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# NON-RESONANT RESPONSE USING STATISTICAL ENERGY ANALYSIS

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When excited acoustically, the response of a panel consists of resonant and non-resonant waves. The non-resonant response is negligible for limp panels. However, it can become significant in the case of thin light structural panels. SEA modelling does not predict the non-resonant response of the structure. This paper discusses the above limitation of SEA and presents a modified SEA formulation by which the non-resonant response can also be estimated. The resonant and the non-resonant contributions to the response are assumed to be arising out of two separate subsystems. In the present formulation, modelling for non-resonant response is similar to the conventional SEA modelling for resonant response but uses different expressions for the coupling loss factors. The classical problem of two reverberant rooms separated by a panel is considered as an example. It is shown that by using this procedure the non-resonant response of the structure can be estimated. Also, the non-resonant sound transmission is obtained exactly. Results of a numerical example are presented to compare the conventional and the modified SEA modelling results.


## 1. INTRODUCTION

Response of a structure to acoustic excitation is due to the contributions from both the resonant and non-resonant waves [1–3]. Resonant part of the response is due to the structural modes caused by the interaction of the free bending waves with the boundaries of the structure. On the other hand, the non-resonant response, also called forced response, is the result of the trace wave generated in the panel by the incident acoustic excitation field. It is to be noted that by non-resonant response, we mean the response due to the trace wave generated in the structure and not the response at frequencies away from the natural frequencies of the structure. In a similar way, the sound field radiated by a panel when excited acoustically is from both the resonant and non-resonant responses.

The resonant response is very small at frequencies below the critical frequency of the structure and is large at the frequencies near and above the critical frequency. The non-resonant response is generally obtained by using the mass law sound transmission characteristics. According to the mass law, the sound power transmission coefficient decreases with the increase in frequency and therefore the non-resonant response also decreases with the increase in frequency. Hence, the non-resonant response contribution is

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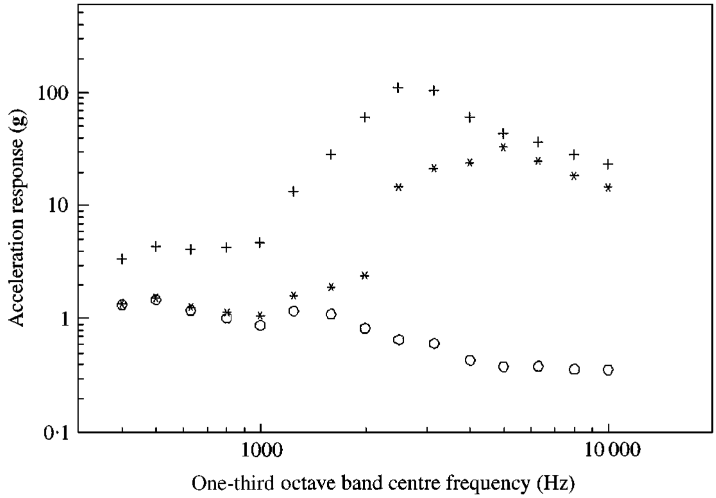


Figure 1. Resonant and non-resonant responses of a plate having finite stiffness: $+$, resonant; $*$, non-resonant; $\circ$, limp panel (mass law sound transmission).

higher at low frequencies than at high frequencies. Even at low frequencies the non-resonant response contribution can be much lower compared to the resonant response. Therefore, the non-resonant response of the structure is not considered as important as resonant response. The above characteristics can be easily understood from Figure 1 in which the resonant and the non-resonant response of a typical plate when subjected to acoustic excitation are shown. The plate is made of aluminium and its thickness is 5 mm. The critical frequency of the plate is estimated to be 2512 Hz. The non-resonant response is calculated using the mass law sound transmission characteristics.

We shall now consider the sound radiated by the structure when it is subjected to acoustic excitation. As mentioned above, the resonant response is small at frequencies below the critical frequency of the structure. The radiation resistance corresponding to the resonant response is also small below the critical frequency. Above the critical frequency both the resonant response and the corresponding radiation resistance are large. Hence, the sound radiation from resonant response is significant only at frequencies above the critical frequency of the structure. Though the non-resonant response contribution can be small compared to the resonant response at frequencies below the critical frequency, the corresponding radiation resistance is large. Hence, the sound radiated by the non-resonant response is significant at frequencies below the critical frequency. Therefore, below the critical frequency the non-resonant sound radiation is significant compared to resonant sound radiation, while above the critical frequency the resonant sound radiation is very much significant compared to the non-resonant sound radiation. This behaviour can be seen from the results presented in section 5.

The above-mentioned behaviour is generally true for panels for which sound transmission characteristics are governed by the mass law. Limp panels, i.e., panels having no flexural rigidity, show mass law sound transmission characteristics. In the strict sense, limp panels do not have resonant responses. But limp panel models are used to determine the sound transmission characteristics. Structural panels possess some amount of stiffness and their sound power transmission characteristics cannot be represented by the mass law. For such panels, the sound transmission characteristics depend on the bending stiffness as well. Expressions for sound power transmission coefficients of such panels are given by

Beranek [4], Reynolds [5] and Fahy [6]. In such cases, the sound power transmission coefficient near the critical frequency of the panel is very large. Also, it is about one order higher at frequencies above the critical frequency compared to that obtained using mass law sound power transmission characteristics. At frequencies below the critical frequency, the sound transmission behaviour is almost the same as that for a limp panel. Since the non-resonant response contribution is directly dependent on the sound power transmission coefficient, it becomes significant for the structural panels near and above their critical frequencies. It is therefore important to consider the non-resonant response contribution of panels for studying their behaviour when subjected to acoustic excitation. The non-resonant response of a plate having finite stiffness is shown in Figure 1. It is clear from the figure that the non-resonant response of a plate can be significant at frequencies near and above its critical frequency. For a thin plate where the modal density is large, the resonant response is generally significant compared to the non-resonant response as seen in the results shown in Figure 1. But in case of relatively thick plates, the non-resonant response can be as significant as resonant response as in the case considered in section 5. It is to be noted that the resonant response of a limp panel is zero since it does not possess any stiffness whereas the non-resonant response is present.

While using statistical energy analysis (SEA) for predicting the structural response to acoustic excitation, Crocker and Price [7] used an indirect coupling loss factor between the two acoustic fields to represent the non-resonant sound transmission. Lyon [8], Eaton [9] and Norton [3] used the above procedure to represent the non-resonant sound transmission. In this type of formulation, the structure is not coupled to the acoustic field and hence SEA does not predict the non-resonant response of the structure. It is pertinent to point out here that while the conventional SEA formulation through an indirect coupling loss factor can take care of the non-resonant sound transmission characteristics it is not capable of estimating the non-resonant structural response. As discussed previously, for structural elements such as panels of spacecraft, the non-resonant response can be significant and hence it becomes necessary to estimate it. Since SEA does not predict the non-resonant response of the structure, alternate modelling techniques are to be considered.

In this paper, an alternate technique to represent the non-resonant response in SEA formulation is presented. The alternate method suggested does not make use of an indirect coupling loss factor as used in the conventional SEA models. The non-resonant response is obtained in a direct manner as is being done for the resonant response The resonant response contribution and the non-resonant response contribution are treated as two separate subsystems in the modified SEA formulation. The sound transmission characteristics between two reverberant rooms separated by a panel and the response of the panel is considered as an example. The modified SEA formulation presented in this paper can estimate the non-resonant sound transmission and also the non-resonant response of the structure. The numerical results presented here demonstrates the modified SEA formulation.

## 2. NON-RESONANT RESPONSE

The sound radiation behaviour of a structure is represented by its radiation resistance denoted by $R_{rad}$ or radiation efficiency denoted by $\sigma$. The sound power radiated, $W$, by a plate having a spatial average of mean square value of velocity $\langle v^2\rangle_x$ is given by

$$W = R_{rad}\langle v^2\rangle_x. \tag{1}$$

If the plate is having an area $A$ and the medium of the acoustic field has a characteristic impedance $\rho c$ where $\rho$ is the density of the medium and $c$ is the speed of the acoustic wave, the sound power radiated by the plate can also be written as

$$W = \sigma \rho c A \langle v^2 \rangle_x. \tag{2}$$

As discussed earlier, when a plate is subjected to acoustic excitation its response is due to contributions from resonant and non-resonant waves. Thus, the velocity of the plate at a point can be expressed as

$$v = v_{free} + v_{forced}, \tag{3}$$

where $v_{free}$ is the velocity due to the resonant response and $v_{forced}$ is that due to the non-resonant response. The spatial average of the velocity can be shown to be

$$\langle v^2 \rangle_x = \langle v_{free}^2 \rangle_x + \langle v_{forced}^2 \rangle_x. \tag{4}$$

As Heckl [2] has pointed out, the cross terms corresponding to $v_{free}$ and $v_{forced}$ vanish when the average is taken over the plate area. The sound power radiated by the plate can then be expressed as

$$W = \rho c A \{ \sigma_{free} \langle v_{free}^2 \rangle_x + \sigma_{forced} \langle v_{forced}^2 \rangle_x \}. \tag{5}$$

The present objective is to determine $\langle v_{forced}^2 \rangle_x$, which is the trace wave generated in the structure.

### 2.1. PLANE WAVE EXCITATION

Consider a plate of infinite extent with an incident acoustic field at an angle $\theta$ as shown in Figure 2. A part of the incident sound energy is reflected while the remaining part of the energy sets the panel into vibration. This part of the energy that causes the plate vibrations is balanced by the energy dissipated within the plate due to internal damping, the sound energy radiated into the space on the incident side and the sound energy radiated into the space on the other side of the plate, called the transmitted sound. The normal velocity $v$ of the plate for harmonic plane wave excitation can be shown to be

$$v = (A_t/\rho c) \cos \theta \, \mathrm{e}^{\mathrm{j}\omega t}, \tag{6}$$

where $A_t$ is the amplitude of the transmitted sound. The spatial average of the mean square velocity is given by

$$\langle v^2 \rangle_x = (A_t^2/2\rho^2 c^2) \cos^2 \theta. \tag{7}$$

This can be considered as entirely due to the non-resonant response as the plate is of infinite extent and hence there is no way of the flexural waves interacting with the boundaries.

Alternatively, we can express

$$\langle v^2 \rangle_x = (p_{rms}^2/\rho^2 c^2) \tau \cos^2 \theta, \tag{8}$$

where $p_{rms}$ is the root mean square value of the incident acoustic wave and $\tau$ is the sound power transmission co-efficient given by

$$\tau = A_t^2/A_i^2 \tag{9}$$

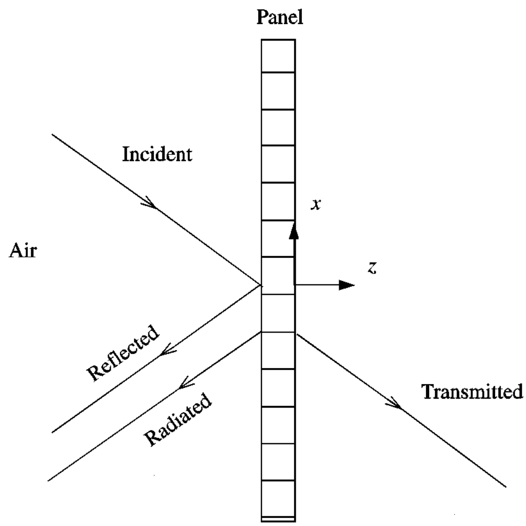


Figure 2. Panel with incident acoustic field.

with $A_i$ being the amplitude of the incident acoustic wave. Sound power transmission co-efficient of a limp panel having mass per unit area $m$ is given by [4]

$$\tau^{-1} = 1 + a^2 \cos^2 \theta, \tag{10}$$

where $a = m\omega/2\rho c$ and $\omega$ is the circular frequency. For a panel having stiffness the sound power transmission coefficient at a frequency $f$ can be obtained using the equation [5]

$$\tau^{-1} = \{1 + \eta a \cos\theta \sin^4\theta (f/f_c)^2\}^2 + \{a \cos\theta (1 - (f/f_c)^2 \sin^4\theta)\}^2. \tag{11}$$

The plate is having a critical frequency of $f_c$ and a loss factor of $\eta$. Either equation (10) or equation (11) could be used to calculate the non-resonant response depending on the characteristics of the plate.

Though the expression for the non-resonant response has been obtained for an infinite plate, the same expression can also be used for calculating the non-resonant response of a finite plate. The difference between the behaviour of finite and infinite plates to acoustic excitation is that the finite plate response contains resonant response in addition to the non-resonant response where the infinite plate does not have resonant response.

### 2.2. REVERBERANT FIELD EXCITATION

The non-resonant response of a plate to a reverberant acoustic excitation can be derived in the following way. The intensity of a reverberant acoustic field acting on a plate is given by [1, 4]

$$I = p_{rms}^2/4\rho c. \tag{12}$$

The intensity of the transmitted sound $I_t$ is [1]

$$I_t = I\tau, \tag{13}$$

where $\tau$ is the field incidence sound power transmission coefficient. From the continuity of the particle velocity at the interface of the plate and the acoustic medium on the transmitted side, the space-averaged mean square velocity of the plate is given by

$$\langle v^2 \rangle_x = I_t/\rho c. \tag{14}$$

Substituting equations (12) and (13) into equation (14) we get

$$\langle v^2 \rangle_x = (p_{rms}^2/4\rho^2 c^2)\tau. \tag{15}$$

Equation (15) represents the non-resonant response contribution as it represents the response due to the trace wave generated by the acoustic excitation.

For a limp panel $\tau = 8\rho^2 c^2/m^2\omega^2$ [2]. Hence, the non-resonant response of a limp panel is given by

$$\langle v^2 \rangle_x = 2p_{rms}^2/m^2\omega^2. \tag{16}$$

The above results is the same as what Heckl [2] had obtained for a limp panel but using a different procedure.

### 2.3. SOUND TRANSMISSION

Consider two reverberant rooms separated by a panel as shown in Figure 3. Norton [3] has shown that the acoustic pressure in the receiving room is given by

$$p_{3,rms}^2 = p_{1,rms}^2\{\tau A/(\tau A + s\alpha + A\alpha_d)\}. \tag{17}$$

In equation (17), $A$ is the area of the partition, $\tau$ is the sound power transmission co-efficient of the panel, $s$ is the surface area of the room which excludes the area of the panel. The absorption co-efficient of the walls of the receiving room is denoted by $\alpha$ and that of the panel is denoted by $\alpha_d$. It is to be noted that equation (17) gives the non-resonant acoustic field.

## 3. CONVENTIONAL SEA MODELLING

Let us now consider the above problem in SEA formalism. For this, the source room, the receiver room and the panel are considered as subsystems 1, 3 and 2 respectively. The

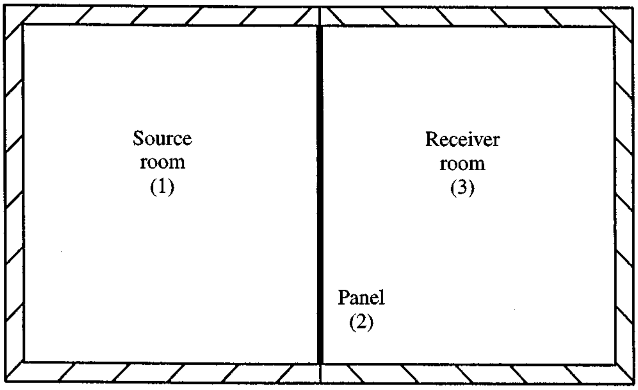


Figure 3. Two reverberant rooms separated by a panel.

non-resonant response is represented by an indirect coupling loss factor given by

$$\eta_{13} = \tau Ac/(8\pi f V_1), \tag{18}$$

as used by Crocker and Price [7]. Accordingly, the power balance equations become

$$\begin{Bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} + \eta_{13} & -\eta_{21} & -\eta_{31} \\ -\eta_{12} & \eta_2 + \eta_{21} + \eta_{23} & -\eta_{32} \\ -\eta_{13} & -\eta_{23} & \eta_3 + \eta_{31} + \eta_{32} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}. \tag{19}$$

In equation (19), $\pi_n$, $E_n$ and $\eta_n$ are the power input, the mean energy and the dissipation loss factor, respectively, of $n$th subsystem. The coupling loss factor (CLF) for $m$th subsystem to the $n$th subsystem is denoted by $\eta_{mn}$. The coupling loss factors $\eta_{21}$ and $\eta_{23}$ can be obtained based on the radiation resistances of the panel [10, 7, 11] and the coupling loss factors $\eta_{12}$ and $\eta_{13}$ can be obtained from the reciprocity relations [8]. The dissipation loss factor of the receiver room is given by

$$\eta_3 = sc\alpha/(8\pi f V_3). \tag{20}$$

In equation (20), $s\alpha$ is the room absorption of the receiver room that includes the absorption of all the walls of the receiver room but does not include the absorption due to the panel. The absorption due to the panel is indirectly accounted in the dissipation loss factor of subsystem 2. Since the total input power to the system should be balanced by the power dissipated by all the three subsystems, the dissipation loss factor of the receiver room should not include the loss factor of the panel which is considered as a separate subsystem in the SEA formulation. The above formulation is followed by Lyon [8], Eaton [9] and Norton [3]. It is seen that the above model does not predict the non-resonant response of the panel. This is discussed below.

From equation (19), the energy of the plate is given by

$$E_2 = \{\eta_{12}E_1 + \eta_{32}E_3\}/\{\eta_2 + \eta_{21} + \eta_{23}\}. \tag{21}$$

It can be seen from the above equation that the energy of the plate is a sum of two terms in which only one term is related to the direct power flow from the source room, that is $\eta_{12}E_1$. The other term, that is $\eta_{32}E_3$, represents the reciprocal power flow from the receiver room. The term representing the direct power flow is concerned with only the resonant response and the term related to the non-resonant response is absent. Hence, the model does not predict the non-resonant response of the structure. It may be argued that since the energy of the receiver room is related to the energy of the source room, the term $\eta_{32}E_3$ also contains power flow from the source room. This can be verified by expressing $E_3$ in terms of $E_1$ which is carried out in the following.

Equation (21) can be further modified as

$$E_2 = \{\eta_{12} + [\eta_{13}\eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})]\}E_1/\{\eta_2 + \eta_{21} + \eta_{23} - [\eta_{23} + \eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})]\}. \tag{22}$$

Since the product of two coupling loss factors is negligible compared to the coupling loss factor, equation (22) reduces to

$$E_2 = \eta_{12}E_1/\{\eta_2 + \eta_{21} + \eta_{23}\}. \tag{23}$$

It can be seen from the above equation that the estimated energy of the plate is dependent only on the coupling loss factors relating to resonant response. If alone the resonant response is present, the power balance equations of the subsystems become

$$\begin{Bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} & -\eta_{21} & 0 \\ -\eta_{12} & \eta_2 + \eta_{21} + \eta_{23} & -\eta_{32} \\ 0 & -\eta_{23} & \eta_3 + \eta_{32} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}. \tag{24}$$

The energy of the plate can then be obtained from the power balance of the subsystems as

$$E_2 = \eta_{12} E_1 / \{\eta_2 + \eta_{21} + \eta_{23} - [\eta_{23}\eta_{32}/(\eta_3 + \eta_{32})]\}. \tag{25}$$

The last term in the denominator is very much negligible since it contains a product of two coupling loss factors. Neglecting the above term, the energy of the plate becomes

$$E_2 = \eta_{12} E_1 / \{\eta_2 + \eta_{21} + \eta_{23}\}, \tag{26}$$

which is the same as equation (23). This means that the response predicted by SEA is only the resonant part. For the above argument, it is necessary that

$$\eta_{12} \gg \{\eta_{13}\eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})\},$$

$$\eta_2 + \eta_{21} + \eta_{23} \gg \{\eta_{23}\eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})\},$$

$$\eta_2 + \eta_{21} + \eta_{23} \gg \{\eta_{23}\eta_{32}/(\eta_3 + \eta_{32})\}.$$

In most of the cases, the above inequalities hold good. Consider the case given in section 5. The coupling loss factors $\eta_{12}$ and $\eta_{13}$ and the dissipation loss factor $\eta_3$ are shown in Figure 4. In this particular case, $\eta_{12} = \eta_{32}$ and $\eta_{13} = \eta_{31}$ since the sizes of both the rooms are equal. The first inequality can be verified from the results shown in Figure 5 and the second inequality can be verified from the results shown in Figure 6. If the second inequality holds good, then the third inequality always holds good.

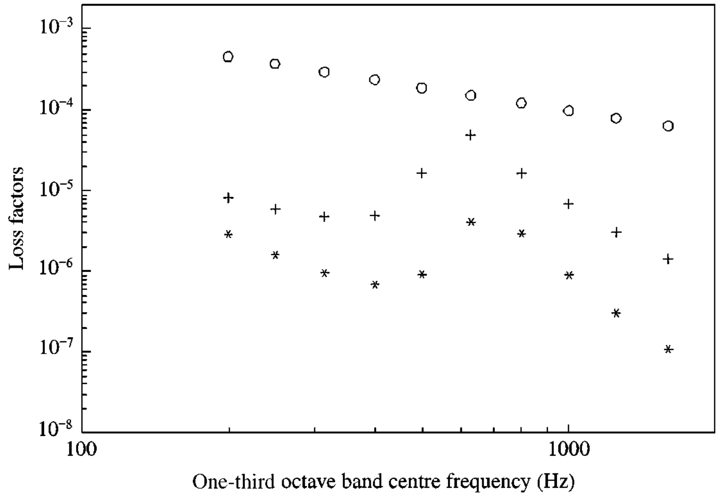


Figure 4. Loss factors of the subsystems: +, $\eta_{12}$; $*$, $\eta_{13}$; ○, $\eta_3$.

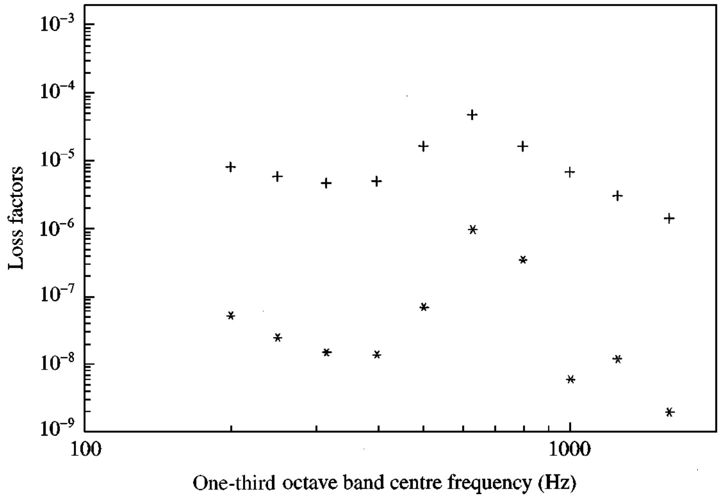


Figure 5. Loss factors of the subsystems: $+$, $\eta_{12}$; $*$, $\eta_{13}\eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})$.

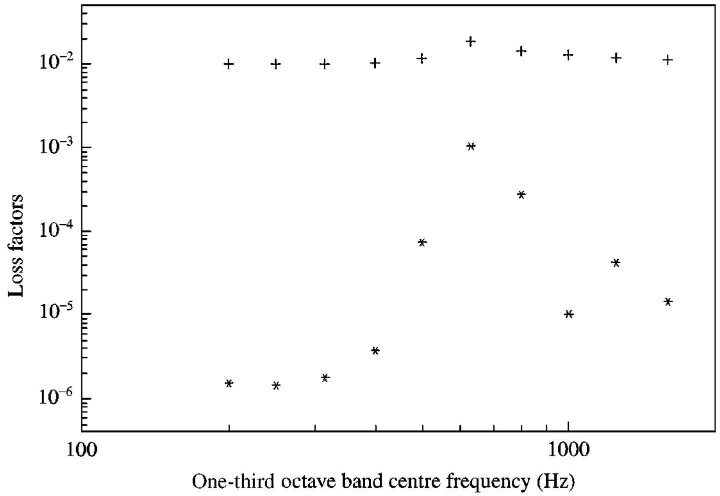


Figure 6. Loss factors of the subsystems: $+$, $\eta_2 + \eta_{21} + \eta_{23}$; $*$, $\eta_{23}\eta_{32}/(\eta_3 + \eta_{31} + \eta_{32})$.

Thus, the results presented show that the above inequalities are valid and hence the response predicted by the conventional SEA is only the resonant part. One can argue that this is true only in cases where the above inequalities are valid. Or one also can argue that the neglected responses are due to the non-resonant responses and hence the non-resonant responses are predicted, but they are negligible. This can be verified by considering the limiting case where only the non-resonant response is present. In this case, the power balance equations become

$$\begin{Bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{13} & 0 & -\eta_{31} \\ 0 & \eta_2 & 0 \\ -\eta_{13} & 0 & \eta_3 + \eta_{31} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}. \tag{27}$$

One can see that in this formulation the panel is not coupled to the acoustic field. Therefore, $E_2 = 0$, that is the response of the panel predicted by SEA is zero. This means that the non-resonant response cannot be estimated using the formulation with indirect coupling loss factors.

The energy of the acoustic field in the receiver room can be derived from equation (27) as

$$E_3 = \eta_{13} E_1 / \{\eta_3 + \eta_{31}\}. \tag{28}$$

Substituting the expressions for the dissipation loss factor and the coupling loss factors, that is equations (18) and (20), the expression for the energy of the acoustic field in the receiver room is given by

$$E_3 = (p_{1,rms}^2 / \rho c^2) V_3 (\tau A / (\tau A + s\alpha)\}. \tag{29}$$

It is interesting to note that the acoustic pressure that is predicted by equation (29) is higher than the classical result given by equation (17). But the difference is not significant owing to the reason that $A\alpha_d$ is smaller compared to other terms. To predict the non-resonant acoustic field exactly, the dissipation loss factor of subsystem 3 should include the panel absorption also. But in such a situation the resonant acoustic field will not be predicted exactly.

In summary, the existing SEA technique does not predict the non-resonant response of the structure but predicts the non-resonant sound transmission, though slightly higher. The indirect coupling loss factor terms present in the expression for the energy of the plate are due to the reciprocal power flow from the receiver room and does not represent the non-resonant response. The numerical example discussed in section 5 will provide further clarity on this.

## 4. MODIFIED SEA EQUATIONS

In this section, an alternate method within the framework of SEA is presented by which both resonant and non-resonant contributions to the response as well as the sound transmission can be estimated. In this formulation, the resonant as well as the non-resonant response of the structure are considered as two separate subsystems. The non-resonant response is modelled in the same way as the resonant part, i.e., the non-resonant response of the structure is coupled to the acoustic field directly and the two acoustic fields are not coupled directly. The coupling loss factors for the non-resonant responses are different from those for resonant response and they are derived here.

### 4.1. STRUCTURE TO AIR CLF

The radiation efficiency of a structure is the ratio of mean sound power radiated by the structure to the sound power radiated by a piston having the same surface area and the same mean square value of velocity as the structure. For an infinite plate carrying bending waves of wavelength $\lambda_b$, the radiation efficiency is given by

$$\sigma = \{1 - (\lambda/\lambda_b)^2\}^{-1/2}, \tag{30}$$

for $\lambda_b > \lambda$, where $\lambda$ is the acoustic wavelength. If $\lambda_b < \lambda$, the radiation efficiency is zero.

We will now obtain the radiation efficiency of the forced acoustic wave. Corresponding to the angle of incidence $\theta$, the wavelength of the travel wave is $\lambda/\sin\theta$. Hence, the wavelength of the non-resonant wave is always greater than the wavelength of the sound and therefore the trace wave radiates sound well. Thus, the radiation efficiency of the non-resonant wave generated by a plane wave excitation is obtained by substituting $\lambda_b = \lambda/\sin\theta$ in equation (30) as

$$\sigma = 1/\cos\theta. \tag{31}$$

For a reverberant field excitation, the radiation efficiency can be obtained as follows. The mean sound power radiated by the non-resonant wave for reverberant field excitation is given by

$$W = I_t A, \tag{32}$$

where $I_t$ is the intensity of the transmitted sound at the interface of the panel and the acoustic field. Using equations (13) and (12), the sound power radiated by the non-resonant wave becomes

$$W = (p_{1,rms}^2/4\rho c)\tau A. \tag{33}$$

The mean square velocity corresponding to the non-resonant response is given by equation (15). The sound power radiated by a piston having the same mean square velocity is given by

$$W_p = (p_{1,rms}^2/4\rho c)\tau A. \tag{34}$$

From equations (33) and (34), the radiation efficiency corresponding to the non-resonant wave for reverberant acoustic excitation can be taken as unity. Heckl [2] had also suggested the use of unity as the radiation efficiency for non-resonant wave. The coupling loss factor for structure to air is now given by

$$CLF = \rho c/m\omega. \tag{35}$$

### 4.2. AIR TO STRUCTURE CLF

In the case of resonant response, the air to structure coupling loss factor and structure to air coupling loss factor are related reciprocally by the relation

$$\eta_{12} n_1 = \eta_{21} n_2, \tag{36}$$

where $n_1$ and $n_2$ are the modal densities of subsystems 1 and 2 respectively. Since the non-resonant response is not related to any resonant mode, the reciprocal relation given by equation (36) is not valid. The reciprocity between the structural response and the acoustic field should be still valid. But for the non-resonant response, it does not reduce to equation (36). Hence, a different approach is followed to determine the acoustic field to structure coupling loss factor.

From the basic definition, the air to structure coupling loss factor is the ratio of power flow to the structure ($\pi_{12}$) from the acoustic field per rad/s to the mean energy of the acoustic field ($E_1$). Hence,

$$\eta_{12} = \pi_{12}/\omega E_1. \tag{37}$$

As discussed earlier, a part of the energy incident on the panel is reflected and the remaining energy sets the panel into vibration. Let $\alpha_r$ be the sound power reflection coefficient. The energy of vibration of the panel is balanced by the energy dissipated within the panel and the sound energy radiated from the panel into both the spaces. If $\alpha_d$ is the sound power dissipation coefficient of the panel, from the energy balance it follows that

$$\alpha_r + \alpha_d + 2\tau = 1. \tag{38}$$

The power flowing into the structure from the acoustic field having intensity $I$ at the acoustic field–structure interface is given by

$$\pi_{12} = I(1 - \alpha_r)A. \tag{39}$$

Using the energy balance given by equation (38) and using equation (12), the power flow becomes

$$\pi_{12} = (p_{1,rms}^2/4\rho c)(\alpha_d + 2\tau)A. \tag{40}$$

The energy of the acoustic field is related to acoustic pressure in the source room by the equation

$$E_1 = (p_{1,rms}^2/\rho c^2)V_1. \tag{41}$$

The expression for the coupling loss factor now becomes

$$\eta_{12} = (\alpha_d + 2\tau)Ac/8\pi f V_1. \tag{42}$$

The sound power dissipation coefficient, denoted by $\alpha_d$, can be written in terms of sound power transmission coefficient as follows. By definition, the sound power dissipation coefficient is the ratio of the power dissipated per unit area to the intensity of the incident energy and is given by

$$\alpha_d = \pi_d/AI, \tag{43}$$

where $\pi_d$ is the power dissipated in the structure. If $\eta$ is the dissipation loss factor of the panel, the rate of energy dissipated will be

$$\pi_d = \eta\omega m A\langle v^2\rangle_x. \tag{44}$$

From equations (43), (44), (15) and (12), the sound power dissipation coefficient becomes

$$\alpha_d = 2\eta a\tau, \tag{45}$$

where $a = m\omega/2\rho c$. Substituting equation (45) into equation (42), the coupling loss factor is given by

$$\eta_{12} = 2\tau Ac(1 + \eta a)/8\pi f V_1. \tag{46}$$

### 4.3. NON-RESONANT TRANSMISSION

Having obtained the expressions for the coupling loss factors, let us now estimate the non-resonant responses. Since the non-resonant response is modelled in the same way as the resonant response, equation (24) still gives the power balance relations with $\eta_{12}$ given by equation (46). Consequently, the non-resonant energies of the subsystems are

$$E_2 = (p_{1,rms}^2/4\rho^2c^2)\tau mA\{s\alpha + 2\tau A(1 + \eta a)\}/\{s\alpha + \tau A(1 + 2\eta a)\}, \tag{47}$$

$$E_3 = (p_{1,rms}^2/\rho c^2)V_3\tau A\{\tau A + s\alpha + A\alpha_d). \tag{48}$$

Let us now compare the results obtained using the modified SEA modelling technique with those predicted by conventional SEA as well as the classical results. From equation (48), the acoustic pressure in the receiving room is

$$p_{3,rms}^2 = p_{1,rms}^2\tau A/(\tau A + s\alpha + A\alpha_d), \tag{49}$$

which is same as equation (17). This means that the modified SEA modelling predicts the non-resonant sound transmission exactly. The non-resonant response of the structure can be estimated using equation (47). The expression for the non-resonant response of the structure derived earlier, i.e., equation (15), is without considering subsystem 3. It can be seen that if the power flow between subsystems 2 and 3 is neglected the non-resonant response predicted by the modified SEA modelling is the same as the result given by equation (15).

Hence, it can be seen that the modified SEA modelling technique presented here predicts the non-resonant response of the structure. Also, the non-resonant sound transmission is predicted correctly.

### 4.4. RESONANT AND NON-RESONANT TRANSMISSION

When both resonant and non-resonant responses have to be represented, they are considered as two different subsystems. The acoustic field is not divided into resonant and non-resonant subsystems since the coupling loss factor is defined with respect to the total acoustic energy. Hence, in this case there are four subsystems in the SEA modelling. Subsystems 1 and 3 are the acoustic field in rooms 1 and 3 respectively. The resonant response of the panel is subsystem 2 and the non-resonant response of the panel is subsystem 4. The power balance equations are

$$\begin{Bmatrix}\pi_1\\ \pi_2\\ \pi_3\\ \pi_4\end{Bmatrix} = \omega\begin{bmatrix}\eta_1+\eta_{12}+\eta_{14} & -\eta_{21} & 0 & -\eta_{41}\\ -\eta_{12} & \eta_2+\eta_{21}+\eta_{23} & -\eta_{32} & 0\\ 0 & -\eta_{23} & \eta_3+\eta_{32}+\eta_{34} & -\eta_{43}\\ -\eta_{14} & 0 & -\eta_{34} & \eta_4+\eta_{41}+\eta_{43}\end{bmatrix}\begin{Bmatrix}E_1\\ E_2\\ E_3\\ E_4\end{Bmatrix}. \tag{50}$$

From the above power balance equations, one can obtain the resonant as well as the non-resonant responses of the panel and the acoustic field in the receiver room. The total response of the panel is the sum of the resonant and the non-resonant responses. In this model, the resonant and the non-resonant responses of the structure are obtained separately but the acoustic field is the net energy. It is to be noted that by using the modified SEA technique, the number of subsystems present in the analysis is increased. Since the total number of subsystems which appears in SEA modelling is very small, the above increase in the number of subsystems is not a concern for computation.

### 4.5. PANEL HUNG IN REVERBERATION ROOM

Though the preceding discussions are carried out on SEA of a system having two reverberant rooms separated by a panel, the concept and the expressions for coupling loss factors derived here can very well be used for other situations. For example, if a panel is hung in a diffuse field, as per the SEA modelling presented here there are three subsystems, namely the acoustic field, resonant response of the panel and the non-resonant response of the panel. The power balance of the system then becomes

$$\begin{Bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{Bmatrix} = \omega \begin{bmatrix} \eta_1 + \eta_{12} + \eta_{13} & -\eta_{21} & -\eta_{31} \\ -\eta_{12} & \eta_2 + \eta_{21} & 0 \\ -\eta_{13} & 0 & \eta_3 + \eta_{31} \end{bmatrix} \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}. \tag{51}$$

The power is supplied only to subsystem 1 that is the source room. Hence, by setting $\pi_3 = 0$, we get

$$E_3 = \{\eta_{13} E_1\}/\{\eta_3 + \eta_{31}\}. \tag{52}$$

In this case, the acoustic structure-coupling loss factor for non-resonant response is as given by equation (46) and hence

$$\eta_{13} = 2\tau Ac(1 + \eta a)/8\pi f V_1. \tag{53}$$

The structure-acoustic coupling loss factor is as given by equation (35). Since the plate radiates sound to both sides, the coupling loss factor will be double [11] of what is given by equation (35). Hence, the expression for coupling loss factor becomes

$$\eta_{31} = 2\rho c/m\omega. \tag{54}$$

Substituting equations (53), (54) and (41) into equation (52) and since the dissipation loss factor for the non-resonant response is denoted by $\eta$, the non-resonant vibrational energy of the plate can be obtained as

$$E_3 = (p_{1,rms}^2/4\rho^2 c^2)\tau m A. \tag{55}$$

Comparing it with equation (15), it can be seen that the modified SEA model predicts the non-resonant response. The conventional SEA procedure does not even allow us to represent non-resonant behaviour since the coupling loss factor is defined between two acoustic fields.

## 5. NUMERICAL RESULTS

To get further clarity on the above results, numerical results for a system of two reverberant rooms separated by a thin plate are presented here. The rooms have dimensions 10·1 m × 7·0 m × 8·4 m each. They are separated by a 22·5 mm thick aluminium plate having dimensions 7 m × 8·4 m. The density of the air is considered to be as 1·21 kg/m$^3$ and the speed of the sound wave is assumed to be 343·0 m/s. The critical frequency of the above plate is estimated as 560·0 Hz. The loss factor of the plate is assumed to be 0·01 at all frequencies. The room mentioned here is representative of a typical reverberation chamber. Corresponding sound absorption values for the walls are used here but not given here for brevity. The sound pressure level (SPL) in the source room is given in Table 1. Results are obtained in one-third octave bands.

The resonant and the non-resonant responses of the panel are estimated and given in Table 1 and Figure 7. The non-resonant response is estimated using the thin-plate model. The results show that the non-resonant response is as significant as the resonant response

TABLE 1

*Comparison of resonant and non-resonant responses*

| One-third octave band centre frequency (Hz) | SPL in source room (dB) | RMS value of acceleration response (*g*) | | |
|---|---|---|---|---|
| | | Resonant | Non-resonant | Conventional SEA |
| 200 | 139·3 | 0·72 | 0·58 (0·54) | 0·72 |
| 250 | 140·1 | 0·83 | 0·67 (0·59) | 0·83 |
| 315 | 137·7 | 0·71 | 0·56 (0·45) | 0·71 |
| 400 | 135·2 | 0·68 | 0·50 (0·34) | 0·68 |
| 500 | 134·9 | 1·4 | 0·80 (0·33) | 1·4 |
| 630 | 132·7 | 2·0 | 1·9 (0·26) | 2·0 |
| 800 | 130·8 | 1·3 | 1·8 (0·20) | 1·3 |
| 1000 | 128·5 | 0·84 | 1·1 (0·16) | 0·84 |
| 1250 | 126·9 | 0·61 | 0·73 (0·13) | 0·61 |
| 1600 | 125·0 | 0·43 | 0·50 (0·10) | 0·43 |

*Note*: Values in brackets are for the limp panel model.

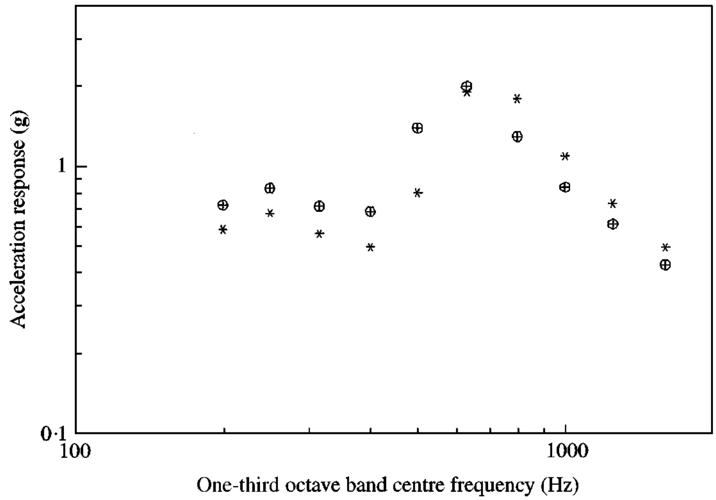


Figure 7. Response of the panel separating the two reverberant rooms: +, resonant; ∗, non-resonant; ○, conventional SEA.

especially at frequencies near and above the critical frequency. Results for the limp panel model are also given in the same table in brackets. It can be seen that the non-resonant response is negligible if the stiffness of the panel is negligible. This could be the reason that the non-resonant response of the structure is not considered important. But the panels of aerospace structure possess stiffness and hence the non-resonant can be significant.

But the conventional SEA modelling does not predict the non-resonant response of the structure. The response predicted by the conventional SEA is only the resonant part. This is illustrated in Table 1 and Figure 7 where the resonant and non-resonant responses are compared with those estimated using the conventional SEA procedure. It can be seen that the response predicted by the conventional SEA modelling does not include the non-resonant part and it is merely the resonant response.

The results show the importance of the non-resonant responses and the conventional SEA modelling technique fails to account for this. The proposed SEA modelling technique does predict the non-resonant response.

The SPL in the receiving room is estimated using both conventional SEA and proposed SEA modelling method. The results, which are given in Table 2, clearly show that both the methods predict approximately the same value of transmitted sound. As pointed out earlier, it can be seen that the conventional SEA predicts slightly larger value for the transmitted sound but the difference is very much insignificant.

It was mentioned in the beginning that the non-resonant sound transmission is significant at frequencies below the critical frequency. This is illustrated by the results given in Table 3 and Figure 8. The SPL in the receiving room is calculated for the resonant transmission and then for the non-resonant transmission. The results show that below the critical frequency of the panel the non-resonant sound transmission is much higher than the resonant acoustic

TABLE 2

*SPL in the receiver room using conventional and modified SEA modelling*

| One-third octave band centre Frequency (Hz) | SPL in source room (dB) | SPL in the receiver room (dB) | |
|---|---|---|---|
| | | Conventional SEA | Modified SEA |
| 200 | 139·3 | 117·3 | 117·3 |
| 250 | 140·1 | 116·5 | 116·4 |
| 315 | 137·7 | 112·9 | 112·9 |
| 400 | 135·2 | 110·3 | 110·2 |
| 500 | 134·9 | 115·3 | 115·3 |
| 630 | 132·7 | 121·8 | 121·3 |
| 800 | 130·8 | 116·9 | 116·2 |
| 1000 | 128·5 | 110·6 | 110·3 |
| 1250 | 126·9 | 105·4 | 105·2 |
| 1600 | 125·0 | 100·1 | 99·97 |

TABLE 3

*Comparison of resonant and non-resonant SPL in the receiver room*

| One-third octave band centre frequency (Hz) | SPL in source room (dB) | SPL in the receiver room (dB) | | |
|---|---|---|---|---|
| | | | Non-resonant | |
| | | Resonant | Thin | Limp |
| 200 | 139·3 | 101·0 | 117·3 | 116·6 |
| 250 | 140·1 | 101·6 | 116·4 | 115·3 |
| 315 | 137·7 | 100·1 | 112·7 | 110·9 |
| 400 | 135·2 | 100·9 | 109·8 | 106·3 |
| 500 | 134·9 | 113·1 | 111·6 | 104·0 |
| 630 | 132·7 | 120·6 | 116·3 | 99·76 |
| 800 | 130·8 | 113·7 | 113·8 | 95·78 |
| 1000 | 128·5 | 107·4 | 107·7 | 91·40 |
| 1250 | 126·9 | 102·2 | 102·5 | 87·72 |
| 1600 | 125·0 | 96·89 | 97·22 | 83·74 |

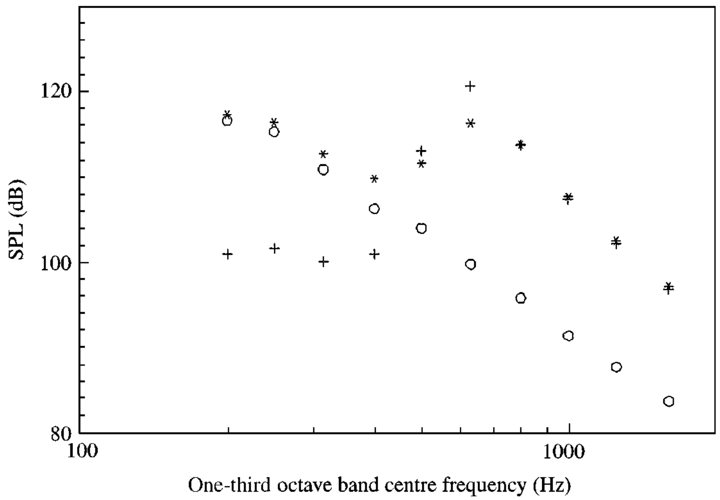


Figure 8. SPL in the receiver room: +, resonant transmission; ∗, non-resonant transmission; ○, mass law transmission.

field. It can be seen from the results that for a limp panel the non-resonant sound is significant only at lower frequencies whereas for a thin structural panel, having some stiffness, it can be significant even at higher frequencies.

## 6. CONCLUSIONS

It is shown that SEA does not predict the non-resonant response of the structure. An alternate SEA modelling technique is presented. In this model, the non-resonant response is represented in the same way as resonant response but uses different expressions for coupling loss factors that are based on the sound power transmission and absorption coefficients of the structure. In this procedure, the resonant and the non-resonant responses have to be considered as two different subsystems. Using this method the non-resonant response of the structure can be estimated. The non-resonant response is not significant for limp panels but very much significant for thin light panels. Also, the proposed modelling technique predicts the non-resonant sound transmission exactly.

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## APPENDIX A: NOMENCLATURE

Symbols not listed here are used only at specific places and are explained wherever they occur. Since the process considered are stationary random, the dynamic variables discussed are the long time-averaged quantities and in such cases the notation for the averaging is dropped. For example $\langle a^2 \rangle_t$ is written as $a^2$.

| | |
|---|---|
| $A$ | area of a plate |
| $A_i$ | amplitude of the incident sound |
| $A_t$ | amplitude of the transmitted sound |
| $c$ | speed of sound in air |
| $E_i$ | mean energy of subsystem $i$ |
| $f$ | frequency, in Hz |
| $f_c$ | critical frequency, in Hz |
| $I$ | intensity of the sound |
| $I_t$ | intensity of the transmitted sound |
| $m$ | mass per unit area |
| $n_i$ | modal density of subsystem $i$ |
| $p$ | acoustic pressure |
| $p_i$ | acoustic pressure in subsystem $i$ |
| $p^2_{rms}$ | mean square value of acoustic pressure |
| $R_{rad}$ | radiation resistance of a structure |
| $s$ | surface area of an acoustic cavity |
| $V_i$ | volume of subsystem $i$ |
| $v$ | velocity of a structure |
| $v_{forced}$ | velocity of the forced wave |
| $v_{free}$ | velocity of the free wave |
| $W$ | sound power radiated by a structure |
| $W_p$ | sound power radiated by a piston |
| $\alpha$ | sound power absorption coefficient of an acoustic cavity |
| $\alpha_d$ | sound power absorption coefficient of a panel |
| $\alpha_r$ | sound power reflection coefficient |
| $\eta$ | dissipation loss factor |
| $\eta_i$ | dissipation loss factor of subsystem $i$ |
| $\eta_{ij}$ | coupling loss factor for subsystem $i$–$j$ |
| $\lambda$ | wavelength |
| $\lambda_b$ | wavelength of the bending wave |
| $\pi_d$ | dissipated power |
| $\pi_i$ | power input to subsystem $i$ |
| $\pi_{ij}$ | power flow from subsystem $i$–$j$ |
| $\omega$ | circular frequency, in rad/s |
| $\rho$ | density of the medium of the acoustic field |
| $\sigma$ | radiation efficiency of a structure |
| $\theta$ | angle of incidence |
| $\tau$ | sound power transmission coefficient of a structure |
| $\langle\ \rangle_x$ | average over the domain $x$ |