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AN APPROXIMATE DISPERSION EQUATION FOR SOUND WAVES IN A NARROW PIPE WITH AMBIENT GRADIENTS

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There is some interest on the effect of temperature gradient on sound propagation in a catalytic monolith. Some results are available in the literature for the effect of a constant temperature gradient on non-isentropic propagation in a narrow pipe modelling a monolith pore, but the effect of mean flow is neglected in these solutions. This paper presents an approximate solution in which the presence of a mean flow, which is assumed to have a uniform velocity profile, is taken into account. The solution also includes the effect of a constant pressure gradient. A dispersion equation is derived by assuming that the spatial variations of the ambient variables can be lumped by using their average values. This approximation limits the range of application of the solution to small ambient gradients or relatively high frequencies. For typical catalytic monolith cylinder dimensions, the present solution can be used to predict the mean flow effects over a useful frequency range.

1. INTRODUCTION

A chemical performance study by Bennett *et al.* [1] has shown that the mean temperature distribution along a monolithic reactor is not uniform, but a substantial temperature increase occurs within about the first half of its length, after which the temperature remains constant. Tests in automobile applications show a wall temperature increase amounting to about 100 K in the first few centimeters of the monolith [2]. There is therefore some interest on sound propagation in the pores of a monolith with a temperature gradient. From previous work on sound transmission in a monolith without a temperature gradient, it is known that a non-isentropic fundamental mode formulation based on the simplification of the general acoustic equations in the manner of Zwicker and Kosten is satisfactory for acoustic modelling of monolith pores as narrow pipes. An extension of this approach for the presence of a constant temperature gradient has been presented by Peat [3], who has proposed an approximate analytical solution neglecting mean flow and the presence of a mean pressure gradient. The present paper describes an approximate analytical solution in which these effects are also taken into account. The mean flow profile is assumed to be uniform and the temperature and pressure gradients are assumed to be constant. In the analysis it is assumed that the spatial variations of the ambient state variables are small enough to be replaced by their average values in the governing acoustic equations. Under this approximation, the present solution can be expected to be accurate for relatively small temperature and mean pressure increases, or for relatively high frequencies. Nevertheless, typical mean temperature increases in automotive applications amount to only about 10% or less, and the mean pressure drop is even less than this. For these conditions, the proposed solution can be used for the prediction of the mean flow effects in a useful frequency range.

2. THEORETICAL FORMULATION

2.1. GOVERNING EQUATIONS AND ASSUMPTIONS

The governing equations are derived from the continuity, momentum and energy equations of fluid dynamics and the state equation for a perfect gas by the usual process of linearization. Several approximations are made in order to simplify the linearized equations into a mathematically tractable form. The pipe is assumed to be uniform and of circular cross-section and the study is restricted to the fundamental mode of propagation. An order magnitude analysis is applied, in the manner of the Zwicker and Kosten theory, in order to dispense with the second order terms arising from the extreme differences in length and velocity scales in axial and radial directions. The mean flow velocity, v_0, is assumed to be axial and have a uniform profile. The mean temperature, T_0, is assumed to be a function of the axial co-ordinate, x, and μ and κ, the shear viscosity coefficient and thermal conductivity, respectively, are assumed to be slowly varying functions of T_0 so that the gradients $\mathrm{d}\mu/\mathrm{d}x$ and $\mathrm{d}\kappa/\mathrm{d}x$ are small to the first order. Under these assumptions, the continuity equation is

$$\frac{\partial \rho}{\partial t} + v_x \frac{\mathrm{d}\rho_0}{\mathrm{d}x} + v_0 \frac{\partial \rho}{\partial x} + \rho \frac{\mathrm{d}v_0}{\mathrm{d}x} + \rho_0 \nabla \cdot \mathbf{v} = 0, \tag{1}$$

where t denotes the time, ρ denotes the acoustic density fluctuations, ρ_0 is the ambient density, $\mathbf{v}$ denotes the particle velocity, v_x being its axial component and

$$\nabla \cdot \mathbf{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r}. \tag{2}$$

Here, v_r is the radial component of the particle velocity and r denotes the radial co-ordinate. The axial component of the momentum equation is

$$\rho_0 \frac{\partial v_x}{\partial t} + \rho_0 v_0 \frac{\partial v_x}{\partial x} + \rho_0 v_x \frac{\mathrm{d}v_0}{\mathrm{d}x} = -\frac{\partial p}{\partial x} + \mu \nabla_s^2 v_x, \tag{3}$$

where p is the acoustic pressure and the Laplacian on the cross-section is defined as

$$\nabla_s^2 = \frac{\partial^2}{\partial r^2} + \frac{\partial}{r \partial r}. \tag{4}$$

The radial component of the momentum equation is simply

$$\frac{\partial p}{\partial r} = 0. \tag{5}$$

The energy equation is

$$\rho_0 c_p \left[\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial x} + v_x \frac{\mathrm{d}T_0}{\mathrm{d}x} \right] + \rho c_p v_0 \frac{\mathrm{d}T_0}{\mathrm{d}x} = \frac{\partial p}{\partial t} + v_0 \frac{\partial p}{\partial x} + \frac{\mathrm{d}p_0}{\mathrm{d}x} v_x + \kappa \nabla_s^2 T, \tag{6}$$

where T denotes the temperature fluctuations, T_0 is the ambient temperature and c_p is the specific heat coefficient at constant pressure, These equations are closed by the state equation for the ambient fluid, which is assumed to be a perfect gas:

$$\rho = \frac{p}{RT_0} - \frac{\rho_0 T}{T_0}. \tag{7}$$

Here, R denotes the gas constant. The boundary conditions for which a solution of the foregoing equations is required are that $\mathbf{v}$ and T have finite values on the pipe center line, $r = 0$, and vanish on the pipe periphery, $r = a$, where a is the pipe radius.

The axial distribution of the mean temperature and pressure is assumed to be linear,

$$T_0(x) = \bar{T}_0(1 + \tau\xi), \quad p_0(x) = \bar{p}_0(1 + \pi\xi), \quad \xi = -1 + \frac{2x}{L}, \tag{8}$$

where p_0 denotes the ambient pressures, L is the length of the pipe, the overbar ($^{-}$) denotes an axial average, and the temperature and pressure change parameters are defined as

$$\tau = \frac{T_0(L) - T_0(0)}{2\bar{T}_0}, \quad \pi = \frac{p_0(L) - p_0(0)}{2\bar{p}_0}. \tag{9}$$

The axial variation of the mean density distribution is then determined by the perfect gas law $p_0 = \rho_0 R T_0$, and the axial variation of the mean flow velocity is determined from the continuity equation for the mean flow, $\rho_0 v_0 = \text{constant}$. Hence, the ambient gradients occurring in the governing acoustic equations can be expressed in terms of the temperature and pressure change parameters by using the relations

$$\frac{\mathrm{d}T_0}{\mathrm{d}x} = \frac{2\tau\bar{T}_0}{L}, \quad \frac{\mathrm{d}p_0}{\mathrm{d}x} = \frac{2\pi\bar{p}_0}{L}, \quad -\frac{\mathrm{d}\rho_0}{\rho_0\mathrm{d}x} = \frac{\mathrm{d}v_0}{v_0\mathrm{d}x} = \frac{\bar{T}_0}{T_0}\frac{2\tau}{L} - \frac{\bar{p}_0}{p_0}\frac{2\pi}{L}. \tag{10}$$

After having substituted these gradients, and assuming $\exp(-\mathrm{i}\omega t)$ time dependence, where ω is the radian frequency and i denotes the unit imaginary number, equations (1), (3) and (6) can be expressed to $O[\tau^2]$ and $O[\pi^2]$ as follows:

$$-\left[\mathrm{i}\omega\frac{1-\pi\xi}{\bar{p}_0}p + \bar{v}_0\left\{\frac{2\pi}{L\bar{p}_0}p - \frac{1-(2\pi-\tau)\xi}{\bar{p}_0}\frac{\partial p}{\partial x}\right\}\right]$$

$$+\left[\mathrm{i}\omega\frac{1-\tau\xi}{\bar{T}_0}T + \bar{v}_0\left\{\frac{2\tau}{L\bar{T}_0}T - \frac{1-\pi\xi}{\bar{T}_0}\frac{\partial T}{\partial x}\right\}\right] - \frac{2(\tau-\pi)}{L}v_x + \nabla\cdot\mathbf{v} = \mathbf{0}, \tag{11}$$

$$-\mathrm{i}\omega\bar{\rho}_0[1-(\tau-\pi)\xi]v_x + \bar{\rho}_0\bar{v}_0\left[\frac{\partial v_x}{\partial x} + \frac{2(\tau-\pi)v_x}{L}\right] = -\frac{\partial p}{\partial x} + \bar{\mu}\left(1 + \frac{\tau\xi}{2}\right)\nabla_s^2 v_x, \tag{12}$$

$$-\bar{\rho}_0\bar{c}_p\left[\mathrm{i}\omega[1-(\tau-\pi)\xi]T + \bar{v}_0\left\{\frac{2\tau T}{L} - \frac{\partial T}{\partial x}\right\}\right] = -\left[\pi - \frac{\bar{\gamma}\tau}{\bar{\gamma}-1}\right]\frac{2\bar{p}_0 v_x}{L}$$

$$-\left[\mathrm{i}\omega p + \bar{v}_0\left\{\frac{2\bar{\gamma}\tau p}{L(\bar{\gamma}-1)} - [1-(\tau-\pi)\xi]\frac{\partial p}{\partial x}\right\}\right] + \bar{\kappa}\left(1 + \frac{\tau\xi}{2}\right)\nabla_s^2 T. \tag{13}$$

Here, γ is the ratio of specific heat coefficients and it is assumed that c_p can be treated as a constant at its value for $\bar{T}_0$ and κ and μ are approximately proportional to the square root of the absolute temperature at sufficiently high temperatures. A solution of these equations could not be found without making the assumption that in the terms involving ξ, ρ_0, v_0, T_0,

p_0 and κ, μ can be replaced by their axial averages. With this approximation, the accuracy of the analysis may no longer be maintained at $O[\tau^2]$ and $O[\pi^2]$ under all conditions. As an examination of equations (11–13) will reveal, this approximation can be justified if τ and π are small compared to unity. However, the same effect can also be obtained if one assumes that the mean value theorem can be invoked for the ξ-dependent terms by taking the axial mean value position at the pipe center, $\xi = 0$. Thus, equations (11–13) may remain accurate to first order in τ and π if these are small compared to unity or if the mean value theorem is applicable as stipulated above.

2.2. THE DISPERSION EQUATION

Under the above-described approximation, solution of equations (11)–(13) can be searched in the form

$$p = A\exp(\mathrm{i}K\bar{k}x), \quad v_x = H(r)p, \quad T = F(r)p. \tag{14}$$

Here, A denotes a constant, K denotes a propagation constant and $\bar{k}$ is the average wavenumber, $\bar{k} = \omega/\bar{c}$, where $\bar{c} = \sqrt{\bar{\gamma}\bar{p}_0/\bar{\rho}_0}$. Substituting these in equations (12) and (13) gives the equations governing the radial functions $H(r)$ and $F(r)$ respectively:

$$\frac{\mathrm{d}^2H}{\mathrm{d}r^2} + \frac{\mathrm{d}H}{r\mathrm{d}r} + \beta^2 H = \beta^2 H_0, \tag{15}$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}r^2} + \frac{\mathrm{d}F}{r\mathrm{d}r} + \sigma^2 F = \sigma^2 F_0 + F_1 H. \tag{16}$$

Here,

$$\sigma^2 F_0 = \frac{\mathrm{i}\omega}{\bar{\kappa}}\left[1 - \bar{M}K + \frac{2\tau\bar{M}\bar{\gamma}}{\mathrm{i}(\bar{\gamma}-1)\bar{k}L}\right], \quad \beta^2 H_0 = \frac{\mathrm{i}K\bar{k}}{\bar{\mu}}, \tag{17}$$

$$F_1 = \frac{2\bar{p}_0}{\bar{\kappa}L}\left[\frac{\tau\bar{\gamma}}{\bar{\gamma}-1} - \pi\right], \tag{18}$$

$$\beta^2 a^2 = \mathrm{i}\Phi(\pi - \tau)\bar{s}^2, \quad \sigma^2 a^2 = \mathrm{i}\Phi(\tau)\bar{s}^2\bar{P}_r, \tag{19}$$

where

$$\bar{M} = \frac{\bar{v}_0}{\bar{c}_0}, \quad \bar{P}_r = \frac{\bar{\mu}\bar{c}_p}{\bar{\kappa}}, \quad \bar{s} = a\sqrt{\frac{\bar{\rho}_0\omega}{\bar{\mu}}}, \tag{20}$$

and the function Φ is defined as

$$\Phi(\alpha) = 1 - \bar{M}K + \frac{2\bar{M}\alpha}{\mathrm{i}\bar{k}L}. \tag{21}$$

Solution of equation (15) can be expressed in terms of Bessel function of zeroth order. Writing down the general solution of equation (15) applying the boundary condition $v_x = 0$ and the finiteness requirement at $r = 0$, one finds that $H(r)$ is given as

$$H(r) = H_0\left[1 - \frac{\mathrm{J}_0(\beta r)}{\mathrm{J}_0(\beta a)}\right], \tag{22}$$

where J_n denotes a Bessel function of order n. An analytical solution of equation (16) can also be found. Briefly, equation (22) is substituted into equation (16) and the right-hand side of the resulting equation is re-arranged as a sum of a constant term and an r-dependent term. Since the left-hand sides of equations (15) and (16) are formally the same, the solution corresponding to the constant part can be written down by emulating the solution of equation (15). The particular solution corresponding to the r-dependent term, on the other hand, is obtained by noting that it must be proportional to $\mathrm{J}_0(\beta r)$. Thus, the solution of equation (16), which follows after invoking the principle of superposition and applying the boundary condition $T = 0$ at $r = a$ and the finiteness condition at $r = 0$, can be expressed as

$$F(r) = \left[F_0 + \frac{F_1 H_0}{\sigma^2}\right]\left[1 - \frac{\mathrm{J}_0(\sigma r)}{\mathrm{J}_0(\sigma a)}\right] - \left[\frac{F_1 H_0}{\sigma^2 - \beta^2}\right]\left[\frac{\mathrm{J}_0(\beta r)}{\mathrm{J}_0(\beta a)} - \frac{\mathrm{J}_0(\sigma r)}{\mathrm{J}_0(\sigma a)}\right]. \tag{23}$$

Now, expressing ρ in terms of F and H, substituting the result in equation (11) and invoking the boundary conditions $v_r = 0$ at $r = 0$ and $v_r = 0$ at $r = a$ gives

$$\int_0^a r\mathrm{i}\bar{k}\left[\frac{\bar{c}_0 \Phi(\pi)}{\bar{p}_0} - \frac{\bar{c}_0 \Phi(\tau) F(r)}{\bar{T}_0} - \left\{K + \frac{2(\pi - \tau)}{\mathrm{i}\bar{k}L}\right\} H(r)\right] \mathrm{d}r = 0. \tag{24}$$

This is the dispersion that determines the propagation constant K. Upon integration, it can be expressed as

$$\bar{\gamma} + \left[\frac{(\bar{\gamma} - 1)C_1}{\Phi(\pi)}\right]\left[\frac{\mathrm{J}_2(\sigma a)}{\mathrm{J}_0(\sigma a)}\right] + \left[\frac{K}{1 - \bar{M}K}\right]^2 \left[\frac{C_2}{C_3}\right]\left[\frac{\mathrm{J}_2(\beta a)}{\mathrm{J}_0(\beta a)}\right] = 0, \tag{25}$$

where

$$C_1 = 1 - \bar{M}K + \left[\frac{2\tau\bar{\gamma}\bar{M}}{\mathrm{i}(\bar{\gamma} - 1)\bar{k}L}\right] - \left[\frac{2\bar{s}^2 K}{(\bar{\gamma} - 1)\bar{k}L}\right]\left[\frac{\tau\bar{\gamma} - \pi(\bar{\gamma} - 1)}{\bar{\gamma}a^2(\sigma^2 - \beta^2)}\right], \tag{26}$$

$$C_2 = 1 - \left[\frac{2(\tau - \pi)}{\mathrm{i}\bar{k}KL}\right] + \left[\frac{2\sigma^2 a^2}{\mathrm{i}\bar{k}KL}\right]\left[\frac{\tau\bar{\gamma} - \pi(\bar{\gamma} - 1)}{\bar{\gamma}a^2(\sigma^2 - \beta^2)}\right], \tag{27}$$

$$C_3 = \frac{\Phi(\pi - \tau)\Phi(\pi)}{(1 - \bar{M}K)^2}. \tag{28}$$

In the case of zero mean flow, equation (25) simplifies to a quadratic equation for the propagation constants:

$$K^2 + A_1 K + A_2 = 0, \tag{29}$$

where

$$A_1 = g_1 - g_2\left[\frac{\mathrm{J}_2(\bar{s}\sqrt{\mathrm{i}\bar{P}_r})}{\mathrm{J}_0(\bar{s}\sqrt{\mathrm{i}\bar{P}_r})}\right]\left[\frac{\mathrm{J}_0(\bar{s}\sqrt{\mathrm{i}})}{\mathrm{J}_2(\bar{s}\sqrt{\mathrm{i}})}\right], \tag{30}$$

$$A_2 = \left[\bar{\gamma} + (\bar{\gamma} - 1)\left\{\frac{\mathrm{J}_2(\bar{s}\sqrt{\mathrm{i}\bar{P}_r})}{\mathrm{J}_0(\bar{s}\sqrt{\mathrm{i}\bar{P}_r})}\right\}\right]\left[\frac{\mathrm{J}_0(\bar{s}\sqrt{\mathrm{i}})}{\mathrm{J}_2(\bar{s}\sqrt{\mathrm{i}})}\right], \tag{31}$$

$$g_1 = g_2\bar{P}_r + \frac{\mathrm{i}2(\tau - \pi)}{\bar{k}L}, \quad g_2 = \mathrm{i}2\left[\frac{\tau\bar{\gamma} - \pi(\bar{\gamma} - 1)}{\bar{\gamma}(1 - \bar{P}_r)\bar{k}L}\right]. \tag{32}$$

As $\tau/\bar{k}L$ and $\pi/\bar{k}L$ tend to zero, the quotients $C_1/\Phi(\pi)$ and C_2/C_3 tend to unity, and the dispersion equations reduces to the uniform temperature case [4].

3. RESULTS AND DISCUSSION

The propagation constants depend on the parameters $\bar{\gamma}$, $\bar{s}$, $\bar{M}$, $\bar{P}_r, \tau/\bar{k}L, \pi/\bar{k}L$. The presence of mean flow may introduce some hydrodynamic modes into the dispersion equation. The present study is concerned only with the effects of mean flow on the two propagation constants of the zero mean flow case. These can in general be distinguished as waves along the positive (forward) and negative (backward) directions of the pipe axis and are denoted, respectively, by K^+ and K^-, as usual. The wave transfer over a pipe of length L can then be expressed as

$$p^+(L) = p^+(0)\exp(\mathrm{i}\bar{k}K^+L), \quad p^-(L) = p^-(0)\exp(\mathrm{i}\bar{k}K^-L), \tag{33}$$

where $p^+(x)$ and $p^-(x)$ denote forward and backward sound wave pressures, respectively, at section x. The propagation constants are usually given as attenuation and phase speed ratio. Attenuation in dB/m is defined as $A^{\pm} = \pm 8{\cdot}686\bar{k}K_I$ and the phase speed ratio is given by $\phi^{\pm} = \pm 1/K_R$, where the subscripts, 'R' and 'I' denote the real and imaginary parts of a complex quantity, respectively.

Ambient gradients are represented in the dispersion equation by the non-dimensional parameters $\tau/\bar{k}L$ and $\pi/\bar{k}L$. As these parameters tend to zero, the dispersion equation reduces to the uniform temperature case. Therefore, the smaller they are, the more accurate should be the representation of the ambient gradient effects in the solution. This shows that the present solution can be expected to be a good approximation for relatively small temperature and pressure change parameters or for relatively high frequencies. On these premises, the following test can be used to estimate the relative accuracy of the present solution: Compute the propagation constants by decreasing L gradually, while all other parameters are kept unchanged. The solution will be most accurate for the largest L and over the range of L for which the results remain substantially the same as those for the largest L.

An application of this test is shown in Figure 1 for a monolith tube of radius $a = 0{\cdot}5$ mm with $\bar{T}_0 = 950\,\mathrm{K}$, $\tau = 0{\cdot}05$, $\bar{p}_0 = 105$ kPa, $\pi = 0$, $\bar{\mu} = 4{\cdot}0 \times 10^{-5}$ Ns/m^2, $R = 288{\cdot}1$ J/kg K, $\bar{\gamma} = 1{\cdot}34$, $\bar{P}_r = 0{\cdot}70$ and $M(0) = 0$, where $M(0)$ is the Mach number of the mean flow velocity at the tube inlet. The lower limit of the frequency range is 20 Hz and, as can be inferred from the figure, the present solution can be expected to be satisfactory in this frequency range for tube lengths greater than about 50 mm. This does not mean that the results will be grossly inaccurate for shorter lengths, they may also be acceptable for the shorter tube lengths, but this cannot be asserted with certainty *a priori*, except when the frequency is sufficiently high. In general, the smaller is $|\tau|$, or $|\pi|$, the smaller is the tube length for which the present solution is a valid approximation in a given frequency range, the effect of low Mach number mean flow on the accuracy being indiscernible.

Peat [3] has presented some results for this monolith tube for $L = 20$ mm and $\tau = 0{\cdot}05$ and 0·1. The present propagation constants for the case of $\tau = 0{\cdot}05$ are contained in Figure 1. A comparison of this case with the results of reference [3] is not presented here, because of the lack of a satisfactory correlation. This may be due to the present solution not being accurate enough for $L = 20$ mm and $\tau = 0{\cdot}05$. However, for the larger tube lengths indicated in the above test, the present results are in fairly good agreement with the approximate analytical solution of reference [3]. Shown in Figure 2 is the comparison of the attenuation and phase speed ratio of the monolith tube under discussion for $L = 60$ mm

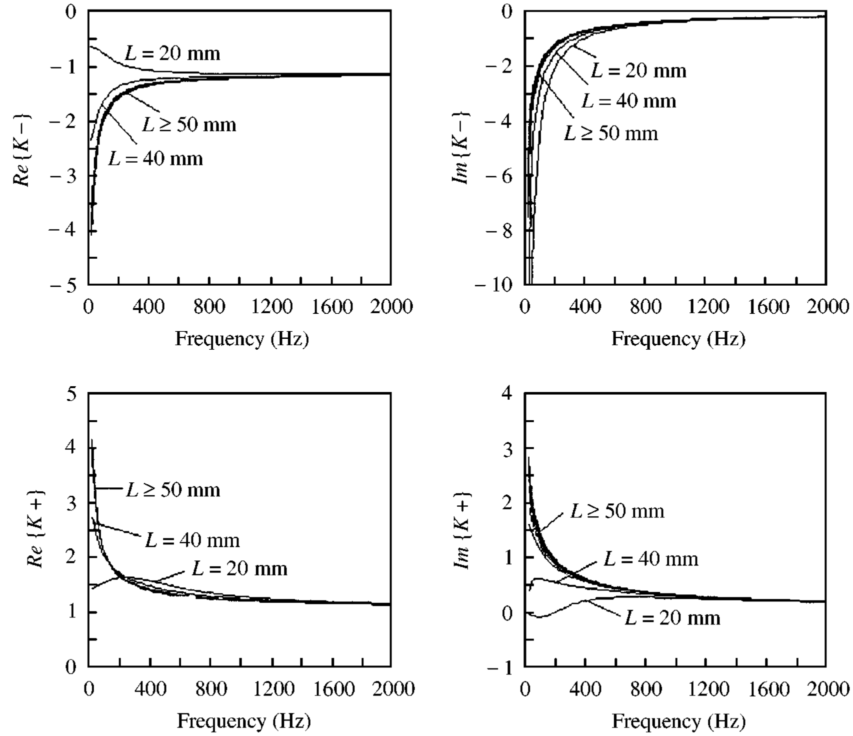

Figure 1. Effect of pipe length on propagation constants in a narrow pipe of radius $a = 0{\cdot}5$ mm, $\bar{T} = 950$ K, $\tau = 0{\cdot}05$, $\pi = 0$ and $M(0) = 0$.

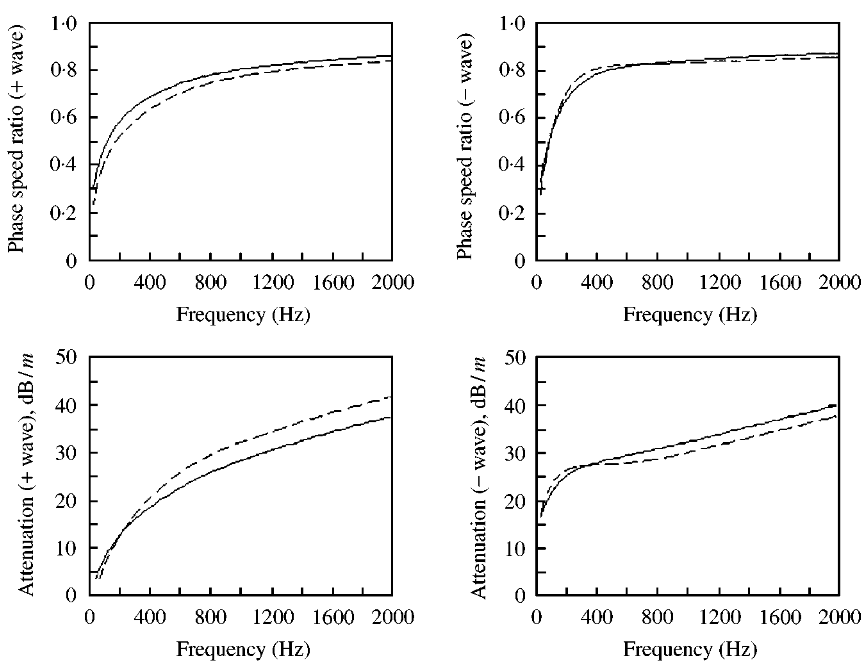

Figure 2. The attenuation and phase speed ratio of a $L = 0{\cdot}06$ m long narrow pipe of radius $a = 0{\cdot}5$ mm, $\bar{T} = 950$ K, $\tau = 0{\cdot}05$, $\pi = 0$ and $M(0) = 0$: (——), present solution; (– – –), computed from wavenumbers given in reference [3] (phase speed ratio corresponds to $\xi = 0$).

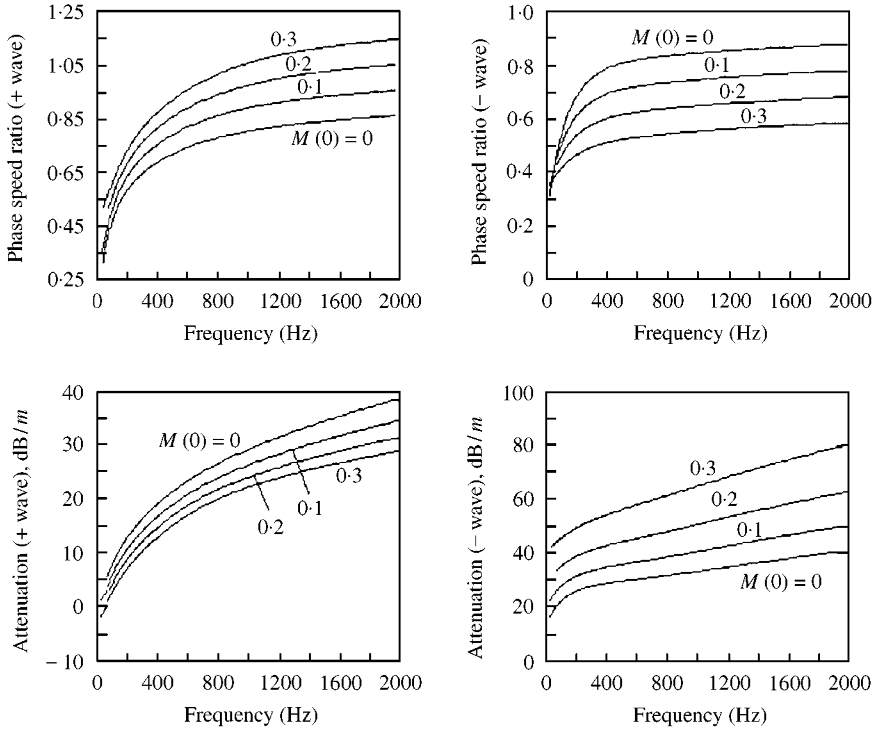

Figure 3. Effect of mean flow on attenuation and phase speed ratio of the forward and backward waves in an $L = 0{\cdot}06$ m long narrow pipe of radius $a = 0{\cdot}5$ mm, $\bar{T} = 950$ K, $\tau = 0{\cdot}05$, $\pi = 0$.

and $\tau = 0{\cdot}05$, as computed by using the present method and the expressions given in reference [3] for the wavenumbers. The observed correlation between the two solutions improves as the tube length increases, or the temperature increase parameter decreases, and for $\tau = 0$ they become identical, as expected. As further validation apparently is needed to confirm the accuracy of the present solution for the relatively shorter lengths of this tube, further results will be presented here for $L = 60$ mm. Tests indicate that the temperature increase in a catalytic monolith occurs over about 40 mm, but this is for the measured wall temperature; increase in the gas temperature may be distributed over a larger distance.

Shown in Figure 3 is the effect of the mean flow on the attenuation and phase speed ratio of the forward and backward waves for $\bar{T} = 950$ K, $\tau = 0{\cdot}05$ and $\tau = 0{\cdot}05$ and $\pi = 0$. As can be seen, increasing the inlet mean flow velocity Mach number, $M(0)$, assists the transmission of the forward wave, and hinders the transmission of the backward wave.

Figure 4 shows the effect of changing the temperature change parameter. The tube is assumed to carry a mean flow of $M(0) = 0{\cdot}1$. The presence of a positive temperature gradient assists the forward wave and obstructs the backward wave.

Shown in Figure 5 is the effect of changing the mean pressure change parameter for $\bar{T} = 950$ K, $\tau = 0{\cdot}05$ and $M(0) = 0{\cdot}3$. It is seen that the effect of the mean pressure drop tends to combine with the effect of the mean temperature gradient; however, this is indiscernible except for the attenuation of the backward wave.

4. CONCLUSION

An approximate solution has been presented for the transmission of sound waves in a narrow pipe with a constant temperature gradient and a mean flow of uniform velocity

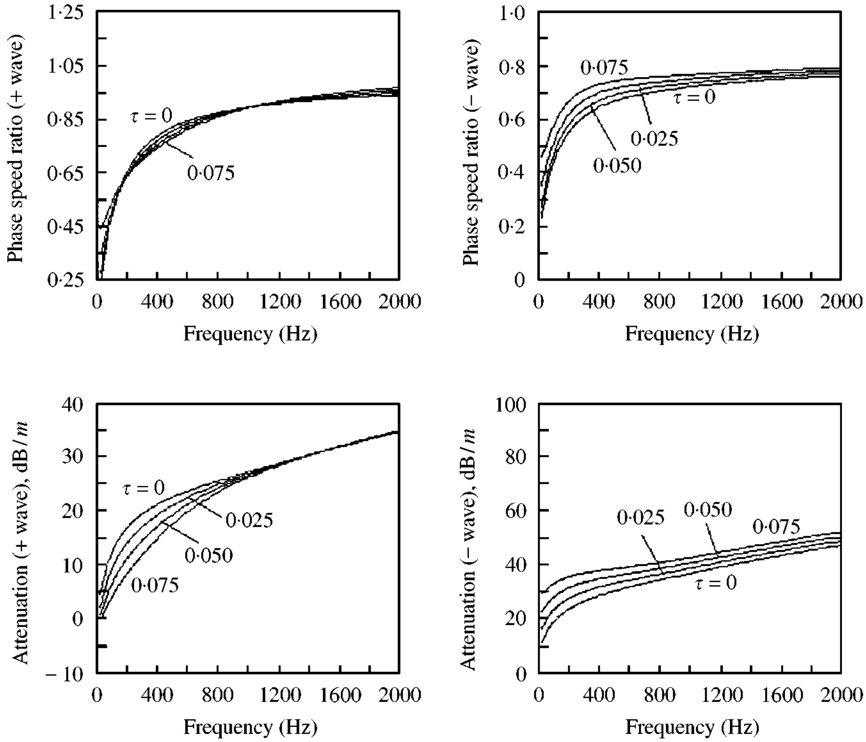

Figure 4. Effect of the temperature rise on attenuation and phase speed ratio of the forward and backward waves in a narrow pipe of radius $a = 0{\cdot}5$ mm, $\bar{T} = 950\,\mathrm{K}$, $\pi = 0$ and $M(0) = 0{\cdot}1$.

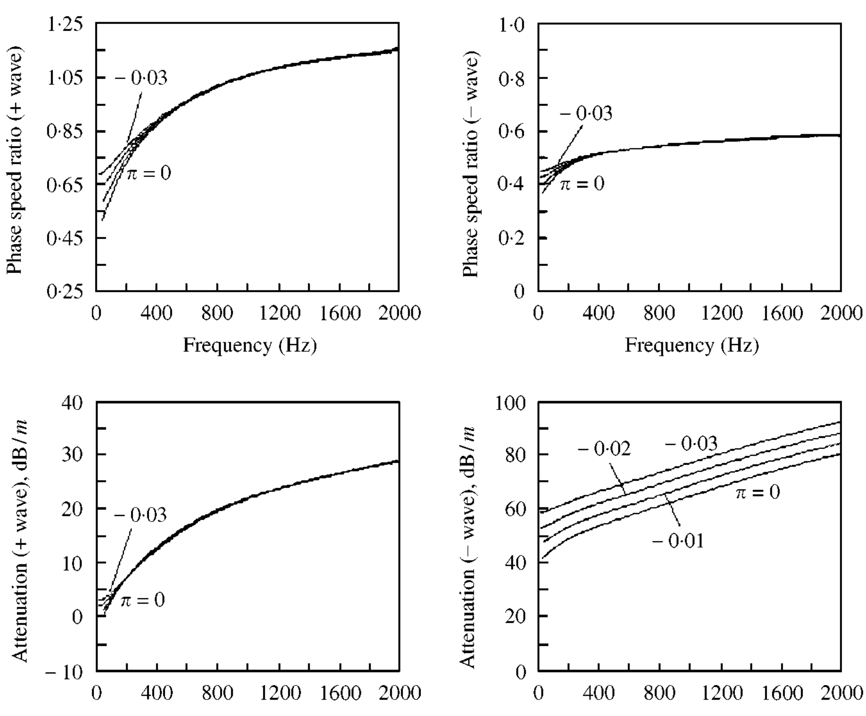

Figure 5. Effect of the mean pressure drop on attenuation and phase speed ratio of the forward and backward waves in a narrow pipe of radius $a = 0{\cdot}5$ mm, $\bar{T} = 950\,\mathrm{K}$, $\tau = 0{\cdot}05$ and $M(0) = 0{\cdot}3$.

profile. The solution also includes the effect of a constant pressure gradient. The dispersion equation gives the sound field in the pipe as a superposition of forward and backward waves. In the presence of mean temperature and pressure gradients, the sound field is continuously reflective and the propagation constants should be dependent on the axial co-ordinate. The present solution assumes that these effects can be lumped by using average values for the ambient variables. This approximation, which is permissible if $\tau/\bar{k}L$ and $\tau/\bar{k}L$ are small enough, limits the range of application of the solution to small ambient gradients or relatively high frequencies. For typical dimensions of a catalytic monolith cylinder, the present solution can provide accurate prediction of the mean flow effects over a useful frequency range.

If the tube is short enough the present solution predicts spatial instability waves at relatively low frequencies. An example of this is displayed in Figure 1 for the case of $L = 20$ mm. For this case, the imaginary part of the propagation constant for the forward wave remains negative up to about 200 Hz, implying an amplifying wave. For higher values of τ, the spatial instability waves occur over larger frequency ranges and in a more complex pattern that requires the application of Briggs' criterion for the identification of the amplifying and evanescent waves. This phenomenon is not pursued in this paper because it occurs for tube lengths for which the low-frequency accuracy of the present solution cannot be asserted on the basis of the test described in the previous section.

The presence of mean flow may introduce some hydrodynamic modes into the dispersion equation; however, no attempt has been made in the present study to search these modes.

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