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LOW FREQUENCY SOUND PROPAGATION IN A COAXIAL CYLINDRICAL DUCT: APPLICATION TO SUDDEN AREA EXPANSIONS AND TO DISSIPATIVE SILENCERS

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A method based on the approximation of the radial pressure profile is developed to analyze the acoustic performance of a sudden expansion and of an expansion chamber at low frequencies. This model is able to predict very accurately the added length of an expansion which includes neither porous material nor a perforated tube. It can also be applied when a bulk-reacting lining and perforated tube are included. This model of a dissipative silencer gives results which compare very favourably with experimental data.

1. INTRODUCTION

The cylindrical dissipative silencer is one of the most commonly used devices in practical flow duct acoustic. Its acoustic performance can be predicted in the general case (with flow, with arbitrary external shape, etc.) by a FEM approach [1] or by mode matching techniques [2, 3]. However, those methods require a considerable numerical effort which limits their usefulness in practice.

The purpose of this paper is to derive a simple model which can be used to predict the acoustic performance of a bulk-reacting dissipative silencer at low frequencies.

With a similar aim in mind, Wang [4] applied the decoupling method to the case of a dissipative silencer with a perforated tube (the decoupling method was derived by Sullivan and Crocker [5] for the case of an expansion chamber with a perforated tube). This method assumes that the acoustic pressure is uniform on either side of the perforated tube and that the difference between those two pressures is linked to the perforated tube impedance. However, in most practical applications, the perforated tube is introduced to avoid erosion of the porous material and is not supposed to have any significant acoustical effect. However, when the impedance of the perforated tube is small or vanishes, the decoupling method fails to give an accurate description of the silencer.

The key point of the model developed in this paper is that it takes into account the actual radial variation of pressure (and then of radial velocity) in an approximate way. In the presentation given here, this effect is introduced via an equivalent admittance linking the difference in mean pressures between the air and the material to the radial velocity at the interface. Then, even in the absence of a perforated tube the model can give an accurate description of acoustical performance.

In section 2, the principle of the method is described. By averaging the Euler and continuity equations for both the air and the porous material, an eigenequation is obtained by assuming that the radial velocity at the interface depends only on the difference in mean pressure. The equivalent admittance is then given by assigning an appropriate shape for the radial velocity profile. The eigenequation displays two kinds of solutions in the lined section; one corresponds to the classical plane or quasi-plane wave, the other takes into account most of the effects of higher order modes. This approach is applied in section 3 to a sudden expansion with lining in the large section. The model is applied to the case with neither porous material nor perforated tube in order to provide an easy comparison with an exact solution. In section 4, a model of the cylindrical dissipative silencer of finite length is given. Predictions based upon the model are then compared to experimental results.

2. LOW FREQUENCIES APPROXIMATION

2.1. AVERAGED EQUATIONS

In this Section, the basic linear equations governing the propagation of axisymmetric fluctuations in a duct of radius r_b are given. This duct consists of an inner cylinder with radius r_a, referred to as region A, and of an outer coaxial cylinder with inner and outer radii of r_a and r_b, respectively, referred to as region B (see Figure 1). Between those two regions, a rigid perforated screen induces a pressure jump proportional to the radial velocity.

The fluctuating variables used here are pressure p, axial velocity u and radial velocity v with subscript a or b depending on the region. In region A, the fluid is characterized by the compressibility κ_a and the density ρ_a. In region B, a porous material with a rigid frame can be present and is described using an equivalent fluid model. The porous medium is then characterized by an effective compressibility $\kappa_b(\omega)$ and an effective density $\rho_b(\omega)$ depending on the frequency ω. These effective quantities are expressed in Appendix A as functions of the characteristics of the material and of the saturating fluid.

Taking a time dependence $e^{j\omega t}$, the propagation equations can be found from the continuity and Euler equations:

$$j\omega\kappa_i p_i = -\nabla\cdot\mathbf{v}_i \tag{1a}$$

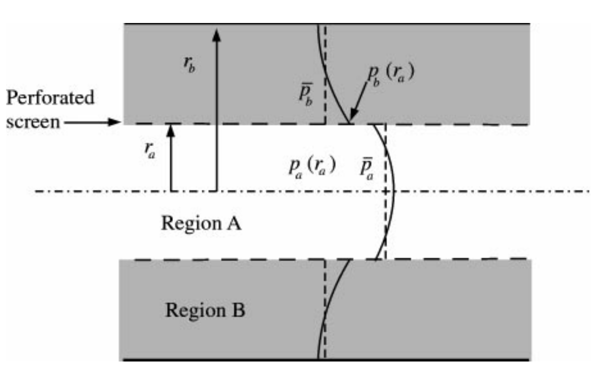

Figure 1. Typical geometry and pressure profile.

$$j\omega \rho_i \mathbf{v}_i = -\nabla p_i \tag{1b}$$

where $i = a$ or b.

If there is no screen, the radial velocity and the pressure are continuous at $r = r_a$. With a perforated screen (the impedance of the screen is z_s), it can be assumed that the radial velocity is still continuous but the pressure jumps from $p_b(r_a)$ to $p_a(r_a)$ with $p_b(r_a) - p_a(r_a) = z_s v_a(r_a) = z_s v_b(r_a)$.

By integrating equation (1a) and projecting equation (1b) on the axial direction, the following averaged equations are obtained:

$$Y_a \bar{p}_a = -\frac{dU_a}{dx} - q, \qquad Z_a U_a = -\frac{d\bar{p}_a}{dx}, \tag{2a}$$

$$Y_b \bar{p}_b = -\frac{dU_b}{dx} + q, \qquad Z_b U_b = -\frac{d\bar{p}_b}{dx}. \tag{2b}$$

where $Y_i = j\omega\kappa_i S_i$ is the admittance per unit length, $Z_i = j\omega\rho_i/S_i$ is the impedance per unit length, $\bar{p}_i$ is the mean pressure over the section S_i ($S_a = \pi r_a^2$ and $S_b = \pi(r_b^2 - r_a^2)$), U_i is the acoustical flow rate over the section S_i (integral of axial velocity over the section) and $q = 2\pi r_a v_a(r_a)$ is the flow rate per unit length through the perforated screen.

The flow rate q is assumed to be linearly related to the difference in mean pressure between the two regions: $q = Y(\bar{p}_a - \bar{p}_b)$. Thus, with an axial dependence of the form e^{-jkx}, two equations for the mean pressures can be found:

$$\begin{bmatrix} k^2 + \Gamma_a^2 + Z_a Y & -Z_a Y \\ -Z_b Y & k^2 + \Gamma_b^2 + Z_b Y \end{bmatrix} \begin{pmatrix} \bar{p}_a \\ \bar{p}_b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{3}$$

where the propagation constants Γ_a and Γ_b in regions A and B are given by $\Gamma_i^2 = Z_i Y_i$ ($i = a, b$).

2.2. DETERMINATION OF THE WAVENUMBERS

The determinant of system (3) must vanish in order to have non-trivial solutions. This gives the eigenequation for the wavenumber k:

$$k^4 + 2(\Gamma_m^2 + Z_m Y)k^2 + (\Gamma_m^2)^2 - \tfrac{1}{4}(\Delta\Gamma^2)^2 + 2(Z_m\Gamma_m^2 - \tfrac{1}{4}\Delta Z \Delta\Gamma^2)Y = 0, \tag{4}$$

where $\Gamma_m^2 = (\Gamma_a^2 + \Gamma_b^2)/2$, $\Delta\Gamma^2 = \Gamma_a^2 - \Gamma_b^2$, $Z_m = (Z_a + Z_b)/2$ and $\Delta Z = Z_a - Z_b$.

The solutions are written

$$k_1^2 = -\Gamma_m^2 - Z_m Y(1 - A),$$

$$k_2^2 = -\Gamma_m^2 - Z_m Y(1 + A),$$

where

$$A = \left(1 + \frac{\Delta\Gamma^2}{2Z_m^2 Y}\left(\frac{\Delta\Gamma^2}{2Y} + \Delta Z\right)\right)^{1/2}. \tag{5}$$

The values of k_1 and k_2 are chosen so that their imaginary parts are negative (obviously, $-k_1$ and $-k_2$ are also solutions of the eigenequation). By substituting k by k_1 and k_2 in equation (3), it can be seen that the mean pressures and the acoustical flow rates in the two regions are related for the first solution by

$$\bar{p}_{b1} = m\bar{p}_{a1} \quad \text{and} \quad Z_b U_{b1} = m Z_a U_{a1}$$

and for the second solution by

$$U_{b2} = -\, m U_{a2} \quad \text{and} \quad m Z_a \bar{p}_{b2} = -\, Z_b \bar{p}_{a2}$$

where

$$m = \frac{(\Delta\Gamma^2 + \Delta Z Y + 2Z_m A Y)}{2Z_a Y}. \tag{6}$$

When $\Delta\Gamma^2$ is equal to zero (i.e., no porous material in the region B), the coefficient m is equal to 1. In this case, the first solution is exactly a plane wave ($\bar{p}_{b1} = \bar{p}_{a1}$) and the second solution corresponds to a transverse wave with a zero total acoustical flow rate ($U_{a2} + U_{b2} = 0$). It means that the mass transmission along the duct is only connected to the plane wave.

When the porous material in region B is not too resistive (i.e., when $\Delta\Gamma^2 \ll \Gamma_m^2$), m is close to 1. In this case, the first wave is not exactly a plane wave ($\bar{p}_{b1} - \bar{p}_{a1} = (m-1)\bar{p}_{a1} \neq 0$, it becomes a quasi-planar wave) and mass will be transmitted by the second wave ($U_{a2} + U_{b2} = (1-m) U_{a2} \neq 0$).

2.3. DETERMINATION OF THE EQUIVALENT ADMITTANCE

The remaining problem is the determination of the admittance

$$Y = \frac{2\pi r_a v_a(r_a)}{(\bar{p}_a - \bar{p}_b)} = \frac{2\pi r_a}{(\mathrm{j}\omega\rho_a\delta_a + z_s + \mathrm{j}\omega\rho_b\delta_b)},$$

where [6]

$$\delta_a = \frac{(\bar{p}_a - p_a(r_a))}{\mathrm{j}\omega\rho_a v_a(r_a)}, \quad \delta_b = \frac{(p_b(r_a) - \bar{p}_b)}{\mathrm{j}\omega\rho_b v_b(r_a)}.$$

To compute the coefficients δ_a and δ_b, the shape of the pressure and of the radial velocity have to be known in regions A and B. For that purpose, approximate profiles of velocity and pressure are needed in the two regions. For the problem studied in this paper (area expansion), a good approximation of the acoustic field downstream from an expansion is needed. Thus the approximate profiles are chosen with this aim in mind.

In region A, the radial velocity vanishes when $r = 0$ and is supposed to increase linearly. Thus, $v_a(r)$ is approximated by $v_a(r) = Ar$. By integration of the radial projection of equation (1b), the pressure $p_a(r)$ is given by $p_a(r) = p_{0a} - \mathrm{j}\omega\rho_a A r^2/2$. In this approximation, the coefficient δ_a can easily be computed and $\delta_a = r_a/4$.

In region B, the radial velocity is chosen (i) to fulfill the boundary condition at the outer wall: $v_b(r_b) = 0$; (ii) to fit the radial velocity just behind an expansion with a large area ratio. Thus the axial velocity is taken in the form $v_b(r) = B(1/r_b^2 - 1/r^2)$ [7] and, by integration, the pressure is $p_b(r) = p_{0b} - \mathrm{j}\omega\rho_b B(r/r_b^2 + 1/r)$. With this approximate shape, the coefficient δ_b is equal to $\delta_b = r_a f(\alpha)$ with $\alpha = r_a/r_b$ and

$$f(\alpha) = \frac{(1-\alpha)(3+\alpha)}{3(1+\alpha)^2}.$$

It can be noted that for moderate area ratio ($\alpha \sim 0{\cdot}5$) a linear radial velocity profile in region B give a value of δ_b very close to the value obtained with the chosen profile. Then the admittance Y is written

$$Y = \frac{2\pi}{j\omega\rho_a}\left[\frac{1}{4} + \frac{\rho_b}{\rho_a}f(\alpha) + \frac{z_s}{j\omega\rho_a r_a}\right]^{-1}. \tag{7}$$

2.4. MODEL WITHOUT POROUS MATERIAL

In this sub-section, region B is filled with the same fluid as region A. The sound velocity in the fluid is c_0 and ρ_0 is the density. Then the admittances per unit length are $Z_a = j\omega\rho_0/S_a$ and $Z_b = j\omega\rho_0/S_b$. The propagation constants are $\Gamma_a = \Gamma_b = \Gamma_m = -j\omega/c_0$ and $\Delta\Gamma^2 = 0$. The wavenumbers are

$$k_1 = j\Gamma_m = \frac{\omega}{c_0} \tag{8}$$

and

$$k_2 = (-\Gamma_m^2 - 2Z_m Y)^{1/2} = \left(\left(\frac{\omega}{c_0}\right)^2 - j\omega\rho_0\left(\frac{1}{S_a} + \frac{1}{S_b}\right)Y\right)^{1/2}. \tag{9}$$

It can easily be seen that $\bar{p}_{a1} = \bar{p}_{b1}$. Thus, the first solution corresponds to the classical plane wave in the duct and is not influenced by the value of the admittance Y. For the second solution, the mean pressures in both regions are related by $S_a\bar{p}_{a2} = -S_b\bar{p}_{b2}$.

Equation (9) is given, for instance, by Pierce [8] when there is a perforated tube between the two regions and can be deduced from the results of Kergomard *et al.* [9] in the case of a perforated tube modelled discretely. The main difference is that the model presented here takes into account, in an approximate form, the difference between the actual pressure and the mean pressure in the two regions. This difference is included in the admittance Y. Thus, this model is also valid when the screen impedance goes to zero.

When the perforated screen is absent $z_s = 0$, the admittance Y is given by

$$Y = \left(\frac{j\omega\rho_0}{2\pi}\left(\frac{1}{4} + f(\alpha)\right)\right)^{-1}$$

and the wave number of the second solution is

$$k_2 = \left(\left(\frac{\omega}{c_0}\right)^2 - \frac{\gamma^2}{r_b^2}\right)^{1/2} \quad \text{where} \quad \gamma^2 = \frac{2}{\alpha^2(1-\alpha^2)(1/4 + f(\alpha))}.$$

This second mode is evanescent as long as the pulsation frequency ω is lower than $\omega_c = \gamma c_0/r_b$. For $\alpha = 0{\cdot}5$, the value of ω_c is $4{\cdot}43\ c_0/r_b$ which is reasonably close to the cut-off pulsation frequency $3{\cdot}83\ c_0/r_b$ for the second axisymmetric mode of a circular duct with radius r_b. This comes from the similarity of the pressure profile of the second approximate solution and of the pressure profile of the exact second mode with $\alpha = 0{\cdot}5$ (see Figure 2a). When α is much smaller than 1 (see Figure 2b), those profiles are very different. It can be seen from this figure that the second approximate solution is especially adapted to take into account most of the effects of the higher order modes near a sudden expansion.

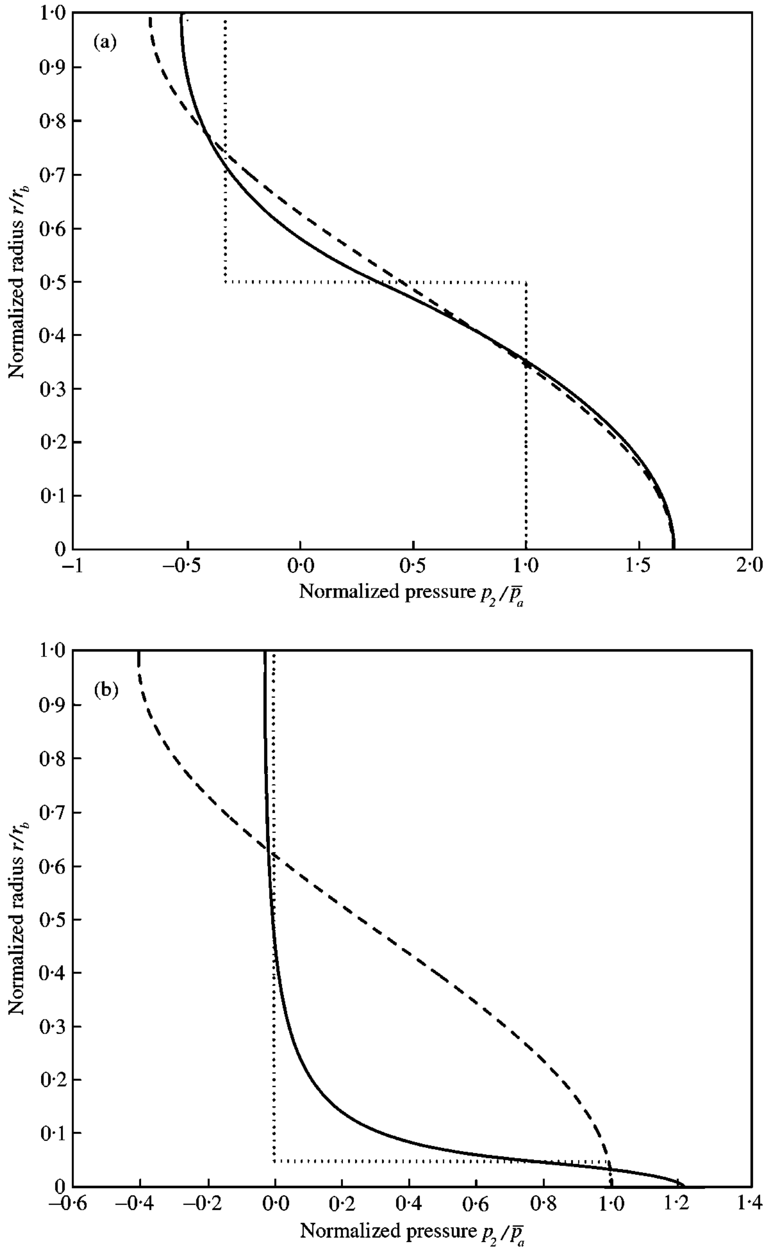

Figure 2. Normalized pressure for the second solution $p_2/\bar{p}_a$ as a function of the normalized radius r/r_b with $\alpha = 0{\cdot}5$ (a) and $\alpha = 0{\cdot}05$ (b); continuous line: approximate solution, dotted line: mean pressure in regions A and B; dashed line: second axisymmetric mode.

3. SUDDEN EXPANSION AT LOW FREQUENCIES

3.1. TRANSFER MATRIX OF A SUDDEN EXPANSION

Two semi-infinite ducts of radius r_a and r_b are joined at $x = 0$. In the small duct ($x < 0$, radius r_a) only a plane wave propagates: $p_0 = p_0^+ \, \mathrm{e}^{-\mathrm{j}k_0 x} + p_0^- \mathrm{e}^{\mathrm{j}k_0 x}$ where $k_0 = \omega/c_0$. This wave induces an acoustical flow rate equal to U_0 at $x = 0$ with $Z_a U_0 = \mathrm{j}k_0(p_0^+ - p_0^-)$. In the lined duct ($x > 0$, radius $r_b > r_a$), the model proposed in section 2 is applied when the frequency is lower than ω_c. In the infinite lined duct, there is no incoming evanescent mode (subscript 2). Thus the wave in this duct can be seen as the sum of three terms with an axial dependence $\mathrm{e}^{-\mathrm{j}k_1x}$, $\mathrm{e}^{\mathrm{j}k_1x}$ and $\mathrm{e}^{-\mathrm{j}k_2x}$ (the imaginary part of $k_{1,2}$ having been taken as negative). Accordingly, the mean pressures in regions A and B of the lined duct are written:

$$\bar{p}_a = p_1^+ \mathrm{e}^{-\mathrm{j}k_1x} + p_1^- \mathrm{e}^{\mathrm{j}k_1x} + p_2^+ \mathrm{e}^{-\mathrm{j}k_2x}$$

$$\bar{p}_b = mp_1^+ \mathrm{e}^{-\mathrm{j}k_1x} + mp_1^- \mathrm{e}^{\mathrm{j}k_1x} - \frac{Z_b}{mZ_a} p_2^+ \mathrm{e}^{-\mathrm{j}k_2x}$$

where m is given by equation (6).

The boundary conditions at $x = 0$ are the continuity of the mean pressure and flow rate for $r < r_a$ and vanishing of the flow rate for $r_a < r < r_b$:

$$p_0^+ + p_0^- = p_1^+ + p_1^- + p_2^+, \tag{10}$$

$$\mathrm{j}k_0(p_0^+ - p_0^-) = \mathrm{j}k_1(p_1^+ - p_1^-) + \mathrm{j}k_2 p_2^+, \tag{11}$$

$$0 = \mathrm{j}k_1 m(p_1^+ - p_1^-) - \mathrm{j}k_2 \frac{Z_b}{mZ_a} p_2^+. \tag{12}$$

From equations (11) and (12), a continuity of volume velocity can be written

$$U_0 = U_1, \tag{13}$$

where $U_0 = (p_0^+ - p_0^-)/z_{c0}$,

$$z_{c0} = \frac{Z_a}{\mathrm{j}k_0} = \frac{\rho_0 c_0}{S_a}$$

and $U_1 = (p_1^+ - p_1^-)/z_{c1}$, where

$$z_{c1} = \frac{Z_a Z_b}{\mathrm{j}k_1(Z_b + m^2 Z_a)}.$$

It can be noticed that U_0 is the real volume velocity in the small tube but that U_1 is not, for $m \neq 1$, the total volume velocity of the first mode in the large tube, as part of the acoustic mass flow for $m \neq 1$ is transmitted by the transverse mode.

The relation between the mean pressure in the small tube $p_0 = p_0^+ + p_0^-$ and the mean pressure associated with the quasi-planar mode in region A of the lined duct $p_1 = p_1^+ + p_1^-$ is

$$p_1 = p_0 - z_{add} U_0 \tag{14}$$

where

$$z_{add} = \frac{m^2 Z_a/Z_b}{(1 + m^2 Z_a/Z_b)} \frac{Z_a}{\mathrm{j}k_2}.$$

Equations 13 and 14 lead to the transfer matrix for the plane waves

$$\begin{pmatrix} p_1 \\ U_1 \end{pmatrix} = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} p_0 \\ U_0 \end{pmatrix}. \tag{15}$$

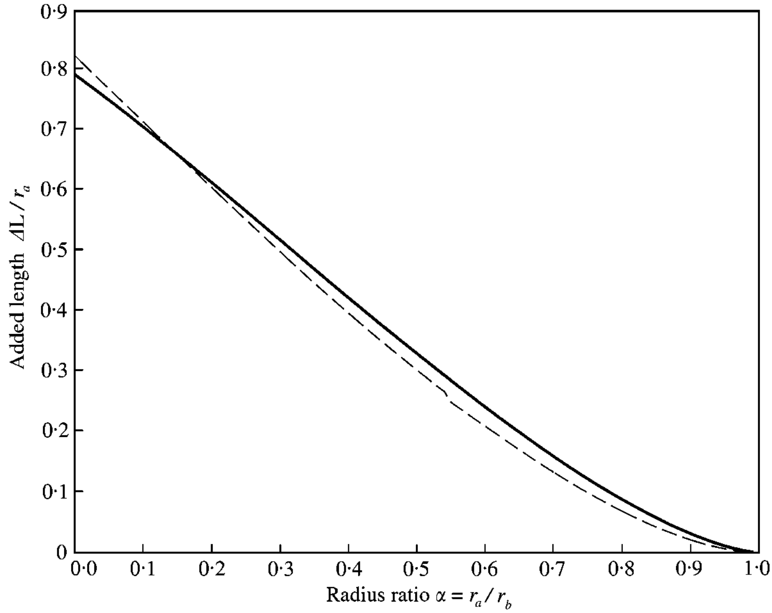

Figure 3. Variation of the added length $\Delta L/r_a$ at the zero frequency limit as a function of the radius ratio $\alpha = r_a/r_b$; continuous line: equation (16), dashed line: reference [10].

With this approximate model, the complex acoustic phenomena in the expansion linked to the interaction of planar and transverse wave is simply described by introducing an additional impedance at the entrance of the sudden expansion.

3.2. EXPANSION WITHOUT POROUS MATERIAL

When region B is filled with the same fluid as region A, the continuity of volume velocities between the plane modes on both sides of the expansion ($U_0 = U_1$) can be applied in this approximate model. It can be noted that this relation is also verified in the exact model (see for instance reference [10]).

From equation (14), the plane mode pressures in the two ducts can be related by $p_1 = p_0 - \mathrm{j}\omega\rho_0 \Delta L(\omega) u_0$ where u_0 is the acoustic velocity of the plane mode in the small tube $u_0 = U_0/S_a$ and $\Delta L(\omega)$ is the added length given by

$$\Delta L = \frac{z_{add} S_a}{\mathrm{j}\omega\rho_0} = \frac{(1-\alpha^2)}{\mathrm{j}k_2},$$

where $\mathrm{j}k_2$ is a real and positive number in the absence of a resistive screen (real part of $z_s = 0$).

Without any screen and when $\omega \to 0$, this added length tends toward

$$\Delta L = r_a \frac{1-\alpha^2}{\alpha\gamma} = r_a \left(\frac{(1-\alpha)^2 (1-\alpha^2)(15 - 2\alpha - \alpha^2)}{24} \right)^{1/2}. \tag{16}$$

This result is compared in Figure 3 with the formulae for the added length given in reference [10] with a precision of 0·1% for the zero-frequency limit. The agreement is good over all the α range.

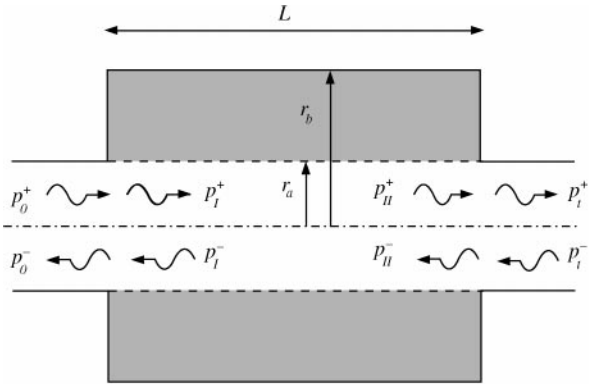

Figure 4. Geometry of the lined expansion chamber.

Thus, this very simple model allows a good approximation of the acoustic behaviour of a sudden expansion at low frequencies.

4. DISSIPATIVE SILENCER

4.1. TRANSFER MATRIX OF AN EXPANSION CHAMBER

An expansion chamber of length L is filled with porous material for $r_b > r > r_a$. This chamber is connected with two pipes of radius r_a (see Figure 4). The approximate model developed above is applied to this chamber. The wave in the lined chamber can be seen as the sum of four terms with an axial dependence $\mathrm{e}^{-\mathrm{j}k_1 x}$, $\mathrm{e}^{\mathrm{j}k_1 x}$, $\mathrm{e}^{-\mathrm{j}k_2 x}$, and $\mathrm{e}^{\mathrm{j}k_2 x}$. The boundary conditions are the vanishing of the mean axial velocity in the porous media at $x = 0$ and L and the continuity of the mean velocity and pressure in zone A at $x = 0$ and L. Thus, the problem can be completely solved in the context of the present approximate model. Nevertheless, significant simplifications appear when the amplitude of the most attenuated mode (wavenumber k_2) created at one side of the chamber can be neglected when it reaches the other side (i.e. $|\mathrm{e}^{-\mathrm{j}k_2 L}| \ll 1$). This assumption is true for most practical applications. In this case, the transfer matrix of an expansion chamber can be seen as the product of the transfer matrices of an expansion, $\mathscr{T}_e$, of quasi plane wave propagation from 0 to L, $\mathscr{T}_p$, and of a contraction, T_c, where

$$\begin{pmatrix} p_I \\ U_I \end{pmatrix} = \mathscr{T}_e \begin{pmatrix} p_0 \\ U_0 \end{pmatrix} \quad \text{with} \quad \mathscr{T}_e = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix},$$

$$\begin{pmatrix} p_{11} \\ U_{11} \end{pmatrix} = \mathscr{T}_p \begin{pmatrix} p_I \\ U_I \end{pmatrix} \quad \text{with} \quad \mathscr{T}_p = \begin{bmatrix} \cosh(\mathrm{j}k_1 L) & -z_{c1}\sinh(\mathrm{j}k_1 L) \\ -\sinh(\mathrm{j}k_1 L)/z_{c1} & \cosh(\mathrm{j}k_1 L) \end{bmatrix}$$

and

$$\begin{pmatrix} p_t \\ U_t \end{pmatrix} = \mathscr{T}_c \begin{pmatrix} p_{II} \\ U_{II} \end{pmatrix} \quad \text{with} \quad \mathscr{T}_c = \begin{bmatrix} 1 & -z_{add} \\ 0 & 1 \end{bmatrix}.$$

The transfer matrix of the chamber can be written:

$$\begin{pmatrix} p_t \\ U_t \end{pmatrix} = \mathscr{T}_c \mathscr{T}_p \mathscr{T}_e \begin{pmatrix} p_0 \\ U_0 \end{pmatrix} = \begin{bmatrix} A & (A^2-1)/C \\ C & A \end{bmatrix} \begin{pmatrix} p_0 \\ U_0 \end{pmatrix}, \tag{17}$$

where $A = \cosh(jk_1 l) + z_{add}\sinh(jk_1 l)/z_{c1}$ and $C = -\sinh(jk_1 l)/z_{c1}$. The transmission and reflection coefficients of this chamber are

$$T = \frac{-2Z_{c0}C}{((Z_{c0}C - A)^2 - 1)} \quad \text{and} \quad R = \frac{(A^2 - (Z_{c0}C)^2 - 1)}{((Z_{c0}C - A)^2 - 1)}.$$

4.2. EXPERIMENTAL VALIDATION

The transmission and reflection coefficients of an expansion chamber with porous material were investigated experimentally in order to test the validity of the approximate model. The chamber was inserted in a tube in which linear acoustical planar waves were excited. On one side (side 0, see Figure 4), an acoustic source provided a wave in the frequency range 50–1500 Hz. The acoustic pressure in tube 1 is written as $p_0 = p_0^+ e^{-j\bar{k}_0 x} + p_0 e^{-j\bar{k}_0 x}$ where p_0^+ and p_0^- are the incident and reflected pressures on side 0, $\tilde{k}_0$ is the wave number in the tube taking into account the viscothermal attenuation and x is the axial distance from side 0 of the chamber. Four microphones in tube 0 allow an overdetermined estimation of p_0^+ and p_0^-. The overdetermination is used, with a least-squares method, to increase the accuracy of the experimental results. On the other side (side t), four other microphones are used so that the transmitted pressure p_t^+ and the pressure reflected from the tube termination p_t^- can be determined on side t of the chamber.

Reciprocity and symmetry of the measured element imply that the transmission T and reflection R coefficients satisfy $p_t^+ = Tp_0^+ + Rp_t^-$ and $p_0^- = Tp_t^- + Rp_0^+$. Thus, the coefficients

$$R = \frac{p_0^+ p_0^- - p_t^+ p_t^-}{p_0^{+2} - p_t^{-2}} \quad \text{and} \quad T = \frac{p_0^+ p_t^+ - p_0^- p_t^-}{p_0^{+2} - p_t^{-2}}$$

can be computed from the microphone data as functions of the frequency:

For the given expansion chamber the inner radius is $r_a = 15$ mm, the outer radius is $r_b = 47$ mm and the length is $L = 360$ mm. The porous material is a mineral wool whose acoustical parameters have been measured elsewhere. The values of the parameters (see Appendix A) are: porosity $\Phi = 0{\cdot}99$, tortuosity $\alpha_\infty = 1{\cdot}1$, resistivity $\sigma = 75\,000\ \mathrm{kg\,m^{-3}\,s^{-1}}$, viscous and thermal characteristic lengths $\Lambda = 1 \times 10^{-4}$ m and $\Lambda' = 2 \times 10^{-4}$ m.

The mineral wool is known to be anisotropic. The resistivity perpendicular to the fibres (radial direction) is bigger than the resistivity along the fibers (axial direction). The ratio between the axial and radial resistivity is chosen to be 0·7 in accordance with reference [11]. The anisotropy can be easily introduced in the approximate model. The effective density to be used depends on the direction on which the Euler equation is projected. The Euler equation in the radial direction only appears in the determination of the equivalent admittance Y. Thus, the effective density in equation (7) is computed with the radial parameters and the other densities are computed with the axial parameters.

The results, measured (circles) and predicted (using the approximate model, continuous lines), are shown in Figure 5. The agreement is very good except on the absolute value of the transmission when the frequency is above 1 kHz. It should be noted that the attenuation is of the order of 60 dB in this region. Some experimental problems, like flanking transmission through the external tube of the chamber, could explain this discrepancy. For comparison, the results for an empty chamber are given by the dashed lines.

Nevertheless, it can be concluded that the approximate model gives an accurate description of the performance of the dissipative silencer.

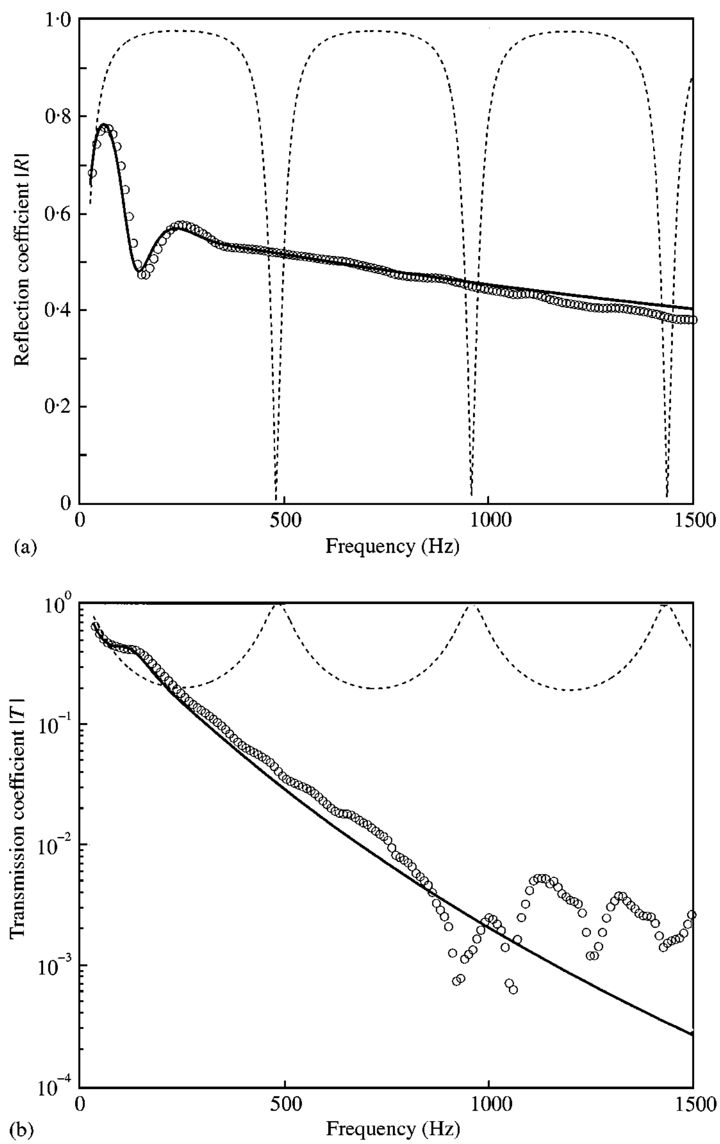

Figure 5. Absolute value of the reflection (a) and transmission (b) coefficients of an expansion chamber with porous material. ○: measurements, continuous line: approximate model, dashed line: results for the same chamber without porous material.

5. CONCLUSIONS

A method based on an approximation of the radial pressure profile is developed to analyze the acoustic performance of a sudden expansion and of an expansion chamber at low frequencies. This model is able to predict very accurately the added length of an expansion where there is neither porous material nor perforated tube. It can also be applied when a bulk-reacting lining and perforated tube are included. This model of a lined expansion chamber gives results which compare very favourably with experimental data. This approach could be extended to the case where a flow is present [12] and it can describe the appearance of hydrodynamic modes in this case.

Owing to its simplicity, this approximate model of a dissipative silencer could be easily implemented as a predictive tool for computing the acoustic performances in flow duct acoustics.

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APPENDIX A

In rigid-framed porous materials, the linear sound propagation can be described by means of using an equivalent fluid with an effective density and an effective compressibility which are complex values depending on the frequency [see, for e.g., references [13, 14]]. The continuity and Euler equations are written as

$$\kappa_e \frac{\partial p}{\partial t} = - \nabla \cdot \mathbf{v},$$

$$\rho_e \frac{\partial \mathbf{v}}{\partial t} = - \nabla p,$$

where p and $\mathbf{v}$ are the macroscopic acoustic pressure and velocity (the macroscopic velocity is chosen as the continuity of velocity applies at an interface between the porous material and air).

The effective characteristics of the material κ_e and ρ_e can be obtained with the help of six parameters: the porosity Φ, the tortuosity α_∞, the viscous and thermal permeabilities k_0 and k'_0, and the viscous and thermal characteristic lengths Λ and Λ'. The effective density is equal to

$$\rho_e = \frac{\rho_0 \alpha_\infty}{\Phi}\left(1 + \frac{1}{\mathrm{j}x}\left[1 + \frac{M}{2}\,\mathrm{j}x\right]^{1/2}\right),$$

where the reduced frequency x is given by

$$x = \frac{\omega \alpha_\infty k_0}{v\Phi}$$

and the shape factor is

$$M = \frac{8 k_0 \alpha_\infty}{\Phi \Lambda^2},$$

where ρ_0 is the density and v is the kinematic viscosity of the fluid. The effective compressibility is equal to

$$\kappa_e = \frac{\Phi}{\rho_0 c_0^2}\left(\gamma - (\gamma - 1)\left(1 + \frac{1}{\mathrm{j}x'}\left[1 + \frac{M'}{2}\,\mathrm{j}x'\right]^{1/2}\right)^{-1}\right)$$

where the reduced frequency x' is

$$x' = \frac{\omega P_r k'_0}{v\Phi}$$

and the shape factor is

$$M' = \frac{8 k'_0}{\Phi \Lambda'^2},$$

where c_0 is the sound velocity and P_r is the Prantl number in the fluid. In this paper, the static thermal permeability is approximated [14] by $k'_0 = \Phi \Lambda'^2/8$ and the static viscous permeability k_0 is related to the air flow resistivity by $\sigma = \rho_0 v/k_0$.