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A SYMMETRICAL FINITE ELEMENT MODEL FOR STRUCTURE–ACOUSTIC COUPLING ANALYSIS OF AN ELASTIC, THIN-WALLED CAVITY

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This paper sets up a symmetrical finite element model for structure–acoustic coupling analysis of an elastic, thin-walled cavity excited both by interior acoustic sources and exterior structural loading. Some relative problems on the symmetrical model, i.e., the non-negativity conditions of eigenvalues, the reduced-order modelling conditions of eigenvalue problem, and the unavailability of the state space method to this model in complex modal analysis, are discussed. Finally, in order to verify the feasibility and correctness of the symmetrical method given in this paper, a calculating example which possesses a rectangular section cavity is given, and the calculated results are compared with the measured ones.

1. INTRODUCTION

An elastic, thin-walled cavity is an important kind of engineering structure, for example, the passenger compartments of automobiles, trains, ships, airplanes, etc. Along with the increasing desire for comfort of the vehicles, the noise in their passenger compartments has been paid more and more attention recently. More detailed research work has now revealed that the low-frequency noise generated in the passenger compartment of a vehicle significantly influences the overall passenger comfort [1–5]. The coupled structure–acoustic vibration problem in low-frequency range of the elastic, thin-walled cavity actually is the crux of the low-noise design or noise control for the vehicles' passenger compartments.

The coupled structure–acoustic vibration problem of the elastic, thin-walled cavity can be dealt with in several ways, such as the finite element method, the boundary element method, the elastic acoustics theory, and the statistical energy analysis, etc. Currently, the fundamental principle of the finite element method has been grasped and generally accepted by most engineers and technicians. A lot of commercial software packages developed for all-purpose finite element analyses can be obtained easily in practical application. This makes the finite element method a relatively simple technique. On the other hand, the finite element method has the internal connecting link with the modal analysis method. At the low frequencies in question, both structural and acoustic modal densities are relatively low, and so are the modal densities of the coupled structure–acoustic vibration system. Therefore, the finite element method is extremely attractive in low-frequency range. But the commonly used finite element model for structure–acoustic coupling analysis is a non-symmetrical set of formulas which choose the structural displacement and the sound pressure as the variables [5, 6]. This is undesirable because it will cause additional error in

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the successive analysis or solving process. Concretely, a realistic but important reason is that only the standard modules based on symmetrical modes have been provided in numerous commercial software packages developed for all-purpose finite element analyses at present. In order to study the coupled structure–acoustic problems by means of the all-purpose finite element analysis software packages, one has to uncouple the non-symmetrical, coupled structure–acoustic finite element model into two symmetrical ones corresponding to the uncoupled structural and acoustic systems, respectively. The characteristics and response of the coupled system can be deduced from the uncoupled systems by means of the modal coupling analysis. The effective uncoupling approach often used here is to ignore the sound pressure put on the structure surface by its interior acoustic space, thus causing the so-called "uncoupling error". Although some software packages can directly provide the processing modules for the non-symmetrical model, the fundamentals of this kind of modules are still based on the uncoupling approach. Obviously, the best way to avoid the uncoupling error is to convert the non-symmetrical, coupled structure–acoustic finite element model into a symmetrical one which can be directly treated with the all-purpose finite element analysis software packages rather than being uncoupled.

The symmetrical finite element model can be obtained by means of introducing a certain fluid potential function as an addition variable [6–10]. Everstine suggested introducing the acoustic velocity potential in addition to the sound pressure as variables in the fluid domain, but the symmetrical model obtained has the drawback of bringing a formal damping matrix though the system is conservative [8]. Ohayon also obtained a symmetrical model with the method of introducing the acoustic displacement potential instead of the acoustic velocity potential, which overcame the drawback of Everstine's model. However, Ohayon's model requires that the number of variables in the field of acoustic displacement potential must be equal to that in the field of sound pressure so that the degrees of freedom in the fluid domain were doubled [9]. Sandberg *et al.* ameliorated Ohayon's approach and gave a symmetrical model in which the number of variables in the field of acoustic displacement potential need not be equal to that in the field of sound pressure any longer. Also, static condensation may be applied to the fluid domain to reduce the degrees of freedom considerably [10]. Nevertheless, Sandberg's model cannot take the excitation of acoustical source into account because the conservation equations of mass and momentum in the fluid domain were introduced into the deducing course.

When one studies the problems of the low-noise design and noise control of a structure, the system response to acoustical sources and structural loading excitations have to be considered sometimes. This paper, consequently, sets up a symmetrical finite element model for the structure–acoustic coupling analysis of an elastic, thin-walled cavity excited by both interior acoustical sources and exterior structural loading. The model given here reserves the virtues of Sandberg's model though both the approaches are different. It should be pointed out that the mass and the stiffness matrices in the symmetrical model set up in this paper lack the good character of being positive definite (the same as those in Sandberg's model) so that some new problems involved in modal analysis come forth, which are also studied in this paper.

2. FINITE ELEMENT MODELS

2.1. NON-SYMMETRICAL MODEL

The wave equation in the area of acoustic source in the fluid domain is [11]

$$\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} + \rho_f \frac{\partial q}{\partial t} = 0, \tag{1}$$

where ∇^2 is the Laplace operator, p the sound pressure, c the speed of sound in the fluid medium, t the time, ρ_f the density of fluid medium, and q the distribution of the volume velocity in the area of acoustic source, which becomes zero outside the source area. For an elastic, thin-walled cavity, the boundary condition is

$$\frac{\partial p}{\partial \mathbf{n}} = -\rho_f \frac{\partial^2 w}{\partial t^2}, \tag{2}$$

where $\mathbf{n}$ is the outward unit normal vector at the fluid–structure interface, and w the normal displacement of the walls of the enclosed cavity.

By using the finite element method, the fluid and the structural domains can be divided into elements. Expanding p and w in different sets of sound pressure and structural displacement interpolation functions, i.e., $\{\mathbf{N}_p\}$ and $\{\mathbf{N}_\delta\}$, one has

$$p = \{\mathbf{N}_p\}\{\mathbf{p}\}^{(e)}, \tag{3}$$

$$w = \{\mathbf{N}_\delta\}\{\delta\}^{(e)}, \tag{4}$$

where $\{\mathbf{p}\}^{(e)}$ is the unknown vector of the sound pressure at the nodes of the fluid element, and $\{\delta\}^{(e)}$ is that of the structural displacement at the nodes of the structural element.

Substituting equations (2)–(4) into equation (1), one can get a set of finite element formulas in the fluid domain with the Galerkin method which takes $\{\mathbf{N}_p\}$ as the weighting function set:

$$[\mathbf{G}]\{\ddot{\mathbf{p}}\} + [\mathbf{H}]\{\mathbf{p}\} = \{\mathbf{I}_q\} - [\mathbf{A}]\{\ddot{\delta}\}, \tag{5}$$

where $[\mathbf{G}]$ is the acoustic mass matrix, $[\mathbf{H}]$ the acoustic stiffness matrix, $[\mathbf{A}]$ the coupling matrix, and $\{\mathbf{I}_q\}$ the vector for acoustic source excitation.

In the structural domain, a set of finite element formulas can be deduced with a relatively straightforward method based on the virtual work theory:

$$[\mathbf{M}_\delta]\{\ddot{\delta}\} + [\mathbf{K}_\delta]\{\delta\} = \{\mathbf{I}_s\} + \rho_f^{-1}[\mathbf{A}]^{\mathrm{T}}\{\mathbf{p}\}, \tag{6}$$

where $[\mathbf{M}_\delta]$ is the structural mass matrix, $[\mathbf{K}_\delta]$ the structural stiffness matrix, and $\{\mathbf{I}_s\}$ the vector for structural loading excitation.

Then, a finite element model of the structure–acoustic coupled system can be set up by means of rewriting equations (5) and (6) into an incorporate form:

$$\begin{bmatrix} [\mathbf{G}] & [\mathbf{A}] \\ [\mathbf{0}] & [\mathbf{M}_\delta] \end{bmatrix} \begin{Bmatrix} \{\ddot{\mathbf{p}}\} \\ \{\ddot{\delta}\} \end{Bmatrix} + \begin{bmatrix} [\mathbf{H}] & [\mathbf{0}] \\ -\rho_f^{-1}[\mathbf{A}]^{\mathrm{T}} & [\mathbf{K}_\delta] \end{bmatrix} \begin{Bmatrix} \{\mathbf{p}\} \\ \{\delta\} \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{I}_q\} \\ \{\mathbf{I}_s\} \end{Bmatrix}. \tag{7}$$

Due to the existence of the coupling matrix $[\mathbf{A}]$, equation (7) is a non-symmetrical model. The elementary matrices corresponding to the submatrices in equation (7) are respectively:

$$[\mathbf{G}]^{(e)} = \int_{\Omega^{(e)}} \frac{1}{c^2}\{\mathbf{N}_p\}^{\mathrm{T}}\{\mathbf{N}_p\}\,\mathrm{d}\Omega, \tag{8}$$

$$[\mathbf{H}]^{(e)} = \int_{\Omega^{(e)}} \{\nabla\mathbf{N}_p\}^{\mathrm{T}}\cdot\{\nabla\mathbf{N}_p\}\,\mathrm{d}\Omega, \tag{9}$$

$$[\mathbf{M}_\delta]^{(e)} = \int_{V^{(e)}} \rho_s\{\mathbf{N}_\delta\}^{\mathrm{T}}\{\mathbf{N}_\delta\}\,\mathrm{d}V, \tag{10}$$

$$[\mathbf{K}_\delta]^{(e)} = \int_{V^{(e)}} \{\mathbf{N}_\delta\}^{\mathrm{T}}[\mathbf{B}]^{\mathrm{T}}[\mathbf{D}][\mathbf{B}]\{\mathbf{N}_\delta\}\,\mathrm{d}V, \tag{11}$$

$$[\mathbf{A}]^{(e)} = \int_{\Sigma_a^{(e)}} \rho_f \{\mathbf{N}_p\}^{\mathrm{T}}\{\mathbf{N}_\delta\}\,\mathrm{d}\Sigma, \tag{12}$$

$$\{\mathbf{I}_q\}^{(e)} = \int_{\Omega^{(e)}} \{\mathbf{N}_p\}^{\mathrm{T}} \rho_f \frac{\partial q}{\partial t}\,\mathrm{d}\Omega, \tag{13}$$

$$\{\mathbf{I}_s\}^{(e)} = \int_{\Sigma_b^{(e)}} \{\mathbf{N}_\delta\}^{\mathrm{T}} f\,\mathrm{d}\Sigma, \tag{14}$$

where ∇ is the nabla operator, ρ_s the density of the structure material, f the distribution of the exterior force loading on the structure, $[\mathbf{B}]$ the structural geometry matrix, $[\mathbf{D}]$ the structural elasticity matrix, $\Omega^{(e)}$ the volume of the fluid element, $V^{(e)}$ the volume of the structural element, $\Sigma_a^{(e)}$ the area of the coupled interface between the fluid and the structural elements, and $\Sigma_b^{(e)}$ the area of the structural element surface suffering the force f.

2.2. SYMMETRICAL MODEL

Introducing the acoustic displacement potential function ψ, one has

$$\mathbf{u}_f = \nabla\psi, \tag{15}$$

where $\mathbf{u}_f$ is the displacement of the fluid particle. Here, the boundary condition can be expressed as

$$\mathbf{u}_f \cdot \mathbf{n} = w. \tag{16}$$

The linearized momentum conservation equation in the fluid domain is

$$\rho_f \frac{\partial^2 \mathbf{u}_f}{\partial t^2} + \nabla p = \mathbf{0}. \tag{17}$$

Substituting equation (15) into equation (17), one has

$$\rho_f \frac{\partial^2(\nabla\psi)}{\partial t^2} + \nabla p = \mathbf{0}. \tag{18}$$

Equation (18) is operated with nabla operator ∇ and rewritten as

$$\nabla^2 p = -\rho_f \frac{\partial^2(\nabla^2\psi)}{\partial t^2}. \tag{19}$$

Substituting equation (19) into equation (1) and then integrating with respect to time t two times (the initial condition is neglected), the following equation can be deduced:

$$\nabla^2\psi + \frac{1}{\rho_f c^2} p - \int_0^t q\,\mathrm{d}t = 0. \tag{20}$$

Compartmentalizing the fluid domain again and expanding ψ in a set of acoustic displacement potential interpolation functions $\{\mathbf{N}_\psi\}$, one has

$$\psi = \{\mathbf{N}_\psi\}\{\psi\}^{(e)}, \tag{21}$$

where $\{\psi\}^{(e)}$ is the unknown vector of the acoustic displacement potential at the nodes of the fluid element corresponding to the second discretization of the fluid domain.

Introducing equations (3), (4), (16) and (21) into equation (20), one can deduce a set of finite element formulas different from equation (5) in the fluid domain with the Galerkin method in which $\{\mathbf{N}_p\}$ is still taken as the weighting function set:

$$[\mathbf{K}_p]\{\mathbf{p}\} = \{\mathbf{I}_a\} + [\mathbf{A}_{p\psi}]\{\psi\} - [\mathbf{A}_{p\delta}]\{\delta\}, \tag{22}$$

where $[\mathbf{K}_p]$ is the sound pressure stiffness matrix, $[\mathbf{A}_{p\psi}]$ the coupling matrix between p and ψ, and $[\mathbf{A}_{p\delta}]$ the coupling matrix between p and δ. It is noticed that the vector of acoustic source excitation in equation (22) is $\{\mathbf{I}_a\}$ which is unequal to $\{\mathbf{I}_q\}$ in equation (5).

Introducing equations (3) and (21), one can also derive another set of finite element formulas in the fluid domain based on equation (18) with the weighted residual method which takes $\{\nabla\mathbf{N}_\psi\}$ as the weighting function set:

$$[\mathbf{M}_\psi]\{\ddot{\psi}\} = -[\mathbf{A}_{p\psi}]^{\mathrm{T}}\{\mathbf{p}\}, \tag{23}$$

where $[\mathbf{M}_\psi]$ is the mass matrix about acoustic displacement potential.

The elementary matrices corresponding to the matrices in equations (22) and (23) are respectively:

$$[\mathbf{K}_p]^{(e)} = \int_{\Omega^{(e)}} \frac{1}{\rho_f c^2}\{\mathbf{N}_p\}^{\mathrm{T}}\{\mathbf{N}_p\}\,\mathrm{d}\Omega, \tag{24}$$

$$[\mathbf{A}_{p\psi}]^{(e)} = \int_{\Omega_a^{(e)}} \{\nabla\mathbf{N}_p\}^{\mathrm{T}}\cdot\{\nabla\mathbf{N}_\psi\}\,\mathrm{d}\Omega, \tag{25}$$

$$[\mathbf{A}_{p\delta}]^{(e)} = \int_{\Sigma_a^{(e)}} \{\mathbf{N}_p\}^{\mathrm{T}}\{\mathbf{N}_\delta\}\,\mathrm{d}\Sigma, \tag{26}$$

$$\{\mathbf{I}_a\}^{(e)} = \int_{\Omega^{(e)}} \{\mathbf{N}_p\}^{\mathrm{T}}\left(\int_0^t q\,\mathrm{d}t\right)\mathrm{d}\Omega, \tag{27}$$

$$[\mathbf{M}_\psi]^{(e)} = \int_{Q^{(e)}} \rho_f\{\nabla\mathbf{N}_\psi\}^{\mathrm{T}}\cdot\{\nabla\mathbf{N}_\psi\}\,\mathrm{d}Q, \tag{28}$$

where $Q^{(e)}$ is the volume of the fluid element corresponding to the second discretization of the fluid domain, and $\Omega_a^{(e)}$ the volume of the coupled space between the fluid elements corresponding to the first and the second discretization respectively.

Comparing equation (12) with equation (26), one has

$$[\mathbf{A}] = \rho_f[\mathbf{A}_{p\delta}]. \tag{29}$$

Substituting equation (29) into equation (6), then equations (6), (22) and (23) can be assembled as

$$\begin{bmatrix} [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{M}_\psi] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{0}] & [\mathbf{M}_\delta] \end{bmatrix} \begin{Bmatrix} \{\ddot{\mathbf{p}}\} \\ \{\ddot{\psi}\} \\ \{\ddot{\delta}\} \end{Bmatrix} + \begin{bmatrix} -[\mathbf{K}_p] & [\mathbf{A}_{p\psi}] & -[\mathbf{A}_{p\delta}] \\ [\mathbf{A}_{p\psi}]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{0}] \\ -[\mathbf{A}_{p\delta}]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{K}_\delta] \end{bmatrix} \begin{Bmatrix} \{\mathbf{p}\} \\ \{\psi\} \\ \{\delta\} \end{Bmatrix} = \begin{Bmatrix} -\{\mathbf{I}_a\} \\ \{\mathbf{0}\} \\ \{\mathbf{I}_s\} \end{Bmatrix}. \quad (30)$$

It is equation (30) that is a symmetrical finite element model for the structure–acoustic coupling analysis of an elastic, thin-walled cavity excited both by interior acoustic sources and exterior structural loading.

For convenient reasons, let

$$[\mathbf{M}] = \begin{bmatrix} [\mathbf{0}] & [\mathbf{0}] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{M}_\psi] & [\mathbf{0}] \\ [\mathbf{0}] & [\mathbf{0}] & [\mathbf{M}_\delta] \end{bmatrix}, \quad (31)$$

$$[\mathbf{K}] = \begin{bmatrix} -[\mathbf{K}_p] & [\mathbf{A}_{p\psi}] & -[\mathbf{A}_{p\delta}] \\ [\mathbf{A}_{p\psi}]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{0}] \\ -[\mathbf{A}_{p\delta}]^{\mathrm{T}} & [\mathbf{0}] & [\mathbf{K}_\delta] \end{bmatrix}, \quad (32)$$

$$\{\mathbf{f}\} = \{-\{\mathbf{I}_a\}^{\mathrm{T}} \quad \{\mathbf{0}\}^{\mathrm{T}} \quad \{\mathbf{I}_s\}^{\mathrm{T}}\}^{\mathrm{T}}, \quad (33)$$

$$\{\mathbf{x}\} = \{\{\mathbf{p}\}^{\mathrm{T}} \quad \{\psi\}^{\mathrm{T}} \quad \{\delta\}^{\mathrm{T}}\}^{\mathrm{T}}. \quad (34)$$

Then, equation (30) can be rewritten as

$$[\mathbf{M}]\{\ddot{\mathbf{x}}\} + [\mathbf{K}]\{\mathbf{x}\} = \{\mathbf{f}\}, \quad (35)$$

where $[\mathbf{M}]$ is the mass matrix and $[\mathbf{K}]$ is the stiffness matrix of the coupled system. Both of them are symmetrical, real-value matrices, but neither of them is positive definite. In fact, $[\mathbf{M}]$ is a positive semi-definite matrix and $[\mathbf{K}]$ is an indefinite matrix.

3. CHARACTERISTIC ANALYSES OF THE SYMMETRICAL MODEL

3.1. NON-NEGATIVITY CONDITIONS OF EIGENVALUES

The generalized eigenvalue problem corresponding to equation (35) is

$$([\mathbf{K}] - \lambda[\mathbf{M}])\{\mathbf{X}\} = \{\mathbf{0}\}, \quad (36)$$

where λ is the eigenvalue and $\{\mathbf{X}\}$ is the eigenvector of the system. The eigenequation is

$$\det([\mathbf{K}] - \lambda[\mathbf{M}]) = 0. \quad (37)$$

According to the algebraic theory, it is known that under the positive definite condition of matrix $[\mathbf{M}]$, all the eigenvalues are positive if matrix $[\mathbf{K}]$ is positive definite, and all the eigenvalues are non-negative if matrix $[\mathbf{K}]$ is positive semi-definite [12, 13]. Obviously, matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ in equation (36) do not satisfy the above conditions. Hence, the non-negativity conditions of the eigenvalues of the system must be investigated.

In fact, all the submatrices in matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ expressed, respectively, in equations (31) and (32) are sparse band matrices. So, equation (37) can be expanded to the following form if all the diagonal elements in submatrix $[\mathbf{K}_p]$ are non-zero:

$$(-\lambda m_{11}^{\psi})(-\lambda m_{22}^{\psi})\cdots(-\lambda m_{n_1 n_1}^{\psi})(k_{11}^{\delta}-\lambda m_{11}^{\delta})(k_{22}^{\delta}-\lambda m_{22}^{\delta})\cdots(k_{n_2 n_2}^{\delta}-\lambda m_{n_2 n_2}^{\delta})=0, \tag{38}$$

where, n_1 is the order of matrix $[\mathbf{M}_\psi]$, n_2 the order of matrix $[\mathbf{K}_\delta]$ and matrix $[\mathbf{M}_\delta]$, m_{ii}^{ψ}, k_{ii}^{δ} and m_{ii}^{δ} are, respectively, the diagonal elements of matrix $[\mathbf{M}_\psi]$, matrix $[\mathbf{K}_\delta]$ and matrix $[\mathbf{M}_\delta]$ corresponding to the order i.

At the same time, equation (38) is also the eigenequation of the following eigenvalue problem:

$$\left(\begin{bmatrix}[\mathbf{0}] & [\mathbf{0}]\\ [\mathbf{0}] & [\mathbf{K}_\delta]\end{bmatrix}-\lambda\begin{bmatrix}[\mathbf{M}_\psi] & [\mathbf{0}]\\ [\mathbf{0}] & [\mathbf{M}_\delta]\end{bmatrix}\right)\{\mathbf{Y}\}=\{\mathbf{0}\}. \tag{39}$$

That means the eigenvalues of equation (36) are equal to those of equation (39), but the eigenvectors of the two equations are different.

In equation (39), matrix $[\mathbf{K}_\delta]$ is positive definite or positive semi-definite, therefore, all the eigenvalues are non-negative if both of matrix $[\mathbf{M}_\psi]$ and matrix $[\mathbf{M}_\delta]$ are positive definite. It is these that are the non-negativity conditions of the eigenvalues of equation (36).

For the system expressed by equation (30), zero eigenvalues correspond to the constant sound pressure and acoustic displacement potential solution in a rigid wall cavity.

3.2. REDUCED ORDER MODELLING CONDITIONS OF EIGENVALUE PROBLEM

Generally, only some of the low order modes of a system in engineering application are taken into account. On the other hand, in order to decrease the calculation work, the order of a large eigenvalue problem must be reduced in practice. Presently, the primary reduced order modelling method is the Rayleigh–Ritz method and its ameliorations. The theoretic basis of the Rayleigh–Ritz method is that all the eigenvalues must be extrema of the Rayleigh quotient which can be defined as [14, 15]

$$\rho(\{\phi\})=\frac{\{\phi\}^{\mathrm{T}}[\mathbf{K}]\{\phi\}}{\{\phi\}^{\mathrm{T}}[\mathbf{M}]\{\phi\}}, \tag{40}$$

where $\{\phi\}$ is an arbitrary vector. Moreover, $[\mathbf{M}]$ and $[\mathbf{K}]$ in equation (40) should be symmetrical, positive definite matrices commonly.

Since both of matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ in equation (36) are symmetrical but not positive definite, one can reduce properly certain restrictions for matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ in formation of the Rayleigh quotient, namely, matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ in equation (40) ought to be symmetrical, and matrix $[\mathbf{M}]$ should be also non-singular. Therefore, the Rayleigh quotient can be constructed with matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$ in equation (36) if the singularity of matrix $[\mathbf{M}]$ can be removed with an appropriate shift technique [12–17]. In fact, the eigenvalues of all order modes of equation (36) are still extrema of the Rayleigh quotient treated with the method discussed here. This shows the possibility of the reduced order to the eigenvalue problem expressed with equation (36). The detailed proof is given as follows:

Assume that equation (36) has already been treated with a certain shift technique so that matrix $[\mathbf{M}]$ in it is non-singular now (the original symbols are still used here for convenience). Supposing that λ_i is the eigenvalue and $\{\mathbf{X}\}_i$ is the relevant eigenvector corresponding to the mode i, vector $\{\phi\}$ can be expanded as

$$\{\phi\} = \sum_{i=1}^{n} \alpha_i \{\mathbf{X}\}_i, \tag{41}$$

where α_i is the constant coefficient and n the order of the eigenvalue problem already treated with the shift technique.

Substituting equation (41) into equation (40) and noticing the weighting orthogonality between eigenvectors $\{\mathbf{X}\}_i$ about matrix $[\mathbf{M}]$ and matrix $[\mathbf{K}]$, one has

$$\rho(\{\phi\}) = \sum_{i=1}^{n} \alpha_i^2 \tilde{K}_i \Big/ \sum_{i=1}^{n} \alpha_i^2 \tilde{M}_i, \tag{42}$$

where $\tilde{K}_i$ is the modal stiffness and $\tilde{M}_i$ the modal mass of the system corresponding to the mode i. It is known that

$$\lambda_i = \frac{\tilde{K}_i}{\tilde{M}_i}. \tag{43}$$

If equation (42) is differentiated with respect to α_k and equation (43) is introduced into it, the following equation can be deduced:

$$\frac{\partial \rho(\{\phi\})}{\partial \alpha_k} = \frac{2\alpha_k \tilde{M}_k}{\sum_{i=1}^{n} \alpha_i^2 \tilde{M}_i} (\lambda_k - \rho(\{\phi\})), \quad k = 1, 2, \ldots, n. \tag{44}$$

It is obvious that the partial derivative on the left-hand side of equation (44) is equal to zero if the Rayleigh quotient $\rho(\{\phi\})$ is equal to eigenvalue λ_k. Therefore, the proof is successful.

3.3. UNAVAILABILITY OF THE STATE SPACE METHOD

In the case of arbitrary viscous damping, a symmetrical, real value damping matrix, $[\mathbf{C}]$, is introduced. Then, the quadratic eigenvalue problem of the vibration system is

$$(\xi^2[\mathbf{M}] + \xi[\mathbf{C}] + [\mathbf{K}])\{\Phi\} = \{\mathbf{0}\}, \tag{45}$$

where ξ is the eigenvalue and $\{\Phi\}$ is the eigenvector. The eigenequation of equation (45) is

$$\det(\xi^2[\mathbf{M}] + \xi[\mathbf{C}] + [\mathbf{K}]) = 0. \tag{46}$$

It should be noticed that the eigenvalues and eigenvectors here may be complex.

Since the damping matrix $[\mathbf{C}]$ usually cannot be uncoupled in the real-valued modal space, equation (45) is linearized with the so-called state space method which is the same as that commonly used for uncoupled systems [14, 15]. Consequently, the generalized eigenvalue problem of the coupled system in the state space is

$$\left(\begin{bmatrix} [\mathbf{C}] & [\mathbf{M}] \\ [\mathbf{M}] & [\mathbf{0}] \end{bmatrix} \xi + \begin{bmatrix} [\mathbf{K}] & [\mathbf{0}] \\ [\mathbf{0}] & -[\mathbf{M}] \end{bmatrix}\right) \begin{Bmatrix} \{\Phi\} \\ \{\Phi\}\xi \end{Bmatrix} = \begin{Bmatrix} \{\mathbf{0}\} \\ \{\mathbf{0}\} \end{Bmatrix}. \tag{47}$$

The corresponding eigenequation is

$$\det\left(\begin{bmatrix} [\mathbf{C}] & [\mathbf{M}] \\ [\mathbf{M}] & [\mathbf{0}] \end{bmatrix}\xi + \begin{bmatrix} [\mathbf{K}] & [\mathbf{0}] \\ [\mathbf{0}] & -[\mathbf{M}] \end{bmatrix}\right) = 0. \tag{48}$$

By examining the characteristics of the submatrices in equations (31) and (32), one can easily find that equation (48) in this case is an identical equation actually. That is to say, equation (48) is valid for arbitrary values of ξ. In fact, the essential reason for the result is still the singularity of matrix $[\mathbf{M}]$ because it means that equation (48) is no longer equivalent to equation (46). Therefore, the eigenvalues of equation (45) cannot be obtained in the state space.

4. EXAMPLE

Consider an elastic, thin-walled cavity with rectangular sections shown in Figure 1. The length of the cavity is 1·2m, the width is 0·5 m, the height is 0·8 m, and all the wallboards consist of steel plates with a thickness of 2 mm. Point O is the origin of co-ordinate. A (0·2, 0·7, 0·5) is an interior point where a loudspeaker is set as a point sound source and B (0·2, 0·1, 0·5) is another interior point of the cavity. S (0·4, 1·1, 0·8) is the excitation point and a random exciting force in the vertical downward direction is exerted here. The cavity is suspended with four elastic ropes hitched at the angle points on its top surface. The shaker is also suspended with an elastic rope.

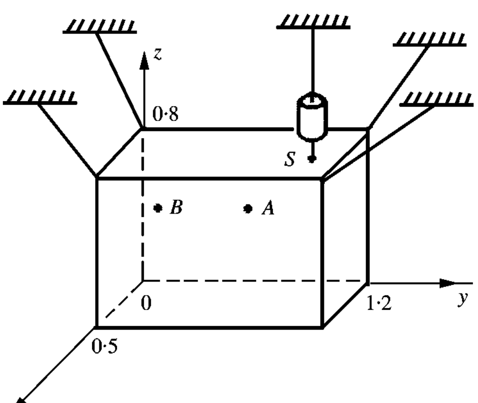

Figure 1. Rectangular section cavity.

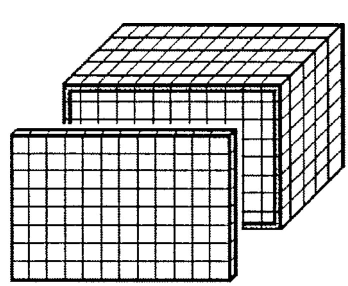

Figure 2. Structural and fluid elements.

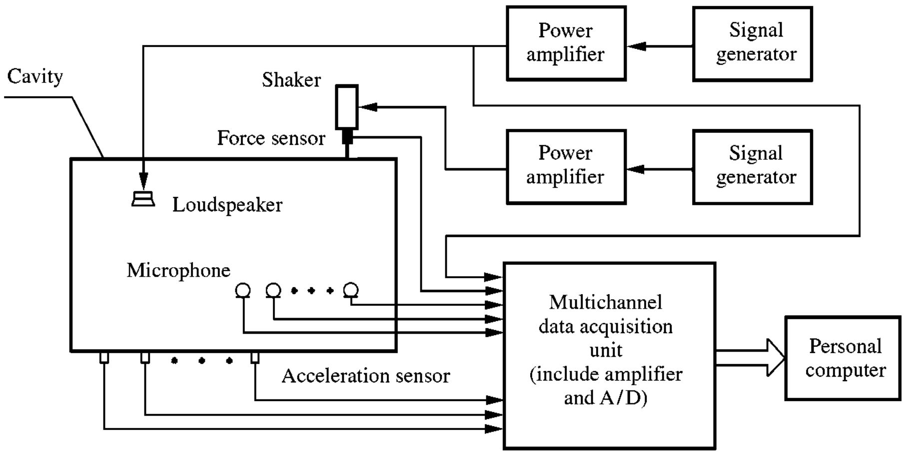

Figure 3. Experimental set-up.

The structural and fluid domains of the cavity are compartmentalized, as shown in Figure 2. The structural element is a rectangular plate element with four nodes and the fluid element is a hexahedral element with eight nodes. Here, the same discrete fashion is taken for the field p and field ψ.

The symmetrical finite element model of the rectangular section cavity is set up with the method given in this paper. Instead of a special application program, it can be solved by any of the all-purpose finite element software packages without causing the uncoupling error. Here the solutions are obtained by using the commercial finite element software package Ansys5.4 as the pre/post processor and the equation and eigenvalue solvers. The number of degrees of freedom that can be endured depends mainly on the memory capacity and computing power of the computer (hardware).

In order to compare with calculating results, the response and characteristics of the cavity are also studied experimentally, as shown in Figure 3. The acoustical source (loudspeaker) was calibrated in advance to obtain the FRF of the radiated volume velocity referenced to the input voltage.

The sound pressure at point B caused by the random excitation of the point sound source at point A is calculated by the symmetrical and non-symmetrical finite element models respectively. The calculated amplitude–frequency curves of the sound pressure are compared with the measured one, as shown in Figure 4. It can be seen that better consistency between the measured curve and the curve based on the symmetrical model is obtained. The difference between the measured curve and the curve based on the non-symmetrical model is relatively larger.

The first 20 low order eigenfrequencies of the coupled system are calculated and contrasted with those measured in the experiment as shown in Table 1 (except the zero eigenfrequencies). It is obvious that the differences between the calculated and the measured values are very small.

It should be emphasized that the data must be sampled in all the fields of p, ψ and δ (or $\ddot{\delta}$) if the eigenfrequencies of the coupled system are determined with the experimental modal analysis method. Otherwise, the experimental analysis model will be defective and bring relatively large errors. Since the currently used apparatuses cannot sample data directly in the field ψ, one has to obtain ψ by means of mathematical treatment through the field p [18].

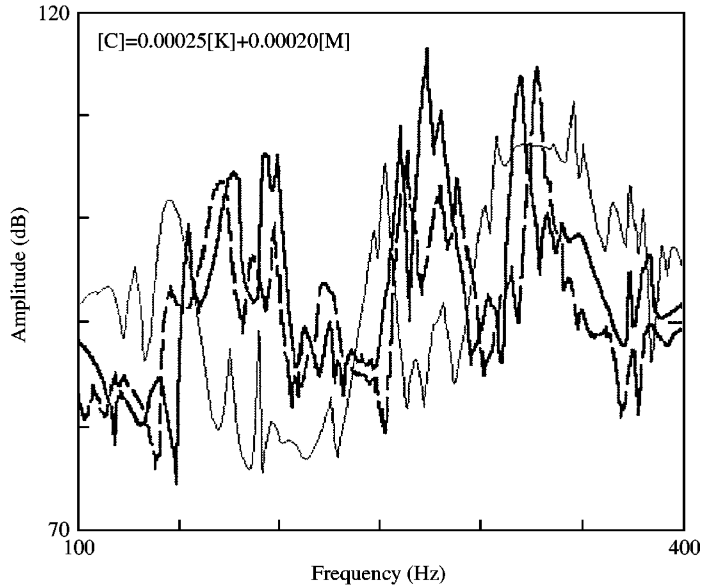

Figure 4. Amplitude–frequency curve: ▬▬, measured curve; — — —, calculated curve (symmetrical model); ———, calculated curve (non-symmetrical model).

Table 1

Eigenfrequencies (Hz)

Mode no. (coupled system)	Coupled system (calculated)	Coupled system (measured)	Acoustic mode in rigid wall cavity (calculated)	Structural mode in vacuum (calculated)
1	107·51	107·36		105·30
2	144·82	145·41	143·67	
3	151·93	155·80		144·02
4	189·06	187·00		183·28
5	214·91	215·37	213·69	
6	219·02	221·72		216·23
7	231·33	228·67		224·45
8	257·45	257·05		248·61
9	257·83	259·94	256·53	
10	281·20	278·63		272·07
11	288·81	287·45	285·82	
12	300·21	301·39		289·00
13	315·77	319·15		303·42
14	324·05	320·83		310·53
15	347·43	348·07	345·25	
16	358·13	355·27		345·62
17	359·04	357·18		351·38
18	360·48	360·52		353·28
19	361·88	362·65	358·42	
20	370·36	367·86		358·73

The acoustic modes (only in field p) in rigid wall cavity and the structural modes in vacuum are also analyzed with Ansys5.4 in order to compare. The results corresponding to the mode number of the coupled system are also listed in Table 1. It can be found that some of the eigenfrequencies of the two instances are close to each other, such as modes 2 and 3, modes 15 and 16, modes 19 and 20. However, for the coupled system, each eigenfrequency undergoes divergence. This phenomenon of eigenfrequency separation was also found in Sandberg's research work [10].

It should be pointed out that the errors between the calculated results based on the symmetrical finite element model and the measured ones are chiefly caused by some limitations of the theoretical model. For example, the cavity is regarded as a perfect airtight system theoretically, but in fact there are some apertures in the wallboards of the cavity for setting the supports and cables of the microphones and loudspeaker; and the loudspeaker at point A actually is not a perfect point source, etc. In addition, the errors also come from the experimentation itself, such as the modal truncation error in the experimental modal analysis and the interfering signals from environment, etc.

5. CONCLUSIONS

A symmetrical finite element model for the structure–acoustic coupling analysis of an elastic, thin-walled cavity excited by both interior acoustical sources and exterior structural loading has been put forward. Then, the coupled structure–acoustic problems can be studied by means of the standard analysis modules based on symmetrical models provided in numerous commercial software packages developed for the all-purpose finite element analysis without the uncoupling error.

Both the mass and the stiffness matrices in the symmetrical model put forward here are not positive definite. However, the eigenvalues can still be non-negative under certain conditions, and the order of the corresponding eigenvalue problem can still be reduced with the Rayleigh–Ritz method if the singularity of the mass matrix can be removed with an appropriate shift technique. Moreover, it is the singularity of the mass matrix that makes the eigenvalues, in the case of arbitrary viscous damping and complex modal analysis, which cannot be determined in the state space.

An example is given to validate the feasibility and correctness of the symmetrical method given in this paper. In fact, the finite element method (with the corresponding modal analysis technique) is more suitable in the relatively low-frequency range because of the relatively low modal densities. A further research work should be done in order to determine the upper frequency limit of the rationality of the finite element method.

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