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OPTIMAL DESIGN OF A TWO-STAGE MOUNTING ISOLATION SYSTEM BY THE MAXIMUM ENTROPY APPROACH

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In this paper, the optimization of the parameters for two-stage mounting isolation system is studied. The maximum entropy approach is applied to deal with the non-differentiable optimization problems. Results of numerical examples confirm the efficiency and accuracy of the proposed method, which is also suitable for the optimization of other kind of isolation system, such as Ruzicka isolation system (Ding wen-jing 1988 *Theory on Vibration Reduction*, Tsinghua University Publishing House, Ruzicka 1967 *Journal of Engineering and Industries ASME* 89), etc

1. INTRODUCTION

As an ideal solution system, two-stage mounting isolation system (TSMIS), is widely used in practice, such as sensitive instruments on ships or planes, high-speed cars, and so on. The effectiveness of a TSMIS depends on its structural parameters (relative mass, damping ratio, frequency ratio). Therefore, it is worth optimizing such parameters of the system. The model of a TSMIS without force is as shown in Figure 1 [1, 2].

The differential equations derived from the Newton's second law that describes the motion of this system are

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 - c_1\dot{x}_2 - k_1x_2 = 0,$$

$$m_2\ddot{x}_2 + (c_1 + c_2)\ddot{x}_2 + (k_1 + k_2)x_2 - c_1\dot{x}_1 - k_1x_1 = c_2\dot{u} + k_2u, \quad (1)$$

where m_1, k_1, c_1, x_1 are the mass, spring constant, viscous friction coefficient, generalized co-ordinate of the mounted equipment, respectively; m_2, k_2, c_2, x_2, the same for the interim block; and u is the position of the supporting structure.

Employing the expressions

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}, \quad \omega_2 = \sqrt{\frac{k_2}{m_2}}, \qquad \xi_1 = \frac{c_1}{2\sqrt{k_1m_1}}, \quad \xi_2 = \frac{c_2}{2\sqrt{k_2m_2}},$$

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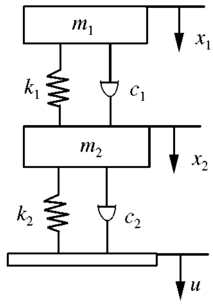

Figure 1. Model for the two-stage mounting isolation system.

$$\mu = \frac{m_2}{m_1}, \quad f = \frac{\omega_2}{\omega_1}, \quad g = \frac{\omega}{\omega_1}$$

the transmissibility is [1, 2]

$$T(g, \xi_1, \xi_2, \mu, f) = \left|\frac{x_1}{u}\right| = \left(\frac{A^2 + B^2}{C^2 + D^2}\right)^{1/2}, \tag{2}$$

where

$$A = f^2 - 4\xi_1\xi_2 fg^2,$$

$$B = 2f(\xi_2 + \xi_1 f)g,$$

$$C = g^4 - \left(\frac{1}{\mu} + f^2 + 1 + 4\xi_1\xi_2 f\right)g^2 + f^2,$$

$$D = 2fg\,(\xi_2 + \xi_1 f) - 2\left(\xi_1 + \frac{\xi_1}{\mu} + \xi_2 f\right)g^3.$$

A known property of TSMIS is that its transmissibility curve, as a function of the frequency ratio g, has two peaks (see Figure 2). The object of the optimization of the parameters for a TSMIS is to minimize the largest ordinate of the curve at the set value μ of the relative mass and the set value ξ_2 of the damping ratio. In order to design a good effect isolation system, it is requisite to calculate its optimum parameters. Two cases involving the optimization to TSMIS are put forward in practice as follows:
(1) For the given parameters relative mass μ, natural frequency ratio f and damping ratio ξ_2, one can determine the optimum value ξ_1^* at which the minimum of the largest ordinate of the transmissibility curve is secured. It is a one-dimensional optimization problem.
(2) For the given relative mass μ and damping ratio ξ_2, the problem is to findout the optimum value f^* and ξ_1^* at which the maximum ordinate of the transmissibility curve reaches the minimum. This is a two-dimensional optimization problem.

Most often a kind of numerical method is investigated to solve such problems. The method is based on the known property of the TSMIS, that is the presence of three invariant points [1, 2] (denoted by A, B, C) within the transmissibility curves, whose ordinates at the fixed value μ and the fixed value ξ_2 do not depend on the damping ratio ξ_1 (see Figure 3). Take problem (2), for example, to illustrate the solving procedure in detail. Using the above property, compute the tangents to the curves at the invariant point A with various damping

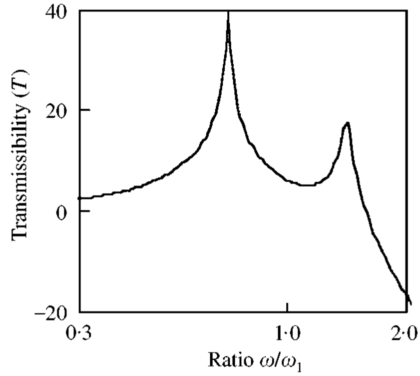

Figure 2. Transmissibility curve for two-stage mounting isolation system.

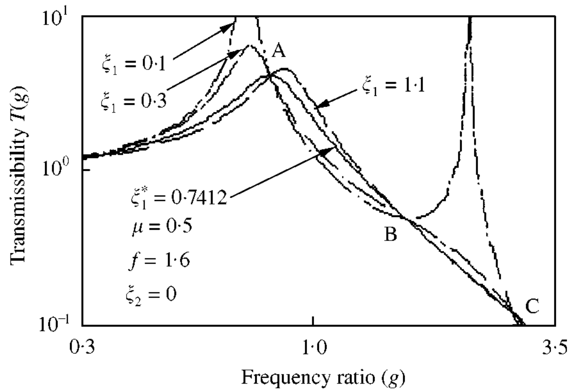

Figure 3. The result of one-dimensional optimization for a TSMIS; both the x-co-ordinate and ordinate are logarithmic scale.

ratio ξ_1, respectively, the damping ratio ξ_1^* at which the tangent to the curve has the minimum inclination is chosen as the optimal parameter wanted. Obviously, it is not a satisfactory method in terms of convergence, effectiveness and accuracy. For this reason, we will treat problems (1) and (2) with a more effective numerical method—the maximum entropy approach (MEA) [3, 7].

2. MAXIMUM ENTROPY APPROACH

Consider the following constrained minimum problem:

$$\begin{aligned} &\min \quad f(x), \\ &\text{s.t.} \quad h_j(x) \leqslant 0, \quad j = 1, 2, \ldots, m, \quad x = x(x_1, x_2, \ldots, x_n) \in R^n. \end{aligned}$$

If the objective function $f(x)$ and the constrained functions $h_1(x)$, $h_2(x)$, ..., $h_m(x)$ are smooth, the problem is called differentiable optimization (DO); else, it is called non-differentiable optimization (NDO). Among NDO problems there exist a special class called semi-infinite minimax problem which can be formulated of the form [3, 4]

Problem A:

$$\begin{aligned} \min \quad & f(x) = \max_{y \in R^1} \varphi(x, y), \\ \text{s.t.} \quad & h_i(x, y) \leqslant 0, \quad i = 1, 2, \ldots, l. \end{aligned}$$

Such problems appear frequently in engineering optimization [5], such as control system design, optimization of mechanism synthesis, and so on. A common feature of such problems is that their objective functions are not differentiable.

Studies [3, 4] show that problems in the form of Problem A can be treated with MEA theoretically. The MEA, an approximating method solving NDO, is based on the approximation of the max function by means of a smooth function (see, for instance, reference [6]) such that the solution of the NDO problem can be obtained by solving the DO. The maximum entropy function model of Problem A is [3, 6]

Problem B:

$$\begin{aligned} \min \quad & f_{\mathrm{p}}(x) = \frac{1}{p} \ln \int_a^b \exp(p\varphi(x, y))\, \mathrm{d}y \\ \text{s.t.} \quad & h_i(x) = \frac{1}{p} \ln \int_a^b \exp(ph_i(x, y))\, \mathrm{d}y \leqslant 0, \quad i = 1, 2, \ldots, l, \end{aligned}$$

where p is a large positive number (generally 10^5–10^8).

According to references [3, 7], the maximum function $f_p(x)$ has the following two properties:

Theorem 2.1. *For every given* $x \in \mathrm{R}^n$, $f_p(x)$ *decreases with increasing* p.

Theorem 2.2. *For every given* $x \in \mathrm{R}^n$, $\max_{y \in [a, b]} \varphi(x, y) \leqslant f_p(x) \leqslant \max_{y \in [a, b]} \varphi(x, y) + \ln(b - a)/p$, and $f_p(x) \to \max_{y \in [a, b]} \varphi(x, y)$ as $p \to \infty$.

Since Problem B including the integral operation which is not suitable for computer, we divide the whole interval $[a, b]$ into a number of smaller ones and approximate

$$f_p(x) \approx \frac{1}{p} \ln \left[\frac{1}{K} \sum_{j=1}^{K} \exp(p\varphi(x, y_j)) \right] = f_p^K(x), \tag{3}$$

$$h_i(x) \approx \frac{1}{p} \ln \left[\frac{1}{K} \sum_{j=1}^{K} \exp(ph_i(x, y_j)) \right] = h_i^K(x), \quad i = 1, 2, \ldots, l, \tag{4}$$

where $y_j = a + (j/K)\ (b - a)\ (j = 1, 2, \ldots, K)$, K denotes the number of all smaller intervals. Substituting equations (3) and (4) into Problem B yields

Problem C:

$$\begin{aligned} \min \quad & f_p^K(x) \\ \text{s.t.} \quad & h_i^K(x) \leqslant 0, \quad i = 1, 2, \ldots, l. \end{aligned}$$

Hence, from Theorems 2.1 and 2.2, we obtain

$$\min f_p^K(x) \to \min f(x) \quad \text{when } k \to \infty \text{ and } p \to \infty.$$

3. ALGORITHM

In order to solve the NDO problems in engineering practice by MEA, one must describe their mathematical models properly. For the previous two optimization problems (see section 1), they can be given by the expression

Problem D:

$$\min_{\xi_1} \max_{g} \quad T(g;\, \mu,\, \xi_1,\, \xi_2, f)$$

$$\text{s.t.} \quad g_0 \leqslant g \leqslant g_1, \ (g_0,\, g_1 \text{ are the lower and the upper limit of } g \text{ respectively}).$$

Problem E:

$$\min_{\xi_1, f} \max_{g} \quad T(g;\, \mu,\, \xi_1,\, \xi_2, f)$$

$$\text{s.t.} \quad g_0 \leqslant g \leqslant g_1, \quad f_0 \leqslant f \leqslant f_1$$

(g_0, g_1 are the lower and the upper limit of g, respectively, the same for f_0, f_1).

Here, $T(g;\, \mu,\, \xi_1,\, \xi_2, f)$ is the same as in section 1.

Algorithm

The following steps can state the MEA (see also Figure 4):

For a given problem in the form of Problem A (see before),

(i) Start with an initial point x_1. Set $P = 10^5$, $k = 1$.

(ii) Divide the interval $[a, b]$ into 2^k small parts with the same size and construct the set

$$D_k = \{y_j \,|\, j = 0, 1, \ldots, 2^k\} \text{ with } y_0 = a,\ y_{2^k} = b.$$

(iii) According to equations (3) and (4), construct the functions $f_p^k(x)$ and $h_i^k(x)$, $i = 1, 2, \ldots, l$.

(iv) Solve the approximating Problem C by penalty function method (PFM) [8], and obtain the solution x_k.

(v) If $\varphi(x_k, y) \leqslant \max_{y \in D_k} \varphi(x_k, y)$

and

$$\max_{1 \leqslant i \leqslant l} h_i\,(x_k, y) \leqslant 0$$

for every $y \in [a, b]$, then stop the procedure by taking

$$x_{opt} \approx x_k$$

Otherwise, set $k = k + 1$ and go to step (ii).

4. NUMERICAL RESULTS AND DISCUSSION

Many numerical examples can be given using the MEA. In this section, two examples on the optimization of the parameters for TSMIS are given to illustrate the validity of the

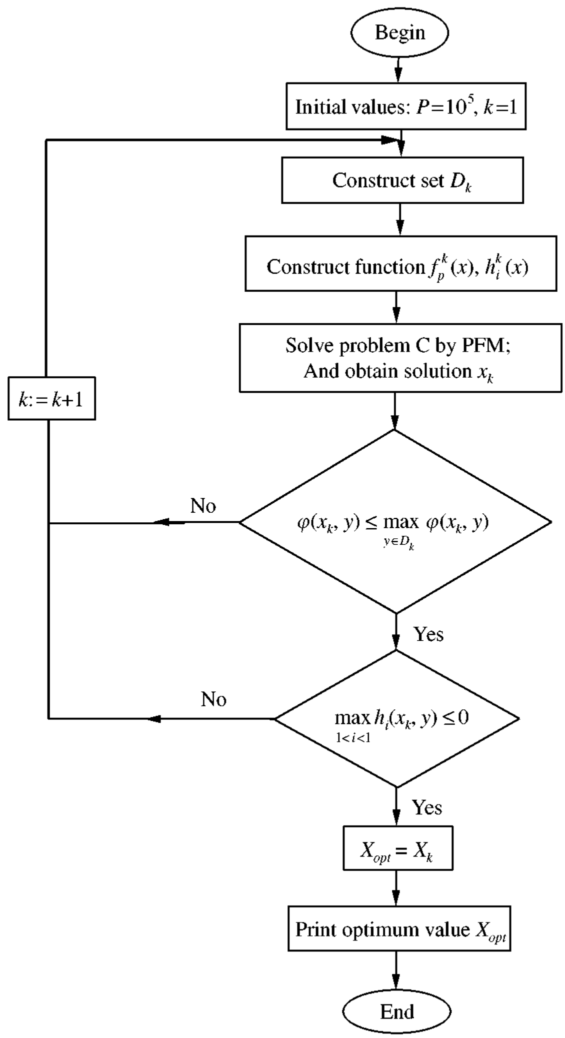

Figure 4. Flowchart of MEA.

proposed method. Note that the numerical operation is implemented on a desktop computer with Pentium I processor (133M).

Figure 3 shows the one-dimensional optimum results for a TSMIS with the following parameters: relative mass $\mu = 0{\cdot}5$, frequency ratio $f = 1{\cdot}6$ and damping ratio $\xi_2 = 0$ (see Problem D). In Table 1, the optimum solutions agree with that in reference [9] ($\xi_1 = 0{\cdot}741$, $T^*(g) = 4{\cdot}136$). Besides, from Table 1, it can be found that both the computing efficiency and accuracy of the proposed method are satisfying.

The second example (see Problem E) is to find the optimum values of the damping ratio ξ_1 and the frequency ratio f of a TSMIS with various relative mass μ under the condition

$$0 < f \leqslant 1{\cdot}93, \quad \xi_2 = 0.$$

The optimum results are as shown in Table 2 (see also Figure 5). In Table 2, the optimal solution at $\mu = 2{\cdot}0$ coincides with that in reference [9] (0·521, 1·830). From Table 2, we can see that the relative mass μ has little impact on the optimal value of f^*, which is the most

TABLE 1

Numerical results obtained with MEA

Initial point $\xi_1^{(1)}$	Optimum value ξ_1^*	Objective value $T^*(g)$	Computing time (s)
0·74	0·7412266839	4·13648137	10
0·5	0·7412289275	4·13648132	12
0·9	0·7412308774	4·13648128	13
1·0	0·7412243882	4·13648141	15

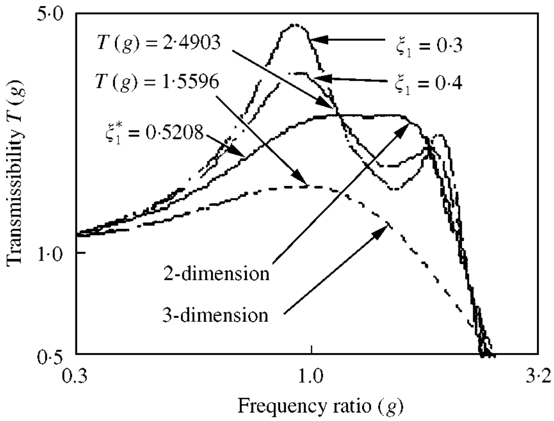

Figure 5. A comparison of two- and three-dimensional optimization with $\mu = 2{\cdot}0$; both the x-co-ordinate and ordinate are logarithmic scale.

TABLE 2

Numerical results obtained with MEA initial point $(\xi_1^{(1)}, f^{(1)}) = (0{\cdot}5, 1{\cdot}5)$

Relative mass μ	Optimum values (ξ_1^*, f^*)	Objective value T^*(g)	Computing time(s)
2·0	(0·5210390499, 1·83022195971)	2·4903013497	48
1·8	(0·5348943261, 1·83012128117)	2·5162042900	74
1·5	(0·5591507951, 1·83012159941)	2·5713904570	58
1·0	(0·6141602886, 1·83013055161)	2·7518876051	60
0·5	(0·7141738611, 1·83018098217)	3·3367609295	70

important point of Problem E. In fact, when increasing the upper boundary of the frequency ratio f, the optimum value f^* also increases (see Table 3, here $0 < f \leqslant 2{\cdot}0$).

Solving the problem again with different damping ratio ξ_2 and different upper boundary of f, we find that the optimum value f^* only depends on the boundary of f, i.e., f^* is equal to the upper boundary of f. Here, we must pay attention to one point, that is, the optimum value f^* is not equal to but less than the upper boundary of f, this is because in the previous algorithm, the penalty function method [8] is used at step (v). Thus, we obtain:

Problem B can be transformed into Problem A if only we assign the upper boundary of the frequency ratio f to the optimum frequency ratio f^*.

As a result, both Problems (A) and (B) can be regarded as one-dimensional optimization problem. Since in Problem A both the object function $f(x)$ and the constrained function

TABLE 3

Numerical results obtained with MEA initial point $(\xi_1^{(1)}, f^{(1)}) = (0{\cdot}5, 1{\cdot}5)$

Relative mass μ	Optimum values (ξ_1^*, f^*)	Objective value T^*(g)	Computing time(s)
2·0	(0·5208427145, 1·9001842271)	2·3502907311	50
1·8	(0·5450014979, 1·9001923301)	2·3718485250	75
1·5	(0·5670773052, 1·9001935629)	2·4264466258	56
1·0	(0·6174429037, 1·9002031211)	2·6003875966	68
0·5	(0·7097867351, 1·9002497308)	3·1512139666	70

$h(x, y)$ are optional, the previous algorithm can be suitable for the optimization of other kind of isolation system such as Ruzicka isolation system [9, 10], etc. The advantage of MEA is obvious. In addition, that the optimum objective value $T^*(g)$ increases as the relative mass μ falls corresponds to the conclusion in references [2, 9]. From this, the validity of the proposed algorithm is demonstrated.

Furthermore, employing the MEA, one can deal with a multi-dimension NDO easily. The outcome ($f^* = 1{\cdot}829979$, $\xi_1^* = 0{\cdot}899993$, $\xi_2^* = 0{\cdot}500074$, object value $T^*(g) = 1{\cdot}559649$) of a three-dimensional optimization problem with $\mu = 2{\cdot}0$ (see Problem F) is as shown in Figure 5 from which one can easily find that the result of the three-dimensional optimization is better than that of the two-dimensional optimization. The reason lies in the more constraint conditions the former considered.

Problem F:

$$\min_{\xi_1, \xi_2, f} \max_{g} \quad T(g; \mu, \xi_1, \xi_2, f),$$

$$\text{s.t.} \quad 0 \leqslant g \leqslant 10, \quad 0 < f \leqslant 2{\cdot}0, \quad 0 \leqslant \xi_1 < 1{\cdot}0 \quad 0 \leqslant \xi_2 < 1{\cdot}0.$$

5. CONCLUSION

This paper has explored the application of the MEA in the optimization of the parameters for TSMIS. An algorithm based on MEA is presented. The given numerical examples have demonstrated its applicability and efficiency.

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