Journal of Sound and Vibration (2001) **239**(4), 885–890
doi:10.1006/jsvi.2000.3220, available online at http://www.idealibrary.com on IDEAL®

THERMOOPTICAL EXCITATION OF SOUND IN LIQUIDS BY MODULATED RADIATION OF AN UNSTABLE-CAVITY LASER

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(*Accepted* 10 *July* 2000)

The thermooptical excitation of sound in a liquid by harmonically intensity-modulated laser radiation with a random distribution of intensity over the laser beam cross-section is considered. Processes are considered which are statistically homogeneous. It is assumed that the spatial spectrum of intensity fluctuations in a laser beam is described by a power (fractal) law. Expressions for the mean-square sound pressure in the acoustic field in a liquid, which is excited by unstable radiation of lasers with apertures shaped as narrow slots and circles, are obtained. It is demonstrated that the fractal structure of unstable chaotic laser radiation affects the structure of the acoustic field in a liquid in the high-frequency range of modulation of radiation intensity when the sound wavelength in a liquid is small as against the correlation length of fluctuations of radiation intensity and the dimensions of the laser beam at the liquid surface. An opportunity for photoacoustic diagnostics of the fractal structure of radiation of an unstable-cavity laser is noted.

1. INTRODUCTION

Theoretical consideration of sound excitation in condensed media, e.g., liquids, is conducted usually under the assumption that the transverse distribution of intensity in a laser beam is quite determinate. It is assumed frequently that it is axially symmetrical and has the Gaussian form [1]. Such intensity distribution is observed when a laser operates in a stable mode. This has been confirmed once more by the results by Kononenko *et al.* [2] published recently. However, the intensity distribution in a laser beam fluctuates frequently. The reason for this is the fact that lasers are non-linear dynamic systems.

Dynamic chaos and fractals occupy an important place among the outstanding discoveries of the 20th century [3, 4]. They are connected closely. Dynamic chaos is chaotic, seemingly random oscillations in non-linear deterministic systems. Fractals are self-similar in a certain sense objects of fractional dimensions.

A lot of attention was given to chaotic generation of lasers in recent years. For example, Loskutov *et al.* [5] studied the scenarios of development and parameters of chaotic generation in an unstable cavity of a fast-flow laser with spatially inhomogeneous pumping. A scheme of optical informational chaos on the basis of lasers operating synchronously in

0022-460X/01/040885 + 06 $35.00/0

a chaotic mode and using chaotic pumping was considered by Napartovich and Sukhorukov [6].

Important parameters of non-linear dynamic systems are scaling and correlation or fractal dimensions [3]. Silin [7] indicated the possible existence of scaling of radiation of harmonics in laser plasma under the effect of powerful pumping. It was demonstrated by Karman and Woerdman [8] that the mode structure of radiation of an unstable-cavity laser is fractal. It was determined that the fractal dimension of the distribution of radiation intensity of a laser with the aperture shaped as a narrow slot has the value $D = 1{\cdot}6$ and in the case of a circular aperture it was equal to $D = 1{\cdot}3$.

It is interesting to consider the particular features of sound excitation in a liquid by unstable chaotic laser radiation and determine whether an opportunity for photoacoustic diagnostics of fractal structure of radiation of unstable-cavity lasers exists.

Further, we consider thermooptical excitation of sound in a liquid by harmonically modulated intensity laser radiation with random fractal spatial distribution of intensity fluctuations in the cross-section of a laser beam. It is necessary to note that the effect of spatial and time fluctuations of intensity of laser radiation on sound excitation in liquids was considered earlier by Bunkin [9]. However, the character of the distribution was not specified.

2. EXCITATION OF SOUND IN A LIQUID BY UNSTABLE LASER RADIATION

Let us assume that a laser beam propagating from the upper half-space (atmosphere) in the positive direction of the z-axis of the rectangular co-ordinate system (x, y, z) is incident upon the free surface of a liquid occupying the lower half-space $z > 0$. Thermal sources of sound result from the absorption of laser radiation in the liquid. The equation of laser thermooptical generation of sound has the form [1]

$$(\Delta + k^2)p = \mathrm{i}\,\frac{\kappa m\omega}{C_p}\,A\mu I(x, y)\exp(-\mu z), \tag{1}$$

where p is the sound pressure, k, C_p, and μ are the coefficient of cubical expansion, specific heat capacity, and absorption coefficient of optical radiation in a liquid, respectively, A is the coefficient of light transmission through the liquid boundary (we assume further that $A = 1$), m is the modulation index, $I(x, y)$ is the distribution of intensity in a laser beam at the liquid surface, $k = \omega/c$, with c being is the sound velocity in a liquid. The time factor $\exp(-\mathrm{i}\omega t)$ is omitted here and further.

Using the reciprocity theorem [10], one can write down the solution of equation (1)

$$p(r) = \mathrm{i}\,\frac{\kappa\omega m}{C_p}\,\mu\int_{\Omega} I(x', y')\exp(-\mu z')\tilde{p}(x', y', z'/x, y, z)\,\mathrm{d}x'\,\mathrm{d}y'\,\mathrm{d}z', \tag{2}$$

where $p(r'/r)$ is the solution of the boundary problem of diffraction of the field of a point source positioned in the point r, where it is necessary to determine the field $p(r)$. We consider the field $p(r)$ in the Fraunhofer zone. In this case, $\tilde{p}(r'/r)$ can be represented as

$$\tilde{p}(r'/r) = \frac{\exp(\mathrm{i}kr)}{4\pi r}\{\exp[-\mathrm{i}(\alpha x' + \beta y' + \gamma z')] - \exp[-\mathrm{i}(\alpha x' + \beta y' - \gamma z')]\}, \tag{3}$$

where $\alpha^2 + \beta^2 + \gamma^2 = k^2$, $r = (x^2 + y^2 + z^2)^{1/2}$.

Let the intensity distribution in the beam be a random function $I(x, y) = I_0 f(x, y)$, where $\langle f(x, y)\rangle = 0$ and random processes are statistically homogeneous.

Taking into account everything said before, substituting expression (3) into expression (2), and integrating with respect to z, one can write down the next expression for the mean-square sound pressure:

$$\langle |p(r)|^2 \rangle = \frac{\kappa^2 \omega^2 m^2}{C_p^2} \frac{1}{4\pi^2 r^2} \frac{\mu^2 \gamma^2}{(\mu^2 + \gamma^2)^2} I_0^2 \sigma \int_\xi \int_\eta B(\xi, \eta) \exp[-\mathrm{i}(\alpha\xi + \beta\eta)] \, \mathrm{d}\xi \, \mathrm{d}\eta, \qquad (4)$$

where $B(\xi, \eta) = \langle f(x', y') f(x'', y'') \rangle$ is the normalized correlation function of fluctuations of intensity of laser radiation, $\xi = |x' - x''|$, $\eta = |y' - y''|$, and σ is the area of the laser spot at the liquid surface. Integration with respect to ξ and η is expanded to the region of action of laser radiation at the liquid surface. However, if $B(\xi, \eta)$ decreases rapidly at the dimensions of the laser beam cross-section and $B(\infty) = 0$, integration may be expanded to the interval from $-\infty$ to $+\infty$.

The properties of statistical fractals are characterized frequently by structural (correlation) functions and their spectra. Their particular feature is the fact that they are described by power laws. This follows from the property of scaling of fractal structures [11].

The parameter of statistical fractals important for wave problems is the power spectrum of fluctuations, which has the form

$$G(\mathbf{k}) \sim \mathbf{k}^\delta, \qquad (5)$$

where the index δ is determined for objects with fractal surface by an expression [12]

$$\delta = D - 2d \qquad (6)$$

with D being the fractal dimension, while d is the dimension of the Euclidean space.

Let us determine the mean-square sound pressure when the laser aperture is shaped as a narrow slot in the x-direction. In this case, one can write down an expression

$$B(\xi, \eta) = B_1(\xi) \, B_2(\eta), \qquad (7)$$

where $B_2(\eta) \approx 1$ since we may consider the distribution of intensity fluctuations in the transverse direction as totally correlated. The normalized correlation function in the transverse direction is represented in the form [13]

$$B(\xi) = \frac{1}{2^{\nu-1}\Gamma(\nu)} \left(\frac{\xi}{\xi_0}\right)^\nu \mathrm{K}_\nu\left(\frac{\xi}{\xi_0}\right), \qquad (8)$$

where $\Gamma(\nu)$ is the gamma-function, $\mathrm{K}_\nu(\xi/\xi_0)$ is the Macdonald function, and ξ_0 is the correlation length of intensity fluctuations of laser radiation in the transverse direction. It is necessary to note that $B(0) = 1$ and $B(\infty) = 0$, while $B(\xi)_{\xi < \xi_0} \sim (\xi/\xi_0)^\nu$, i.e., the correlation function has the power form, and from this point of view it can be used for the description of fractal structure of intensity fluctuations of unstable laser radiation.

Substituting expressions (7) and (8) into expression (4), one obtains after integration,

$$\langle |p(r)|^2 \rangle = \frac{\kappa^2 \omega^2 m^2}{C_p^2} \frac{1}{4\pi^2 r^2} \frac{\mu^2 \gamma^2}{(\mu^2 + \gamma^2)^2} I_0 \sigma \eta_0 G(\alpha), \qquad (9)$$

where η_0 is the transverse dimension of the laser spot at the liquid surface and $G(\alpha)$ is the spectral density of intensity fluctuations of laser radiation,

$$G(\alpha) = \frac{\Gamma(\nu + \frac{1}{2})}{\sqrt{\pi}\,\Gamma(\nu)} \frac{\xi_0}{(1 + \alpha^2 \xi_0^2)^{\nu + 1/2}}. \qquad (10)$$

At $\alpha\xi_0 > 1$ the spectral density $G(\alpha)$ has a power (fractal) form

$$G(\alpha)_{\alpha\xi_0 > 1} \sim \alpha^{-(2\nu+1)}. \tag{11}$$

Now consider the case of a circular aperture. An expression for mean-square fluctuations of sound pressure (4) can be represented in the form

$$\langle |p(r)|^2 \rangle = \frac{\kappa^2 \omega^2 m^2}{C_p^2} \frac{1}{4\pi^2 r^2} \frac{\mu^2 \gamma^2}{(\mu^2 + \gamma^2)^2} I_0 \pi a^2 G(k_\perp), \tag{12}$$

where

$$G(k_\perp) = \int_{-\infty}^{+\infty} B(\rho) \exp(-\mathrm{i} k_\perp \rho)\, \mathrm{d}\rho, \tag{13}$$

$k_\perp$ is the component of the wave vector $\mathbf{k}$ in the horizontal plane, $k_\perp^2 = \alpha^2 + \beta^2$, $\rho = |\rho' - \rho''|$, and a is the radius of laser beam at the liquid surface.

The correlation function $B(\rho)$ is expressed in the form of equation (8) by changing ξ for ρ and ξ_0 for ρ_0, where ρ_0 is the correlation length of fluctuations of laser radiation.

For the spectral density (13) we have an expression

$$G(k_\perp) = \frac{\Gamma(\nu+1)}{\sqrt{\pi}\Gamma(\nu)} \frac{\rho_0^2}{(1 + k_\perp^2 \rho_0^2)^{\nu+1}}. \tag{14}$$

At $k_\perp \rho_0 > 1$ we obtain

$$G(k_\perp) \sim k_\perp^{-2(\nu+1)}. \tag{15}$$

It is necessary to determine specific values of the parameter ν in each case under consideration in order to calculate acoustic field in a liquid.

The value of the dimension of the Euclidean space for the conditions of the numerical experiment by Karman and Woerdman [8] is equal to $d = 2$. We have from expressions (5), (6), (11), and (15) $\nu = 0{\cdot}7$ for the aperture shaped as a slot and $\nu = 0{\cdot}35$ for the circular aperture if we use, respectively, the fractal dimensions $D = 1{\cdot}6$ and $D = 1{\cdot}3$ obtained in the numerical experiment [8].

3. DISCUSSION

Let us consider in more details expressions (9), (10), (12), and (14) characterizing mean-square fluctuations of sound pressure in a liquid and spectra of intensity fluctuations of laser radiation for lasers with the slot and circular apertures respectively. Let us assume that the modulation frequency is relatively small and the conditions $\alpha\xi_0 < 1$ or $\alpha\rho_0 < 1$ are satisfied. These conditions for the observation point located in the plane $x0z$ have the form

$$k \sin\theta \xi_0 < 1 \quad \text{and} \quad k \sin\theta \rho_0 < 1, \tag{16}$$

where θ is the observation angle or the angle between the z-axis and the straight line connecting the observation point with the origin of co-ordinates. Conditions (16) mean that the correlation lengths of fluctuations of laser radiation are small as against the sound wavelength λ in a liquid ($k = 2\pi/\lambda$).

One can see that if conditions (16) are satisfied, the directivity characteristics of laser thermooptical sources of sound in a liquid do not depend on the fractal properties of unstable chaotic radiation of lasers but they are determined basically by the value of the parameter $k\mu^{-1}$. It is necessary to note that the parameter μ^{-1} characterizes the absorption length of laser (optical) radiation in a liquid. If $k\mu^{-1} < 1$, it means that a laser thermooptical sound source in the form of a dipole with the typical directivity pattern arises at the liquid surface. If $k\mu^{-1} > 1$, a laser thermooptical sound source, which is strongly extended along the propagation direction of the laser beam, arises at the liquid surface and the linear dimensions of the source are greater than the sound wavelength. Sound is emitted in a liquid mainly along the liquid–air interface.

Let us consider the characteristics of laser thermooptical sound sources in the case of high frequency of modulation of laser radiation, when the conditions

$$\alpha\xi_0 = k\sin\theta > 1 \quad \text{or} \quad \alpha\rho_0 = k\sin\theta > 1 \tag{17}$$

are satisfied. As one can see from expressions (9)–(16), in this case the fractal structure of intensity fluctuations of laser radiation plays an essential role in the formation of a sound source in a liquid. Let us assume that conditions (17) and an additional condition $k\mu^{-1} > 1$ are satisfied. In this case a laser thermooptical sound source with the dimensions large in comparison with the sound wavelength arises in a liquid. The next expression is valid for mean-square fluctuations of pressure in the acoustic field:

$$\langle |p(r)|^2 \rangle \sim k^{\delta}, \tag{18}$$

where C is the constant determined by the problem parameters.

It follows from expression (18) that

$$\delta = \frac{\log\langle |p(r)|^2 \rangle}{\log k}. \tag{19}$$

Varying the modulation frequency of radiation we obtain the function $\delta = \varphi(k)$ or $D \sim \phi(k)$ respectively. Essentially, expression (19) is the definition of cellular fractal dimension.

One can see that an opportunity of photoacoustic diagnostics of fractal structure of radiation of an unstable-cavity laser exists.

It is necessary to note also that as one can see from the above analysis, the influence of the fractal dimension on the directivity of a thermooptical source is determined by the correlation length. In the case of small laser spots on a liquid surface (a focused laser beam for example) the correlation lengths are small and the effect is essentially in the high-frequency range (about several MHz). However, in real conditions it is necessary often to use powerful lasers with unfocused wide beams (sometimes, even defocused beams) to obtain a sound beam propagating perpendicularly to the surface, for example. In this case, the correlation length is large and the effect should manifest itself in a common ultrasonic range or even at lower frequencies.

4. CONCLUSIONS

Thermooptical excitation of sound in a liquid by harmonically intensity-modulated laser radiation with random fractal distribution of intensity in the beam has been considered. Expressions for mean-square sound pressure in the acoustic field of a laser thermooptical sound source created in a liquid by unstable radiation of an unstable-cavity laser with the aperture shaped as a narrow slot and a circular aperture have been obtained. It is demonstrated that the fractal structure of unstable chaotic radiation of a laser affects the structure of acoustic field in a liquid in the high-frequency range of modulation of radiation intensity, when the sound wavelength in a liquid is small as against the correlation length of fluctuations of radiation intensity and the dimensions of the laser beam at the liquid surface.

Using the expressions obtained for mean-square fluctuations of sound pressure it is possible to monitor the dependence of the directivity pattern of a laser thermooptical sound source on the fractal dimension of intensity fluctuations of laser radiation. In this case it is necessary to select several values of the parameter v within the range, which corresponds to the change of fractal dimension $1 < D < 2$. An opportunity of photoacoustic diagnostics of fractal structure of radiation of an unstable-cavity laser in the conditions, when the transverse and longitudinal dimensions of laser thermooptical source of sound are large as against the sound wavelength, exists. Naturally, the spatial spectrum of intensity fluctuations of laser radiation does not affect directivity patterns of laser thermooptical sound sources in a liquid in the low-frequency range of modulation.

A similar approach may be used to treat the case of sound excitation by pulsed radiation of an unstable-cavity laser. However this is a separate problem, which needs special consideration.

ACKNOWLEDGMENTS

The work is supported by the Russian Foundation for Basic Research, grant No. 99-02-16334 and INTAS, grant No. 97-31680.

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