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AN APPROXIMATE FREQUENCY FORMULA FOR PIEZOELECTRIC CIRCULAR CYLINDRICAL SHELLS

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In this paper, an approach for investigating vibration characteristics of piezoelectric cylindrical shells under transverse vibration modes is presented. It is an extension of the related work for elastic shells. A formula for estimating transverse frequencies of the piezoelectric cylindrical shells is obtained. Because of its simplicity and clarity, the formula can be used to investigate the influence of electromechanical coupling effect and geometry parameters on the natural frequencies conveniently.

1. INTRODUCTION

In recent years, micro-devices utilizing piezoelectric effects have been widely used in many electromechanical applications. The performance of the devices usually depends on dynamic behaviours of piezoelectric elements or structures significantly, which have been studied extensively in literature, see e.g. references [1, 2]. However, due to coupling effect between electrical and mechanical fields, the dynamic problems of piezoelectric structures are more complicated than pure mechanical ones. Therefore, it is necessary to develop simple and effective approximate methods for some piezoelectric dynamic problems in engineering applications.

In the present paper, a simplified method for estimating vibration characters of a piezoelectric cylindrical shell is discussed. This study is introduced due to a research concerning modelling on cylindrical ultrasonic micromotors. The principle of the motors is to utilize the bending vibrations of a closed piezoelectric cylindrical shell under electric power to drive moving piece through friction force [3]. Therefore, understanding the vibration behaviours, such as natural frequencies and modes, of the cylindrical shell is very important in the design of the micromotors.

Since transverse vibration of the piezoelectric cylindrical shell is especially interested in the investigations, the approximate technique by Soedel [4, 5] for elastic cylindrical shells

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can be extended. This approach is to solve a Donnell–Mushtari–Vlasov-type equations [6] by Galerkin method with general beam functions, and a simplification is made that allows various boundary condition cases to be written in terms of a single simple frequency formula which incorporates the roots for the analogous beam problem. The expression of the formula is simple and allows one to assess the various parameter influences on the natural frequencies conveniently. Therefore, it is suitable for practical use in structure designs. In this paper, the approach for the elastic shells is successfully extended to piezoelectric problems, and related equations and formulations are derived and obtained. To the best of the authors' knowledge, the results have not been reported in literature. It provides an alternative simplified method for the predication of dynamic behaviours of piezoelectric shells.

2. GOVERNING EQUATIONS

A piezoelectric cylindrical shell is shown in Figure 1, where a, h and L are respectively, radius, thickness and length of the shell. The cylindrical shell can be defined by cylindrical co-ordinate system with x-, θ- and α_3-axis, in which x defines the longitudinal direction (length), θ the circumferential direction, and α_3 the transverse direction. For the thin piezoelectric shell polarized in radial direction, only electric field E_3 along thickness direction is considered.

According to the assumptions, the electromechanical equations of the piezoelectric cylindrical shell in u_x, u_θ and u_3 displacements can be written as [2]

$$\frac{\partial(N^m_{xx} - N^e_{xx})}{\partial x} + \frac{1}{a}\frac{\partial N^m_{\theta x}}{\partial \theta} - \rho h \ddot{u}_x = 0,$$

$$\frac{\partial N^m_{x\theta}}{\partial x} + \frac{1}{a}\frac{\partial(N^m_{\theta\theta} - N^e_{\theta\theta})}{\partial \theta} + \frac{1}{a}\frac{\partial M^m_{x\theta}}{\partial x} + \frac{1}{a^2}\frac{\partial(M^m_{\theta\theta} - M^e_{\theta\theta})}{\partial \theta} - \rho h \ddot{u}_\theta = 0, \qquad (1)$$

$$\frac{\partial^2(M^m_{xx} - M^e_{xx})}{\partial x^2} + \frac{2}{a}\frac{\partial^2 M^m_{\theta x}}{\partial x \partial \theta} + \frac{1}{a^2}\frac{\partial^2(M^m_{\theta\theta} - M^e_{\theta\theta})}{\partial \theta^2} - \frac{N^m_{\theta\theta} - N^e_{\theta\theta}}{a} - \rho h \ddot{u}_3 = 0.$$

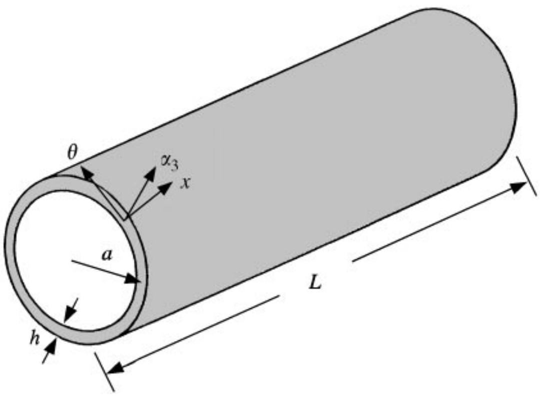

Figure 1. A radially polarized piezoelectric circular cylindrical shell.

The charge equation of electrostatics is given by

$$\frac{\partial}{\partial \alpha_3}(e_{31}S_{xx} + e_{31}S_{\theta\theta} + \varepsilon_{33}E_3)a\left(1 + \frac{\alpha_3}{a}\right) = 0, \tag{2}$$

where e_{31} is the transverse piezoelectric constant, and ε_{33} the dielectric permittivity. With Love's simplifications, the strain–displacement relations are written as

$$S_{xx} = S^0_{xx} + \alpha_3\kappa_{xx}, \quad S_{\theta\theta} = S^0_{\theta\theta} + \alpha_3\kappa_{\theta\theta}, \quad S_{x\theta} = S^0_{x\theta} + \alpha_3\kappa_{x\theta}, \tag{3}$$

where

$$S^0_{xx} = \frac{\partial u_x}{\partial x}, \quad S^0_{\theta\theta} = \frac{1}{a}\frac{\partial u_\theta}{\partial \theta} + \frac{u_3}{a}, \quad S^0_{x\theta} = \frac{1}{a}\frac{\partial u_x}{\partial \theta} + \frac{\partial u_\theta}{\partial x},$$

$$k_{xx} = -\frac{\partial^2 u_3}{\partial x^2}, \quad \kappa_{\theta\theta} = \frac{1}{a^2}\left(\frac{\partial u_\theta}{\partial \theta} - \frac{\partial^2 u_3}{\partial \theta^2}\right), \quad \kappa_{x\theta} = \frac{1}{a}\left(\frac{\partial u_\theta}{\partial x} - 2\frac{\partial^2 u_3}{\partial x \partial \theta}\right). \tag{4}$$

In equation (1), mechanical membrane forces N^m_{ij} and bending moments M^m_{ij} are given by

$$N^m_{xx} = K(S^0_{xx} + \mu S^0_{\theta\theta}), \quad N^m_{\theta\theta} = K(S^0_{\theta\theta} + \mu S^0_{xx}), \quad N^m_{x\theta} = \frac{K(1-\mu)}{2}S^0_{x\theta},$$

$$M^m_{xx} = D(\kappa_{xx} + \mu\kappa_{\theta\theta}), \quad M^m_{\theta\theta} = D(\kappa_{\theta\theta} + \mu\kappa_{xx}), \quad M^m_{x\theta} = \frac{D(1-\mu)}{2}\kappa_{x\theta}, \tag{5}$$

where $K = Eh/(1-\mu^2)$ is the membrane stiffness, and $D = Eh^3/[12(1-\mu^2)]$ the bending stiffness. By using equation (2), electric membrane forces N^e_{ij} and bending moments M^e_{ij} can be obtained as [2]

$$N^e_{xx} = N^e_{\theta\theta} = -\frac{e^2_{31}h}{\varepsilon_{33}}(S^0_{xx} + S^0_{\theta\theta}), \quad M^e_{xx} = M^e_{\theta\theta} = -\frac{e^2_{31}I}{\varepsilon_{33}}(\kappa_{xx} + \kappa_{\theta\theta}), \tag{6}$$

where $I = h^3/12$ is the area moment of inertia per unit.

3. DONNELL–MUSHTARI–VLASOV-TYPE EQUATIONS

In this section, the Donnell–Mushtari–Vlasov equations for elastic shells are extended to piezoelectric ones. Based on the simplifications of the method [5, 6], the bending strains κ_{ij} in equation (4) and the equations of motion (1) can be reduced to

$$\kappa_{xx} = -\frac{\partial^2 u_3}{\partial x^2}, \quad \kappa_{\theta\theta} = -\frac{1}{a^2}\frac{\partial^2 u_3}{\partial \theta^2}, \quad \kappa_{x\theta} = -\frac{2}{a}\frac{\partial^2 u_3}{\partial x \partial \theta} \tag{7}$$

and

$$a\frac{\partial(N^m_{xx} - N^e_{xx})}{\partial x} + \frac{\partial N^m_{\theta x}}{\partial \theta} = 0, \quad a\frac{\partial N^m_{x\theta}}{\partial x} + \frac{\partial(N^m_{\theta\theta} - N^e_{\theta\theta})}{\partial \theta} = 0,$$

$$\left(D + \frac{e^2_{31}I}{\varepsilon_{33}}\right)\nabla^4 u_3 + \frac{N^m_{\theta\theta} - N^e_{\theta\theta}}{a} + \rho h\ddot{u}_3 = 0. \tag{8}$$

Introduce a function ϕ defined by

$$N_{xx}^m - N_{xx}^e = \frac{1}{a^2}\frac{\partial^2\phi}{\partial\theta^2}, \quad N_{\theta\theta}^m - N_{\theta\theta}^e = \frac{\partial^2\phi}{\partial x^2}, \quad N_{x\theta}^m = -\frac{1}{a}\frac{\partial^2\phi}{\partial x\partial\theta}. \tag{9}$$

By substituting these definitions into equation (8), it is found that the first two equations are satisfied and the third one becomes

$$\left(D + \frac{e_{31}^2 I}{\varepsilon_{33}}\right)\nabla^4 u_3 + \nabla_K^2\phi + \rho h\ddot{u}_3 = 0, \tag{10}$$

where

$$\nabla^4(\cdot) = \frac{1}{a^4}\frac{\partial^4(\cdot)}{\partial\theta^4} + \frac{\partial^4(\cdot)}{\partial x^4} + \frac{2}{a^2}\frac{\partial^4(\cdot)}{\partial x^2\partial\theta^2}, \quad \nabla_K^2(\cdot) = \frac{1}{a}\frac{\partial^2(\cdot)}{\partial x^2}. \tag{11}$$

The second differential equation relating to the functions u_3 and ϕ can be obtained from the compatibility equation for a circular cylindrical shell [6]

$$\kappa_{xx} + \frac{\partial}{\partial x}\left(a\frac{\partial S_{\theta\theta}^0}{\partial x} - \frac{1}{2}\frac{\partial S_{x\theta}^0}{\partial\theta}\right) + \frac{1}{a}\frac{\partial}{\partial\theta}\left(\frac{\partial S_{xx}^0}{\partial\theta} - \frac{a}{2}\frac{\partial S_{x\theta}^0}{\partial x}\right) = 0. \tag{12}$$

From equations (5), (6) and (9), we have

$$\begin{aligned} S_{xx}^0 &= \frac{1}{(K+K^e)^2 - (\mu K + K^e)^2}\left[(K+K^e)\frac{1}{a^2}\frac{\partial^2\phi}{\partial\theta^2} - (\mu K + K^e)\frac{\partial^2\phi}{\partial x^2}\right], \\ S_{\theta\theta}^0 &= \frac{1}{(K+K^e)^2 - (\mu K + K^e)^2}\left[(K+K^e)\frac{\partial^2\phi}{\partial x^2} - (\mu K + K^e)\frac{1}{a^2}\frac{\partial^2\phi}{\partial\theta^2}\right], \\ S_{x\theta}^0 &= -\frac{2}{(1-\mu)K}\frac{1}{a}\frac{\partial^2\phi}{\partial x\partial\theta}, \end{aligned} \tag{13}$$

where $K^e = e_{31}^2 h/\varepsilon_{33}$. By substituting the relations into equation (12), it gives

$$\left[K + K^e - \frac{(\mu K + K^e)^2}{K + K^e}\right]\nabla_K^2 u_3 - \nabla^4\phi = 0. \tag{14}$$

Equations (10) and (14) are Donnell–Mushtari–Vlasov-type equations for piezoelectric cylindrical shells, which can be reduced to the equations for elastic problems when $K^e = 0$.

By introducing static transverse electromechanical coupling coefficient as

$$k_{31}^2 = \frac{1-\mu^2}{E}\frac{e_{31}^2}{\varepsilon_{33}}, \tag{15}$$

we can define generalized bending and membrane stiffness appearing in equations (10) and (14) as

$$D^* = \bar{\xi}_D D, \quad K^* = \bar{\xi}_K Eh, \quad \bar{\xi}_D = 1 + k_{31}^2, \quad \bar{\xi}_K = \frac{1}{1+k_{31}^2}\left(1 + \frac{2k_{31}^2}{1+\mu}\right), \tag{16}$$

where $\bar{\xi}_D$ and $\bar{\xi}_K$ are non-dimensional parameters related to the coupling coefficient k_{31}^2. Therefore, the Donnell–Mushtari–Vlasov equations for piezoelectric shells can be further written as

$$D^* \nabla^4 u_3 + \nabla_K^2 \phi + \rho h \ddot{u}_3 = 0, \quad K^* \nabla_K^2 u_3 - \nabla^4 \phi = 0. \tag{17}$$

It is seen that the Donnell–Mushtari–Vlasov equations for piezoelectric as well as elastic shells can be expressed in unified form. When the coupling coefficient k_{31}^2 is set to zero, the differential equations (17) reduce to those for elastic shells.

4. NATURAL FREQUENCIES AND MODES

To obtain the natural frequencies and modes, all external mechanical and electric excitations are set to be zero. It is assumed that the shell oscillates harmonically at a natural frequency, i.e.,

$$u_3(x, \theta, t) = U_3(x, \theta)\mathrm{e}^{\mathrm{j}\omega t}, \quad \phi(x, \theta, t) = \Phi(x, \theta)\mathrm{e}^{\mathrm{j}\omega t}. \tag{18}$$

Substituting these expressions into equation (17) gives

$$D^* \nabla^4 U_3 + \nabla_K^2 \Phi - \rho h \omega^2 U_3 = 0, \quad K^* \nabla_K^2 U_3 - \nabla^4 \Phi = 0. \tag{19}$$

By eliminating Φ from the above equations, one obtains

$$D^* \nabla^8 U_3 + K^* \nabla_K^4 U_3 - \rho h \omega^2 \nabla^4 U_3 = 0. \tag{20}$$

For vibration of circular cylindrical shells that are closed in the θ direction, the solution has the form

$$U_3(x, \theta) = U_{3m}(x) \cos n(\theta - \varphi), \tag{21}$$

where φ is an arbitrary angle according to the fact that there is no preferential direction of the mode shape in circumferential direction. Substituting expression (21) into equation (20) gives

$$D^* \left(\frac{n^2}{a^2} - \frac{\mathrm{d}^2}{\mathrm{d}x^2} \right)^4 U_{3m}(x) + \frac{K^*}{a^2} \frac{\mathrm{d}^4}{\mathrm{d}x^4} U_{3m}(x) - \rho h \omega^2 \left(\frac{n^2}{a^2} - \frac{\mathrm{d}^2}{\mathrm{d}x^2} \right)^2 U_{3m}(x) = 0. \tag{22}$$

This equation can be solved approximately by Galerkin's method [4, 5]. Following the treatment in reference [4] by assuming $U_{3m}(x)$ to be the mode shape of a transverse vibration beam with boundary conditions analogous to the considered shell, the approximate frequencies of the cylindrical shell can be obtained as

$$\omega_{mn}^2 = \frac{1}{\rho h} \left\{ \frac{K^* \lambda_m^4}{a^2 [(n/a)^2 + \lambda_m^2]^2} + D^* [(n/a)^2 + \lambda_m^2]^2 \right\}, \tag{23}$$

where λ_m are the roots of the analogous beam equation For elastic cylindrical shells, it has been verified that under the selection of the beam functions, the results (23) are exact solutions of equation (22) for the simply supported shell and approximate but with good accuracy for other boundary condition cases [4, 5]. These results are also suitable for piezoelectric shells due to the same type Donnell–Mushtari–Vlasov equations. Therefore, equation (23) provides an alternative approach for estimating natural frequencies and modes of piezoelectric cylindrical shells. Because of its simplicity and clarity, the expression can be used to investigate the influence of electromechanical coupling effect on the natural frequencies conveniently.

Defining non-dimensional parameters as

$$\bar{\lambda}_m = \lambda_m L, \qquad \bar{a} = \frac{a}{L}, \qquad \bar{h} = \frac{h}{a}, \tag{24}$$

equation (23) can be further written as

$$\omega_{mn} = \frac{1}{a}\sqrt{E/\rho}\,\bar{\Omega}_{mn}, \tag{25}$$

where

$$\bar{\Omega}_{mn} = \left\{ \frac{\bar{\xi}_K \bar{\lambda}_m^4}{[(n/\bar{a})^2 + \bar{\lambda}_m^2]^2} + \frac{\bar{\xi}_D \bar{h}^2 \bar{a}^4}{12(1-\mu^2)} [(n/\bar{a})^2 + \bar{\lambda}_m^2]^2 \right\}^{1/2} \tag{26}$$

is the non-dimensional frequency, and does not rely on the material modulus and density explicitly. When coupling-effect-related parameters $\bar{\xi}_D = \bar{\xi}_K = 1$, the above expression reduces to the case for elastic shells.

5. NUMERICAL STUDIES

To illustrate the validity of the approximate solution (23) or (26), its results are compared with the FEM solutions obtained from commercial finite element package ABAQUS. Since there is no piezoelectric shell elements in ABAQUS element library, solid elements are used to model the cylinder geometry. For simplicity of element discretization, eigenvalue analysis by FEM for axisymmetric vibration modes of the cylinder is performed to compare with the approximate results, by setting $n = 0$ in equation (23) or (26). Generally, the expression (23) or (26) for $n \neq 0$ can provide better approximations than that for $n = 0$. Therefore, this comparison is reasonable.

In FEM analysis, the cylinder is modeled as an axisymmetric structure utilizing 200 four-node axisymmetric bilinear elements. The comparisons of the results between FEM and the formula (23) are given in Figures 2 and 3, respectively, for elastic cylinder and piezoelectric cylinder. The geometry of the cylindrical shell is given as $L = 200$ mm,

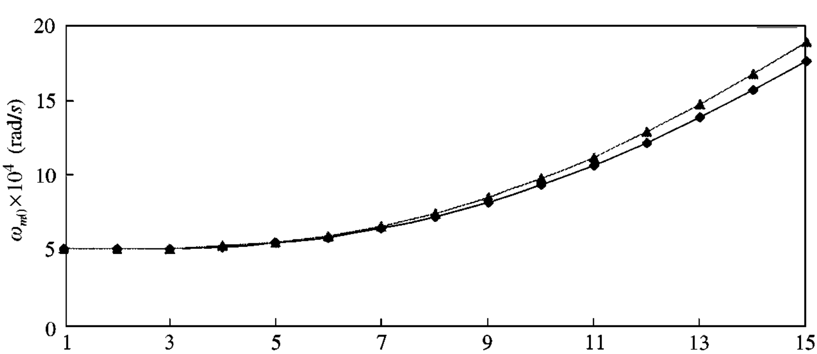

Figure 2. Comparison of approximate formula (23) with FEM solutions for free–free closed elastic cylindrical shell. —◆— FEM; ····▲···· Eq. (23).

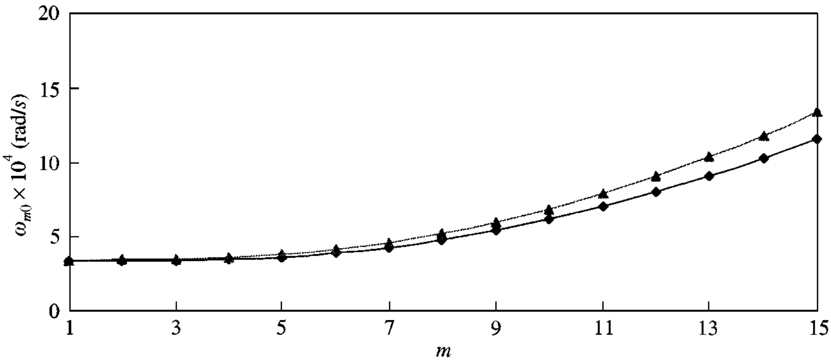

Figure 3. Comparison of approximate formula (23) with FEM solutions for free–free closed piezoelectric cylindrical shell. —◆— FEM; ---▲--- Eq. (23).

$a = 100$ mm and $h = 2$ mm. The material properties for the elastic and the piezoelectric cylindrical shells are given, respectively, as

$$E = 20{\cdot}6 \times 10^{10}\,\mathrm{N/m^2}, \quad \rho = 7{\cdot}85 \times 10^{-9}\,\mathrm{kg/m^3}, \quad \mu = 0{\cdot}3 \quad \text{for elastic shell,}$$

$$E = 8{\cdot}13 \times 10^{10}\,\mathrm{N/m^2}, \quad \rho = 7{\cdot}5 \times 10^{-9}\,\mathrm{kg/m^3}, \quad \mu = 0{\cdot}33,$$

$$e_{31} = 14{\cdot}9\,\mathrm{c/m^2}, \quad \varepsilon_{33} = 13{\cdot}05 \times 10^{-9}\,\mathrm{F/m} \quad \text{for piezoelectric shell.}$$

It is seen that the agreement of the approximate and FEM solutions for lower longitudinal vibration modes is satisfactory for both elastic and piezoelectric shells. When the number of the longitudinal vibration mode increases, the difference of the results between FEM and the approximate formula is also increased. The frequencies by the approximate method are higher than those obtained by FEM. It is partly due to the reason that the approximate method provides extra restrictions on the structures through some assumptions and therefore stiffens the structures. Another possible reason is that in numerical, such as FEM, eigenproblem analysis, lower order eigenvalues obtained are generally more accurate than higher order ones. As it is known, lower order vibration modes and frequencies are most interested in many engineering applications. Therefore, the results of the approximate formula given in this paper are shown to be satisfactory compared with the numerical solutions.

In this section, the non-dimensional frequency expression (26) is taken to investigate how the electromechanical coupling effect as well as geometry parameters influence the natural frequencies.

It can be observed from equation (16) that piezoelectric coupling coefficients contribute to the stiffness constants and so increase the natural frequencies. Figures 4–6 show the dependence of $\bar{\Omega}_{mn}$ on the coupling coefficient k_{31}^2 and roots $\bar{\lambda}_m$ for a clamped–clamped or free–free piezoelectric cylindrical shell. The parameters are $\mu = 0{\cdot}3$, $\bar{a} = 0{\cdot}5$, and $\bar{h} = 0{\cdot}02$. It is seen that with increasing wave number n, the influence of the coupling effect upon the natural frequencies also increases significantly. Therefore, for higher vibration modes, the influence of k_{31}^2 cannot be neglected. Figures 7 and 8 show that the influences of the geometry parameters $\bar{a}$ and $\bar{h}$ on $\bar{\Omega}_{mn}$ are different. $\bar{a}$ only influences the frequencies of lower order vibration modes while the influence of $\bar{h}$ is dramatic as n increases.

For the piezoelectric cylindrical ultrasonic motors, the vibration mode of $n = 1$ and $m = 1$ is most interesting. Figures 9 and 10 show the frequency variations against the

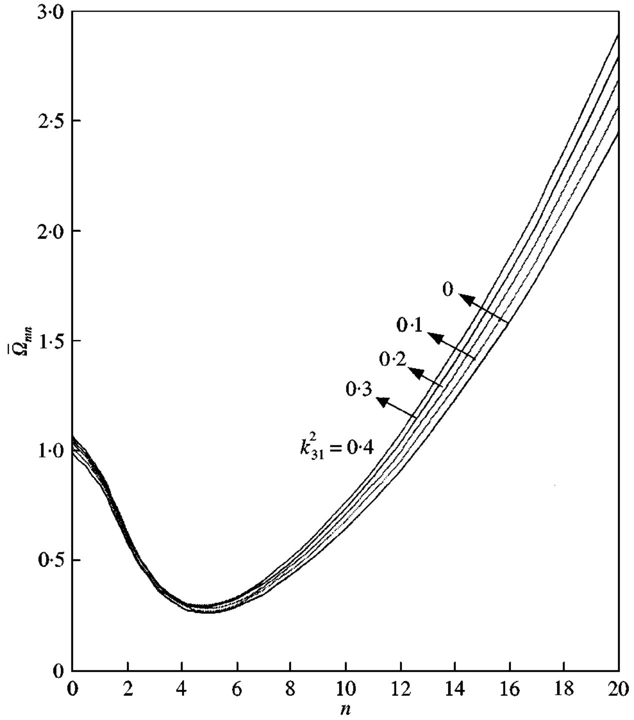

Figure 4. Variations of $\bar{\Omega}_{mn}$ due to changes of k_{31}^2, for $\bar{\lambda}_1 = 4·73$, $\mu = 0·3$, $a/L = 0·5$, $h/a = 0·02$.

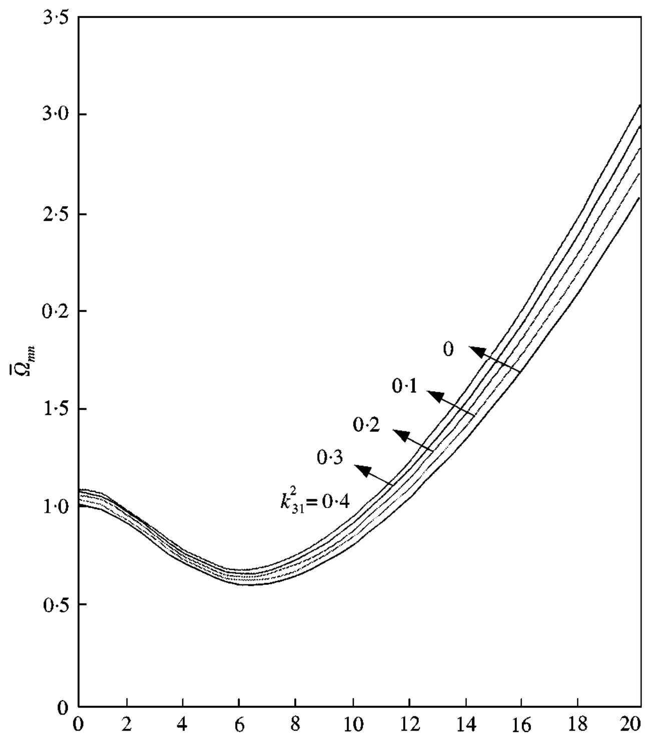

Figure 5. Variations of $\bar{\Omega}_{mn}$ due to changes of k_{31}^2, for $\bar{\lambda}_2 = 7·85$, $\mu = 0·3$, $a/L = 0·5$, $h/a = 0·02$.

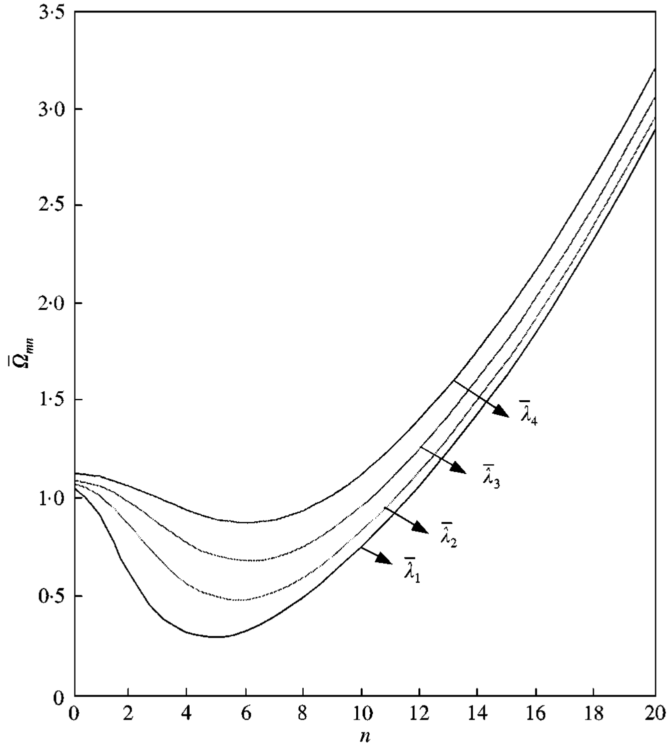

Figure 6. Variety of $\bar{\Omega}_{mn}$ with $\bar{\lambda}_m$ for free–free or clamped–clamped boundary conditions. $\mu = 0{\cdot}3$, $k_{31}^2 = 0{\cdot}4$, $a/L = 0{\cdot}5$, $h/a = 0{\cdot}02$.

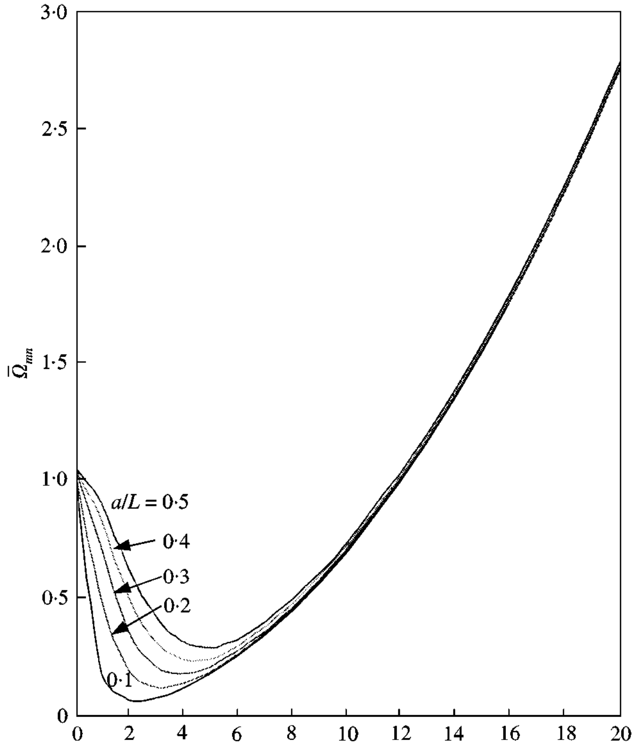

Figure 7. Variations of $\bar{\Omega}_{mn}$ due to changes of a/L, for $\bar{\lambda}_1 = 4{\cdot}73$, $k_{31}^2 = 0{\cdot}3$, $\mu = 0{\cdot}3$, $h/a = 0{\cdot}02$.

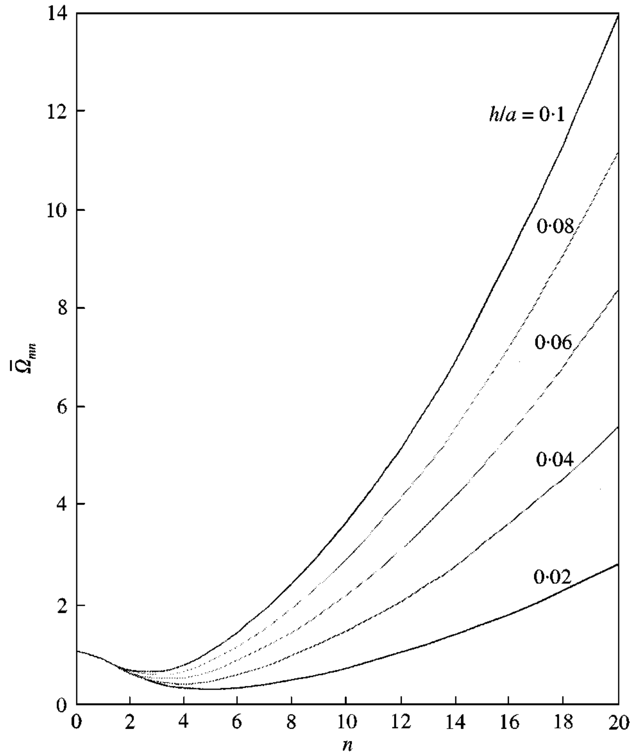

Figure 8. Variations of $\bar{\Omega}_{mn}$ due to changes of h/a, for $\bar{\lambda}_1 = 4{\cdot}73$, $k_{31}^2 = 0{\cdot}3$, $\mu = 0{\cdot}3$, $a/L = 0{\cdot}5$.

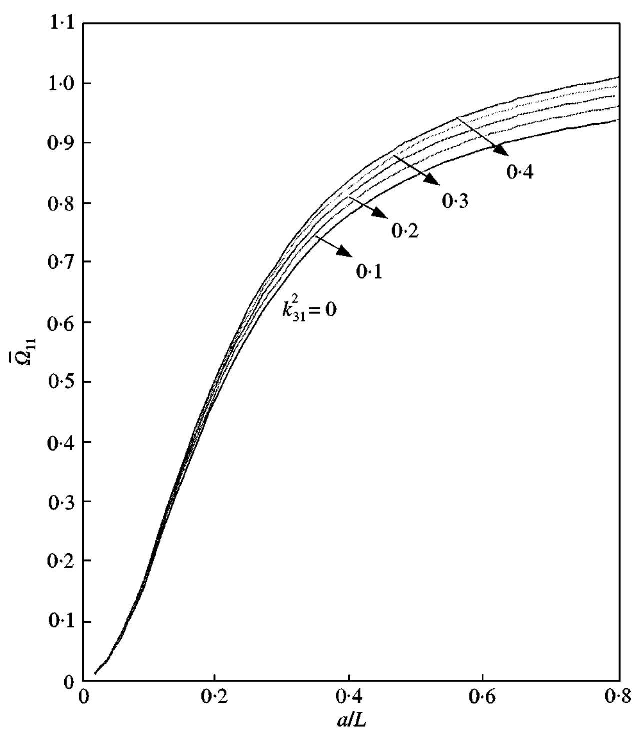

Figure 9. Variety of $\bar{\Omega}_{11}$ with a/L, and k_{31}^2. $\mu = 0{\cdot}3$, $n = 1$, $h/a = 0{\cdot}02$, $\bar{\lambda}_1 = 4{\cdot}73$.

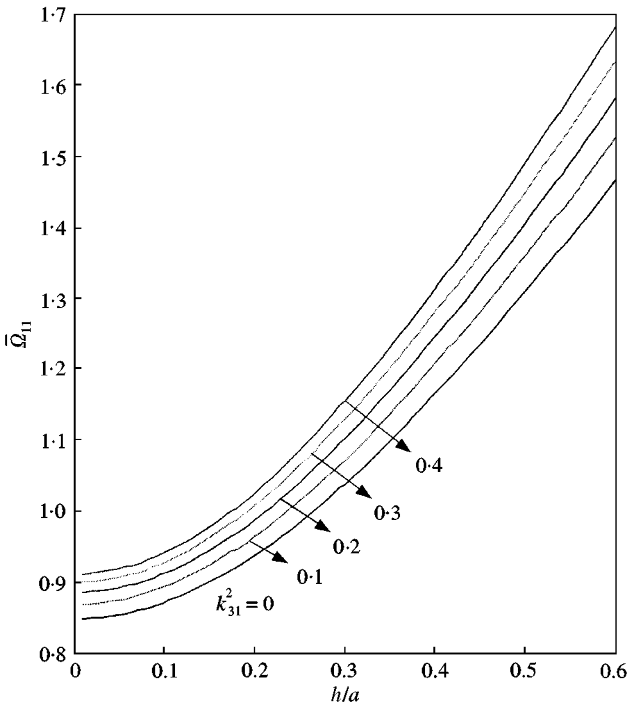

Figure 10. Variety of $\bar{\Omega}_{11}$ with h/a, and k_{31}^2. $\mu = 0{\cdot}3$, $n = 1$, $a/L = 0{\cdot}02$, $\bar{\lambda}_1 = 4{\cdot}73$.

parameters $\bar{a}$ and $\bar{h}$, respectively, due to the changes of the coupling coefficient k_{31}^2. It is noted that the influence of k_{31}^2 on the frequencies is more sensitive with the parameter $\bar{h}$ than $\bar{a}$.

Since the two ends of the piezoelectric cylinder are attached with two steel stators in the cylindrical micromotor structures, the analogous beam can be modelled as a free beam attached with the mass of the stator m_s at each end. The eigenfrequency equation for this beam can be obtained as

$$1 - \cos\bar{\lambda}\cosh\bar{\lambda} + 2\bar{m}^2\bar{\lambda}^2\sin\bar{\lambda}\sinh\bar{\lambda} - 2\bar{m}\bar{\lambda}(\cos\bar{\lambda}\sinh\bar{\lambda} - \sin\bar{\lambda}\cosh\bar{\lambda}) = 0, \qquad (27)$$

where $\bar{m} = m_r/m_c$ is the non-dimensional mass ratio, and m_c the mass of the piezoelectric cylinder. From the equation, the mass ratio related eigenvalues, $\bar{\lambda}_m$, for different modes can be obtained. Figure 11 shows the dependence of the first order bending frequency $\bar{\Omega}_{11}$ on the mass ratio $\bar{m}$. It is observed that the influence of $\bar{m}$ on $\bar{\Omega}_{11}$ tends to be less significant when $\bar{m}$ is larger than a certain amount for different geometry parameters.

6. SUMMARY

Donnell–Mushtari–Vlasov-type equations for elastic shells were extended to the case for piezoelectric shells. By introducing the so-called generalized bending and membrane stiffness, the Donnell–Mushtari–Vlasov equations for piezoelectric as well as elastic shells can be expressed in unified form. Following the treatment by Soedel [4], an approximate formula for estimating transverse vibrating frequencies of closed circular cylindrical shells

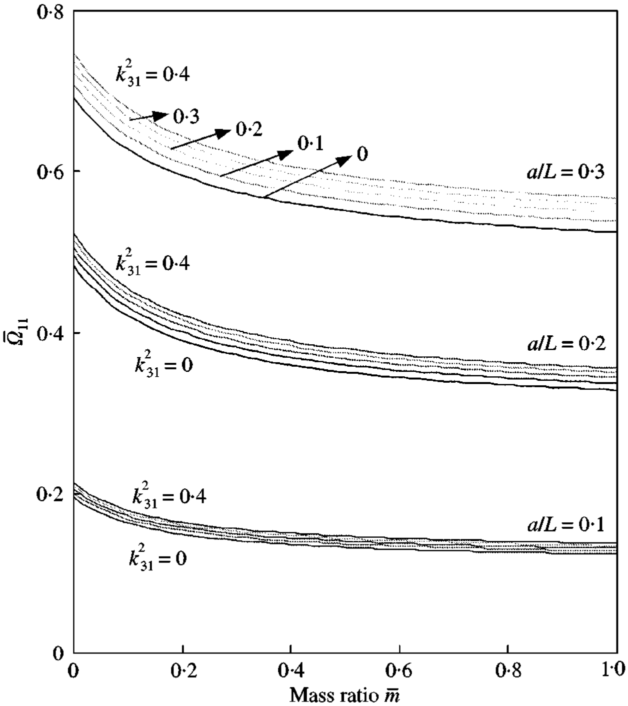

Figure 11. Variety of $\bar{\Omega}_{11}$ with $\bar{m}$, a/L, and k_{31}^2. $\mu = 0·3$, $n = 1$, $h/a = 0·2$, $\bar{\lambda}_1 = 4·73$.

was obtained. Based on the formula, the influences of electromechanical coupling effect as well as geometry parameters on the natural frequencies of cylindrical piezoelectric shells with various boundary conditions were investigated.

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