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THE INFLUENCE OF INITIAL FORCING ON NON-LINEAR CONTROL

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The property of sensitive dependence on initial forcing may be used as an effective trigger of changes in a system response and, thus, used to redistribute energy favorably and efficiently. One unstable periodic orbit was selected from each experiment to demonstrate the role of the initial forcing in an active control strategy to yield improved performance.

1. BACKGROUND

The initial forcing is the origin of the response state. Two slightly different initial forcing parameters, after sufficient time, lead to widely different response states. The initial forcing is known to be the beginning or the origin of the information displayed by non-linearity. In a laboratory, one can verify that each data run is associated with an initial forcing and can show that each run contains different deterministic information [1]. Control of the disturbances in each run is dependent on knowledge of the initial forcing of that run. This type of active control improves the performance of the system being controlled. One issue is related to bifurcation, a change in character of the orbit from stable to unstable. One way of controlling the continuous changes is to prevent or change the character of the bifurcation. This problem has been addressed by the well-known work of Ott *et al.* [2], who postulated that it should be possible to stabilize a system about one of the periodic motions by using small parameter perturbations. This method has since attracted growing interest, and it has been applied in a variety of scientific disciplines. Control of periodic waves and chaos has been achieved in a variety of physical experiments discussed by Ditto *et al.* [3], Hubiger *et al.* [4], Maestrello [5, 6], Bolottin *et al.* [7], Chow and Maestrello [8], and Pyragas and Tamasevicius [9]. From a practical point of view, controlling a fully unstable system from a fixed point is interesting and important. In the systems in the present study, instability is detected from experimental data; the control method used is different from the Ott–Grebogi–Yorke method. It is a very simple method that requires the knowledge of the initial unstable disturbances in terms of frequency, amplitude, and phase or their equivalent temporal values to cancel the growth after several bifurcations from periodic to chaotic states. A simple controller relative to the system being controlled is discussed by Corron *et al.* [10].

Non-linear time-series analysis is becoming a reliable tool for the study of complicated dynamics from measurements and aids in spawning a generation of experiments that are much more quantitative than earlier ones. However, the complexity of these experiments is associated with unpredictability, also known as the generation of information based on the property of sensitive dependence on initial starting parameters (see references [11–15]).

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Present time series from measurements are applied to describe the initial forcing parameters and used in the development of an active control strategy. In addition, time series are used to evaluate the energy variance, Lyapunov exponents, correlation dimension, prediction errors with the intention of identifying the presence of non-linearity, and chaos by Abarbanel [16], Ruelle [17], Grassberger and Procaccia [18], Abraham *et al.* [19], Ditto *et al.* [3], Dowell [20], and Wiggins [21]. The first two experiments reported herein are designed to demonstrate the role of the initial forcing on control strategy, whereas the second two are designed to demonstrate the use of the initial forcing to control screech tone at the nozzle lip and the subharmonic on a panel forced by sound. Although the four experiments are unrelated, they all demonstrate the uniqueness of the method by using their individual initial starting parameters for active input to the controller. In these experiments, large long-term control effects have been achieved.

2. FLOW AND ACOUSTICS LOADING

In engineering, one may come across a situation where it is not necessary to suppress chaos completely, but reducing it to a periodic response is sufficient. In each experiment, the control strategy is to identify the initial forcing in terms of frequencies, amplitude, and phase of the original signal. The procedures can be simple or very difficult. The technique stabilizes the systems around a periodic orbit with knowledge of initial forcing. In the simplest form, non-linearity begins by the period-doubling bifurcation sequences. A single periodic orbit originates with frequency f_1 and, as the level increases, subharmonic frequency $(1/2)\,f_1$ harmonic $2f_1$ form. Repeated bifurcation yields repeated period doubling that gives rise to the frequencies $(1/4)f_1$, $(1/8)f_1$, etc., and harmonics; this accumulation of periodic behaviors continues up to a critical value of the parameters, beyond which non-periodic and chaotic behavior is triggered. Transition to non-linear response depends sensitively on the initial forcing in terms of amplitude, phase and frequency. The initial forcing is highlighted in this section, and active control is highlighted in section 3. The control perturbation is implemented by using a freely suspended low power active device operating at the initial starting condition of the system response.

2.1. PANEL FORCED BY MULTIFREQUENCY SOUND

A two-panel structure is forced by an acoustic source from a state of rest in an anechoic room. The panels are curved airplane fuselages made of aluminum, machined from a plate into two equal size panels separated by a longitudinal tear stopper (Figure 1). The structure is 60·9 cm wide, 101·9 cm long, and 0·0109 cm thick and is forced by an acoustic source at four non-harmonic frequencies at a power level of 138 dB; $f_1 = 387$, $f_2 = 425$, $f_3 = 512$, and $f_4 = 687$ Hz. To being with, the initial forcing amplitudes are kept small. As time progresses, the amplitude is increased gradually to drive the system into different dynamical regimes from the initial period to chaos. These regimes can be observed in time series as well as in power spectra.

In each run, the panel response $g(t)$ (the panel acceleration) and transmitted acoustic pressure $p_r(t)$ are measured. The results show that the chaotic panel response depends strongly on the forcing conditions. The acceleration response $g(t) = \mathrm{d}^2w/\mathrm{d}t^2$ of the panel, as the amplitudes increase in time to a set of constant postchaotic transition values, is shown in Figure 2, where a non-symmetrical panel response $g(t)$ is indicated in the chaotic regime.

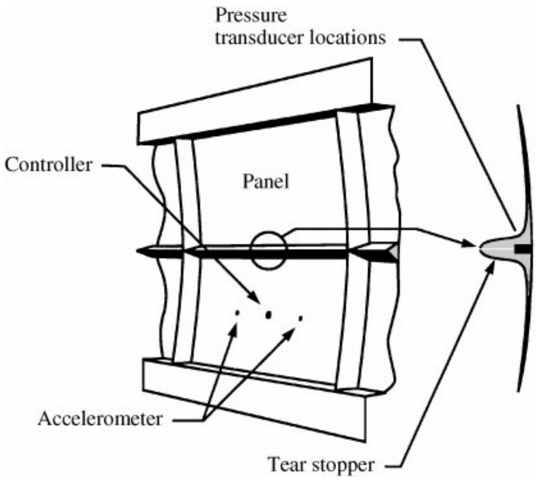

Figure 1. Panel structure.

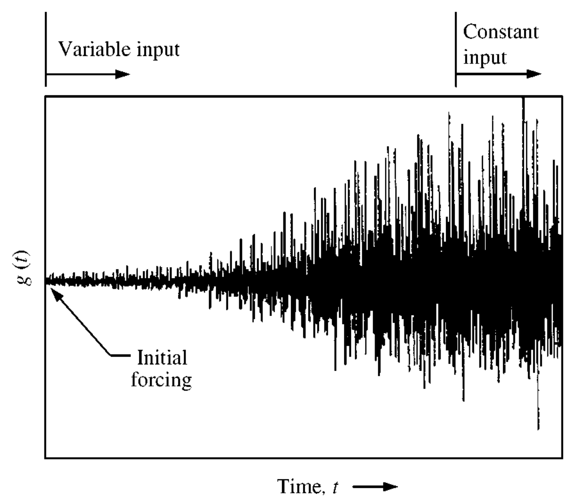

Figure 2. Initial behavior of the panel acceleration response forced by external non-harmonic multifrequency sound.

Based on the measured data, the finite time T, the wall pressure $p(t)$, the panel response $g(t)$, and the transmitted pressure $p_r(t)$ and then, by the Fourier transform, the corresponding spectral density functions of the wall pressure $P(f, T)$, the panel response $G(f, T)$, and the transmitted pressure $P_r(f, T)$ are calculated, where T is chosen so that the experimental run contains the interval T. At the initial start, a comparison of the spectral density is made in Figure 3. As expected, the wall pressure spectrum $P(f, T)$ shows peaks at the four initial frequencies f_1–f_4. The panel response spectrum $G(f, T)$ exhibits robust peaks at the same four forcing frequencies, f_1–f_4 and, in addition, at the subharmonics $(1/2)f_1$ and $(1/2)f_2$ as well as the harmonics $2f_1$ and $2f_2$. The other peak frequencies are incommensurable. The transmitted pressure spectrum $P_r(f, T)$ also shows the same four peaked forcing frequencies with subharmonics and harmonics. Thus, from the experimental evidence it is seen that at lower amplitude after the initial start, the four basic forcing

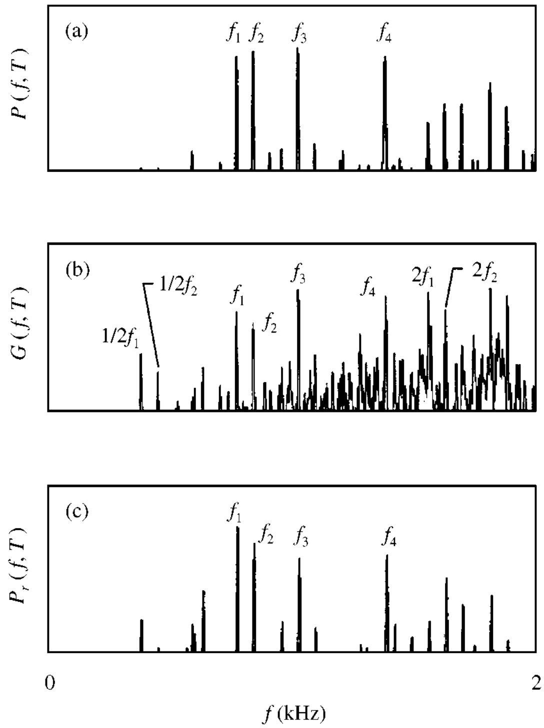

Figure 3. Spectral density of the initial start: (a) input pressure, (b) panel r response, (c) transmitted pressure.

frequencies f_1–f_4 appear prominently in both the panel response and the transmitted pressure power spectra.

To suppress the chaos, in view of the sensitive dependence on the initial forcing parameters, it is sensible to use a weak quasiperiodic forcing $u(\varepsilon, \omega, \theta, t)$ as a control. In particular, u is taken to be

$$u(\varepsilon, \omega, \theta, t) = \sum_{i=1}^{4} \varepsilon_i \cos(\omega_i t + \theta_i),$$

where ω_i, $i = 1, 2, 3, 4$ represents the initial forcing frequencies, and the amplitudes ε_1, ε_2, ε_3 and ε_4, which are small, and the phases θ_1, θ_2, θ_3, and θ_4 are to be adjusted experimentally to suppress the chaos. In practice, the forcing frequencies f_i need not be measured. Then, at least for the low-frequency case, they may be identifiable, amid the subharmonics and harmonics, from the peak frequencies in the panel response and transmitted pressure power spectra $G(f, T)$ and $P_r(f, T)$. Such peak frequencies are selected as possible forcing frequencies as a trial control function u. The possibility is tested and reported in reference [6] as part of the chaos control strategy.

2.2. PANEL FORCED BY TURBULENT BOUNDARY LAYER AND SOUND

In this experiment, panel responses to turbulent boundary layer and sound are studied in a wind tunnel. The structure is mounted in the sidewall of the tunnel and consists of two aluminum aircraft-type panels, joined by a stringer, and the size of the panels is 65 cm long,

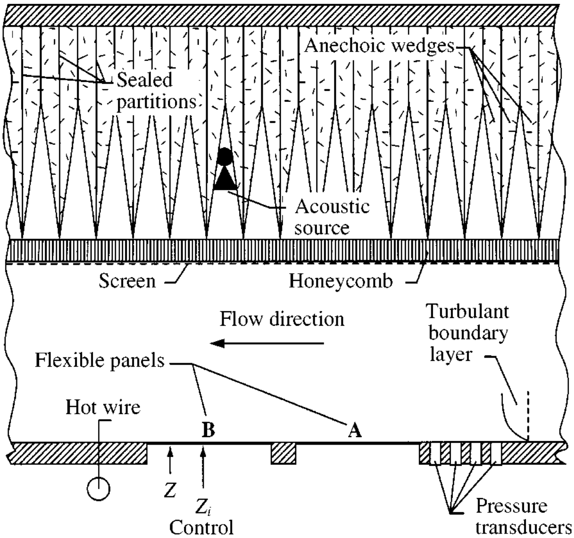

Figure 4. Experimental set-up inside the anechoic wind tunnel.

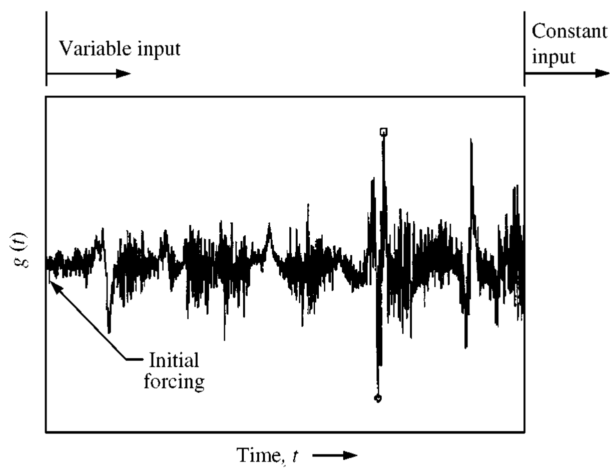

Figure 5. Initial behavior of the panel acceleration response forced for turbulent boundary layer and sound from state of rest to constant speed.

20 cm wide, and 0·01 cm thick (Figure 4). Turbulent boundary layer and pure-tone sound force the panels from a state of rest to 66 m/s. The acoustic source generating a 500 Hz pure-tone sound at a power level of 128 dB is mounted on the opposite sidewall of the tunnel facing the center of the downstream panel. To begin with, the wind tunnel speed is kept low while the acoustic source maintains the constant prescribed amplitude level. As time progresses, the speed increases gradually to drive the panels into a choatic state at constant speed (Figure 5). The panels in the wind tunnel simulate the response of an aircraft fuselage panel forced by turbulent boundary layer and tonal fan noise from takeoff to constant speed flight. In each repeated run, the panel (B in Figure 4) acceleration response *g(t)* shows the dependency on the initial forcing which is unpredictably different in each run [5]. The panel indicates non-symmetrical response and chaotic behavior. The abrupt changes in the time series have been previously observed [6, 22] to be the result of frequency locking among previously incommensurate frequencies. The appearance of chaos, behind

the locking state, was described to be the loss of synchronization between frequency-locked modes. In previous work [5], a panel forced by an external pure-tone sound in the presence of flow exhibited convective instability and high-dimensional complex responses caused by non-linear coupling between convective short-scale waves in the boundary layer and the localized intense long-scale waves of the pure-tone sound and its harmonics. In many engineering problems, chaos is undesirable and, therefore, needs to be controlled. To suppress it, one needs to identify the initial forcing. Section 3 describes the control strategy used for an unstable fixed point on a panel.

3. ACTIVE CONTROL

Two unrelated experiments were selected to highlight active control results. The first experiment is the sound produced by a supersonic jet with screech tones superimposed on broadband. The origin of the screech and harmonics is the screech fundamental. Similarly, in the second experiment for the harmonic response of a structure forced by tonal sound, the origin of the periodicity is the fundamental, a well-known characteristic of non-linear behavior. The favorable control method is the one that uses the initial forcing condition to manipulate the response with a very small amount of power to trigger the changes. The time series from measurements are used to determine the initial starting conditions and used to control the response into steady state. The technique is practical for analyzing non-linear signals in the time or frequency domains as an adjustment to linear methods that are powerful in their own right. The strategy is to achieve the desired response by manipulating the response with small control forces [6].

3.1. CONTROL OF SCREECH TONE IN A SUPERSONIC JET

The pressure from a supersonic jet consists of screech tones with spiral and flapping non-axisymmetric modes superimposed on broadband. Control of the screech tones at the nozzle lip means either (1) the shock is weakened or is converted into an expansion wave, all or in part, or (2) the vibrating lip attachment prevents the characteristics originating in the column of the shear layer from sustaining connection with an out-of-phase surface vibration. Screech from the supersonic jet induces high loading by the fundamental and harmonics tones at different locations on an aircraft structure and it is also an annoyance to community residents.

Powell [23] first recognized that the instability of the jet column due to the feedback loop between fluid flow and sound is the cause of a powerful wave moving upstream with sufficient strength to govern the stability of the boundary layer at the nozzle exit. In addition, a shock caused by transition from supersonic to subsonic as the shear layer expands into the free stream is the cause of broadband noise originating between approximately 9 and 13 nozzle diameters. Later work by Westley and Woolley [24], Norum [25], Seiner [26], Tam [27], Morris *et al.* [28], and Ponton and Seiner [29] indicated that the self-excited oscillation involved the transfer of energy from one wave to another, and the spinning of the shock is associated with large-scale instability and sound. Broadband noise due to shock-flow transition is noticeable in the power spectrum distribution in the presence of a shock caused by the rigid nozzle boundary, whereas for a shock-free or screech-free jet, broadband noises are less resolved by the spectrum pressure distribution.

In the experiment, a convergent nozzle at a pressure ratio of 3 and stagnation temperature of 35°C is used, with corresponding Reynolds and Strouhal numbers of

$1{\cdot}3 \times 10^6$ and 0·28, respectively, based on jet diameter. Time series of the experimental data are used to unravel the complicated dynamics of the shock oscillation. The motion is captured by an array of pressure transducers and used in the development of the screech-tone control device. Control is achieved by placing a ring at the nozzle lip oscillating at the shock fundamental frequency (Figure 6). The ring prevents the shock characteristics originating in the column of the shear layer from sustaining connection with the out-of-phase surface vibration. Screech-free flow is maintained over a large pressure ratio. The peak power level of the series $P(f, T)$ is reduced by 40 dB to a broadband level by redistributing the energy found in the fundamental and harmonics into the broadband level maintaining energy conservation as shown by the power spectra superimposition (Figure 7). Note from Figure 7 that the broadband amplitude level of the energy after control exceeds the broadband level of the uncontrolled run. The integrated power level from each power spectrum is nearly equal. This conclusion was confirmed after several experimental runs were compared. The power required to activate the control system is less than 1/200 of the jet power. Investigations have been carried out to determine how stabilization depends on the duration of the transient response after a step change from the controller input. Using periodic control with a period shorter than the transient response time also led to stabilization, probably because of the very low added dissipation as a result of the changes in response behaviors. This technique has been successful in eliminating shock or screech noise from a supersonic jet with a vibrating ring at the nozzle exit. Data indicate that a shock-free jet or shock-free screech has optimum performance; it is also operational over variable pressure ratio. It was also noted that the shock due to shear layer transition from supersonic to subsonic, an enhancer of broadband noise, behaves differently with and without screech-tone effects. Data indicate that the contribution to broadband noise is less for a screech-free jet tone than for a jet with a screech tone. The data obtained show that a screech-free jet is one with the lower radiating acoustic power for a given momentum thrust and exit diameter.

3.2. CONTROL OF SUBHARMONICS ON A PANEL FORCED BY SOUND

A panel structure that is subjected to time-periodic acoustic forcing at normal incidence might exhibit subharmonic instability. The panel is aluminum and is $30{\cdot}48 \times 20{\cdot}32 \times 0{\cdot}1046$ cm; the response is measured with a strain gauge (Figure 8). The experiment is designed to demonstrate control of the subharmonic by transferring its energy

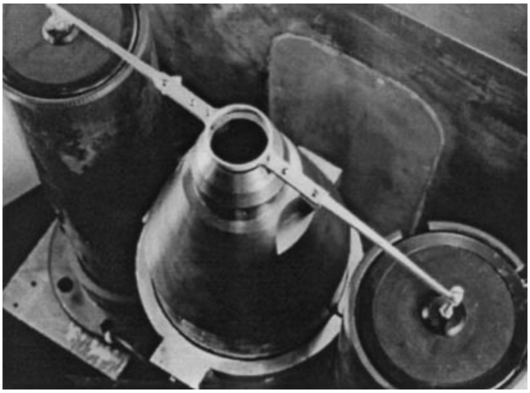

Figure 6. Standard round nozzle with oscillating lip.

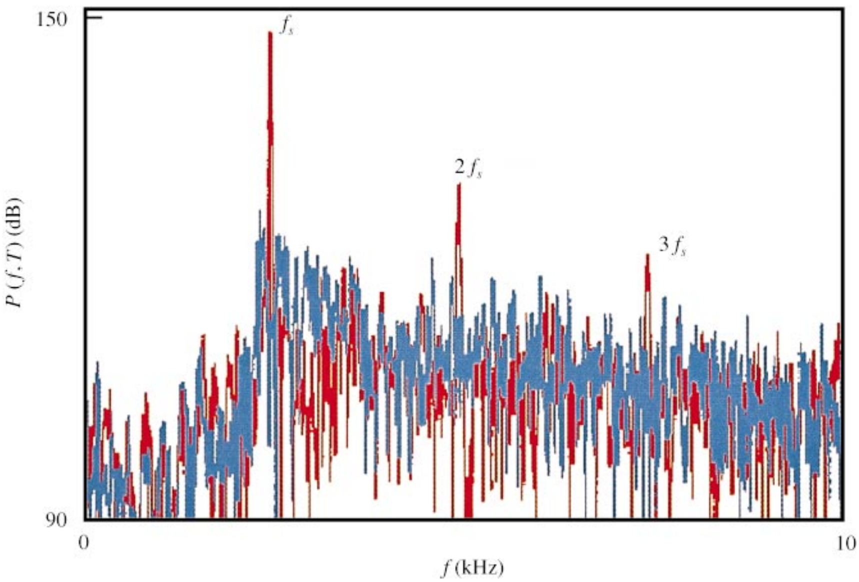

Figure 7. Power spectral density of the shock pressure for uncontrolled and controlled runs: ——, controlled; ——, uncontrolled.

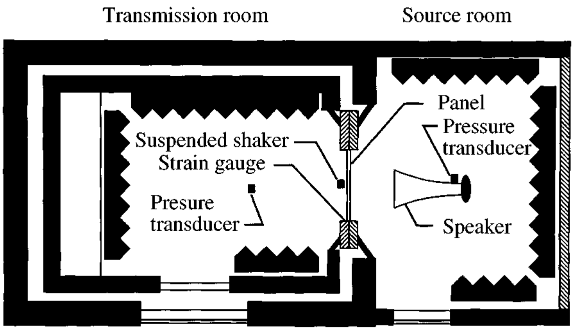

Figure 8. Experimental set-up inside the anechoic chamber.

to the fundamental by using a low-power shaker [5, 30]. The response begins with a single frequency, the fundamental f_1, without any other significant component. As the acoustic input increases, non-linearity is triggered. The oscillation evolves through subharmonic formation with frequency $(1/2)f_1$, and the reconstructed phase depicts a double closed orbit. The response power spectrum of the strain $S(f, T)$ and the phase plot $\dot{s}(t)$ versus $s(t)$ are shown in Figures 9 and 10 for both controlled and uncontrolled runs. In the controlled run the subharmonic $(1/2)f_1$ is eliminated because the energy of the structural bending waves is transferred to the fundamental. The phase plot reduces to a single loop but maintains the same elliptical shape as for the uncontrolled run, which can be attributed to damping and dissipation. The amplitude of the fundamental in Figure 9 increases approximately 3 dB in power because of the energy transfer by the subharmonic, which has the same amplitude of the fundamental. This experiment is an example demonstrating reshaping of the power spectral density by redistributing the response rather than by suppressing it. The dissipation

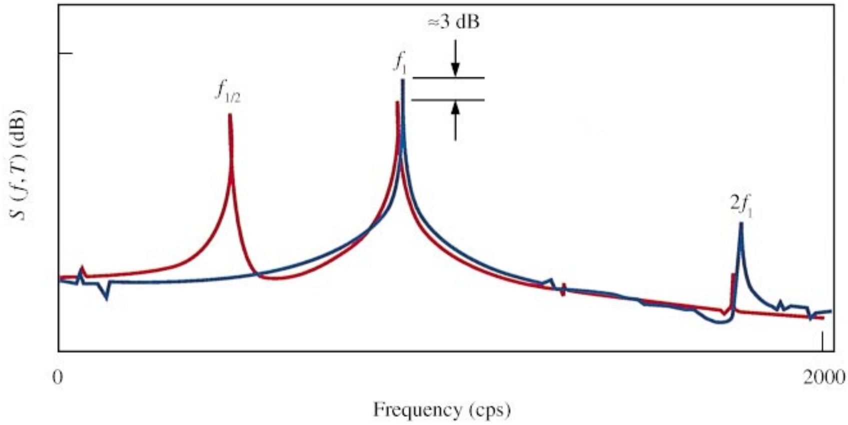

Figure 9. Power spectral density of the panel strain for uncontrolled and controlled runs: ——, controlled; ——, uncontrolled.

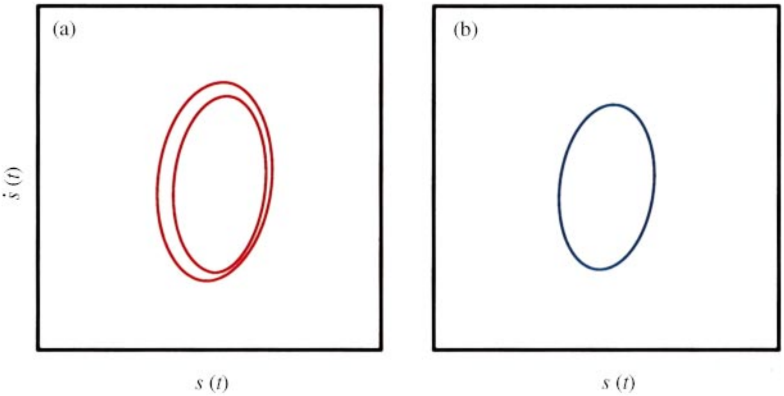

Figure 10. Phase of the panel strain for (a) uncontrolled and (b) controlled runs.

during the control process is very small. The transfer of the bending wave energy from subharmonic to the fundamental affects the sound radiation from the panel in a similar manner. The power required to achieve control varies significantly between experimental runs and on an average is less than 1/10 of the response power, whereas to maintain it, the power required is even less. In conclusion, the periodic control force can preserve energy on certain periodic bifurcations as energy is transferred from one wave mode to another. Furthermore, as evidenced from the phase plots (Figure 9), damping is unaffected by this control method. However, over successive runs, the overall spectra levels of the controlled and uncontrolled runs maintain nearly the same level.

4. CONCLUSIONS

This paper addresses the identification of non-linear systems that exhibit sensitivity to the properties of the initial forcing. The reasoning behind the present work is as follows. Because non-linear systems exhibit sensitive dependence on initial forcing, small control adjustments are likely to exert large long-term effects on the dynamics, thus allowing control. The novelty of this procedure applies to the time series combined with non-linear analysis and active control, demonstrating that the non-linearity and chaos control are extremely simple—even simpler than the physical system being controlled. The method is suitable for experimentalists. The control of the shock wave from a jet and the modal response of a panel transferring energy from one mode wave into another mode wave have been demonstrated. A shock-free supersonic jet is perhaps the most efficient and the most quiet jet flow obtainable. The quality and consistency of the results suggest a full-scale experiment would be relevant.

Small control perturbations were applied to an unstable orbit which yielded an improved performance, such that the orbit is directed to the desired periodic motion or steady state. Initial forcing is used to trigger control of the system response. The time required to achieve control varies; the temporal transients differ greatly for different initial forcing parameters. Controlling a periodic signal with random amplitude, known forcing frequency, and adjustable phase becomes a simpler control strategy. For complex initial forcing, its practical implementation turns out to be far from simple. In general, non-linear systems are intractable with traditional linear methods. Frequently, essentially new phenomena occur in non-linear systems which in principle cannot occur in linear ones. Non-linear vibration control has not been achieved in practice much because engineers and scientists are not familiar with it. The method used is efficient to stabilize highly unstable periodic responses with a low-power activator to manipulate the response to a desired behavior.

It is a great pleasure to dedicate this paper to Professor Phil Doak, my mentor and friend for several decades, on his 80th birthday.

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