



COMMENTS ON “THE EFFECTS OF LARGE VIBRATION  
AMPLITUDES ON THE MODE SHAPES AND NATURAL  
FREQUENCIES OF THIN ELASTIC SHELLS, PART I: ...”

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The recent paper by F. Moussaoui, R. Benamar and R. G. White [1] on large-amplitude vibrations of shells reintroduces an error that has been corrected many years ago by Evensen [2, 3] and many others [4–11].

The paper deals with free, large-amplitude vibrations of infinitely long circular cylindrical shells, neglecting motion in the longitudinal direction and assuming that the generating lines of the shell remain straight after deformation. Thus, the model is suitable for rings but it is not adequate for real shells of finite length. However, the main problem is not there. The system is discretized by using a multi-mode expansion which excludes axisymmetric terms (refer to p. 921 and 927 in reference [1]). In fact all available studies, see e.g. references [2–11], show that the interaction between the asymmetric mode under consideration and the axisymmetric modes is crucial, and that it is the most important of any such intermodal interactions. In particular, Amabili *et al.* [9, 10] have shown that not only the first axisymmetric mode, but also the third should be included for shells of finite length, to correctly predict the trend of non-linearity: if the interaction with axisymmetric modes is neglected, as done by Moussaoui *et al.* [1], a strong hardening-type non-linearity is obtained (see Figure 2 in reference [1]); if it is included, as it ought to be, the non-linearity is softening, for most cylindrical shell geometries. It is very curious that Moussaoui *et al.* [1] used for comparison of their results the single-mode approach used by Chu in 1961 which has been known to give wrong results since 1963 [2].

However, it is not the fact that the erroneous type of non-linear behaviour has been predicted which perturbs most the present writers. It is the fact that the Moussaoui *et al.* analysis reinjects into the literature an omission which was rectified by Evensen and others almost 40 years ago. It is well known that such errors or omissions are often followed by others—with, alas, a long half-life.

Other limitations of the paper [1] are (1) the hypothesis of harmonic oscillations, which eliminates many of the most interesting facets of behaviour of shells (e.g. amplitude modulation, non-stationary and chaotic response) which have been detected in recent studies [9–11], (2) the neglect of the companion mode [3–11], which has the same shape of the driven one but is rotated by  $\pi/(2n)$ , (3) the non-satisfaction of continuity condition of the circumferential displacement [3, 7, 8], (4) a very poor literature review on large-amplitude vibrations of shells which does not include most of the important papers on this topic published in the last 30 years, even missing fundamental papers of the 1960s, and (5) the sentence “Unfortunately the results available in the shell vibration literature are based on the single-mode approach” (refer to p. 931 in reference [1]), which would have been correct if only this were 1961.

## REFERENCES

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