Journal of Sound and Vibration (2001) **242**(5), 829–838
doi:10.1006/jsvi.2000.3388, available online at http://www.idealibrary.com on IDEAL®

CHATTER ONSET IN NON-REGENERATIVE CUTTING: A NUMERICAL STUDY

J. GRADIŠEK, E. GOVEKAR AND I. GRABEC

Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, SI-1000 Ljubljana, Slovenia

(*Received* 12 *June* 2000, *and in final form* 4 *September* 2000)

A non-linear model of non-regenerative cutting is analyzed. When damped, the model has a stable fixed point for small values of chip thickness. As chip thickness is increased, the fixed point becomes unstable via a sub-critical Hopf bifurcation. This transition corresponds to the onset of chatter. When dynamic noise is added to the system, chatter onset can occur in the sub-critical parameter region, although chatter-free cutting is predicted by the stability analysis of the fixed point. A noisy bifurcation diagram is compared qualitatively to a diagram obtained experimentally in regenerative cutting. Similar features regarding the onset of chatter are observed in both diagrams.

1. INTRODUCTION

Chatter denotes self-excited large amplitude vibration of the tool relative to the workpiece during machining. Given its detrimental effect on the workpiece, tool, and machine, chatter has been studied intensively in past decades. One way to cause the onset of chatter is to increase the chip width beyond a certain critical value. It has been found experimentally [1, 2] that this critical value is higher for the transition from chatter-free cutting to chatter than for the reverse transition. This means that when chip width is decreased, chatter also persists across a range of chip widths for which chatter-free cutting is observed when chip width is increased. Such hysteresis is typical for the sub-critical type of bifurcations [3].

Analyses of simple non-linear models of cutting have revealed that the onset of chatter could represent a sub-critical Hopf bifurcation from a stable fixed point to a stable limit cycle [4–7]. These models describe the tool-workpiece assembly as a driven single-degree-of-freedom oscillator, linear or non-linear, in which the driving term depends on both the current and the delayed position of the oscillating mass [8]. In turning, where the rotating cylindrical workpiece is cut by a fixed tool, this means that the current cut is influenced by the cut performed during the previous revolution of the workpiece. Such overlapping of successive cuts is called regeneration and can cause the onset of chatter [9, 10]. Although simple, the delay-models correlate very well with experiments [6, 7], and provide practically useful means for selection of chatter-free cutting parameters.

However, the delay models are not valid for processes in which the successive cuts do not overlap. Examples of such processes include turning of threaded workpieces and planing. Despite the lack of regeneration in these processes, chatter still occurs. Possible causes of chatter onset are: (1) unstable non-linear plastic flow of the cut material, (2) friction between the tool and the cut material, (3) non-linear dependence of the cutting force on cutting velocity, etc.

0022-460X/01/200829 + 10 $35.00/0

In this article, a model of non-regenerative cutting is analyzed [11] in which the tool–workpiece assembly is represented by two linear oscillators coupled by a non-linear driving force. The model thus includes two of the aforementioned possible causes of chatter onset. Previous analyses of the model have shown that during chatter, tool vibration can be harmonic, quasi-periodic, or chaotic, depending on the cutting parameters [11–15]. Here, the interest is only in the *bifurcation* which corresponds to the *onset* of chatter. The purpose is to show that the onset of chatter in non-regenerative cutting represents a sub-critical Hopf bifurcation from a stable fixed point to a stable limit cycle. Therefore, onsets of chatter in both regenerative and non-regenerative cutting represent dynamically similar transitions. Since the dynamics of real cutting are stochastic, the role of dynamic noise is addressed. By using bifurcation diagrams, it is demonstrated that, in the sub-critical parameter region, dynamic noise can cause the onset of chatter, although chatter-free cutting is expected. Finally, a noisy bifurcation diagram is compared qualitatively to a diagram obtained experimentally in regenerative cutting (turning). Similar features regarding the onset of chatter are observed in both diagrams.

2. MODEL OF A CUTTING PROCESS

Consider the simplified orthogonal cutting system shown in Figure 1. The workpiece material flows with velocity v towards the tool in the x direction, becomes plastically deformed in the cutting zone, and slides in the form of chips along the tool face in the y direction.

The material flow excites the elastic tool to oscillate in the (x, y) plane. The cutting system can be described by two coupled oscillators. Their dynamics are governed by [11]

$$m\ddot{x} + d_x\dot{x} + k_x x = F_x, \qquad m\ddot{y} + d_y\dot{y} + k_y y = KF_x, \tag{1}$$

where m represents an inertial mass, k_x and k_y denote spring constants, while d_x and d_y denote damping constants. The cutting force F_x depends non-linearly on the cutting velocity v [11, 16],

$$F_x = F_x^0\left(\frac{h}{h_0}\right)\left[c_1\left(\frac{v}{v_0} - 1\right)^2 + 1\right], \tag{2}$$

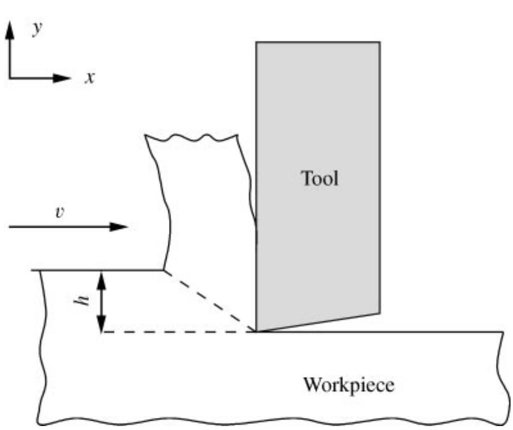

Figure 1. Sketch of the orthogonal cutting system.

and vanishes for negative v and h, i.e., when the tool loses contact with the workpiece. Here, h denotes the chip thickness, F_x^0 is proportional to the chip width, and v_0 and h_0 define the reference state. The friction coefficient is given by [11, 16]

$$K = K_0\left[c_2\left(\frac{v_f}{v_0} - 1\right)^2 + 1\right]\left[c_3\left(\frac{h}{h_0} - 1\right)^2 + 1\right]. \tag{3}$$

The sign of K is chosen so as to oppose the material motion. The friction velocity v_f of the material sliding along the tool face in the y direction is

$$v_f = \frac{v}{R}, \qquad R = R_0\left[c_4\left(\frac{v}{v_0} - 1\right)^2 + 1\right]. \tag{4}$$

Coefficients $c_1 = 0{\cdot}3$, $c_2 = 0{\cdot}7$, $c_3 = 1{\cdot}5$, $c_4 = 1{\cdot}2$, $K_0 = 2{\cdot}2$, and $R_0 = 0{\cdot}36$ are material-dependent. Their values were obtained empirically for a wide class of low carbon-content steels, and for a reference state of $h_0 = 0{\cdot}25$ mm and $v_0 = 6{\cdot}6$ m/s [11, 16].

Following reference [11], the system is non-dimensionalized by using the reference state as

$$X_1 = x/h_0, \quad X_2 = \dot{x}/v_0, \quad X_3 = y/h_0, \quad X_4 = \dot{y}/v_0, \quad V = v/v_0, \quad H = h/h_0. \tag{5}$$

The non-dimensional time is defined by $T = v_0/h_0$. The second order dynamic equations (1) may be converted to a set of four first order differential equations,

$$\dot{X}_1 = X_2, \quad \dot{X}_2 = F - D_x X_2 - E_x X_1, \quad \dot{X}_3 = X_4, \quad \dot{X}_4 = KF - D_y X_4 - E_y X_3, \tag{6}$$

where the dot denotes the derivative with respect to T. The non-dimensional cutting force and friction coefficient are given by

$$F = F_0 H[c_1(V-1)^2 + 1]U(H)U(V),$$

$$K = K_0[c_2(V_f - 1)^2 + 1][c_3(H-1)^2 + 1]\,\mathrm{sgn}(V_f), \tag{7}$$

with the non-dimensional friction velocity expressed as

$$V_f = V - RX_4, \quad R = R_0[c_4(V-1)^2 + 1]. \tag{8}$$

Here, $U()$ and sgn() represent a unit step function and a signum function respectively. They are included in order to account, respectively, for the loss of contact between the tool and the workpiece, and the change of direction of cutting velocity due to tool vibrations. Other dimensionless parameters are written explicitly as

$$F_0 = F_x^0 h_0/(mv_0^2), \quad D_x = d_x h_0/(mv_0), \quad E_x = k_x h_0^2/(mv_0^2),$$

$$D_y = d_y h_0/(mv_0), \quad E_y = k_y h_0^2/(mv_0^2). \tag{9}$$

Finally, the average cutting velocity and chip thickness values need to be replaced by the actual, time-varying, values as

$$V = V_i - X_2, \quad H = H_i - X_3, \tag{10}$$

where V_i and H_i denote the nominal values of V and H.

In the previous investigations of the model [11–15], the equations (6) were integrated for $E_x = 1$, $E_y = 0{\cdot}25$, and $V_i = 0{\cdot}5$, while either H_i (proportional to the chip thickness) or F_0 (proportional to the chip width) was chosen as the control parameter. When the control parameter was increased, harmonic, quasi-periodic, and chaotic tool oscillations were found. Most of the results presented so far were obtained for a deterministic, noise-free, system with zero damping ($D_x = D_y = 0$). The present study extends previous studies by considering non-zero damping, and by introducing additive dynamic noise into the system. In reference [15], the effect of multiplicative dynamic noise on cutting dynamics is also investigated, but in a different context.

3. STABILITY OF CUTTING

In the machining literature, chatter-free cutting is traditionally referred to as a stable cutting regime, while chattering is regarded as unstable [9]. When this terminology is adopted, cutting is stable when the tool rests at a fixed point, and unstable when the tool oscillates. Therefore, in order to check the cutting stability, one first has to determine the fixed points of the system (6), and assess their stability.

Fixed points $\mathbf{X}^*$ can be found by solving $\dot{\mathbf{X}} = \mathscr{F}[\mathbf{X}] = \mathbf{0}$. It is obvious from equations (6) that $X_2^* = X_4^* = 0$. $\mathscr{F}_2$ is a linear function of X_1 and X_3, and $\mathscr{F}_4$ is a third order polynomial depending only on X_3. One can thus solve $\mathscr{F}_4 = 0$ for X_3^* and find the corresponding X_1^* from $\mathscr{F}_2 = 0$. As already shown in reference [12], the cubic equation $\mathscr{F}_4 = 0$ has one real root and a pair of complex conjugate roots for a wide range of parameters considered. This means that the system has only one fixed point of interest.

A general, non-linear, approach to stability analysis requires the use of Lyapunov functions which, in the present case, are not easy to determine. One could also examine stability of the fixed point numerically, by determining its domains of attraction. However, for the sake of simplicity one can apply the linear stability theory which is limited to small disturbances. Stability of the fixed point is determined by the eigenvalues λ_i of the Jacobian matrix $\boldsymbol{J}$,

$$J_{ij} = \frac{\partial \mathscr{F}_i(\mathbf{X})}{\partial X_j} \tag{11}$$

evaluated at the fixed point $\mathbf{X}^*$. Eigenvalues are usually arranged in descending order $\lambda_1 \geqslant \lambda_2 \geqslant \cdots$.

The investigated fixed point has two pairs of complex conjugate eigenvalues, $\lambda_{1,2}$ and $\lambda_{3,4}$, for a wide parameter range. Figure 2 shows the dependence of the real parts of $\lambda_{1,2}$ and $\lambda_{3,4}$ on the nominal chip thickness H_i. For the undamped case, $D_x = D_y = 0$, the fixed point is unstable, representing a saddle in the phase space for $H_i < 0{\cdot}22$, and a source for $H_i > 0{\cdot}22$. For the damped case, $D_x = D_y = 0{\cdot}1$, the fixed point is stable for $H_i < 0{\cdot}35$, and unstable for $H_i > 0{\cdot}35$: a saddle for $0{\cdot}35 < H_i < 0{\cdot}84$, and a source for $H_i > 0{\cdot}84$. The transition observed at $H_i \approx 0{\cdot}35$, where the real part of the complex conjugate eigenvalue pair $\lambda_{1,2}$ changes sign, is known as the Hopf bifurcation from a stable to an unstable fixed point and a stable limit cycle [3]. Below it will be shown that this transition corresponds to the onset of chatter.

4. ONSET OF CHATTER

To simulate a slow and smooth increase and decrease of chip thickness h, the system (6) was solved numerically with H_i slowly increasing and decreasing. The model parameters

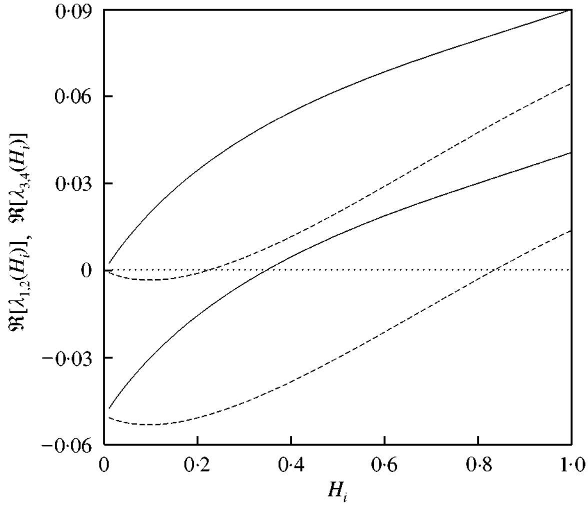

Figure 2. The real parts of the stability coefficients $\lambda_{1,2}$ (—) and $\lambda_{3,4}$ (- - - -) of the fixed point for the undamped ($D_x = D_y = 0$, top curves) and the damped ($D_x = D_y = 0{\cdot}1$, bottom curves) cases.

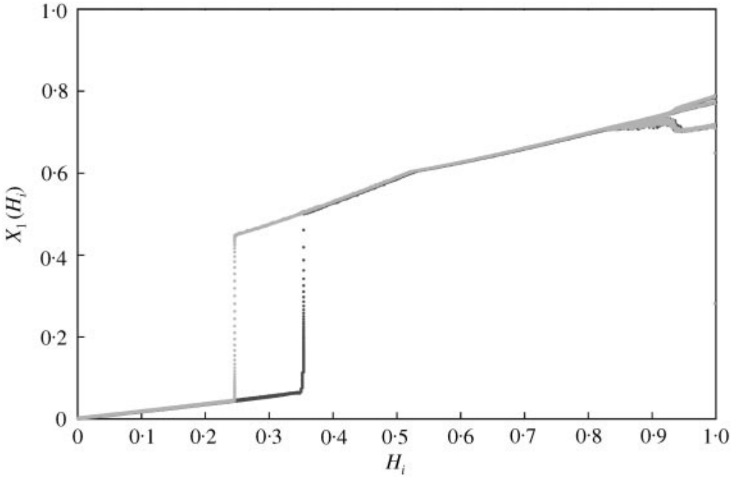

Figure 3. Dependence of X_1 amplitudes on the nominal chip thickness H_i; black dots—H_i smoothly increased, grey dots—H_i smoothly decreased.

were chosen in accordance with references [11, 12], and damping was set to $D_x = D_y = 0{\cdot}1$. In order to facilitate comparison with the numerical solutions of the noisy system discussed in section 5, an Euler integration scheme was used to solve the equations. The integration step of 0·001 was found to be short enough for the solutions to converge reasonably well. Following reference [12], the discontinuous functions $U()$ and sgn() were approximated by $\frac{1}{2}(1 + \tanh x/\varepsilon)$ and $\tanh x/\varepsilon$, respectively, and equations (6) treated as if they were continuous.

The dependence of positive amplitudes of the tool motion in the x direction on the nominal chip thickness H_i is shown in Figure 3. When H_i is increased, the tool rests at the fixed point which is shifted linearly with H_i. This corresponds to increasing elastic deformation of the tool subjected to an increasing load (see Figure 1). When the fixed point becomes unstable ($H_i \approx 0{\cdot}35$) large amplitude oscillations develop. This bifurcation corresponds to the onset of chatter. As H_i is further increased, the amplitude of harmonic oscillations increases. Another bifurcation occurs at $H_i \approx 0{\cdot}84$, but we do not have an experimentally supported explanation for this as yet. When the nominal chip thickness H_i is decreased, the bifurcation scenario is repeated in the reverse order. However, chatter vibrations persist well below the stability limit of the fixed point ($H_i \approx 0{\cdot}35$), and the transition from chatter to chatter-free cutting occurs at $H_i \approx 0{\cdot}25$. Because of different H_i values at the forward and backward bifurcations, a hysteresis appears which is typical of the sub-critical type of bifurcations [3]. One can therefore conclude that the onset of chatter in non-regenerative cutting corresponds to a sub-critical Hopf bifurcation.

Sub-critical bifurcations are often unfavourable in practice because inside the hysteresis (in the sub-critical region) two stable attractors co-exist, and random influences in the process can drive the trajectory from one attractor to the other. In the present case, the two attractors correspond to a fixed point (chatter-free cutting) and a limit cycle (chatter), and random influences could cause the onset of chatter while the fixed point was still stable. Since the dynamics of real cutting are stochastic, and noise is inherent to the process, the situation described is a realistic one.

5. THE INFLUENCE OF NOISE

A possible source of dynamic noise in cutting could be the flow of non-homogeneous cut material. In the first approximation, this effect could be accounted for by adding a random term to the model equations (6) which now assume the form of a Langevin equation:

$$\dot{X}_i(T) = \mathscr{F}_i[\mathbf{X}(T)] + \sum_j g_{ij}[\mathbf{X}(T)]\Gamma_j(T). \tag{12}$$

Here, $\Gamma_j(T)$ denotes uncorrelated noise, $\langle \Gamma_i(T)\Gamma_j(T')\rangle = \delta_{ij}\delta(T-T')$, with vanishing mean, $\langle \Gamma_j(T)\rangle = 0$, for each *i*, *j*. $\mathbf{g}$ denotes the noise amplitude. For the present study, Gaussian distributed Γ_i are adopted, with $g_{22} = g_{44} = g$ and the rest of g_{ij} set to zero. With such a matrix of noise amplitudes $\mathbf{g}$, a random term is added only to the equations for the accelerations of the two oscillators (see equations (6)). In the present case, this is similar, but not equivalent, to disturbing randomly the cutting forces. The disturbances could be caused by variable grain size of the cut material. Since $\mathbf{g}$ is diagonal, noise in the x direction is uncoupled from noise in the y direction. This simplification was found not to be particularly important, because coupled noise did not introduce considerable changes to the cutting dynamics when compared to the dynamics with uncoupled noise.

Figure 4(a) shows the dependence of maximal X_1 amplitudes on the nominal chip thickness H_i for $\mathbf{g} = 0{\cdot}003$. Although noisy, the dependences are similar to the corresponding ones in Figure 3. Note, however, that the hysteresis is narrower in the noisy than in the noise-free diagrams, since the values of H_i, at which the forward and backward bifurcations occur, have shifted to $H_i \approx 0{\cdot}32$ and $H_i \approx 0{\cdot}27$ respectively. As the noise amplitude g is further increased, the hysteresis vanishes (Figure 4(b)). For noise amplitudes of approximately $g > 0{\cdot}01$, the bifurcation cannot be distinguished clearly because, in the sub-critical parameter region, the tool motion trajectory is driven frequently from one attractor to the other. The threshold value of $g \approx 0{\cdot}01$ was found from numerical experiments. It may be considered as a characteristic parameter which describes when the effect of dynamic noise becomes overwhelming so that it masks the abrupt transition to

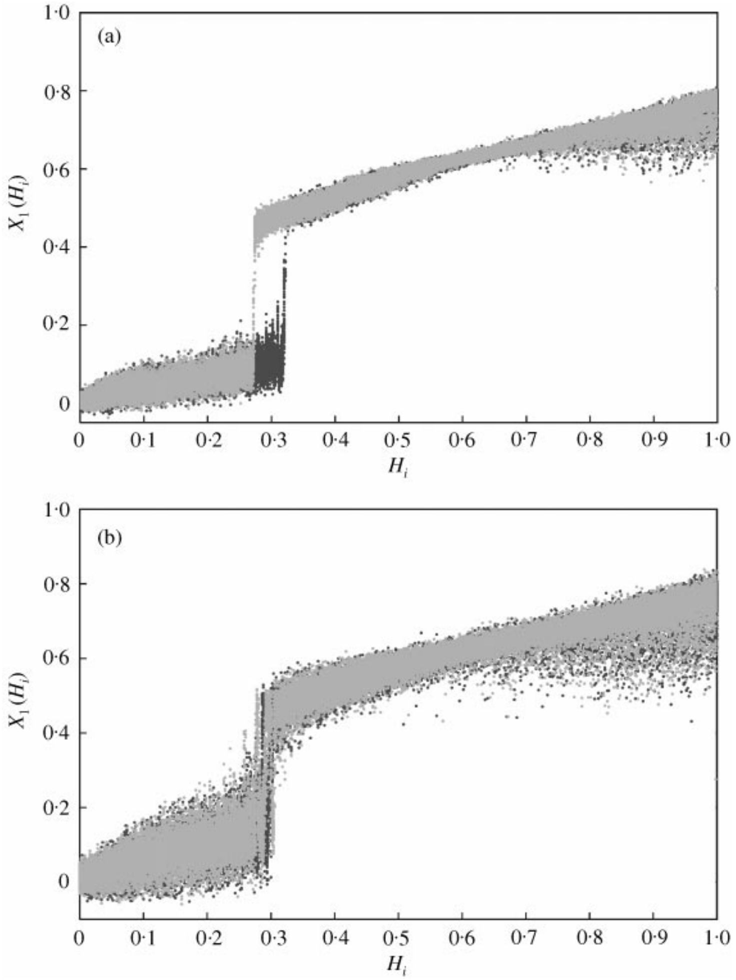

Figure 4. Dependence of X_1 amplitudes on the nominal chip thickness H_i in the presence of dynamic noise with amplitude (a) $g = 0{\cdot}003$ and (b) $g = 0{\cdot}006$; black dots—H_i smoothly increased, grey dots—H_i smoothly decreased.

chatter. It is difficult to compare directly the noise amplitudes in the model to those observed in real cutting. Based on our experience with regenerative cutting, we suspect that noise amplitudes in real cutting are lower than the equivalent threshold mentioned above, since a bifurcation at the chatter onset can usually be distinguished.

We are at present not aware of experiments with non-regenerative cutting in which the cutting parameters would be changed analogously to the above simulations. However, dependences similar to the ones in Figure 4(b) have been obtained experimentally in regenerative cutting [17]. An example of such an experimental diagram is presented in Figure 5, where maxima of the cutting force amplitudes during turning are plotted against the increasing chip width w, which is proportional to the parameter F_0 of the model used in this study. In the diagram, the onset of chatter at $w \approx 0{\cdot}6$ mm is observed clearly as a marked transition between two different cutting regimes. The average amplitude of cutting force oscillations is significantly smaller in the chatter-free than in the chatter regime. Wide scattering of the maxima, especially in the chatter regime, can be attributed in part to the presence of measurement noise in the data, and to non-Gaussian distribution of

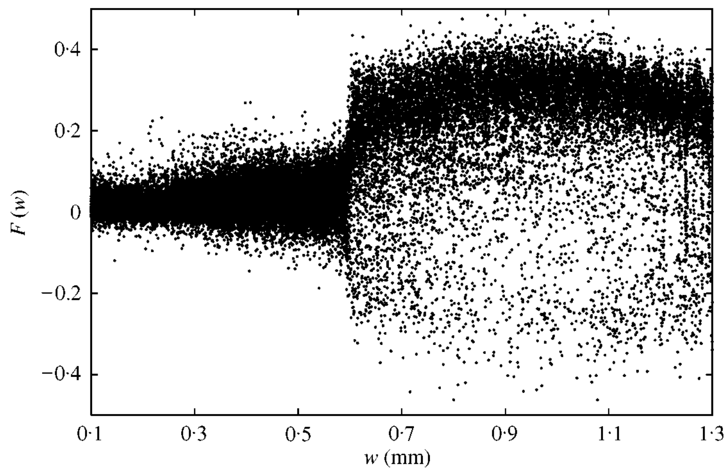

Figure 5. Experimental dependence of the cutting force F amplitudes on the increasing chip width w in regenerative cutting (turning).

dynamic noise. Although this diagram cannot be compared quantitatively to the corresponding diagram in Figure 4(b), qualitative similarities between the two diagrams are obvious. This confirms that, in order to improve the modelling of cutting process dynamics, stochastic models should be considered.

6. CONCLUSIONS

A non-linear model of non-regenerative cutting is analyzed. Chip thickness H_i is selected as the control parameter. When chip thickness is gradually increased, a stable fixed point that represents chatter-free cutting loses its stability via a Hopf bifurcation to a limit cycle that represents chatter. When chip thickness is gradually decreased, the bifurcation scenario is repeated in the reverse order, such that a hysteresis is obtained. The appearance of hysteresis reveals the sub-critical nature of the bifurcation corresponding to the chatter onset. Upon introducing the dynamic noise into the system, the hysteresis becomes narrower at small noise amplitudes, and vanishes when the noise amplitudes are increased sufficiently. This confirms the experimentally observed fact that, in the sub-critical parameter region, random influences in the process can cause chatter onset although chatter-free cutting is predicted by the stability analysis of the fixed point. Finally, due to the lack of experimental results about non-regenerative cutting, the bifurcation diagram obtained in regenerative cutting is compared qualitatively to the calculated noisy diagram. Similar features regarding the onset of chatter are observed in both diagrams.

The model analyzed in this study consists of two linear oscillators which are coupled by a non-linear driving term. Possible coupling of oscillators in the x and y directions, and non-linear properties of the machine-tool assembly, which would be represented by non-linear damping or restoring terms, are neglected for the sake of simplicity. Although simplified, the model nevertheless reproduces an experimentally observed phenomenon very well. As there is still no widely accepted explanation for the causes of chatter onset in non-regenerative cutting, our results indicate that (a) non-linear dependence of cutting force on cutting velocity, and (b) friction between the tool and the cut material can cause the onset of chatter. While non-linear properties of the machine-tool may have an important effect on cutting dynamics, they are not necessary for the chatter to occur. Similar results have been obtained for the regenerative cutting, where a linear oscillator with non-linear regenerative driving is sufficient to model the chatter onset remarkably well [4, 6].

Dynamic noise can have an important influence on the process dynamics, especially in the vicinity of the bifurcation points [18]. Furthermore, noise can broaden a limit cycle attractor, or distort a chaotic attractor to such an extent that, when analyzing time series generated by stochastic processes, one might be misled into searching for a complicated structure in a trivial, but noisy, attractor, and to overlook the fingerprints of chaos in a noisy chaotic attractor [19]. In cutting processes, there are various possible sources of noise. Although incorporating noise into the model makes an analytical treatment difficult or impossible, it seems worthwhile to consider noise in numerical studies. In our study, uncorrelated Gaussian distributed additive dynamic noise was used. Such a type of dynamic noise may not be the most realistic in the case of cutting dynamics. Correlated multiplicative noise in the driving term, i.e., the cutting force, would probably match the conditions of real cutting better [15, 20]. However, for the dynamics of stochastic processes described by the Langevin equation with uncorrelated Gaussian noise, some interesting analytical results are available which have recently led to new methods for analysis of noisy time series [19, 21].

The results of this study may be relevant not only to non-regenerative machining but also to other mechanical processes in which sub-critical bifurcations take place. For example, flutter of airplane wings seems to be an unfavourable phenomenon dynamically analogous to chatter in machining [22].

ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the Volkswagen Foundation, the Ministry of Science and Technology of Slovenia, and the EU COST Action P4.

REFERENCES

1. C. J. HOOKE and S. A. TOBIAS 1963 *Proceedings of the 4th International Machine Tools Design and Research Conference*, 97–109. Manchester: Pergamon Press. Finite amplitude instability—A new type of chatter.
2. N. H. HANNA and S. A. TOBIAS 1974 *ASME Journal of Engineering for Industry* **96**, 247–255. A theory of nonlinear regenerative chatter.
3. S. H. STROGATZ 1994 *Nonlinear Dynamics and Chaos*. Reading, MA: Addison–Wesley.
4. G. STÉPÁN and T. KALMÁR-NAGY 1997 *Proceedings of the 16th ASME Biennial Conference on Mechanical Vibration and Noise*, 1997 ASME Design and Technical Conferences, 1–11 *Sacramento*, *CA*. Nonlinear regenerative machine tool vibrations.
5. A. H. NAYFEH, C.-M. CHIN and J. R. PRATT 1997 *American Society of Mechanical Engineers Journal of Manufacturing Science and Engineering* **119**, 485–493. Perturbation methods in nonlinear dynamics—Applications to machining dynamics.
6. T. KALMÁR-NAGY, J. R. PRATT, M. A. DAVIES and M. D. KENNEDY 1999 In *Proceedings of the 17th ASME Biennial Conference on Mechanical Vibration and Noise*, 1999 *American Society of Mechanical Engineers Design and Technical Conferences*, *Las Vegas*, *Nevada*. 1–9. Experimental and analytical investigation of the subcritical instability in metal cutting.
7. J. R. PRATT, M. A. DAVIES, C. J. EVANS and M. D. KENNEDY 1999 *Annals of the CIRP* **48**, 39–42. Dynamic interrogation of a basic cutting process.
8. G. STÉPÁN 1989 *Retarded Dynamical Systems*: *Stability and Characteristic Functions*. Pitman Research Notes in Mathematics, Vol. 210 Longman Scientific & Technical.
9. S. A. TOBIAS 1965 *Machine Tool Vibration*. London: Blackie & Son, Ltd.

10. H. E. MERRITT 1965 *American Society of Mechanical Engineers Journal of Engineering for Industry* **87**, 447–454. Theory of self-excited machine-tool chatter.
11. I. GRABEC 1986 *Physics Letters* **117**, 384–386. Chaos generated by the cutting process.
12. R. F. GANS 1995 *Journal of Sound and Vibration* **188**, 75–83. When is cutting chaotic?
13. J. GRADIŠEK, E. GOVEKAR and I. GRABEC 1996 *International Journal of Machine Tools and Manufacture* **36**, 1161–1172. Chaotic cutting process and determining optimal cutting parameters using neural networks.
14. M. WIERCIGROCH 1997 *American Society of Mechanical Engineers Journal of Vibration and Acoustics* **119**, 468–475. Chaotic vibration of a simple model of the machine tool-cutting process system.
15. M. WIERCIGROCH and A. H.-D. CHENG 1997 *Chaos, Solitons, & Fractals* **8**, 715–726. Chaotic and stochastic dynamics of orthogonal metal cutting.
16. W. F. HASTINGS, P. MATHEW and P. L. B. OXLEY 1980 *Proceedings of Royal Society London* **A 371**, 569–587. Machining theory for predicting chip geometry, cutting forces, etc., from work-material properties and cutting conditions.
17. J. GRADIŠEK, E. GOVEKAR and I. GRABEC 1998 *Journal of Sound and Vibration* **214**, 941–952. Using coarse-grained entropy rate to detect chatter in cutting.
18. H. HAKEN 1993 *Advanced Synergetics*. Berlin: Springer-Verlag, third edition.
19. J. GRADIŠEK, S. SIEGERT, R. FRIEDRICH and I. GRABEC 2000 *Physical Review* **E 62**, 3146–3155. Analysis of time series from stochastic processes.
20. G. STÉPÁN and T. INSPERGER 2000. Private communication.
21. S. SIEGERT, R. FRIEDRICH and J. PEINKE 1998 *Physics Letters* **A 243**, 275–280. Analysis of data sets of stochastic systems.
22. E. MOSEKILDE 2000. Private communication.