimization into a unified methodology. The design variables include, for example, the addition of masses or multiply-tuned resonators to the structure as well as distributions of stiffeners or constrained damping layers. In all cases, the design variables are introduced as external forces (via their impedances) in the equation for the structure that is given as a series expansion of eigen functions. This step eliminates the need for solution of large matrix eigenvalue problems. An acoustic program POWER is used to assess the radiated sound power as a function of the design variables. Various search engines are used within the computer program MATLAB® to determine which design variables give the 'best fit' to the acoustic specifications. To illustrate the design method, a wine glass is tuned optimally to move the first four eigenvalues into harmonic relationships. The design variables are small masses that are added to the upper surface of the wine glass. Comparison of the wine glass's radiated sound power with and without the optimal masses indicates an excellent agreement between the specified and measured spectra.

1. INTRODUCTION

Through the years, the Journal of Sound and Vibration (JSV) has provided the sound and vibration research community with an excellent compendium of articles focused mainly on advancing the understanding of sound and vibration in terms of generation, propagation and boundary interaction phenomena. Being widely interdisciplinary, the research reported in the journal has spawned applications that are continually evolving, e.g., jet noise suppressors, algorithms for active noise and vibration control, and machinery health monitoring, to name but a few. Theories and their refinements have evolved as well, and many advances leading to commercially available software, e.g., finite element analysis (FEA) or boundary element method (BEM) programs, can be charted in a progression of JSV publications. Samples of these papers are given in chronological order in references [1–23]. Today, structural/acoustical designers benefit enormously from these earlier papers and are presently refining the synthesis of the disciplines of acoustics, structural dynamics and optimization into a unified design methodology. This process is of course greatly accelerated by the development of computer speed, memory and more recently, graphics that allow nearly instant visualization of wave fields within and surrounding complex structures.

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The design strategy outlined in this paper builds on much of the work reported in those earlier JSV publications. It describes a method for designing acoustically tailored structures and illustrates how available computational and experimental tools may be integrated into a unified design methodology. For illustration purposes, a simple wine glass is used as the structure to tailor acoustically. Specifically, its sound spectrum is altered to produce a series of musical tones that are related harmonically rather than atonally, the latter resulting from solutions of fourth order differential equations governing the vibration response of glass rather than second order ones common to most musical instruments. The tailoring is accomplished by adding a series of selected masses to the glass at specific points, a process that requires an optimal search routine. The paper will take the reader through a series of steps leading to the optimal solution. First, a description of how the design variables are incorporated into the governing equations of the structure is given. Next, a brief description of the method of wave superposition that is used to compute radiated acoustic power is presented, followed by the optimization method used in the search routine. The paper concludes with a description of the experiment and results.

2. THE STRUCTURAL MODEL

A common goal for all optimization problems of this type is to construct a structural/acoustical model that will accommodate changes in the design variables without the need for solving large matrices for each iteration. Commonly, in FEM/BEM optimization models, the design variables (e.g., masses or stiffnesses) are imbedded in the matrices of the governing equations and thus finding the optimum set of variables in the search process requires that the entire matrices be solved repeatedly. On very large structural/acoustical optimization problems, the programs become unwieldy, computationally intensive and highly inefficient. To circumvent this problem, it is expedient to introduce the design variables in the form of external impedances containing terms of mass, stiffness and damping or combinations of these, e.g., tuned resonators (see, for example, Constans *et al.* [24]). A modal description of the structural model nicely accommodates this approach and thus its response can be given in terms of basis functions (usually in the form of eigenvectors), resonance frequencies and damping coefficients subjected to exciting forces that also contain the impedance terms. If a physical model of the structure exists, these basis functions can be obtained with modal analysis techniques. However, for solely conceptual designs, numerical models provide good approximations of these quantities although, the computed eigenvalues often have to be "adjusted" to fit those of the eventual physical structure. It should be noted that, for modelling existing structures with complex geometries, the eigenvectors obtained via simplified numerical models form a useful set of basic functions on which to construct the overall response, provided that their boundary conditions are good approximations to the physical ones. To illustrate these ideas, consider the following modal description of the response of a typical structure given as

$$v(x_r) = \mathrm{i}\omega \sum_n^N \frac{\varphi_n(x_r)\{\sum_m^M \varphi_N(x_m)F(x_m) - \sum_c^C \varphi_n(x_c)Z(x_c)v(x_c)\}}{\{\omega^2 - \omega_{dn}^2(1+\mathrm{i}\eta_n)\}}, \tag{1}$$

where $v(x_r)$ is the surface velocity at position x_r, ω is the forcing frequency in rad/s, $\varphi_n(x_r)$ is the nth eigenvector (mass normalized) evaluated at position x_r, $F(x_m)$ is the external driving force at position x_m, $Z(x_c)$ is the point impedance of the design variable, ω_{dn} is the damped modal frequency, and η_n is the loss factor of the nth mode. The terms in brackets in the numerator give the generalized forces and when combined with those in the denominator are often referred to as the modal participation factor.

The impedance of the design variable can be distributed over a surface or it can be applied at a point. For example, the impedance of a tuned absorber attached at position x_c is

$$Z(x_c) = \mathrm{i}\omega k(1 + \mathrm{i}\eta)/\{k(1 + \mathrm{i}\eta) - m\omega^2\}, \tag{2}$$

where k, m and η denote stiffness, mass and loss factor respectively.

If the impedance is distributed over an area as in the case of a stiffener or a constrained layer damping material, it is then necessary to include cross terms in the summation $\sum_c^C$ given in equation (1) as $\sum_c^C \varphi_n(x_c) \sum_{c'}^{C'} Z(x_c, x_{c'}) v(x_{c'})$, where C and C' are the total number of points where the impedance is joined to the structure.

It should be noted that in the above formulation, the resonance frequencies of the optimized structure are influenced by the impedances within the generalized force terms and thus differ from the damped modal frequencies. The above modal formulation gives the vibration response for any linear structural system subjected to external forces. In the following section, it is shown how the system response is combined with a solution to the wave equation to quantify its radiated sound power at any frequency.

3. THE ACOUSTIC RADIATION MODEL

In most acoustic tailoring studies, the overall quantity to be minimized is the radiated sound power over a specified bandwidth. To compute this quantity, we have developed the numerical program POWER based on the principle of superposition. Since the details of this program have already been published extensively (see Koopmann and Fahnline [25]), only an outline will be given of the steps that lead to a relation between the velocity response of a vibrating structure and its radiated sound power. It begins with a variation of the Kirchhoff–Helmholtz equation that uses a Green function of the second kind (rather than the more common free-space version) with the property that $\mathbf{\nabla}_s G(x/x_s) = 0$ is satisfied on the radiating surface. The pressure is then given as

$$p(x) = -\frac{\mathrm{i}\kappa\rho c}{4\pi} \iint_s G(x/x_s) v(x_s) \cdot \mathbf{n}_s \, \mathrm{d}S(x_s), \tag{3}$$

where $p(x)$ is the pressure at the field point x (outside the boundary surface), κ is the acoustic wave number and ρc is the characteristic acoustic impedance. The integration is taken over the radiating surface S with the integrand given as the product of the Green function G and the normal velocity of the surface $\mathbf{v}(x_s) \cdot \mathbf{n}_s$. This reduced form of the Kirchhoff–Helmholtz equation is useful for deriving a lumped parameter radiation model because the scattering from the boundary surface is included in the definition of the Green function. To express this equation in a lumped element form (see illustration of wine glass in Figure 1), the boundary surface is divided into N elemental surfaces of area S_μ, each having a volume velocity u_v. The space-average pressure on the element μ is

$$p_\mu = \frac{1}{S_\mu} \int_{S_\mu}\!\!\int p(x_s) \, \mathrm{d}S_\mu(x_s). \tag{4}$$

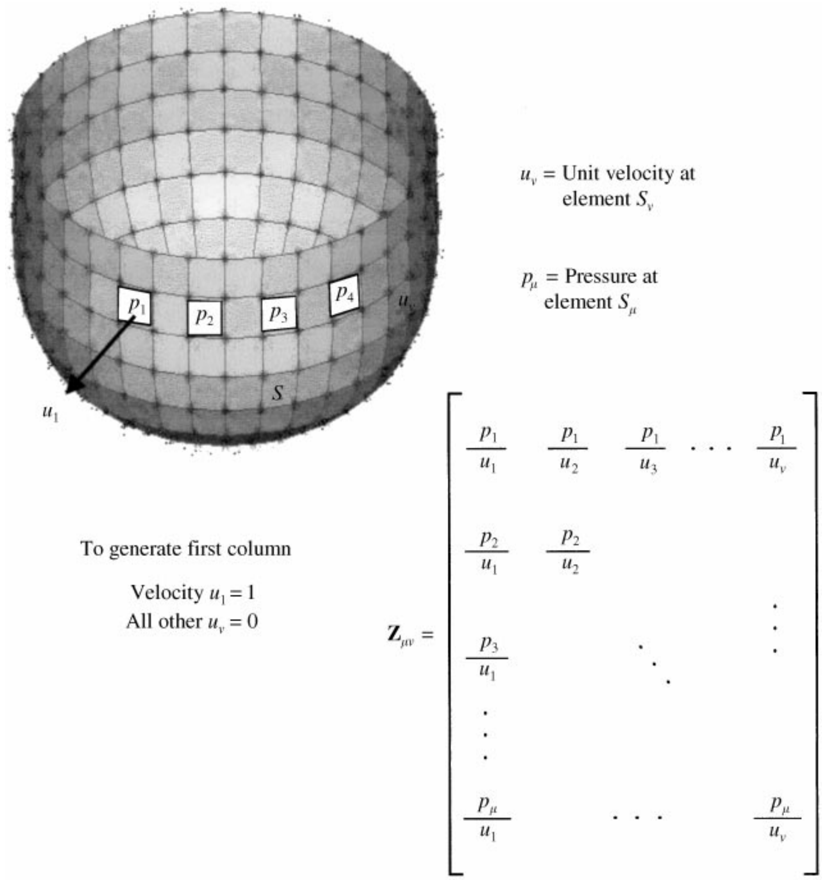

Figure. 1. Lumped element representation of wine glass surface.

With these approximations, the space-average pressure on element μ can be written as

$$p_\mu \approx -\frac{\mathrm{i}\kappa\rho c}{4\pi S_\mu}\sum_{v=1}^{N}\frac{u_v}{S_v}\int_{S_\mu}\!\!\iint_{S_v}\!\!\int G(x/x_s)\,\mathrm{d}S_v(x_s)\,\mathrm{d}S_\mu(x). \tag{5}$$

Taking the radiation impedance $Z_{\mu v}$ to be the ratio of the space-average pressure on element μ to the volume velocity source of element v, then

$$p_\mu = \sum_{v=1}^{N} Z_{\mu v}u_v, \tag{6}$$

where

$$Z_{\mu v} = -\frac{\mathrm{i}\kappa\rho c}{4\pi S_\mu S_v}\int_{S_\mu}\!\!\iint_{S_v}\!\!\int \{Gx/x_s\}\,\mathrm{d}S_v(x_s)\,\mathrm{d}S_\mu(x). \tag{7}$$

The matrix $\mathbf{Z}_{\mu v}$ for the wine glass geometry used in this paper is shown in Figure 1.

The time-average sound power radiated by a structure is

$$\Pi_{av} = \tfrac{1}{2} \sum_{\mu=1}^{N} \mathrm{Re}\{u_\mu^* p_\mu\}. \tag{8}$$

Substituting the expression for p_μ (equation (5)) in Π_{av}, and invoking reciprocity arguments for the impedance, i.e., $Z_{\mu v} = Z_{v\mu}$ and noting that the radiation resistance $\Re_{\mu v} = \mathrm{Re}\{Z_{\mu v}\}$, the lumped parameter approximation for the time-average sound power output is written as

$$\Pi_{av} = \tfrac{1}{2} \sum_{\mu=1}^{N} \sum_{v=1}^{N} u_\mu^* u_v \Re_{\mu v}, \tag{9}$$

where

$$\Re_{\mu v} = \frac{\Re_0}{S_\mu S_v} \int_{S_\mu} \int \int_{S_v} \int \mathrm{Im}\{G(x/x_s)/\kappa\}\, \mathrm{d}S(x_s)\, \mathrm{d}S(x) \tag{10}$$

with $\Re_0 = \kappa^2 \rho C/4\pi$. The computer program POWER is used to compute the integrals of the Green function over the elemental surfaces S_μ and S_v. POWER is an equivalent source method based on the lumped parameter model that uses volume velocity matching as a means of satisfying the boundary conditions on the surface of a radiating structure. Combinations of simple, dipole and tripole sources on each of the elemental surfaces of a structure generate volume velocities on each of those surfaces that are equivalent to the specified boundary condition. In turn, the strengths of these sources are used to calculate the space-average pressure on each elemental surface. For example, to generate $\Re_{v1}$, the first column of the resistance matrix, the volume velocity u_1 is unity and the volume velocity of the remaining surfaces is zero. Solution of the corresponding matrix equation yields simple, dipole and tripole source strengths that satisfy this boundary condition and are used to calculate the space-averaged surface pressure at each of the surfaces, p_μ. A plot of the first column of the acoustic resistance matrix corresponding to the location of the volume velocity source given in Figure 1 is shown in Figure 2. The full resistance matrix is computed by iterating through all the elemental surface volume velocities in this manner. It should be noted that $\Re_{\mu v}$ only depends on the frequency (wavelength) of the radiated sound and the geometry of the vibrating structure. Once the velocity of the structure is available, (e.g., that given by equation (1)), the average power output is computed by carrying out a straightforward operation of pre- and post-multiplying the $\Re_{\mu v}$ matrix with the volume velocity vectors u_μ^* and u_v. The ability to compute sound power directly without a matrix inversion makes the computational steps within the optimization search highly efficient. However, each term in the resistance matrix is frequency dependent, and thus must be evaluated over the bandwidth required in the optimization. Since the resistance terms are slowly varying with frequency, it is possible to reduce this computation effort by spline-fitting a coarse set of points within the frequency band to provide a resistance term at each frequency. In practice, the terms $\Re_{\mu v}(\omega)$ are computed and stored prior to the optimization process and recalled only when required. The final step in an optimal acoustic design process is to combine the above structural and acoustical models with an optimization search routine as given in Section 3.

4. THE OPTIMIZATION STRATEGY

All optimization searches include an objective (or cost) function to be minimized (or maximized) that contains a series of design variables that can be changed subject to a set of

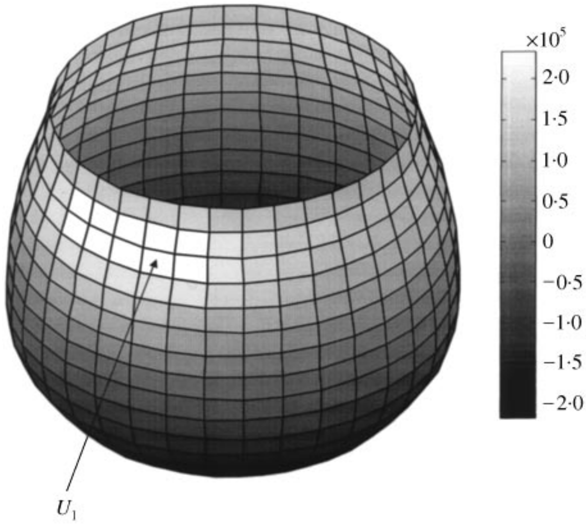

Figure 2. Plot of the first column of the acoustic resistance matrix corresponding to the location of the volume velocity source given in Figure 1.

design constraints that limit the domain of the search. The objective constraint functions must be formulated in mathematical terms to fit the particular search algorithms. These may contain multi-variables often with higher order, non-linear relations and thus, a specific optimization strategy is needed for each problem considered. To illustrate the strategy for optimal acoustic design, recall the expression for the modal response of the structure given in equation (1). The velocity $v(x_r)$ is required to compute radiated sound power. Each such computation for $v(x_r)$ in the optimization search begins with selecting a set of trial design variables (e.g., a set of c impedances, $Z(x_c)$) so that the unknown velocities $v(x_c)$ that appear on the right side of equation (1) can be solved for explicitly. This initial calculation thus requires a set of c equations to be solved simultaneously for the $v(x_c)$'s. For example, with two absorbers with impedances $Z(x_1)$ and $Z(x_2)$, the velocity analysis proceeds as follows: the first equation in the set would have the form

$$v(x_1) = \mathrm{i}\omega \sum_n^N \frac{\varphi_n(x_1)\{\sum_m^M \varphi_n(x_m)F(x_m) - [(\phi_n)(x_1)Z(x_1)v(x_1) + \phi_n(x_2)Z(x_2)v(x_2)]\}}{\{\omega^2 - \omega_{dn}^2(1 + \mathrm{i}\eta_n)\}}. \quad (11)$$

The second equation for $v(x_2)$ would have a similar form. These equations are then solved simultaneously for the velocities at the absorber locations, $v(x_1)$ and $v(x_2)$. In general, with the $v(x_c)$'s in hand, they are incorporated into equation (1) which gives the surface velocity $v(x_r)$ at all points. Combining $v(x_r)$ with $\Re_{\mu v}$ (here $u_\mu = \int_{S_\mu}\int \mathbf{v}(x_r)\cdot \mathbf{n}_r \,\mathrm{d}S(x_r)$) and summing over all elemental surfaces of the radiating structure, the sound power corresponding to each set of design variables can be calculated as

$$\Pi_{av} = \tfrac{1}{2} \sum_{\mu=1}^{N} \sum_{v=1}^{N} u_\mu^* u_v \Re_{\mu v}. \quad (12)$$

This expression for sound power is one form of the objective function used in optimal acoustic design. The choice of search algorithms used for the optimization depends heavily on the number of design variables and their characteristics, e.g., whether locations of the absorbers are to be considered as design variables along with the absorber parameters, and whether the objective function is non-convex with several peaks and valleys. In optimal acoustic design, a hybrid strategy is employed where a gradient-based algorithm is used to select the design variables of a localized impedance while a simulated annealing algorithm is used to optimize their locations on the radiating structure. The concomitant constraint equations place limits on the design variables. For example, if $Z(x_c)$ represents a tuned absorber, its allowable weight, size, and loss factor would have a specified upper limit. For more commercially oriented applications, additional constraint equations may link cost, aesthetics, reliability, etc., in mathematical terms that can be addressed in the search routine.

5. AN APPLICATION OF AN OPTIMAL ACOUSTIC DESIGN

An example is now given to illustrate the steps required for optimal acoustic design. The modal response of a wine glass will be examined experimentally and its spectrum acoustically tailored by adding small masses to its surface, i.e., the design variables. The objective function is a set of preferred resonance frequencies (harmonically related) within its frequency response that must be met within a given error band. The constraint equation simply places a limit on the total weight of the all allowable added masses. The block diagram in Figure 3 illustrates the logic flow of the search routine.

The wine glass used in this experiment is shown in Figure 4 along with the laser vibrometer. The glass is excited with a small piezoceramic (PZT) layer bonded near its rim. A high-voltage amplifier drives the PZT with a signal from a real-time analyzer. The response of the glass is recorded with a laser vibrometer that senses velocity. Since the glass is axisymmetric, a complete velocity response can be synthesized from measurements taken around its rim and vertically along a plane coincident with the PZT exciter. Reflecting tape is attached on these surfaces to enhance the laser signal. To obtain the terms required to generate equation (1) analytically, i.e., φ_n, ω_{dn} and η_n, a modal analysis was performed on the glass using the quadrature method. A typical mobility response function is shown in Figure 5 and includes the first eight modal frequencies (note the light modal damping, $\eta \sim 0.001$). To obtain the excellent agreement between theory and experiment, it was necessary to use the method of added residuals to compensate for the errors introduced in truncating the series expansion. At each frequency, the mode shapes were obtained by rotating the glass in small increments (at least six measurement points/structural wavelength) and by recording the velocity response. Similar measurements were taken in the vertical direction. Care was taken to record the normal surface velocity by adjusting the laser beam accordingly since this is the form of surface velocity required in the sound power calculation. Typical modal frequencies and shapes are shown in Figure 6. Note that most of the vibration response takes place near the upper part of the glass and that the modes are purely circumferential occurring in n multiples of structural wavelengths beginning with $n = 2$. It should also be noted that each of the modes has a corresponding degenerate mode occurring in spatial quadrature, but these are weakly excited due to the small size of the PZT patch. With the terms φ_n, ω_{dn} and η_n in hand from the measurements, the surface velocity of the glass $v(x_r)$ was synthesized using equation (1) and the corresponding radiated sound power was computed for each mode using the numerical program POWER. Figure 7 shows the relative sound powers of the first four modes of the glass.

Attention is now given to the optimization part of the experiment. The objective function is defined in terms of a prescribed power spectrum shown in Figure 8. The dominant components in the spectrum are prescribed in terms of multiples of the fundamental frequency f_1 of the glass. Thus, when tapped, the glass will produce musical sounds that are

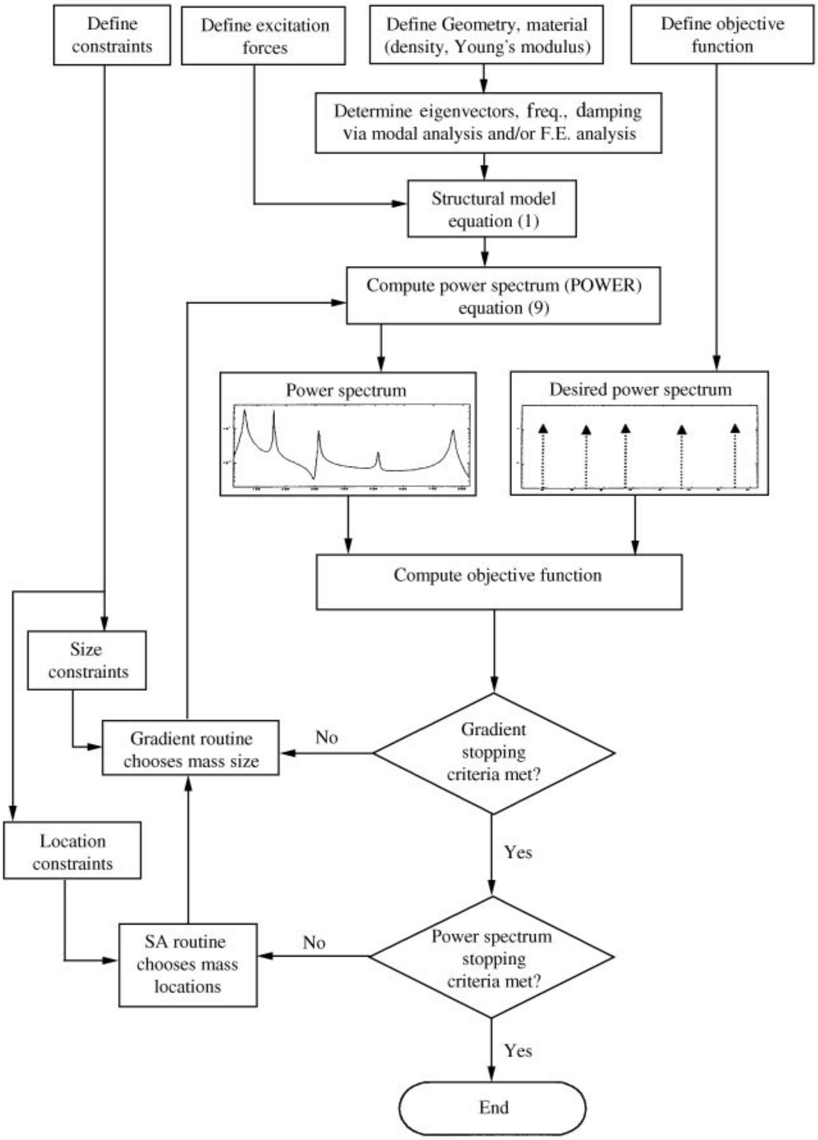

Figure 3. Logic flow of search routine for selecting optimal size and position of tuning mass.

predominantly harmonic in character. Error bands of $\pm 2\%$ provide a criterion for deciding when the optimal search can be concluded. In this experiment, the design variables are the placement and size of n small masses and thus the impedance given in equation (1) is simply $Z(x_c) = \mathrm{i}\omega m(x_c)$, where $m(x_c)$ denotes the size and placement of the mass at point x_c. The constraint placed on these design variables is a limit on allowable total mass, M. Mathematically, the optimization procedure can be written as

Design variables: masses m_c and their placement (x_c)

Objective function: $|(f_n^{opt} - nf_1^{opt})/nf_1^{opt}| \leqslant 2\%$ $\quad n = 2$, 4 and 6.

Constraint equations: $\sum_c^C m_C \leqslant M$. For this study, $M = 10$ g.

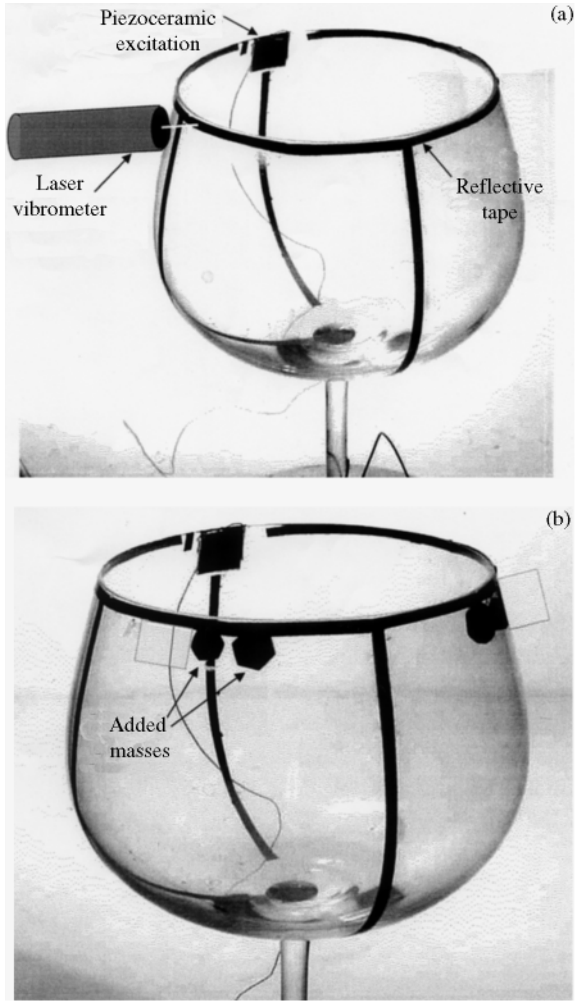

Figure 4. (a) View of wine glass showing PZT layer and laser vibrometer; (b) View of wine glass showing typical locations of added masses.

The search process follows the logic flow given in Figure 3. The program begins with a trial set of three masses at three locations, say, $m(x_1) \rightarrow m(x_3)$. Using a routine MATLAB® program, these impedances are then inserted into equation (11) and the corresponding set of (3×3) equations is solved for the surface velocities $v(x_c)$ at the points where the masses are applied. These velocities are then used in equation (1) to compute the surface velocities $v(x_r)$ at all grid points on the glass. After converting these into volume velocities u_μ, the sound power spectrum $\Pi(\omega)$ is computed via the resistance matrix $\Re_{\mu v}$ provided by the program POWER given in equation (12). With the spectrum in hand, a second MATLAB® program extracts the peaks of the computed spectrum f_n^{opt} and compares these with those prescribed nf_1^{opt}. If the difference is greater than the allowable error of 2%, then the search for the optimal placement and size of the masses continues. The search process is carried out with a third MATLAB® program that runs two search engines simultaneously. The gradient method is used for searching the optimal mass sizes while a simulated annealing method is

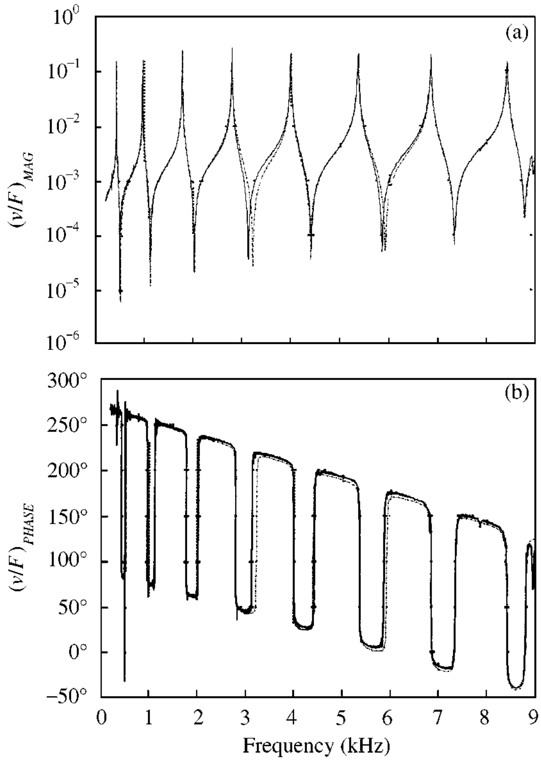

Figure 5. Magnitude and phase plot of point mobility function; experimental (——), theory (-----).

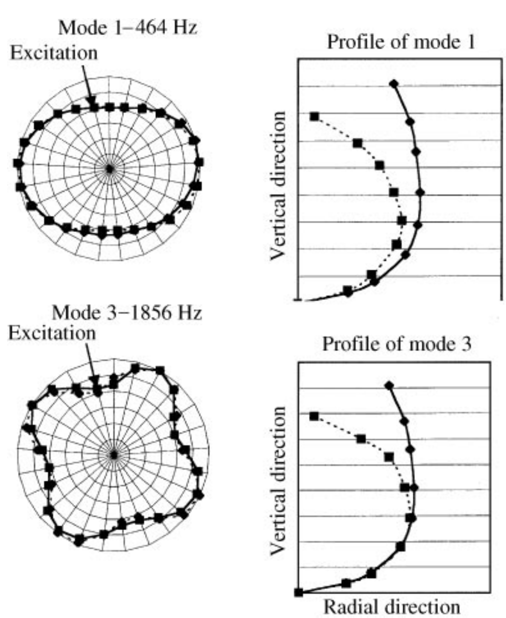

Figure 6. Typical modal frequencies and mode shapes of the wine glass.

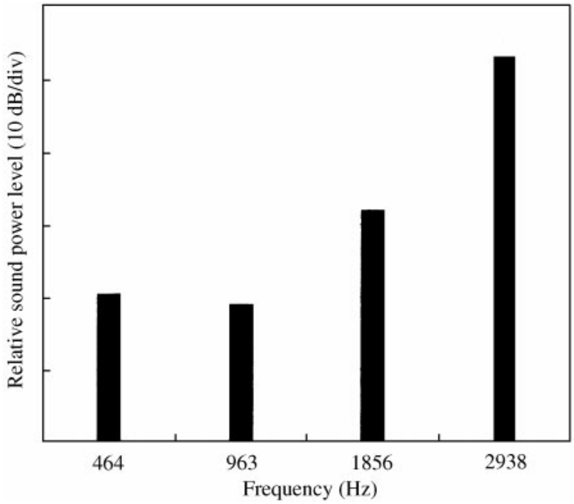

Figure 7. Relative sound powers of the first four modes of the wine glass.

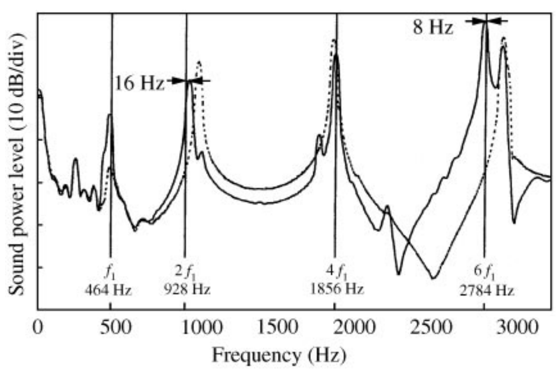

Figure 8. Sound power spectrum before (– – – – –) and after (——) optimal tuning of glass with added masses. nf_1 ($n = 2, 4$, and 6) gives the objective function.

used to optimize the mass placements. As these optimization techniques are well known, we will not elaborate on them. The recent book by Belegundu and Chandrupatla [26] contains several algorithms with source code programs. When the objective function is finally met, the search ends with a prescription of the optimal size and placement of the masses on the wine glass.

6. DESIGN VALIDATION

Optimal acoustic design studies are generally a part of an overall design process where the structure is modelled entirely at the virtual level. Designers rely more and more on such numerical models to provide a basis for decision making, e.g., whether or not to proceed with particular design features that must meet certain mechanical, electrical and acoustical requirements. However, if a validation of the acoustic optimization model is required along the way, it is of course necessary to construct a physical model for an experiment. This study began with a physical model, i.e., the wine glass, and thus it was possible to characterize its dynamic properties (φ_n, ω_{dn} and η_n) directly via experiments. To validate the optimization procedure that specified a set of masses in terms of their sizes and placements, the following experiments were conducted. Based on the results of the optimization study, small masses ($\sim$2–3 g) were bonded to the wine glass at the prescribed points as shown in Figure 4(b). To minimize exciting degenerate modes, the masses were placed symmetrically around the rim of the glass relative to the location of the PZT layer. With the masses in place, the glass was driven again with the same PZT layer and its sound power spectrum was measured, this time in the far field of an anechoic chamber. Figure 8 gives the sound power spectrum of the wine glass before and after its modal frequencies have been optimized to exhibit harmonic relationships. (A direct computation of the radiated sound power was bypassed at this point since the presence of the masses on the glass surface upsets the axisymmetry of its circumferential modes thus necessitating a modal analysis of its entire surface.) It is interesting to note that the introduction of the weights excites the degenerate modes somewhat, even though care was taken to locate the weights symmetrically to the PZT layer. This is particularly evident at the fourth mode (2784 Hz). For all practical purposes, the design requirement has been met. Modes $2f_1$, $4f_1$ and $6f_1$ fall within the 2% error bands. For example, the second modal frequency (944 Hz) is within 1.7% of the desired $2f_1$ frequency (928 Hz). The next two higher harmonics are well within the 2% error band. When tapped, the glass sounds distinctively musical and the predominance of the tone at 464 Hz and its octaves are quite pleasing to the ear. Thus, it can be concluded that both the objective and subjective results of this simple experiment provide a validation point for the method outlined in this paper for the optimal acoustic design of structures.

7. CONCLUSIONS

The method described in this paper, i.e., the optimal acoustic design of a structure, uses the highly efficient feature of introducing the design variables as external forces (via their impedances) in the equation for the structure. This allows the structural response to be written as a series expansion of basis functions that are independent of the design variables. For each design iteration generated with the computer program MATLAB®, the structural response is linked with the computer program POWER to assess the corresponding radiated sound spectrum. Various search algorithms are used within MATLAB® to determine which design variables give the “best fit” to the acoustic specifications. To illustrate the design method, a wine glass was tuned optimally to move the first four eigenvalues into harmonic relationships. The design variables were small masses that were added to the upper surface of the wine glass according to the results of the optimization search. Comparison of the wine glass’s sound power spectrum with and without the optimally placed and sized masses indicated excellent agreement between the specified and measured spectra thereby providing a strong validation point for the design method.

The subject of this paper seems particularly appropriate to pay special tribute to the celebrated and recent octogenarian Philip Doak, Editor-in-Chief of the Journal of Sound and Vibration, because of his enduring curiosity about the subtlety of musical sounds.

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