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THE SOUND RADIATION EFFICIENCY OF FINITE LENGTH CIRCULAR CYLINDRICAL SHELLS UNDER MECHANICAL EXCITATION II: LIMITATIONS OF THE INFINITE LENGTH MODEL

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The acoustic radiation from circular cylindrical shells is of fundamental and applied interest. However, in previous studies, in order to obtain an analytical solution for practical applications, the cylindrical shell is normally assumed infinite in length. Obviously, this assumption would cause error in the analysis for a finite length circular cylindrical shell, especially as the length of the shell becomes comparable to the radius. In this study, the end effects of the length and the boundary conditions on the acoustic behavior of a circular cylindrical shell is discussed. It is found that the boundary conditions would affect the modal radiation efficiencies very much in the subsonic region. However, it has been shown that there exists a condition under which the end effects could be neglected for modal radiation efficiencies so that the infinite model could be used with fair accuracy. Also, it is found that if the length (l) of a circular cylindrical shell with radius a and thickness h is much greater than $2\pi a\sqrt{a/h}$, beam-bending modes would dominate the vibration response below the cut-off frequency of the second circumferential mode and the cylindrical shell can be treated as a beam with reasonable accuracy.

1. INTRODUCTION

Cylindrical shells are one of the most widely used structures in industries, for example, the casing of small- and medium-sized electric machines, and pipes with relatively small radius used in various constructions. Since in engineering all these cylindrical shells are used with finite length and various end conditions, the corresponding acoustic behavior is therefore of particular interest to noise control engineers. The early contribution to the acoustic radiation from finite length cylindrical shells was made by Williams [1] who analyzed the radiation of finite length cylinders with a uniform radial vibration velocity profile and compared the sound field with that of the infinite length model. Williams [1] found that infinite length models may overestimate the acoustic radiation from finite length cylinders, and the error would increase as the length of the cylinder becomes shorter. Then, Schenck [2] presented a surface Helmholtz integral formulation for obtaining approximate solutions of acoustic radiation problems for an arbitrary surface, which improved the accuracy of William's results in the axial direction. In order to take the reaction of the fluid inside the cylindrical shell into account, Sandman [3] examined the model of a baffled finite length cylindrical shell. It was found that the model is a reasonable approximation in determining the acoustic radiation from the surface of finite length cylindrical shells, and is applicable to arbitrary radial vibration distribution. Based on this model, Stepanishen [4] discussed the radiation impedance of different vibration modes of a finite length cylindrical shell, and Zhu [5] examined the "relative sound intensities" for different vibration modes. Also, Stepanishen [6] and Laulagnet *et al.* [7] investigated the effects of fluid on the acoustic radiation of finite length cylindrical shells. However, since most of the studies chose simply supported ends to facilitate their analyses [4–8], the effects of boundary conditions on the acoustic radiation are still not very clear.

In engineering, the infinite length model is often adopted in dealing with the acoustic problems associated with finite length cylindrical shells. This is because the corresponding analytical solutions are easy to obtain and apply. For example, Jeyapalan and Richards [8] developed a simple expression for the modal averaged radiation efficiency of a circular cylindrical shell in beam-bending motion by using an infinite length model. However, the end effects would become more important as the length becomes shorter and comparable to the radius. Under this condition, the vibration behavior of a circular cylindrical shell could change dramatically with frequency and infinite length models would not be valid. Recently, Wang and Lai [9] derived the modal-averaged radiation efficiency for a finite length circular cylindrical shell and showed that it can be modified significantly by the boundary conditions. Their theoretical results have been verified by acoustic boundary element calculations and experiments. It would be beneficial to determine a condition for which the results of infinite length models can be employed with reasonable accuracy.

The objective of this study was, therefore, to investigate the effects of the boundary conditions and length on the modal radiation efficiency of a finite length circular cylindrical shell. The condition for which the corresponding infinite length model could apply will be determined. Also, the condition under which a circular cylindrical shell could be treated like a beam in bending motion will be established.

2. THE MODAL RADIATION EFFICIENCY OF FINITE LENGTH CIRCULAR CYLINDRICAL SHELLS

Modal radiation efficiency is commonly used to characterize the acoustic radiation properties of structures. According to Wang and Lai [9], the modal radiation efficiency associated with a vibration mode (m, n) of a finite length circular cylindrical shell with arbitrary boundary conditions can be predicted by using the following equation:

$$\sigma_{mn}(\omega) = \frac{k}{\pi^2} \frac{1}{\int_0^l |\gamma_m(z)|^2 a \, \mathrm{d}z} \int_{-k}^{k} \frac{|\Gamma_m(k_z)|^2}{k_r^2 |\mathrm{dH}_n^{(2)}(k_r a)/\mathrm{d}(k_r a)|^2} \, \mathrm{d}k_z, \tag{1}$$

where m is the mode number in the axial direction, n is the mode number in the circumferential direction, a, l are the radius and the length of the cylindrical shell, respectively, k is the acoustic wavenumber, $k_r = \sqrt{k^2 - k_z^2}$ in which k_z is the axial Fourier wavenumber component, $\mathrm{H}_n^{(2)}(\cdot)$ is Hankel function of the second kind, and $\Gamma_m(k_z)$ is the mode shape of the cylindrical shell in the wavenumber domain obtained by applying the spatial Fourier transform to the mode shape $\gamma_m(z)$ in the axial direction. Although equation (1) was derived using a baffled cylindrical shell model, it has been shown [9] that the modal-averaged radiation efficiency obtained by using this equation compared reasonably well with the experimental and boundary element results of the corresponding unbaffled cylindrical shell. For cylindrical shells simply supported at both ends, the mode shape in the axial direction is described exactly by

$$\gamma_m(z) = \sin\left(\frac{m\pi}{l} z\right), \quad m = 1, 2, 3, \ldots. \tag{2}$$

Thus, the modal radiation efficiency of a simply supported circular cylindrical shell for mode (m, n) can be derived from equations (1) and (2) as [9]

$$\sigma_{mn}(\omega) = \int_{-k}^{k} \frac{2kl}{\pi^2 a k_r^2 |\mathrm{dH}_n^{(2)}(k_r a)/\mathrm{d}(k_r a)|^2} \left[\frac{m\pi/l}{k_z + m\pi/l}\right]^2 \frac{\sin^2[(l/2)(k_z - m\pi/l)]}{[(l/2)(k_z - m\pi/l)]^2} \,\mathrm{d}k_z. \tag{3}$$

For other boundary conditions, it has been demonstrated [10] that the mode shapes of cylindrical shells in the axial direction can be determined numerically with good approximations by using the beam functions. For example, the beam functions for the clamped–clamped, and free–free boundary conditions are [11]

$$\gamma_m(z) = \cosh(k_{zm}z) - \cos(k_{zm}z) - \frac{\cosh(k_{zm}l) - \cos(k_{zm}l)}{\sinh(k_{zm}l) - \sin(k_{zm}l)}[\sinh(k_{zm}z) - \sin(k_{zm}z)],$$

$$k_{zm} \approx \frac{m\pi}{l} - \frac{\pi}{2l}, \quad m = 1, 2, 3, \ldots. \tag{4}$$

and

$$\gamma_0(z) = 1, \qquad \gamma_1(z) = \frac{z}{l} - \frac{1}{2},$$

$$\gamma_m(z) = \cosh(k_{zm}z) + \cos(k_{zm}z) - \frac{\cosh(k_{zm}l) - \cos(k_{zm}l)}{\sinh(k_{zm}l) - \sin(k_{zm}l)}[\sinh(k_{zm}z) + \sin(k_{zm}z)],$$

$$k_{zm} \approx \frac{m\pi}{l} - \frac{\pi}{2l}, \quad m = 2, 3, 4, \ldots. \tag{5}$$

respectively. Thus, by using equation (1) and the corresponding beam functions (such as equations (4) and (5)), the effects of different boundary conditions on the modal radiation efficiencies can be assessed.

Figure 1 shows the modal radiation efficiencies of various vibration modes of a simply supported circular cylindrical shell with length to radius ratio (l/a) of 3. It can be seen that, like flat plates, all vibration modes would become supersonic when the acoustic wavenumber k is greater than the structural wavenumber k_s, and the corresponding modal radiation efficiency is of the order of unity. When the vibration modes are subsonic, the modal radiation efficiencies are less than 1. This result supports the use of the index ΔL for determining the acoustic behavior of a vibration mode [9]. However, one should bear in mind that, unlike flat plates, cylindrical shells do not have a unique "critical frequency" for all vibration modes, as discussed in reference [9]. Furthermore, it can be seen from Figure 1 that in the subsonic region, for a given axial mode number m, the modal radiation efficiency is quite complex as the circumferential mode number n increases. In the very low subsonic region, the modal radiation efficiency decreases as n increases but the opposite trend is observed in the higher subsonic region. Thus, the modal radiation efficiencies of higher

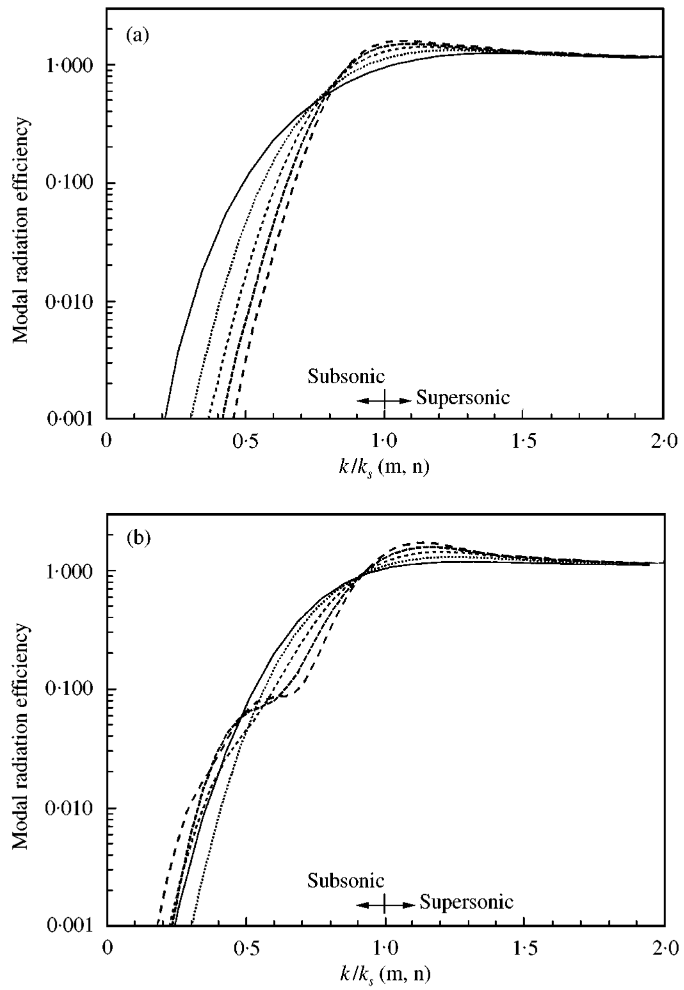

Figure 1. Modal radiation efficiencies of a circular cylindrical shell: $l/a = 3$. (a) ——, $\sigma(2, 1)$; · · · · ·, $\sigma(2, 2)$; - - - - - -, $\sigma(2, 3)$; - - - - - - $\sigma(2, 4)$; – – – – $\sigma(2, 5)$. (b) —— $\sigma(1, 2)$; · · · · · $\sigma(2, 2)$; - - - - - - $\sigma(3, 2)$; – – – – $\sigma(4, 2)$; – – – – $\sigma(5, 2)$.

order modes are not always less than those of the lower order modes. This result is quite different from plates for which the higher order modes would have lower modal radiation efficiencies in the subsonic region due to less radiation areas.

3. VALIDITY OF USING AN INFINITE LENGTH MODEL

The effect of the length of cylindrical shells on the modal radiation efficiency is always of interest to noise control engineers. In many textbooks on acoustics, the modal radiation efficiencies of a cylindrical shell are analyzed by assuming infinite length so that a simple expression can be obtained. Therefore, it would be helpful to obtain a condition under which the infinite length model can be used for finite cylindrical shells without much error.

For convenience, a cylindrical shell with two ends simply supported is considered. For an infinite length cylindrical shell, the modal radiation efficiency has a simple, closed form as given by [12]

$$\sigma_{mn}(\omega) = \begin{cases} 0 & k \leqslant k_z, \\ \dfrac{2k^2}{\pi a k_r^3 |\mathrm{dH}_n^{(2)}(k_r a)/\mathrm{d}(k_r a)|^2}, & k > k_z, \end{cases} \tag{6}$$

where $k_r = \sqrt{k^2 - k_z^2}$. If one applies equation (6) to calculate the modal radiation efficiency of finite length cylindrical shells, k_z has to be set to $m\pi/l$ as determined by using the beam function given in equation (2) for simply supported boundary conditions. According to equation (6), it can be seen that for infinite length cylindrical shells, there is a cut-off frequency at which the corresponding acoustic wavenumber k is equal to k_z $(=m\pi/l)$ for each vibration mode. Below this cut-off frequency, the modal radiation efficiency is zero, and above this frequency, the modal radiation efficiency increases to unity when the acoustic wavenumber k is equal to the structural wavenumber $k_s(=\sqrt{(m\pi/l)^2 + (n/a)^2})$. This result is not obvious because if one uses infinite flat plates as an analogy, the modal radiation efficiencies of infinite length cylindrical shells would be expected to be zero when $k < k_s$ due to the intercell cancellation. However, for cylindrical shells the structural wave speed in the axial direction is faster than that in the circumferential direction due to curvature effects below the ring frequency. Therefore, when the frequency increases, the axial wave would catch up the acoustic wave much faster than the circumferential wave. Hence, there is a frequency range within which the axial component of a vibration mode is supersonic while the circumferential component is still subsonic. The modal radiation efficiency would continue to increase with frequency until both components are supersonic. The frequency at which the axial component becomes supersonic is the cut-off frequency of this mode. Generally, the larger the number m, the higher the cut-off frequency.

In order to examine the effects of the length of a cylindrical shell on its modal radiation efficiency, the modal radiation efficiencies calculated using equation (6) are compared with those obtained using equation (3) in Figures 2(a)–2(e) for five different modes (2, 1), (2, 3), (2, 5), (4, 3) and (4, 5) with different l/a ratios respectively. It can be seen that for low l/a, severe errors would occur from applying the results of infinite length model, i.e., equation (6). This result can be interpreted by analogy with finite/infinite plates. For finite length cylindrical shells, there are no cut-off frequencies because the intercell cancellation does not occur at the ends. Thus, the radiation efficiency of a finite length model is higher than the infinite model below and around the cut-off frequency for the infinite model. Around the demarcation of subsonic and supersonic modes ($k/k_s = 1$), as the acoustic wave speed and structural wave speed are almost the same, there should be more energy accumulated along the infinite length cylindrical shell than that along the finite length model just like for flat plates [13]. Consequently, the radiation efficiency of an infinite length model is higher than that of a finite length model in that region. All these results in Figure 2 show that the larger the length/radius ratio (l/a), the smaller the difference between the two results given by equations (3) and (6).

For a given axial mode number m (for example 2), as the circumferential mode number n increases from 1 in Figure 2(a) to 3 in Figure 2(b), the length/radius ratio for good agreement between infinite and finite length models decreases from $l/a = 20$ for (2, 1) mode to $l/a = 7$ for (2, 3) mode. On the other hand, for a given circumferential mode number n (for example 3), as the axial mode number m increases from 2 in Figure 2(b) to 4 in Figure 2(d), the length/radius ratio for good agreement between infinite and finite length models increases from $l/a = 7$ for (2, 3) mode to $l/a = 14$ for (4, 3) mode. Thus, the length/radius ratio for good agreement between infinite and finite length shells decreases as the circumferential mode order n increases, and as the axial mode order m decreases. Consequently, the condition for applying the results of an infinite length model to a finite length cylindrical shell can be stated as

$$\frac{l}{m}\frac{n}{a} \geqslant \text{Constant}. \tag{7}$$

Obviously, the larger the constant, the closer the two results. From Figure 2, this constant has been estimated to be 10. As equations (3) and (6) indicate that the modal radiation efficiencies are only influenced by the length l, radius a, and mode numbers m, n, the conclusion that $(l/m)n/a \geqslant 10$ should apply to all circular cylindrical shells. Generally, when

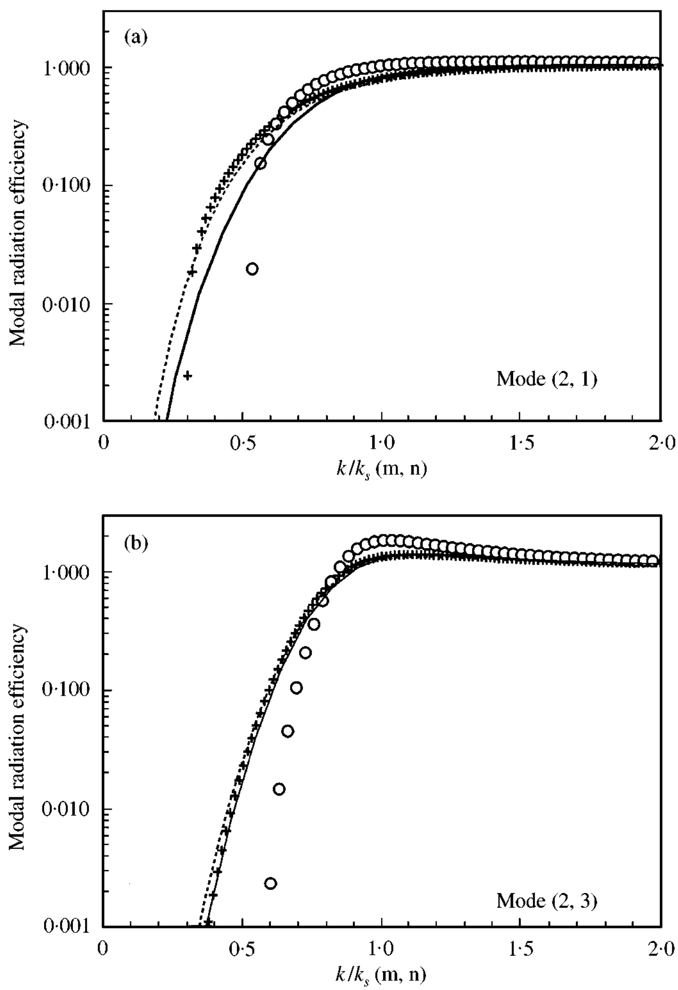

Figure 2. Comparisons of the modal radiation efficiencies of infinite and finite length cylindrical shells. (a) —— finite model $l/a = 10$; ------ finite model $l/a = 20$; ○ infinite model $l/a = 10$; + infinite model $l/a = 20$; (b) —— finite model $l/a = 3$; ------ finite model $l/a = 7$; ○ infinite model $l/a = 3$; + infinite model $l/a = 7$; (c) —— finite model $l/a = 2$; ------ finite model $l/a = 4$; ○ infinite model $l/a = 2$; + infinite model $l/a = 4$; (d) —— finite model $l/a = 7$; ------ finite model $l/a = 14$; ○ infinite model $l/a = 7$; + infinite model $l/a = 14$; (e) —— finite model $l/a = 4$; ------ finite model $l/a = 8$; ○ infinite model $l/a = 4$; + infinite model $l/a = 8$.

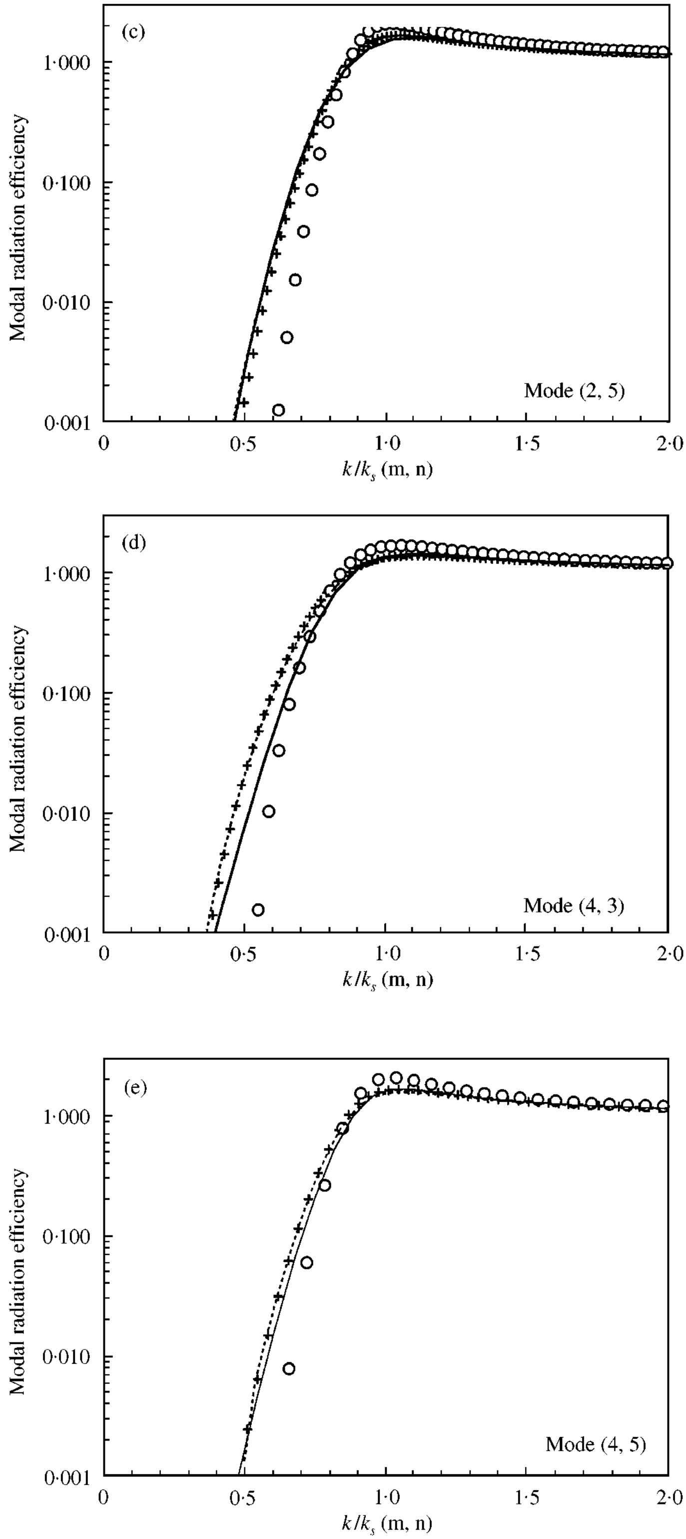

Figure 2. Continued.

m increases, the areas of the "edges" and "corners" would decrease so that the modal radiation efficiency of a finite structure would decrease approaching the results of an infinite length model. However, for finite length cylindrical shells, increasing m not only would reduce the radiation efficiency, but also would increase the cut-off frequency of the corresponding infinite length model. Thus, the difference between the results obtained using an infinite length model (equation (6)) and a finite length model (equation (3)) become greater as m increases. In fact, equation (7) basically implies that when the wavenumber in the circumferential direction is much greater than that in the axial direction, the acoustic behavior of a cylindrical shell is mainly determined by the wavenumber in the circumferential direction. Under this condition, the length of a cylindrical shell is no longer important, and the results of the two models would tend to be the same.

4. EFFECTS OF BOUNDARY CONDITIONS ON MODAL RADIATION EFFICIENCIES

According to equation (1), the modal radiation efficiency depends on the spatial Fourier transform of the mode shape in the axial direction. Basically, different mode shapes $\gamma_m(z)$ would result in different modal radiation efficiencies. Since generally the mode shapes of cylindrical shells in the axial direction are determined by the boundary conditions at both ends, the effects of boundary conditions of the cylindrical shell on its modal radiation efficiencies are therefore of interest. In order to illustrate the effects of boundary conditions, modal radiation efficiencies obtained by using equations (1) and (4) for clamped–clamped ends and equation (3) for simply supported ends for modes (1, 2), (2, 2), (4, 2) and (1, 8) are compared in Figure 3 for $l/a = 1{\cdot}5$. It can be seen that the modal radiation efficiencies of the clamped–clamped shell are normally lower than those of the simply supported shell in the region $k/k_s(m, n) < 1$. This is due to the effect of near fields at the two ends. Generally, for a clamped end, the vibration amplitude near the end would be smaller than that for a simply supported end because of the additional zero bending moment condition. As a result, in the subsonic region, the acoustic power radiated from a clamped–clamped shell, which is acutally due to the radiation from the cells at both ends, would be smaller than that from a simply supported shell, thus resulting in lower radiation efficiency. Furthermore, it can be observed that when m increases, the differences between the two boundary conditions increase. This is because as m increases, the intercell cancellation for simply supported condition is more complete so that the end effects for clamped condition become more important. In the region $k/k_s(m, n) \geqslant 1$, all the modal radiation efficiencies approach unity as expected. Here, although only the clamped–clamped and simply supported boundary conditions are examined, they have been demonstrated to affect the acoustic behavior of a cylindrical shell. Hence, equation (1) generally has to be used for finite length cylindrical shells with different boundary conditions.

It can be observed from Figure 3 that for the clamped–clamped boundary condition, and $k/k_s(m, n) < 1$, the modal radiation efficiency decreases as m increases. This is because modes with small m are normally more efficient in acoustic radiation than those with larger m.

It has been argued in section 3 that provided that $(l/m)n/a \geqslant 10$, the circumferential waves would dominate the overall acoustic performance and the length of a cylindrical shell is not so important. Since the boundary conditions at both ends of a cylindrical shell only affect the wavenumber (k_{zm}) in the axial direction, the effects of boundary conditions can be neglected provided that $(l/m)n/a \geqslant 10$. In Figure 3(a), the radiation efficiencies of mode (1, 8) which satisfies equation (7) for simply supported and clamped boundary conditions are plotted. It can be seen that the two curves agree with each other reasonably well. Thus, it can be concluded that equation (7) is also a condition for which the effects of boundary conditions are not important.

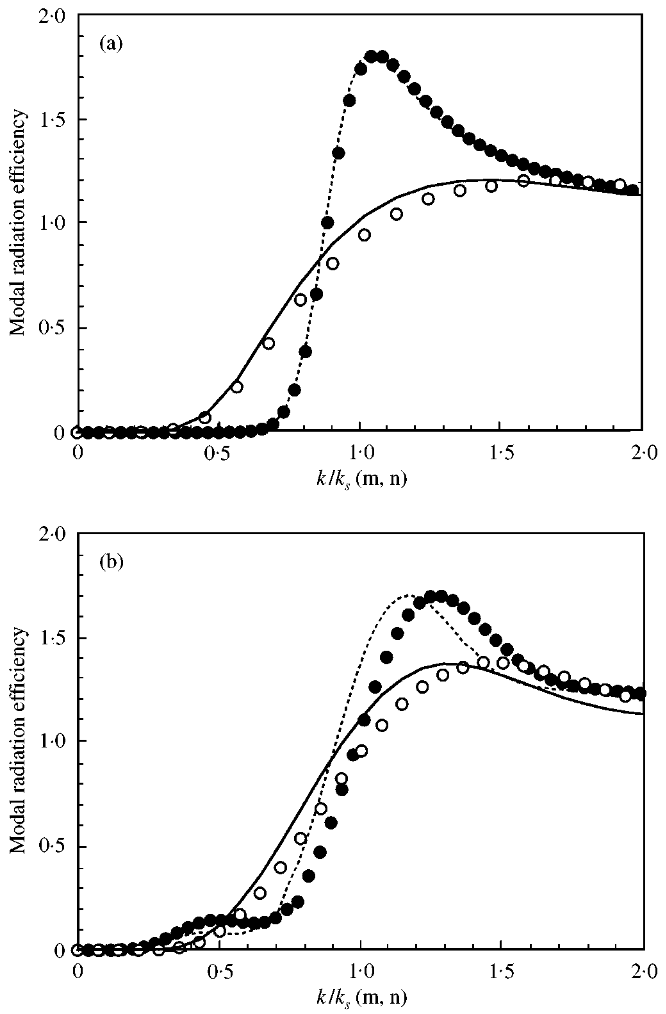

Figure 3. Comparisons of modal radiation efficiencies of simply supported and clamped–clamped cylindrical shells: $l/a = 1{\cdot}5$. (a) —— simply supported mode (1, 2); ------ simply supported mode (1, 8); ○ clamped mode (1, 2); ● clamped mode (1, 8); (b) —— simply supported mode (2, 2); ------ simply supported mode (4, 2); ○ clamped mode (2, 2); ● clamped mode (4, 2).

It should be pointed out that although sometimes the effects of boundary conditions on modal radiation efficiencies can be neglected, the effects of boundary conditions on the modal-averaged radiation efficiency cannot be neglected, because the difference of natural frequencies due to different boundary conditions would also cause some difference in the modal-averaged radiation efficiency, as has already been discussed by Wang and Lai [9].

5. RADIATION EFFICIENCY OF FINITE LENGTH CYLINDRICAL SHELLS IN BENDING MOTION

A special case associated with the sound radiation from cylindrical shells is that the shell is in beam-bending motion. The beam-bending motion refers to a cylindrical shell vibrating like a beam, which corresponds to the circumferential mode number $n = 1$. Generally, it is thought that a long cylindrical shell, such as water pipes, would vibrate in this manner. Johnston and Barr [14] and Richards *et al.* [15] studied this particular case independently by assuming that the cylindrical shell is infinite in length and in bending motion over the whole frequency band. A simple expression for the modal-averaged radiation efficiency was obtained by Richards *et al.* [15]:

$$\sigma_b(\omega) = \begin{cases} 0, & f \leqslant f_{bc}, \\ 1{\cdot}55(ka)^3[1-(f_{bc}/f)], & f_{bc} < f < 6{\cdot}5 f_{bc}, \\ [1-(f_{bc}/f)]^{-1/2}, & f > 6{\cdot}5 f_{bc}, \end{cases} \tag{8}$$

where $f_{bc} = \sqrt{2}c^2/4\pi^2 a^2 f_r$ is the critical frequency for beam bending cylindrical shells, f_r is the ring frequency of the shell. From equation (8), it can be seen that the radiation efficiency is zero below the critical frequency f_{bc}, and increases to unity as the frequency increases to around $6{\cdot}5 f_{bc}$. This equation has been verified by experiments [15]. However, it should be noted that a cylindrical shell would not be in beam-bending motion in the whole frequency range because the vibration behavior of the shell changes with frequency. There must exist a condition under which it can be applied with reasonable accuracy.

In reference [10], exact solutions of an infinite length circular cylindrical shell for longitudinal, torsional, and flexural vibrations were presented. It was shown that when the flexural vibration dominates the overall behavior of an infinite length circular cylindrical shell, in the axial direction there are a series of flexural waves corresponding to different circumferential modes, in which the wave of the circumferential mode number $n = 1$ is the bending wave [10]. Figure 4 displays the variation of the non-dimensional frequency parameter of the flexural waves with $k_z a$ for an infinite length shell with $a/h = 5$ and circumferential mode numbers $n = 0$, 1, 2 and 3. Here Ω is defined as the ratio of the frequency to the ring frequency and k_z is the axial wavenumber. It can be found that corresponding to each of these waves, there is a cut-off frequency (which is different from the cut-off frequency discussed in section 3) above which the corresponding wave can occur. As these cut-off frequencies are actually the natural frequencies of the modes with zero axial wavenumber for free–free cylindrical shells, a simple expression [16] can be used here:

$$f_{vcn}^2 = \frac{h^2}{12a^2}\frac{n^2(n^2-1)^2}{n^2+1} f_r^2. \tag{9}$$

Therefore, the cut-off frequency for $n = 2$ ($f_{vc2} = \sqrt{\frac{3}{5}} f_r h/a$) divides the higher order waves from the beam bending wave in the frequency domain. But below f_{vc2}, as the cut-off frequencies for wave $n = 0$ and $n = 1$ are all zero; $n = 0$ wave would always occur in addition to bending modes even at low frequencies. However, for a given frequency, as the axial wavenumber for wave $n = 0$ is much smaller than that of wave $n = 1$ as shown in Figure 4, the effect of wave $n = 0$ might be small so that the bending wave would dominate the behavior of the cylindrical shell. Actually Cremer *et al.* [12] showed that below the cut-off frequency of $n = 2$, the point input impedance of an infinite length cylindrical shell takes the form of an equivalent beam in bending motion. This result shows that for an infinite length cylindrical shell, below f_{vc2}, it can be simply treated as a beam in the corresponding vibration and acoustics analysis.

For finite length circular cylindrical shells, however, the case is different. According to linear vibration theory, the vibration response of a structure with finite dimensions is the

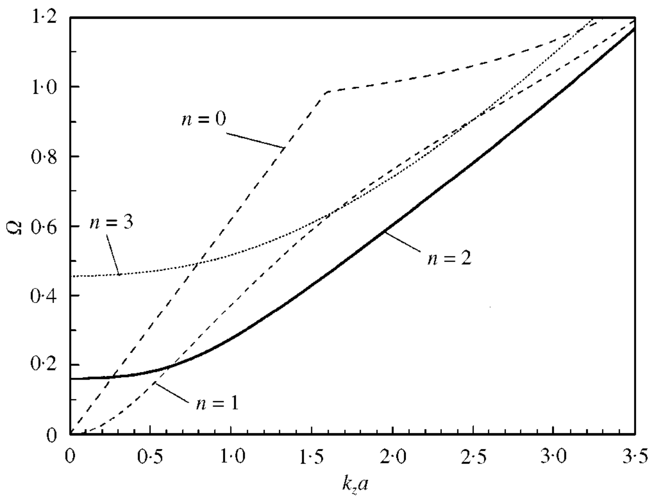

Figure 4. Non-dimensional frequency Ω of flexural waves versus $k_z a$ for an infinite length shell with $a/h = 5$.

superposition of all the vibration modes. At frequencies well below the first natural frequency of the structure, the vibration response is dominated by the behavior of the first mode. Therefore, it can be expected that for a finite length circular cylindrical shell with both ends free, if there is no vibration mode below f_{vc2}, the behavior of the shell at low frequencies is mostly dependent on the behavior of the mode $n = 2$, which obviously cannot be treated as a beam at low frequencies. Only when the cylindrical shell is long enough that bending modes occur below f_{vc2}, could the cylindrical shell behave like a beam at low frequencies. To obtain a quantitative criterion, a cylindrical shell with two ends clamped is taken as an example. The wavenumber of the first bending mode in the axial direction can be approximately written as

$$k_z \approx \frac{1{\cdot}5\pi}{l}. \tag{10}$$

The relationship between the frequency and wavenumber for the bending wave is [13]

$$f = \frac{\sqrt{2}}{2}(k_z a)^2 f_r. \tag{11}$$

By using equations (9)–(11), the condition for the natural frequency of the first bending mode being much smaller than f_{vc2} is

$$l \gg 1{\cdot}5\pi a\sqrt{\frac{a}{h}} \sim 2\pi a\sqrt{\frac{a}{h}} \quad \text{or} \quad \frac{l}{2\pi a\sqrt{a/h}} \gg 1. \tag{12}$$

Equation (12) indicates that only when the dimensions of a finite length cylindrical shell satisfy this relationship, the low-frequency behavior of the cylindrical shell would be dominated by the bending modes. As the length becomes longer, there would be more bending modes below f_{vc2}, and the results obtained by the beam model would be more accurate. Since the natural frequencies of a clamped–clamped cylindrical shell are greater than those of the shell with other boundary conditions, equation (12) should also apply for other boundary conditions. Actually, for a cylindrical shell in bending motion, combining equations (11), (9) with (7), a frequency (f_l) below which the infinite length model could be used for corresponding acoustic analysis can be obtained:

$$f_l < \frac{\pi^2 f_r}{\text{Constant}}. \tag{13}$$

It can be seen that by choosing the constant as 10, this frequency is always greater than f_{vc2}. Therefore, for a finite length cylindrical shell in bending motion, equation (8) can be used directly without taking the effects of boundary conditions into account.

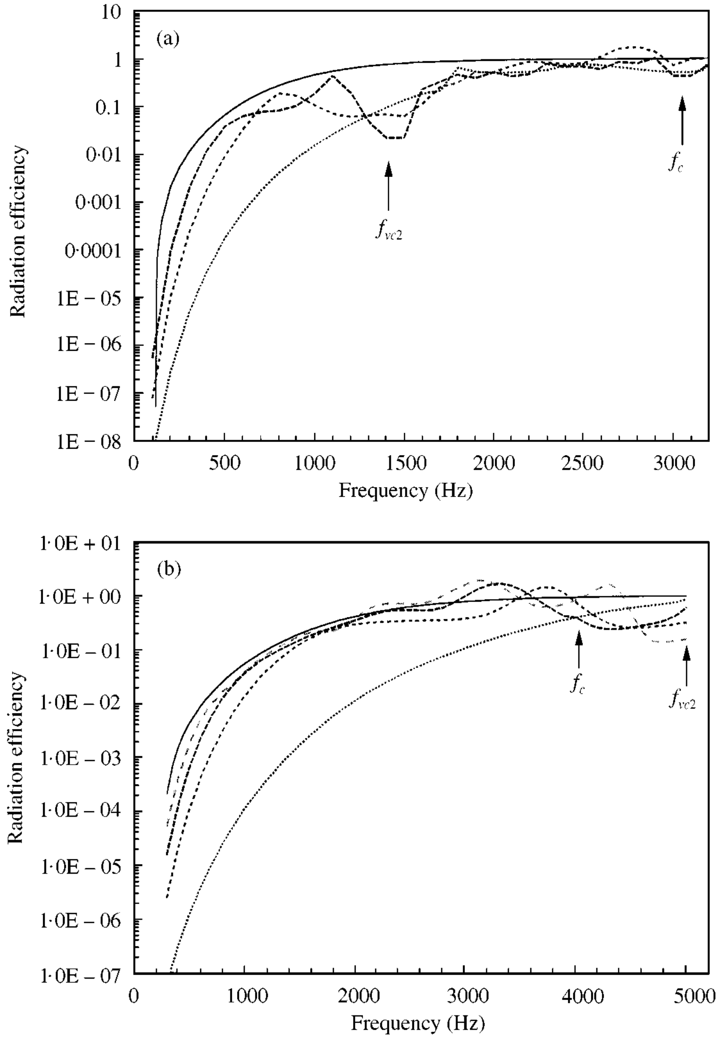

Figure 5. Radiation efficiencies of finite length circular cylindrical shells. (a) $a = 42$ mm; $h = 4$ mm; $f_{vc2} = 1·4$ kHz; $f_c = 3$ kHz; $2\pi a\sqrt{a/h} = 0·8$; —— Richards results; ······· $l = 0·5$ m; - - - - - - $l = 0·8$ m; – – – – $l = 1·1$ m; (b) $a = 19·5$ mm; $h = 3$ mm, $f_{vc2} = 5$ kHz; $f_c = 4$ kHz; $2\pi a\sqrt{a/h} = 0·3$; —— Richards results; ······· $l = 0·2$ m; - - - - - - $l = 0·4$ m; - - - - - - $l = 0·6$ m; – – – – $l = 0·8$ m.

To verify the above result, two groups of steel circular cylindrical shells of different radius/thickness ratios were investigated. By using equations (1) and (5), the radiation efficiencies of the shells with different lengths have been calculated and compared with Richards' results in Figure 5. It can be seen that as the length of the shell increases, the radiation efficiencies of finite length cylindrical shells approach Richards' results. Also, when the cut-off frequency for $n = 2$ is lower than the critical frequency (Figure 5(a)), Richards' results only apply below f_{vc2}. When f_{vc2} is greater than the critical frequency (Figure 5(b)), Richards' results can be used for the whole frequency range. However, if the length of the shell does not satisfy equation (12), the shell cannot be treated as a beam.

6. CONCLUSIONS

The end effects of a finite length cylindrical shell on the modal radiation efficiencies have been discussed in this paper. It is found that generally the boundary conditions affect the modal radiation efficiencies very much in the subsonic region of the modes. However, if a vibration mode (m, n) of a circular cylindrical shell with length l and radius a satisfies the relationship, $(l/m)n/a \geqslant 10$, the end effects could be neglected, and thus the corresponding modal radiation efficiency can be predicted by using an infinite length model. Furthermore, when the length (l) of a circular cylindrical shell with radius a and thickness h is much greater than $2\pi a\sqrt{a/h}$, the results of Richards *et al.* [15] for the modal-averaged radiation efficiency of a cylindrical shell in bending motion can apply below the vibration cut-off frequency of mode $n = 2$. For a finite length cylindrical shell in bending motion, end effects are negligible.

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