Journal of Sound and Vibration (2001) **243**(4), 625–640
doi:10.1006/jsvi.2000.3433, available online at http://www.idealibrary.com on IDEAL®

THE DEVELOPMENT OF A RAPID SINGLE SPECTRUM METHOD FOR DETERMINING THE BLOCKAGE CHARACTERISTICS OF A FINITE LENGTH DUCT

M. H. F De Salis and D. J. Oldham

Acoustics Research Unit, School of Architecture and Building Engineering, University of Liverpool, P. O. Box 147, Liverpool, L69 3BX, England. E-mail: m.de-salis@liv.ac.uk

(*Received* 17 *January* 2000, *and in final form* 27 *June* 2000)

A method for the determination of the blockage area function of a duct or pipe from a single measurement of its transfer function is presented. The technique extends a method developed by de Salis and Oldham (*Journal of Sound and Vibration* **221**, 180–186) [1] where the duct blockage area function reconstruction is achieved as a function of the resonance and anti-resonance frequencies of the unblocked and partially blocked duct. In each case the resonance and anti-resonance frequencies are determined from maximum length sequence measurements (MLS). The advantage of using MLS analysis is that it has inherently high noise immunity and as such gives a sufficiently large signal-to-noise ratio to reveal the positions of the anti-resonance frequencies in the duct transfer function. In the current paper the technique is further simplified as the resonance and anti-resonance frequencies of the unblocked and the partially blocked duct are determined from a single MLS measurement in the partially blocked duct using an approximation suggested by Wu 1994 (*Applied Acoustics* **41**, 229–236) [2]. However, Wu's approach utilized resonance frequency information alone and was thus limited to blockage area functions with no degree of symmetry in the longitudinal plane. In earlier work Wu and Fricke 1990 (*Journal of the Acoustical Society of America* **87**, 67–75) [3] have demonstrated that the method using resonance frequencies measured under two sets of boundary conditions could be applied to functions of all degrees of symmetry. In this paper it is shown that the additional determination of the duct anti-resonance frequencies enables Wu's limited reconstruction technique (Wu 1994 *Applied Acoustics* **41**, 229–236) [2] to achieve results equivalent to the two boundary condition technique (Wu and Fricke 1990 *Journal of the Acoustical Society of America* **87**, 67–75) [3]. Accuracy is enhanced as filtration of the measured transfer function is used to further reduce extraneous noise thus improving the resolution of the transfer function. The determination of the duct area function is achieved utilizing resonance and anti-resonance frequency information alone which renders the determination of the duct length unnecessary. The accuracy of the shift approximation method in achieving a reconstruction technique for the blockage area function of a finite length duct or pipe from a single measurement is demonstrated.

1. INTRODUCTION

Where the accurate determination of the variation of the cross-sectional area with distance along a pipe or duct arising from the development of a blockage is required yet impractical using conventional visual techniques an alternative non-intrusive method may be sought. The work described in this paper outlines a technique where the blockage area function of a duct or pipe may be obtained for a single acoustic broad band measurement of short duration. (The blockage are a function may be defined in this context as the variation in the

0022-460X/01/240625 + 16 $35.00/0

ratio of cross-sectional area of a blockage within a duct to the cross-sectional area of the unblocked duct as a function of duct length.) The method described is a development of a previous technique proposed as a possible blockage detection mechanism for sensitive cooling systems such as those used in the nuclear industry [4], and while the effects of flow and losses are not examined in the current paper, the method described could have applications in similar piped fluid systems. Applications to inherently lossy systems such as vocal tracts or ear canals may be limited due to the losses incurred in these systems, although fundamentally similar work in vocal tract sizing using impedance testing has showed some degree of success [5, 6].

The paper builds on earlier work by the authors [1] in which the internal blockage area function of a length of duct was reconstructed by utilizing the resonance and anti-resonance frequencies measured in its unblocked and partially blocked condition. The resonance and anti-resonance frequencies were determined from measurements made in a duct under a single set of boundary conditions; i.e., closed–open terminations. Prior to this work the internal area function of a duct had been obtained using only the resonance frequencies of the duct, however, these needed to be determined under two separate sets of boundary conditions, i.e. in the closed–open and closed–closed duct [3]. Although a single measurement reconstruction method has been proposed by Wu [2] utilizing only the resonance frequencies of the duct determined under one set of termination boundary conditions, the loss of the resonance frequency information from a second set of boundary conditions means that the technique is only applicable to limited types of blockage area function. The method recently proposed by the authors to employ a single set of measurements [1] overcame the need for measurements under two sets of duct termination boundary conditions by incorporating anti-resonance frequencies as well as resonance frequencies of the unblocked and partially blocked duct in the reconstruction process.

With knowledge of the original resonance and anti-resonance frequencies of the duct in its unblocked state, the blockage area function could now be determined from a single measurement in the blockage perturbed duct [1]. Hence the method proposed was termed a ‘single measurement technique’. However, in cases where the resonance and anti-resonance frequencies of the duct in the “Unblocked” condition are not available, this single measurement method has no reference values and therefore cannot be employed. This problem was partially solved by Wu [2] who employed a limited reconstruction method using a single measurement of the resonance frequencies of a partially blocked closed-open ended duct. In Wu’s work the unknown resonance frequency distribution of the unblocked duct condition was approximated by using qualified assumptions about its relationship with the known resonance frequency distribution of the partially blocked duct. From this approximation the reconstruction of various duct blockage area functions could be determined from a single measurement.

However, without the completion of the expansion by using the additional resonance frequency information from the unblocked and partially blocked closed–closed duct as utilized in Wu’s earlier work with Fricke [3], the reconstruction process could only be applied to blockage area functions which displayed no symmetry about the longitudinal mid-point of the duct. This was obviously a major deficiency in a technique which was initially developed in response to requirements for blockage detection in sensitive cooling systems [4]. Wu’s technique [3] could also only be applied to constrictions as any negative portion of the area function had to be assigned a zero value to eliminate the asymmetrical blockage “image” generated when reconstructing the blockage area function using the closed–open ended duct resonance frequencies alone [3, 7]. The technique introduced by de Salis and Oldham [1] which incorporates anti-resonance frequency information eliminates such errors and allows the determination of the complete blockage area function.

The current paper combines the approach of Wu [2] for obtaining an approximation for the resonance frequencies for the unblocked duct conditions with the de Salis and Oldham technique for measurements under a single set of boundary conditions [1] to achieve a true single measurement method for the reconstruction of the blockage area function of a length of duct. The reconstruction utilizes a single broad band maximum length sequence (MLS) measurement signal to determine the required resonance and anti-resonance frequencies of the partially blocked duct. Filtration routines are introduced to suppress extraneous noise where levels are likely to affect the accurate determination of the anti-resonance frequencies. A similar measurement method has proved a rapid and accurate technique for determining the required resonance and anti-resonance frequencies for use in the reconstruction process using two measurements made under a single set of boundary conditions [1, 7]. In the current paper this approach is applied to the proposed single spectrum method.

2. THEORETICAL ANALYSIS

2.1. DETERMINATION OF DUCT BLOCKAGE AREA FUNCTION FROM TWO MEASUREMENTS UNDER A SINGLE SET OF BOUNDARY CONDITIONS

Wu and Fricke [3], employing a perturbation technique adapted from Bellman [8], presented an expression for the blockage area function of a duct as a function of the resonance frequencies of the unblocked and partially blocked duct measured under two sets of duct termination boundary conditions: i.e., closed–closed and closed–open ends. A similar expression had been developed previously by Mermelstein [5] for reconstruction of the internal area function in vocal tracts. De Salis and Oldham [1], influenced by earlier work of the Schroeder [6] on the vocal tract impedance function, developed Bellman's perturbation theory [8] to express the duct blockage area function as a function of the blockage-induced resonance frequency shifts and anti-resonance shifts measured under only one set of boundary conditions to obtain the following expression:

$$A_b(x)/A_0(x) = \left[1 - \exp\left(\sum_{n=1} \left[\frac{L_e}{n\pi}\right]^2 \mu_{(a)n} \cos\left(\frac{2n\pi x}{L_e}\right) - \sum_{n=1} \left[\frac{2L_e}{(2\pi - 1)\pi}\right]^2 \chi_n \cos\left(\frac{(2n-1)\pi x}{L_e}\right) - a_0\right)\right]. \quad (1)$$

Here $A_b(x)$ is the blockage cross-sectional area function within the duct, $A_0(x)$ is the cross-sectional area function of the unblocked duct, χ_n is the nth resonance value shift and $\mu_{(a)n}$ is the nth anti-resonance value shift due to blockage perturbation of the closed–open duct, L_e is the end-corrected duct length for the closed–open duct and a_0 is an added DC component equal to the ratio of blockage to duct volume [3]. The blockage area function $A_b(x)/A_0(x)$ in equation (1) is thus described in terms of the resonance value shifts χ_n and anti-resonance value shift $\mu_{(a)n}$ obtained under a single set of boundary conditions. As noted by Wu and Fricke [3], this type of perturbation analysis is valid only for relatively small blockages where $A_b(x)/A_0(x) < 0{\cdot}5$ and length of blockage $L_b < L_e/4$. However, while reconstructions of blockage area functions $A_b(x)/A_0(x)$ which transcend these limits tend to lose accuracy, the method is still valid as a means of blockages detection.

The need for measurement of the duct length can be eliminated by introducing a frequency based approach [7]. Given c, the value of the speed of sound in the fluid contained in the duct (typically air), the blockage reconstruction may be written as

$$A_b(x)/A_0(x) = \left(1 - \exp\left[\sum_{n=1} \left[\frac{c}{2\pi\ f^0_{(a)n}}\right]^2 \mu_{(a)n} \cos\left(\frac{4\pi\ f^0_{(a)n}}{c}\right) - \sum_{n=1} \left[\frac{c}{2\pi f_n^0}\right]^2 \chi_n \cos\left(\frac{4\pi f_n^0}{c}\right) - a_0\right]\right) \quad (2)$$

where $f^0_{(a)n}$ is the nth anti-resonance frequency of the unblocked closed–open duct, f_n^0 is the nth resonance frequency of the unblocked closed–open duct, and c in air is estimated as a function of ambient temperature T as described in reference [9] where

$$c = 331{\cdot}45\ \sqrt{(1 + T/373{\cdot}16)}. \quad (3)$$

2.2. DETERMINATION OF DUCT BLOCKAGE AREA FUNCTION FROM A SINGLE MEASUREMENT IN THE PARTIALLY BLOCKED DUCT UNDER A SINGLE SET OF BOUNDARY CONDITIONS USING RESONANCE AND ANTI-RESONANCE FREQUENCY SHIFT APPROXIMATIONS

The expression in equation (2) eliminates the need to measure the response of the duct under a second set of termination boundary conditions [1], and therefore greatly enhances the practicality of the technique. However, a knowledge of the acoustic characteristics of the unblocked duct is still required before any reconstruction of a partially blocked duct internal area function is possible. It may be noted that in practical situations where the frequency response of the "unblocked" condition has not been previously determined, the technique will become impractical as no reference values of resonance and anti-resonance frequencies are available. In attempting to overcome this difficulty, Wu [2] noted that the blockage-induced shifts in duct resonance frequency tended to vary quasi-periodically about zero such that the sum of the shifts about the unblocked condition tended to zero as the number of shifts increased; i.e.,

$$N \to \infty, \quad \sum_{n=1}^{N} (f_n - f_n^0) \to 0, \quad (4)$$

where f_n is the nth resonance frequency of the partially blocked due and f_n^0 is the nth resonance frequency of the unblocked duct. The relationship between the sum of the blocked and unblocked duct resonance frequencies is such that

$$\sum_{n=1}^{N} f_n = \sum_{n=1}^{N} f_n^0 + \sum_{n=1}^{N} (f_n - f_n^0). \quad (5)$$

Substituting into equation (4) from equation (5) yields

$$N \to \infty, \quad \sum_{n=1}^{N} f_n = \sum_{n=1}^{N} f_n^0\,. \quad (6)$$

Thus, the summation of the blocked resonance frequencies tends to the sum of the unblocked or uniform cross-sectional area duct resonance frequencies as the number of shifts increases. With a knowledge of the termination conditions and the resonance frequencies for the partially blocked duct, thus the "unblocked" or uniform duct resonance frequencies can be ascertained. For an unblocked closed–open duct of uniform cross-section,

$$f_n^0 = (2n-1)\, f_1^0, \quad n = 1, 2, 3, \dots . \tag{7}$$

It may also be shown that for the unblocked closed–open duct of uniform cross-section the following relationships exists:

$$f_1^0 = \sum_{n=1}^{N} f_n^0 / N^2. \tag{8}$$

Equations (6)–(8) can be manipulated to yield

$$N \to \infty, \quad f_n^0 = \frac{(2n-1)}{N^2} \sum_{n=1}^{N} f_n, \quad n = 1, 2, 3, \dots \tag{9}$$

Thus, from equation (9) resonance frequencies of the unblocked duct may be ascertained from the resonance frequencies of the partially blocked duct. The resonance value shifts χ_n in equation (2) may then be calculated where

$$\chi_n = \frac{4\pi^2}{c^2} \left([f_n]^2 - [f_n^0]^2 \right). \tag{10}$$

In Wu's work [2] this approach was applied to only the duct resonance frequencies and only the measured and approximated resonance frequencies of the partially blocked and unblocked ducts, respectively, were used in the subsequent reconstruction of the blockage area function. The reconstruction technique described from a single measured spectrum in the partially blocked duct [2] was thus limited as it was identical in essence to the technique used in previous work [3], and thus still required measurements under a second set of boundary conditions in order to obtain a full reconstruction of the blockage area function.

However, de Salis and Oldham [1] have showed that the complete reconstruction for any blockage area function within the duct can be obtained if the anti-resonance frequencies of the unblocked and partially blocked duct are included to complete the Fourier expansion. Now for the anti-resonance frequencies of the closed–open duct of uniform cross-section the relationship

$$f_{(a)n}^0 = n\, f_{(a)1}^0, \quad n = 1, 2, 3, \dots \tag{11}$$

exists where $f_{(a)n}^0$ is the nth order anti-resonance frequency for the unblocked closed–open duct and $f_{(a)1}^0$ is the fundamental unblocked duct anti-resonance frequency.

For the anti-resonance frequencies of the closed–open duct it may be shown that

$$f_{(a)1}^0 = \frac{\sum_{n=1}^{N} f_{(a)n}^0}{N!}. \tag{12}$$

Equation (6) holds for the anti-resonance frequencies of the closed–open duct and thus manipulating equations (6), (11) and (12) yields the following relationship for the anti-resonance frequencies:

$$N \to \infty, \quad f_{(a)n}^0 = \frac{n}{N!} \sum_{n=1}^{N} f_{(a)n}, \quad n = 1, 2, 3, \dots . \tag{13}$$

Here $f_{(a)n}$ is the nth order anti-resonance frequency for the partially blocked duct and N tends to infinity. Thus, by using equation (13) the anti-resonance frequencies of the unblocked duct may be calculated from the anti-resonance frequencies of the partially blocked duct. The anti-resonance value shifts $\mu_{(a)n}$ used in equation (2) may then be calculated where

$$\mu_{(a)n} = \frac{4\pi^2}{c^2}([f_{(a)n}]^2 - [f^a_{(a)n}]^2). \quad (14)$$

Thus, by substituting equations (10) and (14) into equation (2) the internal area function of a partially blocked pipe can be determined from a single measurement under one set of boundary conditions by incorporating the approximated resonance frequency and anti-resonance frequencies for the unblocked duct.

3. EXPERIMENTAL ANALYSIS

3.1. MEASUREMENT TECHNIQUE USING MAXIMUM LENGTH SEQUENCE ANALYSIS

To obtain the blockage area reconstruction from equation (2) from the resonance value and anti-resonance value shift calculated by using equations (10) and (14) requires measurement of the loudspeaker voltage to duct pressure frequency transfer function in the closed–open duct in its partially blocked state. Measurement techniques of relatively long duration such as swept sine or random noise analysis can provide transfer functions of sufficient noise immunity to reveal the modal characteristics of this type of system providing it is not excessively damped. However, in addition to this, accurate information about the residual or anti-resonance condition is required to complete the reconstruction via equation (2). This will often require a higher degree of noise immunity than is possible with traditionally utilized measurement techniques [10].

A high degree of noise immunity for measurements in linear systems can be obtained by using maximum length sequence or MLS analysis [10, 11]. As discussed in reference [10] this type of excitation signal can produce a degree of noise immunity similar to a frequency

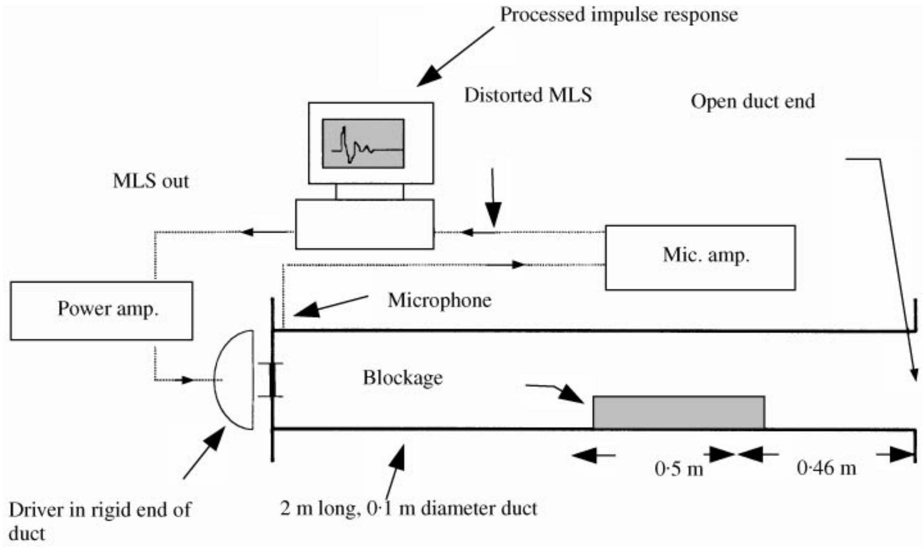

Figure 1. Test rig for closed-open end duct with blockage of area ratio $A_b(x)/A_0(x) = 0{\cdot}23$.

sweep of many times its duration. The pseudo-random MLS signal used to process the system impulse response has the maximum possible ratio of mean to maximum amplitude or crest factor of any signal: i.e., 0 dB. Once the system-distorted MLS is recorded and the impulse response processed by using a Fast Hadamard Transform [12] the system impulse response retains its maximum possible signal-to-noise ratio and therefore yields a comparatively high noise immunity when compared to other methods of obtaining impulse response: e.g., pulse analysis [10]. In addition, the system impulse response is concentrated towards the beginning of the time history whereas the background noise is spread out evenly over the entire duration. By concentrating on the early part of the time history much of the background noise may be windowed out on fast Fourier transformation realizing an even higher degree of noise immunity in the frequency domain. This gives MLS a further advantage over the more traditional measurement techniques cited above, which, while having high crest factors, have reduced signal-to-noise levels due to background noise contamination which cannot be removed from the measured signal retrospectively [7]. With ensemble averaging the noise immunity of the MLS process can be improved still further.

The rapid technique of acquiring the transfer function of the duct when using maximum length sequence analysis is further outlined in reference [7] where successful application of the analysis technique to the blockage reconstruction is described when using measured resonance frequencies in the unblocked the partially blocked duct determined under two sets of duct termination boundary conditions. The measurement technique has also been successfully applied to blockage reconstruction when using resonance and anti-resonance frequencies obtained from similar measurements of frequency transfer function under a single set or boundary conditions [1, 7].

The experimental set-up for a partially blocked duct is shown in Figure 1. The ducts chosen for use in the analysis were hard-walled plastic pipes of circular cross-section. The ducts were terminated at one end with a piezo-electric driver mounted in a Perspex termination plate and left open at the other. A microphone was mounted in the wall of duct close to the driver end. Hardwood blocks were used as blockages and were positioned at various locations within the ducts. The MLS source was set at a 2 kHz bandwidth with the acquisition sampling rate of 8 kHz. The impulse response time history was processed using an MLS of 16834 points giving a 2 s time history with the system impulse response concentrated in the first $\frac{1}{2}$ second [7].

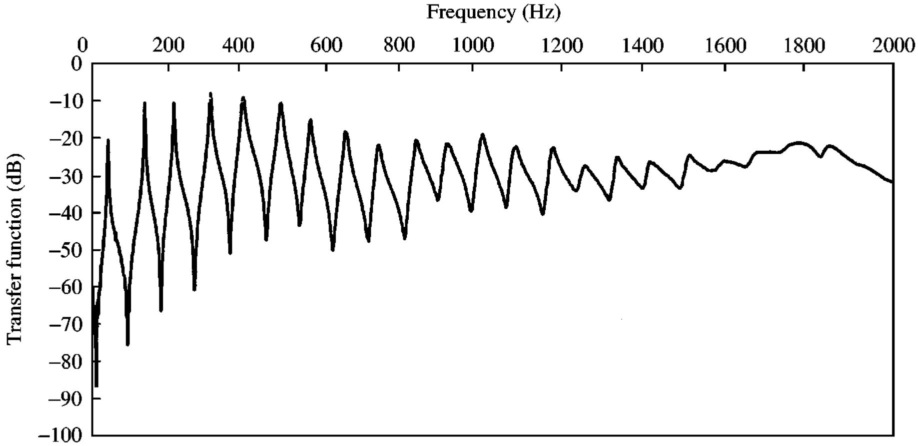

Figure 2. Measured transfer function of partially blocked duct shown in Figure 1.

Early measurements suggested that in general a significant degree of noise immunity was available to reveal the anti-resonance and resonance frequencies of the duct without ensemble averaging. However, to suppress any intermittent background noise contamination ensemble averaging was implemented. The impulse response of the duct shown in Figure 1 was thus obtained by using 16 averages of a 2 s MLS measurement. Once the impulse response time history had been generated, a fast Fourier transform window function was applied to the early part of the time history where the system impulse response was concentrated. The windowing process avoided truncation of the signal while eliminating a large proportion of the background noise as described earlier. The frequency transfer function amplitude distribution obtained with a frequency resolution of 1 Hz is shown in Figure 2. As can be seen it exhibits a strongly defined modal response which clearly shows the anti-resonance frequencies in addition to the prominent resonance frequencies.

3.2. ESTIMATION OF RESONANCE AND ANTI-RESONANCE FREQUENCIES OF THE UNBLOCKED DUCT FROM A SINGLE MEASUREMENT WITHIN THE PARTIALLY BLOCKED DUCT

An approximation of the resonance and anti-resonance frequencies of the unblocked duct from the resonance and anti-resonance frequencies measured in the blockage perturbed duct has been described in section 2.2. The unbroken curves in Figures 3(a) and (b) show the blockage induced resonance value and anti-resonance value shifts obtained from equations (10) and (14) for the duct of Figure 1 obtained without using the approximation method but using the actual measured spectra for the blockage perturbed and unblocked ducts [1]. The dotted curves show the approximated blockage induced resonance value and anti-resonance value shifts calculated by using equations (10) and (14) but incorporating the approximations of anti-resonance and resonance frequencies for for the unblocked duct in equations (9) and (13) determined from the single spectrum measured in the partially blocked duct and shown in Figure 2. The correlation between the estimated and actual shifts is shown to be almost exact for the lower order shifts. While the very lowest order shift may not necessarily be essential for the reconstruction of smaller obstacles (which alleviates some worries for their accurate determination in longer ducts [7]) the other lower order shifts will otherwise be the most important in determination of the blockage function [3]. The calculated higher-order shifts (which are generally less important for the reconstruction) show a similar pattern to the measured shifts although the correlation is not as good. Comparing with previous work undertaken in this area [1–3, 7], the correlation of the actual shifts and estimated shifts in Figure 3 suggests that the estimated shift information could be successfully applied to the reconstruction technique described in section 2 and reference [1]. Successful implementation of these estimated shifts into equation (2) in this manner would thus allow reconstruction of the blockage area function of the duct from a single measurement in the blockage perturbed duct without requiring prior knowledge of the unblocked duct condition.

3.3. POST PROCESSING

With the input of the single spectrum measured in the partially blocked duct and the ambient air temperature (see equation (3)) the entire reconstruction for a 2 m length duct of 0·1 m diameter can be achieved in a matter of seconds on a 90 MHz Pentium PC using

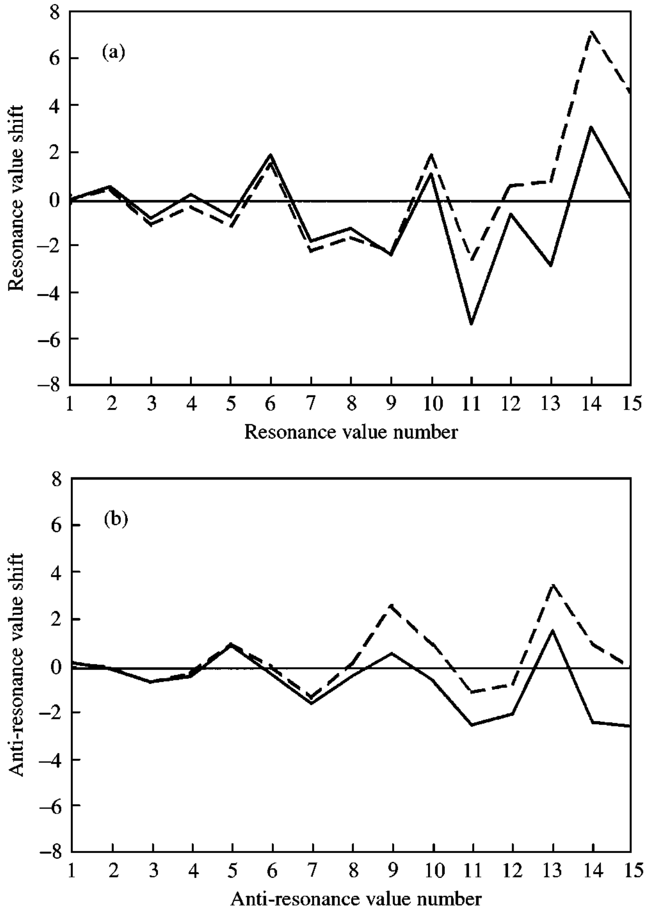

Figure 3. Actual and calculated blockage induced resonance and anti-resonance value shifts. (a) First 15 resonance value shift χ_n for blockage in Figure 1; (b) first 15 anti-resonance value shift $\mu_{(a)n}$ for blockage in Figure 1. (----) Calculated; (———), measured.

a single integrated post processing routine written in a numerical processing package [13]. The resonance and anti-resonance value shifts calculated from the resonance and anti-resonance frequencies, respectively, for the single measured transfer function in the partially blocked duct by using equations (9), (10), (13) and (14) are shown in Figure 3. The reconstruction process was subsequently completed by using equation (2).

3.3.1 *The blockage area function*

The computational post-processing routine used to obtain the reconstructions is similar to those described in references [1] and [7], the difference being that the resonance and anti-resonance frequencies are obtained from a single spectrum from the partially blocked duct and processed using shift approximation routines as described in section 2.2. For the selection of the anti-resonance frequencies, a minimum value comparative window was adapted from the maximum value window routine originally used to pick the duct resonance frequencies [7]. As shown, the locations of the resonance and anti-resonance frequencies are well defined in the frequency transfer function amplitude distribution in

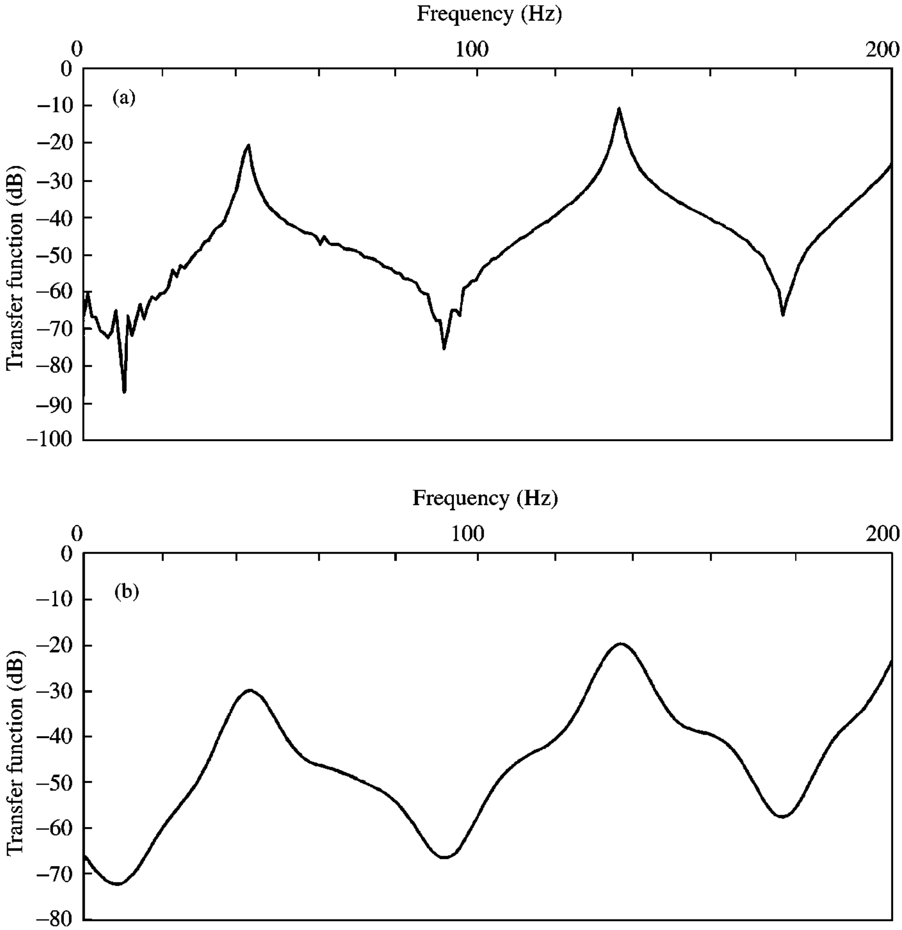

Figure 4. Measured transfer function 0–200 Hz of partially blocked duct shown in Figure 1. (a) Unfiltered; (b) filtered.

Figure 2 and as such are well suited to the window selection process. Localized fluctuation in the frequency distributions of the isolated individual complex components of the transfer function precluded their use as an alternative means of obtaining this information is this manner.

3.3.2. *Filtration*

The resonance frequency peaks in the measured duct transfer functions were generally strong enough to eliminate background noise distortion problems which could possibly contaminate the modal selection process described in the previous passage. However, while the anti-resonance frequencies were generally also strong enough to be selected in a similar manner, in some cases distortion at lower frequencies caused contamination in the "picking window" resulting in selection errors. This distortion may have been due to poor response of the loudspeaker and microphone at low frequencies leading to low signal-to-noise ratio and/or distortion effects. In general distortion and low signal-to-noise ratios did not present a problem for the duct of 2 m length shown in Figure 1. However, in Figure 4(a) the expansion of the lower end of the spectrum shown in Figure 2 highlights a small degree of distortion around the lowest anti-resonance trough. Filtration routines [13] were developed to smooth the frequency transfer functions of such localized distortions to prevent contamination of the resonance and anti-resonance frequency selection process. The filtration process employed automatically corrected any phase shift incurred due to the filtration process. The smooth filtered curve is shown in Figure 4(b) and the anti-resonance trough can be seen to remain clearly identifiable while localized fluctuation is negligible. In each case the filtration routine was applied to the frequency transfer function before implementation of the reconstruction routines.

A high frequency cut-off routine was required to ensure that (a) the resonance and anti-resonance frequencies picked were below the cut-on frequency of the duct thus avoiding confusion with cross-modes, and (b) that the peaks and troughs analyzed were strong enough to be recognized by the computational selection routine. The lack of definition of the peaks and troughs in the transfer function at higher frequencies is illustrated in Figure 2. As described in references [1] and [5], the signal was cut-off at a proportion of the calculated cross-modal cut-on frequency: i.e., for the 2 m length duct of 0·1 m diameter this cut-off point was approximately 1700 Hz which encompassed the first 15 resonance and anti-resonance frequencies which were generally of sufficiently good resolution to use in the reconstruction. A low frequency cut-off limit was also required due to spurious initial troughs arising at the lowest end of the frequency spectrum which may have affected the selection of the anti-resonance frequencies. An example of a spurious trough occurring at very low frequency is shown in Figure 4(a) and (b). Such troughs may have arisen due to the proximity of relative peaks in the low-frequency "noise" region. This very low-frequency region was filtered out prior to the reconstructions as any activity occurring here was superfluous to the analysis.

4. EXPERIMENTAL RESULTS

4.1. RECONSTRUCTION OF THE BLOCKAGE AREA FUNCTION OF THE DUCT

Figure 5 shows the reconstruction of the blockage area function of Figure 1 incorporating the first 15 resonance frequency shifts determined by using four frequency transfer functions

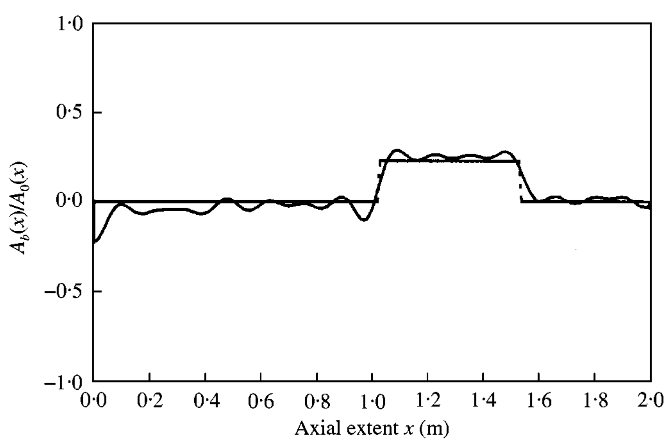

Figure 5. Reconstruction for blockage in Figure 1 by using resonance value shifts determined from four spectra measured under two sets of termination boundary conditions [3]. (——), Reconstructed; (- - - -), actual function.

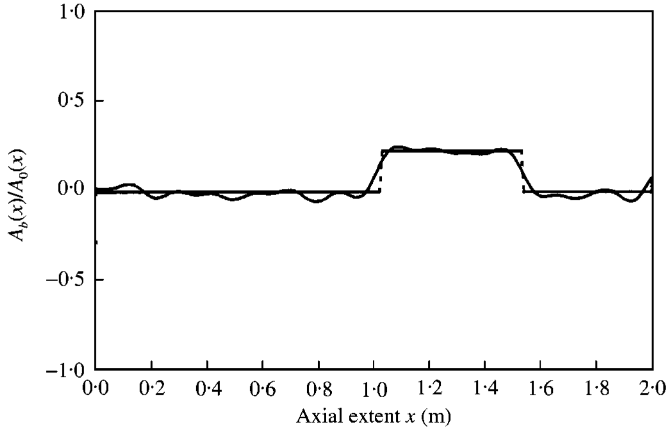

Figure 6. Reconstruction for blockage in Figure 1 by using resonance value and anti-resonance value shifts shown in Figure 3 from two spectra measured under two set of boundary conditions [1]. (——), Reconstructed; (- - - -), actual function.

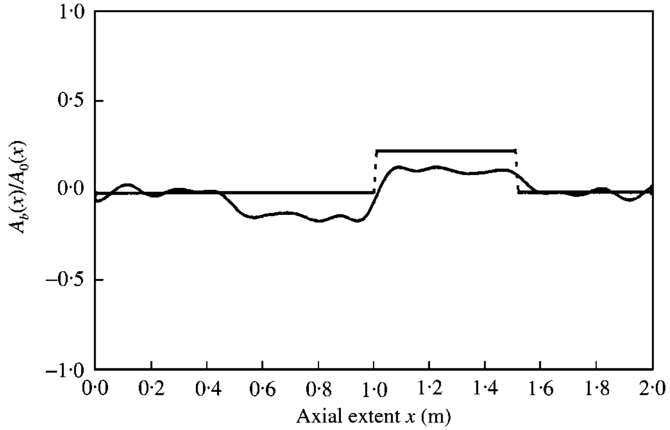

Figure 7. Reconstruction for blockage in Figure 1 by using calculated resonance value shifts from Figure 3 from single spectrum after Wu [2]. (——), Reconstructed; (- - - -), actual function.

measured under two sets of boundary conditions as by Wu and Fricke [3]: i.e., closed–open duct, blocked and unblocked and closed–closed duct, blocked and unblocked. The full reconstruction process is described in reference [7]. Figure 6 shows the reconstruction when using resonance and anti-resonance value shift determined by using two frequency transfer functions measured under a single set of boundary conditions: i.e., measurements in the closed–open duct in its unblocked and partially blocked state [1]. The reconstruction is almost identical to Figure 5 showing the accuracy of this single boundary condition method as described by de Salis and Oldham [1].

The pseudo single spectrum method developed by Wu [2] incorporating the first 15 approximated resonance value shifts χ_n from the single spectrum measured in the partially blocked duct was applied to the reconstruction and the results are shown in Figure 7. The resonance value shifts were calculated from the partially blocked duct resonance values by using equation (10). While the resonance value shift estimation process eliminates the need

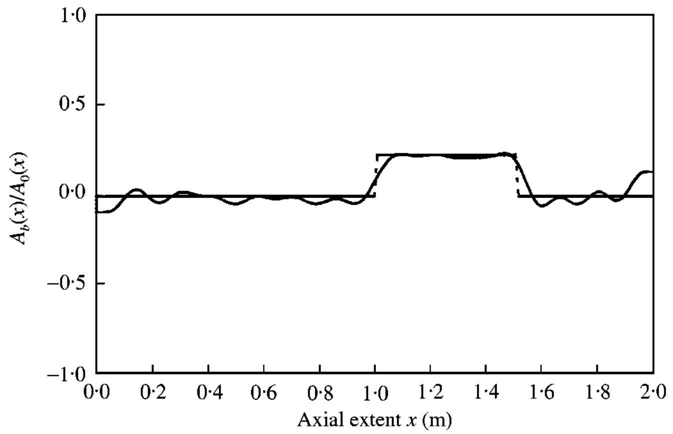

Figure 8. Reconstruction for blockage in Figure 1 by using calculated resonance value and anti-resonance value shifts shown in Figure 3 obtained from single spectrum. (——), Reconstructed; (- - - -), actual function.

for measurement in the unblocked duct to accurately evaluate the shifts, it may be noted that using the single set of closed–open duct resonance value shifts alone in the reconstruction causes an asymmetrical "image" blockage to appear. Wu attempted to overcome this problem by defaulting the blockage area function to zero at any point where the blockage area reconstruction resulted in a negative value. Wu and Fricke [3] had noted in their earlier work that the actual and image blockages realized by using one set of shifts were each of the same length but half the cross-sectional area of the actual blockage, and this is apparent in Figure 7. Thus, if the reconstructed function in Figure 7 in multiplied by 2 and negative portions are set to zero then although errors are increased over the rest of the reconstruction (due to the no "cancelling" effect from non-existent second set of resonance value shifts in equation (2)), a fairly accurate reconstruction of the image blockage will be realized for this type of blockage function. However, this approach means that any dilations in the duct will not be represented by the reconstruction and, more importantly for a blockage detection system, blockages symmetrical about the mid-point of the duct will not register at all due to cancellation of the image and actual blockage representations in the reconstruction. In fact, blockages with any degree of symmetry about the mid-point of the duct would experience cancellation in this way. However, since reconstructions of blockage functions of this nature were not attempted in Wu's work [2] such problems were not reported.

The reconstruction in Figure 8 is shown to overcome these problems by incorporating the anti-resonance value shifts calculated from equation (14) into equation (2) alongside the resonance value shifts calculated from equation (10). In this case the image blockage shown in Figure 7 is cancelled due to the calculated anti-resonance value shifts being utilized alongside the calculated resonance value shifts in the reconstruction. Thus, the complete blockage area function for the duct has been acquired from a single measurement of the frequency transfer function spectrum in the partially blocked closed–open duct.

4.2. FILTRATION EFFECTS

While the frequency transfer function in the previous sections were of sufficient noise immunity to enable accurate selection of the anti-resonance locations for use in the

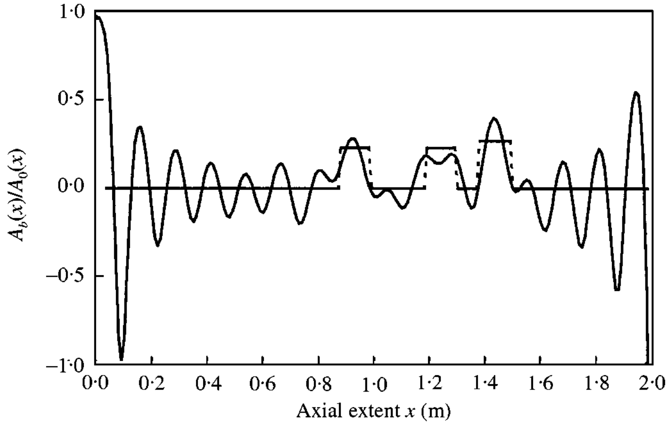

Figure 9. Reconstruction of multiple blockage in 2 m long 0·1 m diameter duct using estimated resonance value and anti-resonance value shifts from single unfiltered spectrum. (——), Reconstructed; (- - - -), actual function.

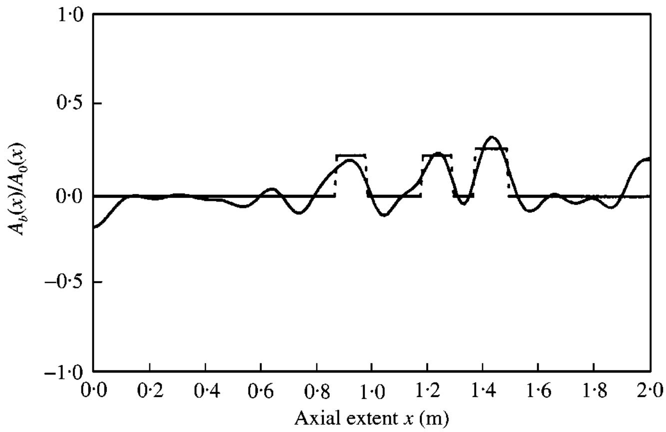

Figure 10. Reconstruction of multiple blockage in 2 m long 0·1 m diameter duct by using estimated resonance value and anti-resonance value shifts from single filtered spectrum. (——), Reconstructed; (- - - -), actual function.

reconstruction process described in section 3, spurious background noise and/or other signal distortion can affect the form of the anti-resonance troughs in the transfer function especially at low frequency. Filtration of the measured transfer functions was implemented as described in section 3.3 with a view to suppressing these spurious signal components prior to the reconstruction of the blockage area function. Figure 4(b) shows an example of the effect of the filtration process when applied to a measured duct transfer function.

The effects of the filtration process on the accuracy of the reconstruction of the blockage area function of a duct using the single spectrum technique are shown in Figures 9–12. The reconstruction shown in Figure 9 from an unfiltered transfer function displays large errors due to noise and/or other signal distortion of the anti-resonance troughs in the transfer function at low frequency. Filtration removes this distortion allowing successful reconstruction of the blockage transfer function as shown in Figure 10.

The problem of background noise and other distortion affecting measurements is potentially more serious for longer ducts for which the first resonance or anti-resonance

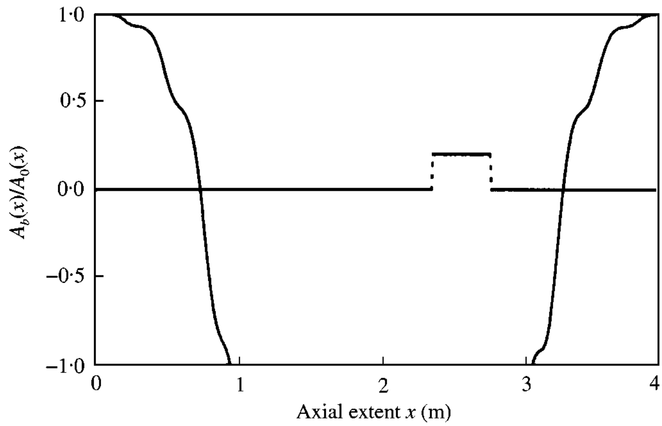

Figure 11 Reconstruction of multiple blockage in 4 m long 0·2 m diameter duct by using estimated resonance value and anti-resonance value shifts from single unfiltered spectrum. (——), Reconstructed; (- - - -), actual function.

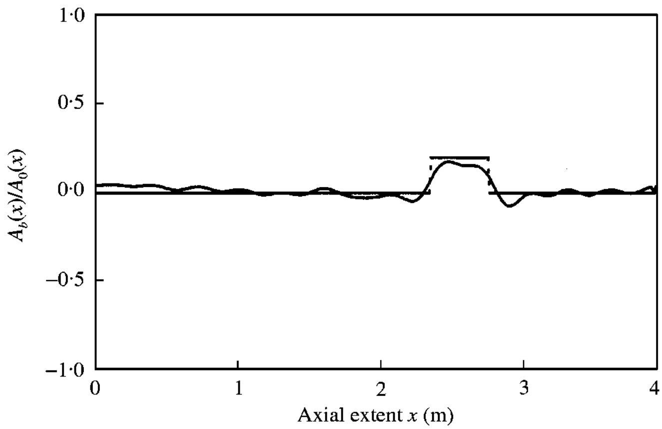

Figure 12. Reconstruction of multiple blockage in 4 m long 0·2 m diameter duct by using estimated resonance value and anti-resonance value shifts from single filtered spectrum. (——), Reconstructed; (- - - -), actual function.

frequency will occur at very low frequencies where the response of driver and sensor may be poor. Figure 11 shows a reconstruction in a 4-m-length duct of 0·2 m diameter where low-frequency background noise and/or other signal distortion has caused the selection process to completely miss the initial anti-resonance trough in the measured transfer function. Such an error throws the entire shift order out such that equation (2) becomes completely aberrant. Figure 12 shows the successful reconstruction where filtration of the transfer has suppressed the localized distortion at low frequency allowing correct selection of the initial anti-resonance trough.

5. CONCLUSIONS

A technique has been described to determine the blockage area characteristics of a length of duct or pipe from a single measurement of its acoustic response in its partially blocked state. The technique utilizes the blockage-induced shifts in resonance and anti-resonance value from the unblocked duct condition which are calculated from the resonance and anti-resonance frequency distribution of the partially blocked duct. The reconstruction utilizes maximum length sequence analysis to obtain rapid broad band measurement of the transfer function of the duct. The inherently high noise immunity of the maximum length sequence technique allows a rapid measurement which reveals the anti-resonance frequency residuals along with the resonance frequencies for the partially blocked duct for use in the analysis. Filtration routines have been incorporated which prevent extraneous noise from impeding the accurate selection of the anti-resonance frequencies. The rapid single spectrum technique described represents a significant improvement in practicality from earlier techniques which were often slow and laborious to undertaken and required multiple measurements under either one [1] or two [3] sets of termination boundary conditions.

ACKNOWLEDGMENTS

The authors would like to thank the Engineering and Physical Sciences Research Council of the United Kingdom for their support of this work.

REFERENCES

1. M. H. F. De Salis and D. J. Oldham 1999 *Journal of Sound and Vibration* **222**, 180–186. Determination of the blockage area function of a finite duct from a single pressure response measurement.
2. Wu Qunli 1994 *Applied Acoustics* **41**, 229–236. Reconstruction of blockage in a duct from single spectrum.
3. Qunli Wu and Fergus Fricke 1990 *Journal of the Acoustical Society of America* **87**, 67–75. Determination of blocking locations and a cross sectional area in a duct by resonance frequency shifts.
4. M. Antonopoulos-Domis 1980 *Journal of Sound and Vibration* **72**, 443–450. Frequency dependence of acoustic resonances on blockage position in a fast reactor subassembly wrapper.
5. P. Mermelstein 1967 *Journal of the Acoustical Society of America* **41**, 1283–1294. Determination of the vocal-tract shape from measured formant frequencies.
6. M. R. Schroeder 1967 *Journal of the Acoustical Society of America* **41**, 1089–1100. Determination of the geometry of the human vocal tract by acoustic measurements.
7. M. H. F. De Salis 1998 *Ph.D. thesis, School of Architecture and Building Engineering, University of Liverpool*, Acoustic sizing and location of blockages in ducts.
8. R. Bellman 1968. *Perturbation Techniques in Mathematics, Physics and Engineering*, New York: Holt, Rinehart and Winston, pp. 25–26.
9. Qunli Wu and Fergus Fricke 1989 *Journal of Sound and Vibration* **133**, 289–301. Estimation of blockage dimensions in a duct using measured resonance frequency shifts.
10. Douglas D. Rife and R. Vanderkooy 1989 *Journal of the Audio Engineering Society* **37**, 419–444. Transfer function measurement with maximum-length sequences.
11. *MLSSA Users Manual*, DRA Associates, 1995.
12. J. Borish and B. Angell 1983 *Journal of the Audio Engineering Society* **31**, 478–487. An efficient algorithm for measuring the impulse response using pseudorandom noise.
13. *Matlab High Performance Numeric Computation and Visualization Software—Users Guide*, The Maths Works Inc., 1992.