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IMPROVEMENT OF THE STRUCTURE DAMPING PERFORMANCE BY INTERCONNECTION

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The procedure of improving the structures damping performance was proposed by interconnecting two structures with a connecting member consisting of a spring and a damper. The modal equations of the first mode of both framed structures interconnected were shown using equations for the motion of a two-degrees-of-freedom (2d.o.f.) system, with two masses and three springs. The tuning method of a connecting element in the above 2d.o.f. system, by which the damping performance of the two systems in the 2d.o.f. system was equalized and maximized, was proposed, and the approximate tuning method of the connecting member, by which the damping performance of the first mode of the two framed structures interconnected was equalized and the maximized, was proposed using the tuning method of a connecting element in the 2d.o.f. system. In numerical investigations, the usefulness of the approximate tuning method and the effectiveness of the interconnecting member were shown.

1. INTRODUCTION

It is well known in general use of the flexible structures that the serious problems due to free vibration over a long period of time after action of the external forces, sometimes occur because of the slight damping performance. Therefore, it was suggested that a free vibration should be damped quickly. Many improvement procedures of the structure damping performance have been contrived in designing many kinds of structures. In this paper, an optimum improvement procedure of the damping performance is proposed by interconnecting the two structures with a connecting member, which consists of a spring and a damper. Many examples, in which passive and active members are used as a connecting member, are found to control vibrations passively and actively in buildings [1]. However, systematical investigation and discussion for the improvement of the structure damping performance by applying the interconnecting member, has not been reported in literature. In this paper, we proposed an approximate tuning method of a connecting member by which the damping performance of the first vibration mode of the two structures interconnected by a connecting member is equalized and maximized.

In the analysis, the structures were treated as a discrete system and it was supposed that the two structures were connected to each other with one connecting member. The approximate tuning method of a connecting member was deduced by the following procedure: (1) modal equations were derived from the equations for motion of two structures interconnected, using the modal matrices of each structure; (2) modal equations for the first vibration mode have proven to be equivalent to equations for the motion of a two-degrees-of-freedom (2d.o.f.) system with two masses and three springs; (3) the tuning condition of a connecting element of a 2d.o.f. system, from which two modal damping ratios of the above 2d.o.f. system were equalized to each other and maximized, was given when two natural circular frequencies are equal to each other and two modal damping ratios are equal; and (4) the approximate tuning method of a connecting member, by which the modal damping ratios of the first vibration mode of both framed structures interconnected are equalized to each other and maximized, was derived from the above tuning condition of the connecting element of the 2d.o.f. system. The tuning method proposed here is an approximate method, because the modal damping ratio of only the first vibration mode of each framed structure interconnected, is maximized by this method. Finally, the applicability of an approximate tuning method of the connecting member, and the usefulness of the connecting member, were shown using numerical investigations for two columns and two framed structures.

2. EQUATIONS OF MOTION AND MODAL EQUATIONS OF TWO STRUCTURES INTERCONNECTED

In this study, space-framed structures shown in Figure 1 were chosen. It was supposed that the natural circular frequency of the first mode of “Structure 1” in Figure 1 is larger than the natural circular frequency of the first mode of “Structure 2”. “Structure 1” is a Md.o.f. system and “Structure 2” is a Nd.o.f. system. The equations for the motion of interconnected framed structures shown in Figure 1 and modal equations of those structures in the modal co-ordinates of each structure were shown in the following.

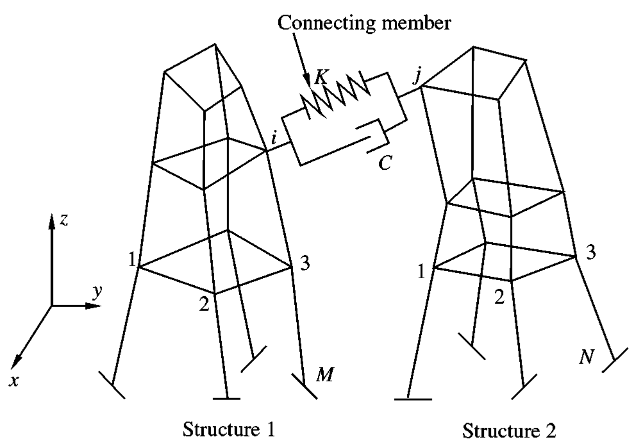

Figure 1. Interconnected structures.

2.1. EQUATIONS OF MOTION

Suppose that the joint i of "Structure 1" connected to the joint j of "Structure 2" with a connecting member and that both of the structures were undamped systems. The equations for the motion of interconnected framed structures shown in Figure 1 are expressed as

Equation for motion of "*Structure* 1"

$$\mathbf{M}_1\ddot{\mathbf{d}}_1 + \mathbf{K}_1\mathbf{d}_1 + K(\mathbf{H}_1\mathbf{d}_1 - \mathbf{H}_2\mathbf{d}_2) + C(\mathbf{H}_1\dot{\mathbf{d}}_1 - \mathbf{H}_2\dot{\mathbf{d}}_2) = \mathbf{0}. \tag{1}$$

Equation for motion of "*Structure* 2"

$$\mathbf{M}_2\ddot{\mathbf{d}}_2 + \mathbf{K}_2\mathbf{d}_2 + K(\mathbf{H}_3\mathbf{d}_2 - \mathbf{H}_4\mathbf{d}_1) + C(\mathbf{H}_3\dot{\mathbf{d}}_2 - \mathbf{H}_4\dot{\mathbf{d}}_1) = \mathbf{0}, \tag{2}$$

where $\mathbf{M}_1$, $\mathbf{M}_2$ are the mass matrices of "Structure 1" and "Structure 2", $\mathbf{K}_1$, $\mathbf{K}_2$ the stiffness matrices of both structures, $\mathbf{d}_1$, $\mathbf{d}_2$ the displacement vectors of both structures, $\mathbf{H}_1$, $\mathbf{H}_2$, $\mathbf{H}_3$, $\mathbf{H}_4$ are the matrices which indicate the position of connecting joints of both structures. $\mathbf{H}_1$ consists of $M \times M$ elements, $\mathbf{H}_2$ is $M \times N$ elements, $\mathbf{H}_3$ is $N \times N$ elements, $\mathbf{H}_4$ is $N \times M$ elements and K, C the spring constant and damping coefficient of connecting member.

2.2. MODAL EQUATIONS

In deriving the modal equations, it is considered that the rigidity of a connecting member is small, therefore, the first vibration mode shapes of both structures interconnected are similar to that of each structure, which is not interconnected.

Suppose that the natural circular frequency and eigenvector of the pth mode of "Structure 1" are denoted by ω_{1p} and $\boldsymbol{\phi}_{1p}$, respectively, and those of the qth mode of "Structure 2" are denoted by ω_{2q} and $\boldsymbol{\phi}_{2q}$ respectively. The orthogonal condition of the eigenvectors and the relationship between the natural circular frequencies and the eigenvectors for both structures are expressed as follows:

$$\phi_{1r}^{\mathrm{T}}\mathbf{M}_1\,\phi_{1p} = \begin{cases} M_{1r}, \ldots r = p, \\ 0, \ldots r \neq p \end{cases} \tag{3}$$

and

$$-\,\omega_{1p}^2\mathbf{M}_1\boldsymbol{\phi}_{1p} + \mathbf{K}_1\boldsymbol{\phi}_{1p} = \mathbf{0} \tag{4}$$

for "Structure 1",

$$\phi_{2s}^{\mathrm{T}}\mathbf{M}_2\,\phi_{2q} = \begin{cases} M_{2s}, \ldots s = q, \\ 0, \ldots s \neq q \end{cases} \tag{5}$$

and

$$-\,\omega_{2q}^2\mathbf{M}_2\boldsymbol{\phi}_{2q} + \mathbf{K}_2\boldsymbol{\phi}_{2q} = \mathbf{0} \tag{6}$$

for "Structure 22".

Displacement vectors in the vibrating state of the interconnected structures under the assumption mentioned above can be approximated by eigenmatrices of both structures as follows:

$$\mathbf{d}_1 = \boldsymbol{\Phi}_1\boldsymbol{\rho}_1, \qquad \mathbf{d}_2 = \boldsymbol{\Phi}_2\boldsymbol{\rho}_2. \tag{7}$$

Here, $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ are modal matrices of "Structure 1" and "Structure 2" when both structures are not interconnected and those are expressed as follows:

$$\mathbf{\Phi}_1 = \{\boldsymbol{\phi}_{11}, \boldsymbol{\phi}_{12}, \dots, \boldsymbol{\phi}_{1p}, \dots, \boldsymbol{\phi}_{1M}\},$$
$$\mathbf{\Phi}_2 = \{\boldsymbol{\phi}_{21}, \boldsymbol{\phi}_{22}, \dots, \boldsymbol{\phi}_{2q}, \dots, \boldsymbol{\phi}_{2N}\}. \tag{8}$$

$\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ are unknown vectors of the time of "Structure 1" and "Structure 2", and those are expressed as follows:

$$\boldsymbol{\rho}_1 = \{\rho_{11}(t), \rho_{12}(t), \dots, \rho_{1p}(t), \dots, \rho_{1M}(t)\}^{\mathrm{T}},$$
$$\boldsymbol{\rho}_2 = \{\rho_{21}(t), \rho_{22}(t), \dots, \rho_{2q}(t), \dots, \rho_{2N}(t)\}^{\mathrm{T}}. \tag{9}$$

Substitution of equation (7) into equations (1) and (2) and their rearrangement according to equations (3)–(6), give the following modal equations:

$$M_{1r}\ddot{\rho}_{1r} + \omega_{1r}^2 M_{1r}\rho_{1r} + KZ_{1rij} + C\dot{Z}_{1rij} = 0, \tag{10}$$

$$\begin{aligned} Z_{1rij} = (lU_{1ir} + mV_{1ir} + nW_{1ir})\Bigg\{ & l\left(\sum_{p=1}^{M} U_{1ip}\,\rho_{1p} - \sum_{q=1}^{N} U_{2jq}\,\rho_{2q}\right) \\ & + m\left(\sum_{p=1}^{M} V_{1ip}\,\rho_{1p} - \sum_{q=1}^{N} V_{2jq}\,\rho_{2q}\right) \\ & + n\left(\sum_{p=1}^{M} W_{1ip}\,\rho_{1p} - \sum_{q=1}^{N} W_{2jq}\,\rho_{2q}\right)\Bigg\}, \quad r = 1, 2, \dots, M, \end{aligned} \tag{11}$$

$$M_{2s}\ddot{\rho}_{2s} + \omega_{2s}^2 M_{2s}\,\rho_{2s} + KZ_{2sij} + C\dot{Z}_{2sij} = 0, \tag{12}$$

$$\begin{aligned} Z_{2sij} = (lU_{2js} + mV_{2js} + nW_{2js})\Bigg\{ & l\left(\sum_{q=1}^{N} U_{2jq}\,\rho_{2q} - \sum_{p=1}^{M} U_{1ip}\,\rho_{1p}\right) \\ & + m\left(\sum_{q=1}^{N} V_{2jq}\,\rho_{2q} - \sum_{p=1}^{M} V_{1ip}\,\rho_{1p}\right) \\ & + n\left(\sum_{q=1}^{N} W_{2jq}\,\rho_{2q} - \sum_{p=1}^{M} W_{1ip}\,\rho_{1p}\right)\Bigg\}, \quad s = 1, 2, \dots, N. \end{aligned} \tag{13}$$

In equations (11) and (13), U_{1ir}, V_{1ir} and W_{1ir} are the displacements in the directions of x-, y- and z-axis at the joint i in the rth vibration mode of "Structure 1". U_{2js}, V_{2js} and W_{2js} are the displacements at the joint j in the sth vibration mode of "Structure 2". l, m and n are direction cosines of the connecting member ($i \sim j$) by which the joint i and the joint j are connected.

3. APPROXIMATE TUNING METHOD OF THE CONNECTING MEMBER

Improvement of the damping performance of the lower order vibration modes, especially that of the first mode, is eagerly desired for actual structures. This is because the vibrating displacements of the high order vibration modes are small and converge rapidly, more than those of the low order modes. In this paper, the improvement of the structure damping performance is carried out by increasing the damping performance of the first vibration mode of the structures. Consequently, the tuning method of the connecting member, which is proposed in this study, is the maximizing method of the damping performance of the first vibration mode of the structures. An approximate tuning method of the connecting member was derived based on the following assumptions: (1) the natural circular frequencies of each structure, which is not interconnected, are not very close to each other; (2) the displacements at a connecting joint in the first mode of each structure are considerably large compared with those of the other modes of the same structure; and (3) the connecting member is attached to the position near the loop of the first vibration mode. Under the above assumptions, the displacements at the connecting joint in the first vibration mode of each structure occupy most of the total displacements in a freely vibrating state of each structure, and the occupying ratio of the displacements of high order modes is low. Therefore, the following relationships are satisfied in equations (11) and (13):

$$U_{1i1} \gg U_{1ip}, \quad U_{2j1} \gg U_{2jq}, \quad V_{1i1} \gg V_{1ip}, \quad V_{2j1} \gg V_{2jq}, \quad W_{1i1} \gg W_{1ip}, \quad W_{2j1} \gg W_{2jq}.$$

These relationships enable us to pick out the terms concerning only the first vibration mode from equations (10)–(13).

In the following, the tuning method of the spring constant and the damping coefficient of the connecting member, by which two modal damping ratios in the coupling state of the first vibration mode of both structures become equal to each other and are given a maximum value, is shown.

3.1. MODAL EQUATIONS AND THE 2d.o.f. SYSTEM

When the terms concerning only the first vibration mode of each structure in equations (10)–(13), are adopted for the reason mentioned above, the modal equations are approximately given as follows:

$$M_{11}\ddot{\rho}_{11} + \omega_{11}^2 M_{11}\rho_{11} + KZ_{11ij} + C\dot{Z}_{11ij} = 0, \tag{14}$$

$$\begin{aligned} Z_{11ij} = (lU_{1i1} + mV_{1i1} + nW_{1i1})\{(lU_{1i1} + mV_{1i1} + nW_{1i1})\,\rho_{11} \\ - (lU_{2j1} + mV_{2j1} + nW_{2j1})\rho_{21}\}, \end{aligned} \tag{15}$$

$$M_{21}\ddot{\rho}_{21} + \omega_{21}^2 M_{21}\rho_{21} + KZ_{21ij} + C\dot{Z}_{21ij} = 0, \tag{16}$$

$$\begin{aligned} Z_{21ij} = (lU_{2j1} + mV_{2j1} + nW_{2j1})\{(lU_{2j1} + mV_{2j1} + nW_{2j1})\,\rho_{21} \\ - (lU_{1i1} + mV_{1i1} + nW_{1i1})\rho_{11})\}. \end{aligned} \tag{17}$$

The above modal equations can be rewritten as

$$M_{11}\ddot{\rho}_{11} + \omega_{11}^2 M_{11}\rho_{11} + \alpha C(\dot{\rho}_{11} - \beta\dot{\rho}_{21}) + \alpha K(\rho_{11} - \beta\rho_{21}) = 0, \tag{18}$$

$$\frac{M_{21}}{\beta^2}(\beta\ddot{\rho}_{21}) + \frac{\omega_{21}^2 M_{21}}{\beta^2}(\beta\rho_{21}) + \alpha C(\beta\dot{\rho}_{21} - \dot{\rho}_{11}) + \alpha K(\beta\rho_{21} - \rho_{11}) = 0, \tag{19}$$

in which

$$\alpha = D_{1i1}^2, \quad \beta = \frac{D_{2j1}}{D_{1i1}}, \tag{20}$$

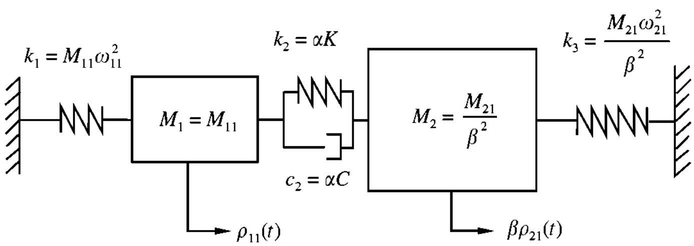

Figure 2. Two-degrees-of-freedom system with two masses and three springs.

where

$$D_{1i1} = lU_{1i1} + mV_{1i1} + nW_{1i1}, \quad D_{2j1} = lU_{2j1} + mV_{2i1} + nW_{2j1}. \tag{21}$$

These equations correspond to those of the motion of a 2d.o.f. system shown in Figure 2. Damping performance of the first vibration mode of "Structure 1" and "Structure 2" is improved by maximizing the damping performance of $\rho_{11}(t)$ and $\rho_{21}(t)$ in Figure 2, as is obvious from equation (22).

$$\mathbf{d}_1 = \phi_{11}\, \rho_{11}(t), \qquad \mathbf{d}_2 = \phi_{21}\, \rho_{21}(t). \tag{22}$$

In the next section, the estimation method of a spring constant and a damping coefficient of the connecting element in the 2d.o.f. system shown in Figure 2 is described, and then the tuning method of the connecting member of the framed structure interconnected, by which the damping performance of the first mode of each structure is maximized, is explained.

3.2. TUNING METHOD OF A CONNECTING ELEMENT IN THE 2D.O.F. SYSTEM

When the masses, spring constants, damping coefficients and displacements in Figure 2 are replaced as follows:

$$\begin{aligned} &M_1 = M_{11}, \qquad M_2 = \frac{M_{21}}{\beta^2}, \qquad k_1 = \omega_{11}^2 M_{11}, \qquad k_2 = \alpha K, \\ &c_2 = \alpha C, \qquad k_3 = \frac{\omega_{21}^2 M_{21}}{\beta^2}, \qquad x_1 = \rho_{11}, \qquad x_2 = \beta\rho_{21}, \end{aligned} \tag{23}$$

modal equations (18) and (19) are rewritten as

$$M_1\ddot{x}_1 + k_1x_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) = 0, \tag{24}$$

$$M_2\ddot{x}_2 + k_3x_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0. \tag{25}$$

The 2d.o.f. system described in equations (24) and (25) has two natural circular frequencies, ω_1 and ω_2, and two modal damping ratios, ξ_1 and ξ_2. Natural circular frequencies and modal damping ratios of the above 2d.o.f. system with variation in the spring constant k_2 and the damping coefficient c_2 of the connecting element have the same behavior as those shown in Figures 3 and 4 respectively. Those figures show the behaviors of modal damping ratios and natural circular frequencies of the 2d.o.f. system with constant masses, $M_1 = 6{\cdot}268$ and $M_2 = 6{\cdot}187$ kg, constant spring constants $k_1 = 1{\cdot}132$ and $k_3 = 8{\cdot}954$ N/cm,

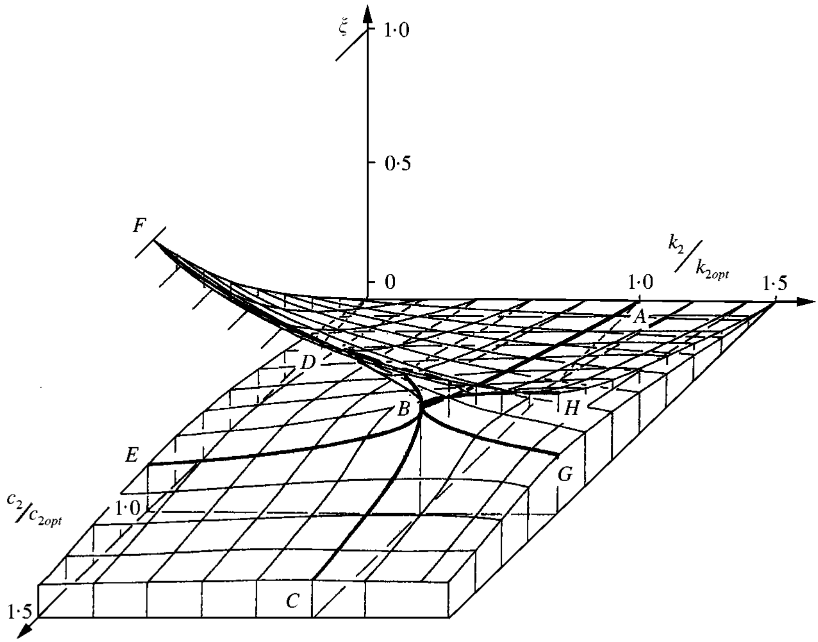

Figure 3. Behavior of modal damping ratios with variation in connecting spring constant and damping coefficient in 2d.o.f. system.

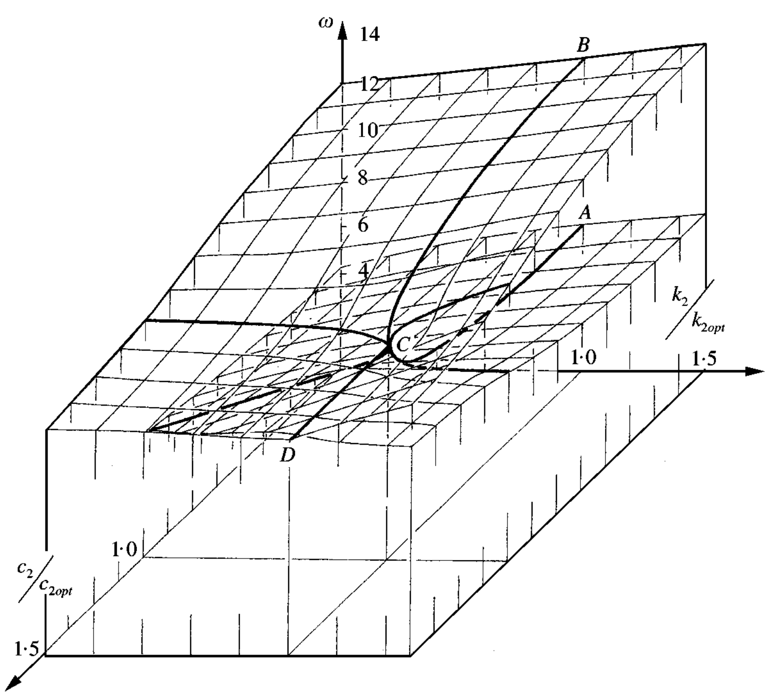

Figure 4. Behavior of natural circular frequency with variation in connecting spring constant and damping coefficient in 2d.o.f. system.

varying spring constant k_2 and damping coefficient c_2. Figure 3 indicates the behavior of the modal damping ratio, and Figure 4 indicates that of the natural circular frequency. It was revealed from those behaviors that the 2d.o.f. system with two masses and three springs has the following characteristics.

(1) When the spring constant k_2 takes a certain value k_{2opt} ($k_2/k_{2opt} = 1{\cdot}0$) and the damping coefficient c_2 takes a value less than a certain value c_{2opt} ($c_2/c_{2opt} \leqslant 1{\cdot}0$), two modal damping ratios, ξ_1 and ξ_2, are equal to each other and they increase with an increase of c_2 (corresponding to line AB in Figure 3). When $k_2/k_{2opt} = 1{\cdot}0$ and $c_2/c_{2opt} = 1{\cdot}0$, $\xi_1 = \xi_2 = \xi_{max}$. When c_2 exceeds a certain damping coefficient c_{2opt} ($c_2/c_{2opt} \geqslant 1{\cdot}0$), ξ_1 for the first mode decreases from ξ_{max} and ξ_2 for the second mode increases (corresponding to lines BC and BD in Figure 3).
(2) When the damping coefficient c_2 takes a certain value c_{2opt} ($c_2/c_{2opt} = 1{\cdot}0$) and the spring constant k_2 takes a value less than a certain value k_{2opt} ($k_2/k_{2opt} < 1{\cdot}0$), $\xi_1 < \xi_{max}$ (corresponds to line EB in Figure 3) and $\xi_2 > \xi_{max}$ (corresponds to line FB in Figure 3), and when c_2 takes a certain damping coefficient c_{2opt} ($c_2/c_{2opt} = 1{\cdot}0$) and k_2 takes a value more than a certain spring constant k_{2opt} ($k_2/k_{2opt} > 1{\cdot}0$), $\xi_1 < \xi_{max}$ (corresponds to line BH in Figure 3) and $\xi_2 > \xi_{max}$ (corresponds to line BG in Figure 3). Consequently, the damping performance of the 2d.o.f. system has a maximum when $k_2 = k_{2opt}$ and $c_2 = c_{2opt}$.
(3) When k_2 takes a certain spring constant k_{2opt} ($k_2/k_{2opt} = 1{\cdot}0$) and c_2 takes a value less than a certain damping coefficient c_{2opt} ($c_2/c_{2opt} \leqslant 1{\cdot}0$), two natural circular frequencies ω_1 and ω_2 have different values respectively (corresponding to lines AC and BC in Figure 4). When c_2 exceeds a certain damping coefficient c_{2opt} ($c_2/c_{2opt} \geqslant 1{\cdot}0$), ω_1 and ω_2 agree precisely (correspond to line CD in Figure 4).
(4) In other words, the damping performance of the 2d.o.f. system is at maximum when $\xi_1 = \xi_2$ and $\omega_1 = \omega_2$. The above behaviors of natural circular frequency and modal damping ratio are similar to those of the 2d.o.f. system with two masses and two springs, which is the 1d.o.f. system with a tuned mass damper (TMD) [2, 3].

In the following, an optimum spring constant k_{2opt} and an optimum damping coefficient c_{2opt} are estimated from the condition which $\xi_1 = \xi_2$ and $\omega_1 = \omega_2$.
When the solutions of equations (24) and (25) are given by

$$x_1 = \bar{x}_1 e^{\lambda t}, \qquad x_2 = \bar{x}_2 e^{\lambda t} \tag{26}$$

the characteristic equation is expressed as

$$\begin{vmatrix} M_1\lambda^2 + c_2\lambda + (k_1 + k_2) & -(c_2\lambda + k_2) \\ -(c_2\lambda + k_2) & M_2\lambda^2 + c_2\lambda + (k_2 + k_3) \end{vmatrix} = 0, \tag{27}$$

in which λ is the characteristic exponent.

Equation (27) is rewritten as

$$\lambda^4 + 2hv_1(1+\mu)\lambda^3 + v_1^2\{f_2^2(1+\mu) + (1+f_3^2)\}\lambda^2 + 2hv_1^3(1+\mu f_3^2)\lambda$$
$$+ v_1^4\{f_2^2(1+\mu f_3^2) + f_3^2\} = 0, \tag{28}$$

where

$$v_1^2 = \frac{k_1}{M_1}, \qquad v_2^2 = \frac{k_2}{M_2}, \qquad v_3^2 = \frac{k_3}{M_2}, \qquad \mu = \frac{M_2}{M_1}, \qquad h = \frac{c_2}{2M_2 v_1}, \qquad f_2 = \frac{v_2}{v_1}, \qquad f_3 = \frac{v_3}{v_1}. \tag{29}$$

The solutions of equation (28) are expressed as the following conjugate complex form:

$$\lambda_n = a_n \pm \mathrm{i} b_n \quad (n = 1, 2). \tag{30}$$

In this case, undamped natural circular frequencies and modal damping ratios are given as

$$\omega_n = \sqrt{a_n^2 + b_n^2}, \quad \xi_n = \frac{-a_n}{\sqrt{a_n^2 + b_n^2}} \quad (n = 1, 2). \tag{31}$$

The condition of the maximum damping performance, i.e., $\xi_1 = \xi_2$ and $\omega_1 = \omega_2$, corresponds to the condition $\lambda_1 = \lambda_2$ as is obvious from equation (31). Accordingly, k_{2opt} and c_{2opt} on the maximum damping performance, are given from the condition that equation (28) has a multiple root.

When the multiple root is expressed as

$$\lambda = a \pm \mathrm{i} b, \tag{32}$$

the characteristic equation with a multiple root (32) is expressed as follows:

$$\{\lambda - (a + \mathrm{i}b)\}^2 \{\lambda - (a - \mathrm{i}b)\}^2 = 0. \tag{33}$$

The above equation can be rewritten as

$$\lambda^4 - 4a\lambda^3 + (6a^2 + 2b^2)\lambda^2 - 4a(a^2 + b^2)\lambda + (a^2 + b^2)^2 = 0. \tag{34}$$

When the characteristic equation (28) has a multiple root, coefficients a and b of root (30) are obtained by equalizing the coefficients of the term with an equal exponent in equations (28) and (34) as follows:

$$\begin{aligned} -4a &= 2hv_1(1 + \mu), \\ 6a^2 + 2b^2 &= v_1^2\{f_2^2(1 + \mu) + (1 + f_3^2)\}, \\ -4a(a^2 + b^2) &= 2hv_1^3(1 + \mu f_3^2), \\ (a^2 + b^2)^2 &= v_1^4\{f_2^2(1 + \mu f_3^2) + f_3^2\}. \end{aligned} \tag{35}$$

When the coefficients a and b are eliminated in the above equations, the following relationships are given:

$$f_2^2 = \frac{(1 - f_3^2)(1 - \mu^2 f_3^2)}{(1 + \mu)^2(1 + \mu f_3^2)}, \tag{36}$$

$$h^2 = \frac{\mu(1 - f_3^2)^2}{(1 + \mu)^3(1 + \mu f_3^2)} \tag{37}$$

and a modal damping ratio ξ and a natural circular frequency ω are obtained from equation (31) as

$$\xi = \frac{(1 - f_3^2)\sqrt{\mu}}{2(1 + \mu f_3^2)}, \quad \omega = \sqrt{\frac{1 + \mu f_3^2}{1 + \mu}}. \tag{38}$$

Equations (36) and (37) are conditions which maximize the damping performance of the 2d.o.f. system shown in Figure 2. When the spring constants k_1 and k_3 of the two vibration systems in the 2d.o.f. system and masses M_1 and M_2 of them are given, a spring constant $k_2(= k_{2opt})$ and a damping coefficient c_2 $(= c_{2opt})$ of the connecting element in the 2d.o.f. system with the maximum damping performance are obtained from the following procedures:

(1) v_1^2 and v_3^2 are calculated from the first and third equations of equation (29) with known quantities k_1, k_3, M_1 and M_2, and then, f_3 is estimated from the seventh equation of equation (29) with v_1^2 and v_3^2. The modal damping ratio ξ is estimated from the first equation of equation (38) using a mass ratio μ given by the fourth equation of equation (29).
(2) Optimum spring constant k_2 $(= k_{2opt})$ is given from the first, second and sixth equations of equations (29) and (36) as the following expression with two known quantities, μ and f_3:

$$k_2\ (= k_{2opt}) = \frac{(1 - f_3^2)(1 - \mu^2 f_3^2)}{(1 + \mu)^2(1 + \mu f_3^2)}\mu k_1. \tag{39}$$

(3) Optimum damping coefficient c_2 $(= c_{2opt})$ is given from the first and fifth equations of equations (29) and (37) as the following expression with two known quantities μ and f_3:

$$c_2\ (= c_{2opt}) = \frac{2\mu(1 - f_3^2)\sqrt{M_2 k_1}}{(1 + \mu)\sqrt{(1 + \mu)(1 + \mu f_3^2)}}. \tag{40}$$

3.3. TUNING PROCEDURE OF CONNECTING MEMBER

The tuning procedure of the connecting member, by which the modal damping ratios of the first mode of both framed structures, which are interconnected, are equalized to each other and then maximized, is explained using the above-mentioned tuning procedure of the connecting element in the 2d.o.f. system with two masses and three springs.

(1) When the material and geometrical constants and boundary conditions of both of the structures which are interconnected ("Structures 1 and 2") are given, the mass matrices, $\mathbf{M}_1$ and $\mathbf{M}_2$, and the stiffness matrices, $\mathbf{K}_1$ and $\mathbf{K}_2$, are calculated.
(2) Natural circular frequencies of the first mode of both structures, ω_{11} and ω_{21} $(\omega_{11} \geqslant \omega_{21})$, eigenvectors of the first mode of them, $\boldsymbol{\phi}_{11}$ and $\boldsymbol{\phi}_{21}$, and generalized masses of the first mode of them, M_{11} and M_{21}, are estimated using the mass matrices and the stiffness matrices calculated above.
(3) A connecting joint of "Structure 1", i, and a connecting joint of "Structure 2", j are set. The direction cosine of the connecting member $(i \sim j)$, l, m and n, are estimated by the co-ordinates of the connecting joints i and j. The displacements U_{1i1}, V_{1i1} and W_{1i1} at the joint i in the first vibration mode of "Structure 1" are decided from the modal vector $\boldsymbol{\phi}_{11}$, and the displacements U_{2j1}, V_{2j1} and W_{2j1} at the joint j in the first vibration mode of "Structure 2" are decided from modal vector $\boldsymbol{\phi}_{21}$.

(4) The following quantities of the 2d.o.f. system in Figure 2 are estimated using the values calculated above:

$$\mu = \frac{M_2}{M_1} = \frac{M_{21}}{\beta^2 M_{11}}, \quad k_1 = \omega_{11}^2 M_{11}, \quad k_3 = \frac{\omega_{21}^2 M_{21}}{\beta^2},$$

$$\nu_1^2 = \frac{k_1}{M_1} = \frac{\omega_{11}^2 M_{11}}{M_{11}} = \omega_{11}^2, \quad \nu_3^2 = \frac{k_3}{M_2} = \frac{\omega_{21}^2 M_{21}}{\beta^2} \times \frac{\beta^2}{M_{21}} = \omega_{21}^2,$$

$$f_3^2 = \frac{\nu_3^2}{\nu_1^2} = \frac{\omega_{21}^2}{\omega_{11}^2}.$$

(5) The modal damping ratio ξ, with which the damping performance of the first mode of each framed structure, which is interconnected, is equal to each other and is at its maximum, is estimated by the following expression using the first equation of equation (38):

$$\xi = \frac{(1 - \omega_{21}^2/\omega_{11}^2)}{2[1 + \{M_{21}/(\beta^2 M_{11})\}(\omega_{21}^2/\omega_{11}^2)]} \sqrt{\frac{M_{21}}{\beta^2 M_{11}}}. \tag{41}$$

(6) Optimum spring constant K_{opt} of the connecting member is given from equation (39) as

$$K_{opt} = \frac{(1 - \omega_{21}^2/\omega_{11}^2)[1 - [M_{21}/(\beta^2 M_{11})\}^2(\omega_{21}^2/\omega_{11}^2)]}{\alpha\{1 + M_{21}/(\beta^2 M_{11})\}^2[1 + \{M_{21}/(\beta^2 M_{11})\}(\omega_{21}^2/\omega_{11}^2)]} \times \frac{\omega_{11}^2 M_{21}}{\beta^2}. \tag{42}$$

(7) Optimum damping coefficient C_{opt} of the connecting member is given from equation (40) as

$$C_{opt} = \frac{2(1 - \omega_{21}^2/\omega_{11}^2)\{M_{21}/(\beta^2 M_{11})\}}{\alpha\{1 + M_{21}/(\beta^2 M_{11})\}} \times \sqrt{\frac{M_{11} M_{21} \omega_{11}^2/\beta^2}{\{1 + M_{21}/(\beta^2 M_{11})\}[1 + \{M_{21}/(\beta^2 M_{11})\}(\omega_{21}^2/\omega_{11}^2)]}}. \tag{43}$$

3.4. RESTRICTION ON TUNING METHOD

When the damping performance of the 2d.o.f. system and that of the first mode of each framed structure, which is interconnected, are at a maximum, the modal damping ratio of them are given by equations (38) and (41) respectively. When natural circular frequencies of each system in the 2d.o.f. system, ν_1 and ν_3, are equal to each other and the first natural circular frequencies of both of the structures which are interconnected, ω_{11} and ω_{21}, are equal to each other, modal damping ratio ξ is zero as is obvious from equations (38) and (41). Consequently, the proposed improvement method of the damping performance is not effective in the above cases. However, the damping performance increases when the difference between the first natural circular frequencies of both of the systems in the 2d.o.f. system and the difference between the first natural circular frequencies of both structures, which are interconnected, increase.

The optimum spring constant of a connecting element in the 2d.o.f. system and that of a connecting member in the interconnected structures are expressed by equations (39) and

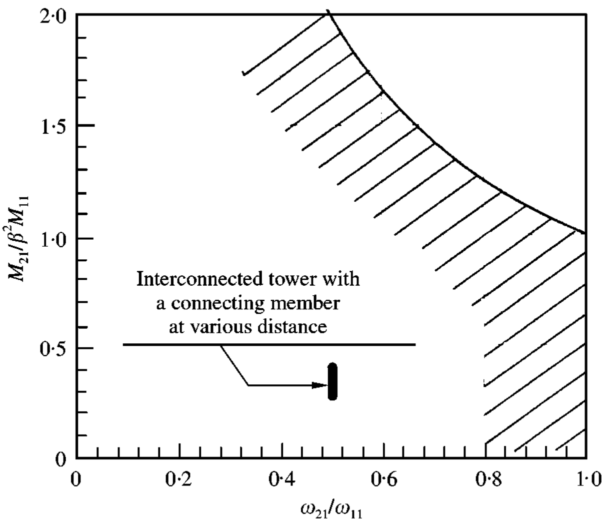

Figure 5. Feasible region of approximate tuning method.

(42) respectively. The values of equations (39) and (42) must be positive because the spring constant of the actual spring is a positive value. Therefore, the dynamic characteristics of both systems in the 2d.o.f. system and the dynamic characteristics of both structures, which are interconnected, have to satisfy the following conditions:

$$1 - f_3^2 \geqslant 0, \quad 1 - \mu^2 f_3^2 \geqslant 0 \quad \text{for the 2d.o.f. system,} \tag{44}$$

$$\frac{\omega_{21}}{\omega_{11}} \leqslant 1, \quad \frac{M_{21}}{M_{11}} \leqslant \frac{\beta^2}{\omega_{21}/\omega_{11}} \quad \text{for the interconnected structure,} \tag{45}$$

in which the first conditions $1 - f_3^2 \geqslant 0$ and $\omega_{21}/\omega_{11} \leqslant 1$ are always satisfied under the assumption described previously, that the first natural circular frequency of “Structure 1” is larger than that of “Structure 2”.

The domain to which the second condition in equation (45) is applied, is indicated by the hatched region in Figure 5. On that region, the actual connecting spring, which maximizes the damping performance of the first mode of both structures which are interconnected, exists and the approximate tuning method proposed here can be applied for maximizing the damping performance of them.

4. NUMERICAL INVESTIGATION

The two columns shown in Figure 6 were adopted as a numerical example for two-dimensional structures, and two framed structures shown in Figure 11 were chosen as numerical examples for three-dimensional structures. The usefulness of the approximate tuning method proposed here is confirmed numerically and the effectiveness of the interconnecting member is assessed. The results of the investigation for those structures were shown in the following.

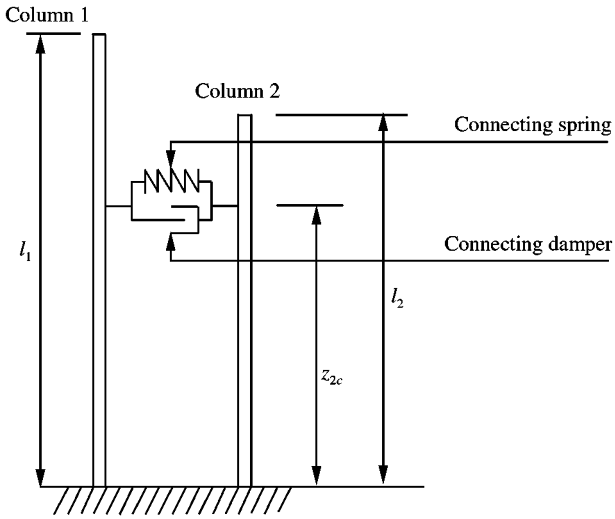

Figure 6. Interconnected columns and their co-ordinates.

TABLE 1

Geometrical constants, generalized masses and natural circular frequencies

	Column 1	Column 2
l_i (m)	30·0	20·0
EI_i (Nm^2)	$4{\cdot}0 \times 10^8$	$4{\cdot}0 \times 10^7$
m_i ($N\,s^2/m^2$)	$1{\cdot}0 \times 10^3$	$2{\cdot}0 \times 10^3$
M_{i1} ($N\,s^2/m$)	0·49799	256·46
ω_{i1} (rad/s)	2·4708	1·2431

4.1. INTERCONNECTION OF TWO COLUMNS STOOD SIDE BY SIDE

The geometrical constants of two columns shown in Figure 6 are shown in Table 1 with the generalized masses and the natural circular frequencies of the first vibration mode of them.

It was confirmed by equation (45) that the optimum spring constant and the optimum damping coefficient of a connecting member at various distances from the bottom of the tower can be estimated. On the domain shown in Figure 5, data of the interconnected columns with a connecting member at the various distances were plotted. Those optimum values were estimated from equations (42) and (43). Then, the natural circular frequencies and modal damping ratios of the interconnected columns with a connecting member tuned were exactly calculated by the complex eigenvalue analysis method.

4.1.1. *Usefulness of approximate tuning method*

The natural circular frequencies and modal damping ratios of the first, second, third and fourth modes of the two types of interconnected columns were calculated by the complex eigenvalue analysis method when the spring constant K of a connecting member is equal to K_{opt} and the damping coefficient C changes. One of the interconnected columns has

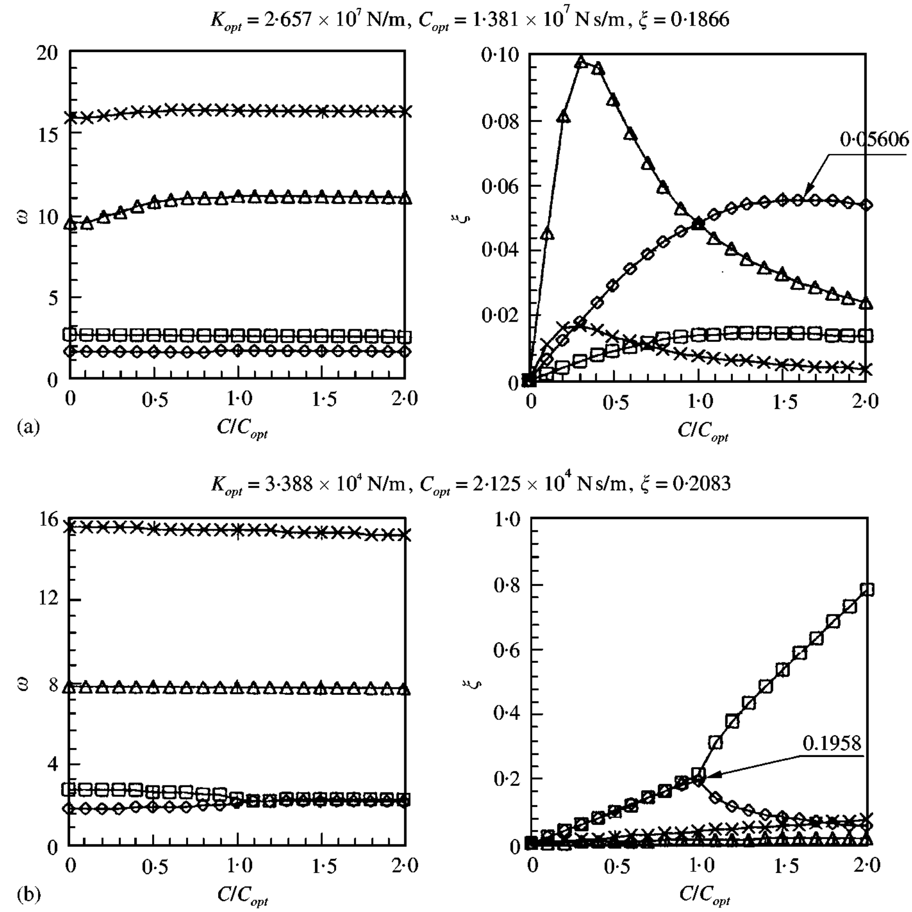

Figure 7. Behavior of natural circular frequencies and modal damping ratios. (a) Attaching point of connecting member = $0{\cdot}25l_2$; (b) attaching point of connecting member = $0{\cdot}85l_2$: —◇—, first mode; —□—, second mode; —△—, third mode; —×—, fourth mode.

a connecting member at the distance, $0{\cdot}25\,l_2$, from the bottom, and the other has a connecting member at the distance, $0{\cdot}85l_2$. The behaviors of the natural circular frequencies and the modal damping ratios of them were shown in Figure 7. Figure 7(a) corresponds to the behavior of the interconnected columns with a connecting member at the distance, $0{\cdot}25l_2$, from the bottom, and Figure 7(b) corresponds to that with a connecting member at the distance, $0{\cdot}85l_2$. In these figures, an optimum spring constant K_{opt} and an optimum damping coefficient C_{opt} for each interconnected column, were indicated with the modal damping ratio, ξ, which is calculated from equation (41). The maximum modal damping ratio of the first mode, which corresponds to a peak on the curve of modal damping ratio of the first mode, which is calculated by the complex eigenvalue analysis method, was also indicated.

The following facts were obvious from the previous figures: in the case of a connecting member attached to the vicinity of the top, (1) the natural circular frequencies of the first and second modes agree well in the vicinity of $C/C_{opt} = 1{\cdot}0$; (2) the modal damping ratios of the first and second modes agree preciously between $C/C_{opt} = 0{\cdot}0$ and $1{\cdot}0$, and they diverge at the vicinity of $C/C_{opt} = 1{\cdot}0$; (3) the modal damping ratio obtained by equation (41) agrees with the maximum one (exact modal damping ratio of the first mode) calculated numerically by the complex eigenvalue analysis method; therefore, (4) the approximate tuning method is useful for the interconnected column with a connecting member in the vicinity of the top. In the case where a connecting member is attached to the vicinity of the bottom, (1) the natural circular frequencies of the first and second modes do not approach each other; (2) the modal damping ratios of the first and second modes do not agree, between $C/C_{opt} = 0{\cdot}0$ and $1{\cdot}0$; (3) the modal damping ratio of the third mode is at a maximum in the vicinity of $C/C_{opt} = 0{\cdot}3$ and that is far larger than the maximum modal damping ratio of the first mode; (4) the modal damping ratio obtained by equation (41) does not agree with the maximum one (exact modal damping ratio of the first mode) calculated numerically by the complex eigenvalue analysis method; therefore, (5) the approximate tuning method cannot be applied to the interconnected column with a connecting member in the vicinity of the bottom. This is the reason that the optimum spring constant K_{opt} and the optimum damping coefficient C_{opt} are very large and the connecting member is attached to the position in the vicinity of the loop of the first vibration mode of each column, at which the displacement of the first mode of each column occupies most of the total displacement in a free vibration.

4.1.2. *Effect of position of connecting member*

The damping performance of the interconnected columns with a connecting member at various distances from the bottom, was investigated by the approximate tuning method proposed here. The behaviors of an optimum spring constant K_{opt} and optimum damping coefficient C_{opt} of the connecting members, estimated by equations (42) and (43), were shown in Figure 8, and the behavior of modal damping ratios estimated by equation (41) and the behavior of the first modal damping ratios at $C/C_{opt} = 1{\cdot}0$, which were estimated by the complex eigenvalue analysis method, were shown in Figure 9. The behaviors of natural circular frequency and modal damping ratio of the first, second, third and fourth modes of the interconnected columns with a connecting member at the various distances from the bottom were also investigated by the complex eigenvalue analysis method when the spring constant K of a connecting member equal to K_{opt} and the damping coefficient C changes. Those behaviors of the interconnected columns with a connecting member attached to the upper part ($0{\cdot}6 \leqslant z_{2c} \leqslant 1{\cdot}0$) were similar to the behaviors shown in Figure 7(b) and those behaviors of the interconnected columns with a connecting member attached to the lower part ($0{\cdot}0 \leqslant z_{2c} \leqslant 0{\cdot}6$) were similar to the behaviors shown in Figure 7(a).

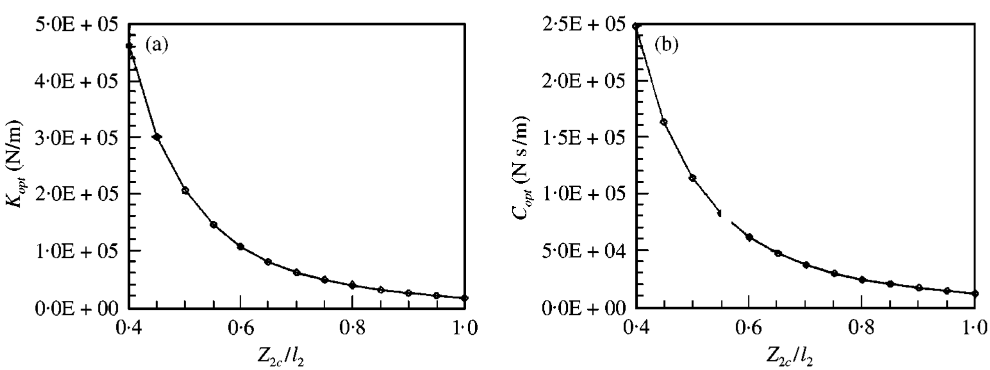

Figure 8. Optimum spring constants and damping coefficients of a connecting member: (a) spring constant; (b) damping coefficient.

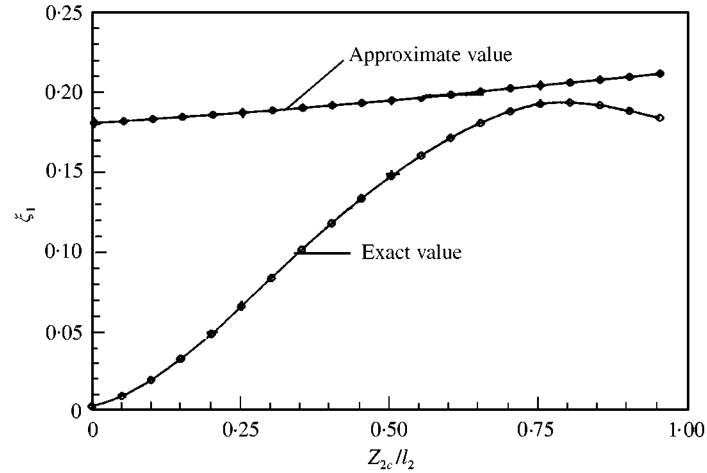

Figure 9. Maximum modal damping ratio of first mode.

The following facts were proven by the above figures and calculated results: (1) the optimum spring constant and the damping coefficient of a connecting member near the top, are smaller than those of a connecting member near the bottom; (2) the damping performance due to a connecting member near the top is better than that due to a connecting member near the bottom; (3) the modal damping ratios, calculated by equation (41), of the interconnected column with a connecting member at the upper part of the tower, agree comparatively with the exact modal damping ratios obtained by the complex eigenvalue analysis method. However, the modal damping ratios, obtained by equation (41), of the interconnected column with a connecting member attached to the lower part, are different from the exact modal damping ratios. Therefore, the approximate tuning method can only be applied to the columns with a connecting member at the upper part; and (4) the best damping performance is given by a connecting member at a distance of $0{\cdot}85l_2$ from the bottom and can be accurately estimated by the approximate tuning method.

From the above investigation, the following facts are desired in designing the connecting member: (1) the assumptions that the rigidity of the connecting member is soft and the connecting member is attached to the position, at which the displacement of the first vibration mode occupies most of the total displacement in free vibration, are satisfied; and (2) the modal damping ratios of the interconnected structure with the connecting member having the optimum spring constant K_{opt}, and the optimum damping coefficient C_{opt}, are calculated by the complex eigenvalue analysis method for estimation of the exact damping performance.

4.1.3. *Effect of internal viscous damping of tower*

The effect of the internal viscous damping of a column on the approximate tuning method was investigated for the interconnected column with a connecting member at the distance of $0{\cdot}85l_2$ from the bottom. First, the optimum spring constant K_{opt} and the damping coefficient C_{opt} of the connecting member, by which the columns without internal viscous damping were interconnected, were estimated from an approximate tuning method. The behaviors of the natural circular frequencies and the modal damping ratios of the interconnected column with a connecting member having the optimum spring constant

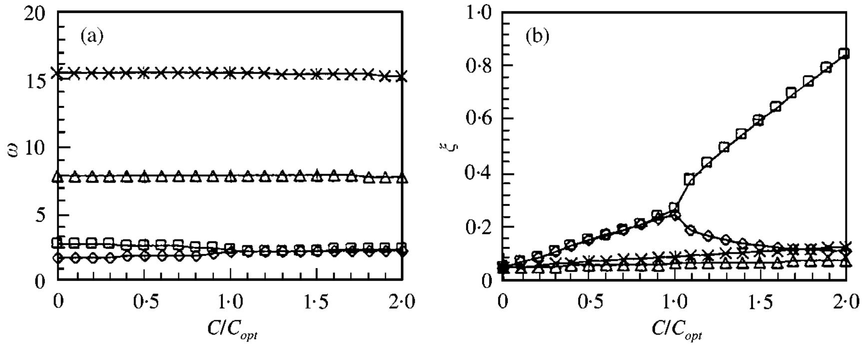

Figure 10. Behavior of natural circular frequencies and modal damping ratios in case of including the viscous damping (damping ratio = 0·05): —◇—, first mode; —□—, second mode; —△—, third mode; —×—, fourth mode.

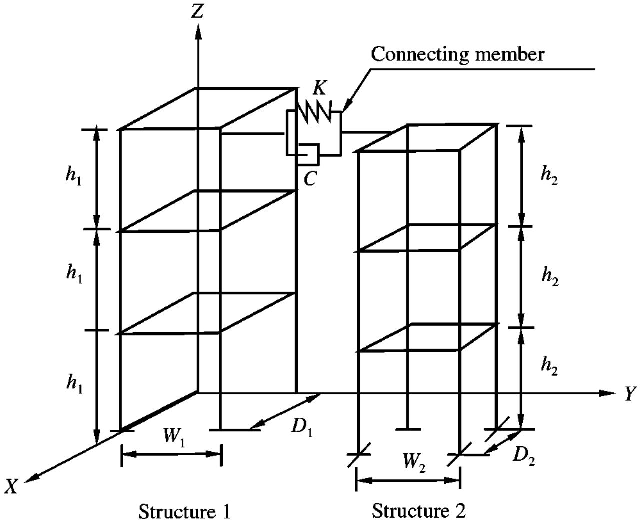

Figure 11. Interconnected framed structures.

K_{opt} and the various damping coefficient C, were estimated by the complex eigenvalue analysis method, when the damping ratios for each vibration mode of each column were equal to $\xi = 0{\cdot}05$. The behaviors of natural circular frequencies and modal damping ratios of the first, second, third and fourth modes were shown in Figure 10.

The following facts became clear from the above figure: (1) the natural circular frequencies do not change much, in comparison with those of the interconnected column without internal viscous damping, which are shown in Figure 7; and (2) the modal damping ratios for each mode are nearly equal to those which added 0·05 to each modal damping ratio of the interconnected column without internal viscous damping.

4.2. INTERCONNECTION OF TWO FRAMED STRUCTURES STOOD SIDE BY SIDE

Dimensions and dynamic characteristics of two framed structures with three stories shown in Figure 11 are indicated in Table 2. In this investigation, a connecting member was

TABLE 2

Dimensions and dynamic characteristics of framed structures

Dimensions of framed structures			
		Structure 1	Structure 2
	Height (m)	$3 \times h_1 = 12$	$3 \times h_2 = 12$
	Width (m)	$W_1 = 4$	$W_2 = 4$
	Depth (m)	$D_1 = 6$	$D_2 = 4$
Natural circular frequencies and predominant modes			
	Structure 1	Structure 2	Predominant mode
First mode (rad/s)	$\omega_{11} = 26{\cdot}23$	$\omega_{21} = 20{\cdot}65$	Bending in y-direction
Second mode (rad/s)	$\omega_{12} = 36{\cdot}46$	$\omega_{22} = 40{\cdot}61$	Bending in x-direction
Third mode (rad/s)	$\omega_{13} = 40{\cdot}90$	$\omega_{23} = 43{\cdot}39$	Torsion

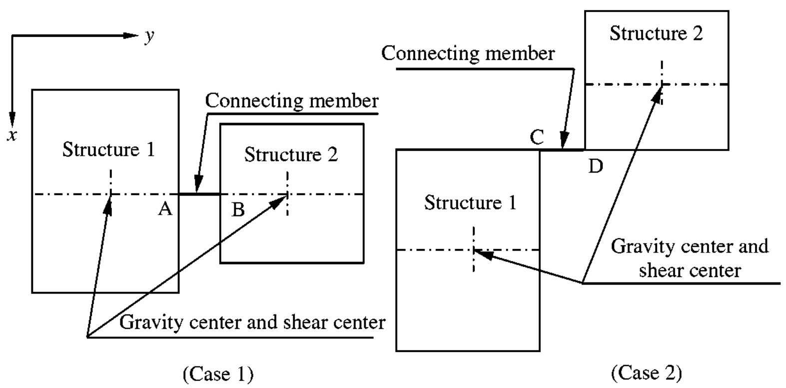

Figure 12. Disposition of framed structures in ground plan and position of a connecting member.

attached to the top story of each frame. Disposition of two framed structures in the ground plan and the attaching position of a connecting member were shown in Figure 12. In the connecting state of Case 1 shown in Figure 12, the bending vibration mode with the predominant displacements in the y-direction are affected by interconnection. Case 1 can be investigated as a two-dimensional structure like a column. In the connecting state of Case 2 shown in Figure 12, the above bending vibration mode and torsional vibration mode are affected.

Optimum spring constant K_{opt} and optimum damping coefficient C_{opt} of a connecting member in each case were calculated by equations (42) and (43), and modal damping ratio ξ in each case was calculated by equation (41). K_{opt}, C_{opt} and ξ of case 1 were $K_{opt} = 3{\cdot}396 \times 10^6$ N/m, $C_{opt} = 6{\cdot}820 \times 10^5$ N s/m and $\xi = 0{\cdot}1141$ respectively. The values in case 2 were almost equal to those of case 1. This is the reason that displacements of two attaching points (points A and C) of framed structure 1 in the first mode are almost equal to each other and displacements of two attaching points (points B and D) of framed structure 2 in the first mode are also almost equal to each other. Two values of α in cases 1 and 2 are equal to each other and two values of β in both of the cases are equal to each other, as is obvious from equations (20) and (21).

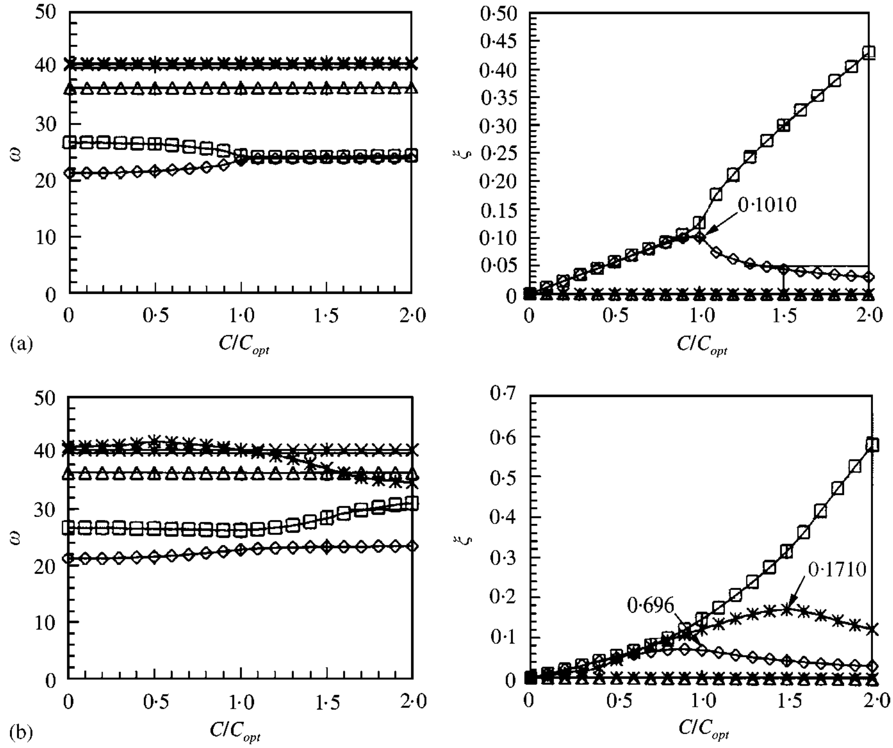

Figure 13. Behavior of natural circular frequencies and modal damping ratios of interconnected framed structures. (a) Case 1; (b) case 2: —◇—, first mode; —□—, second mode; —△—, third mode; —×—, fourth mode; —✱—, fifth mode.

The natural circular frequencies and modal damping ratios of the first, second, third, fourth and fifth modes in two cases of the framed structures interconnected with a connecting member with the optimum spring constant K_{opt} and the various damping coefficients C were calculated by the complex eigenvalue analysis method. The behaviors of the natural circular frequencies and modal damping ratios were shown in Figure 13. Figure 13(a) corresponds to case 1 of the framed structures interconnected and Figure 13(b) corresponds to case 2 of those. The maximum modal damping ratio of the first mode, which corresponds to a peak on the curve of the first modal damping ratio, was also indicated in those figures. The maximum modal damping ratio of the fifth mode, in which torsional displacements are predominant, was also indicated in Figure 13(b).

The following facts were obvious from those figures: in case 1, (1) the behaviors of natural circular frequencies and modal damping ratios are similar to those of the interconnected columns with a connecting member attached to the upper part of column; (2) the modal damping ratio obtained by equation (41) agrees with the maximum one, calculated by the complex eigenvalue analysis method, within an error of about 10%; therefore, (3) the approximate tuning method is useful in this case. In case 2, (1) the natural circular frequencies of the first and second modes do not approach each other; (2) the modal damping ratio obtained by equation (41) does not agree with the maximum one of the first mode, which is calculated by the complex eigenvalue analysis method; (3) the modal damping ratio of the fifth mode (torsional mode) increases as the damping coefficient of the connecting member increases; therefore, (4) the approximate method proposed here is not useful for tuning the connecting member interconnected the framed structures arranged like in Case 2 in which the torsional vibration occurs; and (5) the modal damping ratio of torsional mode increases considerably.

5. CONCLUDING REMARKS

A procedure for improving the structure damping performance of the first vibration mode was proposed by using the interconnection of the two structures with a connecting member which consists of a spring and a damper. This procedure was proposed based on the following assumptions: (1) the rigidity of the connecting member is soft; (2) the natural circular frequencies of each structure, which is not interconnected, are not very close to each other; and (3) the connecting member is attached to the position at which the displacement of the first vibration mode of each structure occupies most of the total displacement in free vibration.

Modal equations of the first vibration mode of each structure, which were interconnected with a connecting member, were shown to be approximately equivalent to the equations for the motion of the 2d.o.f. system with two masses and three springs. It was found that the damping performances of the 2d.o.f. system is at maximum when the two modal damping ratios are equal to each other, and also the two natural circular frequencies are equal. The tuning method of the connecting element in a 2d.o.f. system with two masses and three springs, was proposed using the above characteristics.

The approximate tuning method of a connecting member, by which the modal damping ratios of the first mode of both framed structures interconnected were equalized to each other and then maximized, was proposed using the tuning method of the connecting element in the above 2d.o.f. system. Moreover, the conditions for which the tuning method proposed here can be applied, were also shown. It was also shown that the connecting member is not effective for the improvement of the damping performance, when the first natural circular frequencies of both framed structures, which are interconnected, are equal to each other, and the effectiveness of the connecting member increases as the difference between the first natural circular frequencies mentioned above increases.

The following facts became clear from the numerical investigation: in the case of the interconnected column, (1) when the connecting member is attached to the position in the vicinity of a loop of the first mode of structure, the damping performance of the interconnected structure greatly improves, and the approximate tuning method is accurate; (2) when the connecting member is attached to the position at a distance from the loop of the first mode of the structure, it is not useful, and the degree of improvement of the damping performance is low. Consequently, the approximate tuning method is not effective; and (3) the damping ratio, due to the internal viscous damping in the structure, is added to the modal damping ratio of the interconnected structure. In the case of the framed structure interconnected, (1) when three-dimensional framed structures interconnected by a connecting member vibrate two-dimensionally, the approximate tuning method is effective in the same way as a interconnected column; however, (2) when the framed structures interconnected vibrate three-dimensionally and torsional vibration is caused by this connecting member, the accuracy of the approximate tuning method is not satisfactory. In these cases, freely vibrating displacement vectors $\mathbf{d}_1$ and $\mathbf{d}_2$ denoted by equations (7) should be expressed by the sum of the first, second and third mode vectors in deriving the modal equations. As for such a case, a description will be made in a separate paper.

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REFERENCES

1. K. SETO 1991 *Journal of the Acoustics Society of Japan* **47**, 669–677. Active control of vibration (in Japanese).
2. H. YAMAGUCHI 1988 *Transactions of Japanese Society of Mechanical Engineers*, *Series C* (*Mechanical System*, *Machine Elements and Manufacturing*) **54**, 561–568. Damping of transient vibration by a dynamic absorber (in Japanese).
3. Y. FUJINO and M. ABE 1993 *Earthquake Engineering and Structural Dynamics* **22**, 833–854. Design formulas for tuned mass dampers based on perturbation technique.