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DYNAMICS OF A FLEXIBLE MULTI-LINK COSMIC ROBOT-MANIPULATOR

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The problem of dynamic and kinematic control of the spatial movements of a flexible multi-link manipulator, mounted on a platform freely moving in cosmic space, is defined. Allowance for distributed properties of elasticity and inertia of the manipulator links is made, based upon the design model of the Euler–Bernoulli beam on the assumption that each element is in compound motion. A technique for the numerical construction of solutions for an essentially non-linear hybrid-type system of constituent equations is proposed. Examples of computer simulation of the dynamics of a two-link cosmic robot, manipulating a useful body, are considered. Comparisons are made between the results of calculations and data on the dynamic behaviour of a rigid cosmic manipulator with equivalent geometrical and inertial parameters.

1. INTRODUCTION

The design of technically reliable multi-link robots and manipulators for cosmic assignments requires mathematical models of the dynamics of mechanical systems with variable configurations, which include interconnected solid and elastic parts. Taking into account the distributed properties of elasticity and inertia of the elongated links of cosmic manipulators enables the influence of elastic oscillations, induced by their fast manoeuvring, on the dynamics of the systems studied to be investigated and the precision of their positioning to be improved. However, in this case, a number of difficulties of a theoretical character arise. Elements of elastic links comprising a kinematic chain undertake several forms of motion simultaneously and gyroscopic interaction between the rotary and linear components of these motions complicates their isolation and separate analysis. At the manipulator configuration variation, a change in its inertial characteristics also takes place, which is accompanied by evolution of its spectrum of frequencies and modes of free vibrations and provides the system with an opportunity to self-tune to the resonant interactions between partial modes of motions and the external disturbance in separate intervals of its motion trajectory. The necessity for mobility of the carrying platform, on which the manipulator is established, and the presence of cyclic coordinates in the system engender additional difficulties for theoretical modelling of the cosmic robots dynamics.

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In this connection, the attempts to construct design models of the dynamics of robots with elastic beam links are restricted, as a rule, to cases of elementary structures of one- and two-link industrial manipulators [1–9]. Their finite d.o.f. models are based on the special assumptions of the inertia and rigidity properties of links and modes of their deformation. A review of the modern approaches to the study of peculiarities and regularities of dynamical behaviour of the controlled cosmic manipulators is presented in reference [10]. Examples of their applications for calculation of an elastic one-link manipulator with a tip concentrated mass are presented. Elastic models of multi-link manipulators of cosmic basing are considered in references [11, 12].

The computing difficulties when simulating the dynamics of real manipulators with the distributed parameters, are associated with the necessity to solve the evolutionary problems via partial differential equations. In references [1–6] the procedures of spatial discretization of elastic displacements were overcome with the help of spectral expansion, and methods of finite difference and finite element are also used.

The application of methods for immediate mathematical modelling is considered to be most convenient for the study of program-controlled motions of multi-link flexible cosmic robots. In this paper, the mathematical model of the dynamics of an elastic multi-link controlled cosmic manipulator as a system with distributed parameters, and which is influenced by the action of inertial forces of relative elastic vibrations and compound motion, is proposed. A set of non-linear differential hybrid-type equations with the ordinary and partial derivatives of the required variables bound by the conditions of connection at the joining hinges is constructed. A step-by-step algorithm for the construction of solutions of these equations is developed. This allows, at each step of the computing procedures, the non-linear part of the problem of determination of the angles of relative slewing in the hinges of the adjacent links to be separated from the linear multipoint-like boundary value problem of determination of relative elastic movements of the links as a whole pivotal chain. The problems of dynamic and kinematic control of the elastic robot are set up. The technique developed for the specified problem-solving is based on the joint use of the method on initial parameters [13, 14], the algorithm of discrete orthogonalization [13], the implicit finite-difference scheme by Houbolt, and the one- and multi-step-by-step methods of numerical integration [15].

2. MATHEMATICAL MODEL OF DYNAMICS OF THE MULTI-LINK COSMIC ROBOT-MANIPULATOR

Let the cosmic robot-manipulator consist of a chain of N rectilinear elastic rods of length $l_n (n = 1, \dots, N)$, joined together by ideal cylindrical hinges, which is attached to a mobile platform (see Figure 1). This system of spatial motion can be either dynamically or kinematically controlled. The links are numbered in the order of their connection to each other, assuming the first one to be pivoted to the mobile platform. Introduce the inertial reference frame $OXYZ$. Let the local system of co-ordinates $O_n x_n y_n z_n$ with the orthogonal triad of the unit vectors $\mathbf{i}_n, \mathbf{j}_n, \mathbf{k}_n$ be connected with the nth $(n = 1, \dots, N)$ link in such a way that its origin O_n coincides with this rod's initial end, the axis $O_n x_n$ is coincident with its axial line and the corresponding axes $O_n y_n$ and $O_n z_n$ of all the links are parallel, when the whole kinematic chain of the manipulator is extended in one direct line and the axes $O_n x_n$ are aligned and parallel to the axis OX. Every axis of the cylindrical hinges connecting the adjoining links is considered to be in line with one of the axes of the local frame of reference. The angles of rotation of the subsequent nth link around one of the axes $O_n x_n$, $O_n y_n$ or $O_n z_n$ of the preceding one are designated φ_n, ψ_n or ϑ_n respectively. The angle is considered to be

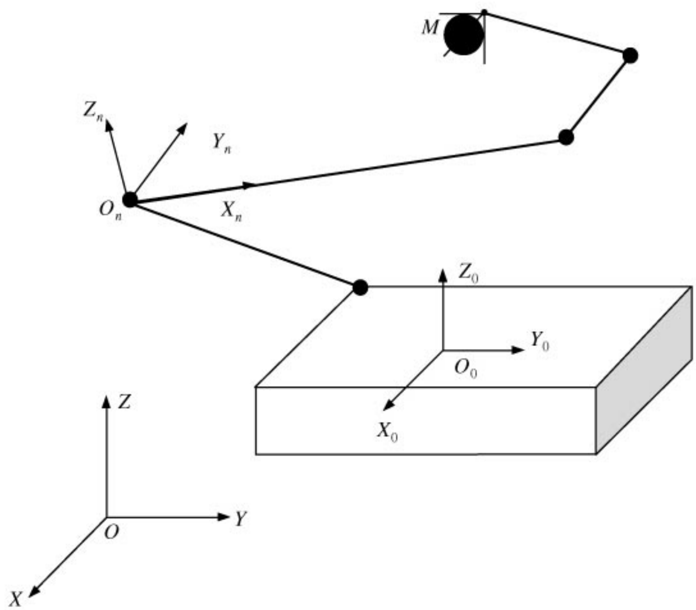

Figure 1. Mechanic model of a space multi-link robot manipulator.

positive, if from the end of the corresponding axis of rotation it is seen occurring against the course of an hour pointer.

In the case of dynamical control of the robot the prescribed controlling moments $M_n^e = M_n^e(t)\,(n = 1, \ldots, N)$ are applied to the links in the hinges. The moments are assumed to be positive, if they tend to slew the links in the positive directions. The angles of relative slewing of the links are considered to be required. At the kinematic control of the system, the set of N functions $\varphi_k = \varphi_k(t)$, $\psi_l = \psi_l(t)$, $\vartheta_m = \vartheta_m(t)$ ($k \neq l$, $l \neq m$, $m \neq k$) are given and the moments at hinges are to be computed.

Let the nth ($n = 1, \ldots, N$) link of the manipulator be conventionally separated. Using the d'Alembert principle, one can write the equations of the link dynamical equilibrium as follows:

$$E_n F_n \frac{\partial^2 u_n}{\partial x_n^2} - \rho_n F_n a_{x,n} = 0,$$

$$E_n J_{zn} \frac{\partial^4 v_n}{\partial x_n^4} - \frac{\partial}{\partial x_n}\left[N_{x,n}(x_n) \frac{\partial v_n}{\partial x_n} \right] + \rho_n F_n a_{y,n} = 0,$$

$$E_n J_{yn} \frac{\partial^4 w_n}{\partial x_n^4} - \frac{\partial}{\partial x_n}\left[N_{x,n}(x_n) \frac{\partial w_n}{\partial x_n} \right] + \rho_n F_n a_{z,n} = 0,$$

$$G_n J_{pn} \frac{\partial^2 \gamma_n}{\partial x_n^2} - \rho_n J_{pn} \varepsilon_{x,n} = 0. \tag{1}$$

Here u_n, v_n, w_n are the functions of elastic displacements of the moving rod along the axes $O_n x_n$, $O_n y_n$, $O_n z_n$ respectively; γ_n is the torsion angle; $E_n F_n$, $E_n J_{zn}$, $E_n J_{yn}$, $G_n J_{pn}$ are the tensile, bending and torsional stiffness factors; $N_{x,n}(x_n) = E_n F_n \partial u_n / \partial x_n$, the internal longitudinal force; ρ_n is the material density; F_n is the cross-section area; $a_{x,n}$, $a_{y,n}$, $a_{z,n}$ are the components of the vector of an absolute acceleration of the rod element; $\varepsilon_{x,n}$, $\varepsilon_{y,n}$, $\varepsilon_{z,n}$ are the components of the vector of an absolute angular acceleration of the rod element.

It is considered that each element of the rod is in the state of compound motion so its absolute acceleration $\mathbf{a}_n(x_n)$ may be represented as [16]

$$\mathbf{a}_n(x_n) = \mathbf{a}_n^e(x_n) + \mathbf{a}_n^c(x_n) + \mathbf{a}_n^r(x_n), \tag{2}$$

where $\mathbf{a}_n^e(x_n)$, $\mathbf{a}_n^c(x_n)$, $\mathbf{a}_n^r(x_n)$ are the vectors of translational, Coriolis and relative accelerations respectively.

The acceleration $\mathbf{a}_n^e(x_n)$ is calculated with the formula

$$\mathbf{a}_n^e(x_n) = \mathbf{a}_n(O) + \boldsymbol{\varepsilon}_n(O)\times x_n\mathbf{i}_n + \boldsymbol{\omega}_n(O)\times(\boldsymbol{\omega}_n(O)\times x_n\mathbf{i}_n), \tag{3}$$

where $\mathbf{a}_n(O)$ is the absolute acceleration of the point O_n; $\boldsymbol{\omega}_n(O)$, $\boldsymbol{\varepsilon}_n(O)$ are the angular velocity and angular acceleration vectors of the movable system of coordinates $O_nx_ny_nz_n$ relative to the reference frame $OXYZ$. For the calculation of $\mathbf{a}_n^e(x_n)$ the assumption of infinitesimal elastic displacements of the rod elements in comparison with its geometrical sizes, accepted in the geometrically linear theory of rods and permitting one to neglect the specified displacements in the calculation of the radius vectors of the rod elements, has been used.

The acceleration $\mathbf{a}_n^c(x_n)$ is represented in the form

$$\mathbf{a}_n^c(x_n) = 2\boldsymbol{\omega}_n(O)\left(\frac{\partial u_n}{\partial t}\,\mathbf{i}_n + \frac{\partial v_n}{\partial t}\,\mathbf{j}_n + \frac{\partial w_n}{\partial t}\,\mathbf{k}_n\right). \tag{4}$$

It is worth noting that at real values of the angular velocity $\boldsymbol{\omega}_n(O)$ and small elastic displacements u_n, v_n, w_n this one may be disregarded.

The relative acceleration $\mathbf{a}_n^r(x_n)$ is defined as

$$\mathbf{a}_n^r(x_n) = \frac{\partial^2 u_n}{\partial t^2}\,\mathbf{i}_n + \frac{\partial^2 v_n}{\partial t^2}\,\mathbf{j}_n + \frac{\partial^2 w_n}{\partial t^2}\,\mathbf{k}_n. \tag{5}$$

According to reference [16], the absolute angular velocity $\boldsymbol{\omega}_n(x_n)$ of an element of a rod, participating simultaneously in several rotational motions, can be presented by the equality

$$\boldsymbol{\omega}_n(x_n) = \boldsymbol{\omega}_n(O) + \frac{\partial \gamma_n}{\partial t}\,\mathbf{i}_n - \frac{\partial}{\partial t}\left[\frac{\partial w_n}{\partial x_n}\right]\mathbf{j}_n + \frac{\partial}{\partial t}\left[\frac{\partial v_n}{\partial x_n}\right]\mathbf{k}_n, \tag{6}$$

and its absolute angular acceleration $\boldsymbol{\varepsilon}_n(x_n)$ can be obtained by differentiation of expression (6) in the movable system of co-ordinates $O_nx_ny_nz_n$:

$$\boldsymbol{\varepsilon}_n(x_n) = \boldsymbol{\varepsilon}_n(O) + \frac{\partial^2 \gamma_n}{\partial t^2}\,\mathbf{i}_n - \frac{\partial^2}{\partial t^2}\left[\frac{\partial w_n}{\partial x_n}\right]\mathbf{j}_n + \frac{\partial^2}{\partial t^2}\left[\frac{\partial v_n}{\partial x_n}\right]\mathbf{k}_n. \tag{7}$$

In formula (5) the first summand is associated with longitudinal vibrations of rods, which happen with higher frequency in comparison with the lateral ones and occur with small amplitudes. Therefore, the process of longitudinal perturbations propagation in the rod may be conceived to be instantaneous for the considered dynamic phenomena, the acceleration $\partial^2 u_n/\partial t^2$ may be neglected and the longitudinal force $N_{x,n}(x_n)$ can be counted from the first equation of system (1), which for the assumptions adopted takes on a quasi-static form. The structure of system (1) fourth equation, in which the inertial forces of torsional vibrations of the rod elements are not taken into account, is defined with the same reasoning.

With allowance made for equations (2)–(5) and (7) system (1) is reduced to the form

$$E_n F_n \frac{\partial^2 u_n}{\partial x_n^2} = \rho_n F_n [a_{x,n}(O) - (\omega_{y,n}^2(O) + \omega_{z,n}^2(O)) x_n],$$

$$E_n J_{zn} \frac{\partial^4 v_n}{\partial x_n^4} - \frac{\partial}{\partial x_n}\left[N_{x,n}(x_n)\frac{\partial v_n}{\partial x_n}\right] = -\rho_n F_n \left[a_{y,n}(O) + \varepsilon_{z,n}(O) x_n + \omega_{y,n}(O)\omega_{x,n}(O) x_n + \frac{\partial^2 v_n}{\partial t^2}\right],$$

$$E_n J_{yn} \frac{\partial^4 w_n}{\partial x_n^4} - \frac{\partial}{\partial x_n}\left[N_{x,n}(x_n)\frac{\partial w_n}{\partial x_n}\right] = -\rho_n F_n \left[a_{z,n}(O) - \varepsilon_{y,n}(O) x_n + \omega_{z,n}(O)\omega_{x,n}(O) x_n + \frac{\partial^2 w_n}{\partial t^2}\right],$$

$$G_n J_{pn} \frac{\partial^2 \gamma_n}{\partial x_n^2} = \rho_n J_{pn} \varepsilon_{x,n}(O). \tag{8}$$

In the statement of the boundary conditions for system (8) it is necessary to take into account that their total order equals 12. Furthermore, six unknown functions of time $a_{x,n}(O)$, $a_{y,n}(O)$, $a_{z,n}(O)$, $\varepsilon_{x,n}(O)$, $\varepsilon_{y,n}(O)$, $\varepsilon_{z,n}(O)$ describing the acceleration of the origin O_n of the movable co-ordinate system $O_n x_n y_n z_n$ and its angular acceleration are included in the equations, besides the unknown field variables $u_n(x_n, t)$, $v_n(x_n, t)$, $w_n(x_n, t)$, $\gamma_n(x_n, t)$. Therefore, for the closure of system (8) it is necessary to formulate 18 equations for the joining of the $(n-1)$th and nth links.

3. BOUNDARY AND INITIAL CONDITIONS

Without reducing the generality of the problem statement, it is accepted for definiteness, that the $(n-1)$th and nth rods are connected by a cylindrical hinge, the axis of which coincides with the axis $O_n z_n$.

As the system $O_n x_n y_n z_n$ is rigidly connected with the nth rod edge at $x_n = 0$, one has the following equalities:

$$u_n(O) = v_n(O) = w_n(O) = 0 \quad v_n'(O) = w_n'(O) = 0, \quad \gamma_n(O) = 0. \tag{9}$$

Here the prime designates differentiation with respect to x_n.

The following equations stem from the condition of a pin joint of the $(n-1)$th rod end with nth rod beginning.

The equation of opposite directions of the reaction vectors equal in modulus,

$$\mathbf{R}_{n-1}(l_{n-1}) = -\mathbf{R}_n(O), \tag{10}$$

where $\mathbf{R}_{n-1}(l_{n-1}) = N_{x,n-1}(l_{n-1})\,\mathbf{i}_{n-1} - Q_{y,n-1}(l_{n-1})\,\mathbf{j}_{n-1} - Q_{z,n-1}(l_{n-1})\mathbf{k}_{n-1}$ and $\mathbf{R}_n(O) = -N_{x,n}(O)\,\mathbf{i}_n + Q_{y,n}(O)\,\mathbf{j}_n + Q_{z,n}(O)\,\mathbf{k}_n$ are respectively the reactions at the end $x_{n-1} = l_{n-1}$ of the $(n-1)$th rod and at the beginning $x_n = 0$ of the nth rod.

The equation of opposite directions of the reaction moment vectors equal in modulus,

$$\mathbf{M}_{n-1}(l_{n-1}) = -\mathbf{M}_n(O), \tag{11}$$

where $\mathbf{M}_{n-1}(l_{n-1}) = M_{x,n-1}(l_{n-1})\,\mathbf{i}_{n-1} - M_{y,n-1}(l_{n-1})\,\mathbf{j}_{n-1} + M_{z,n-1}(l_{n-1})\mathbf{k}_{n-1}$, and $\mathbf{M}_n(O) = -M_{x,n}(O)\,\mathbf{i}_n + M_{y,n}(O)\,\mathbf{j}_n - M_{z,n}(O)\mathbf{k}_n$ are respectively the reactional moments at the end $x_{n-1} = l_{n-1}$ of the $(n-1)$th rod and at the beginning $x_n = 0$ of the nth rod.

The equation of equality of the absolute acceleration vectors,

$$\mathbf{a}_{n-1}(l_{n-1}) = \mathbf{a}_n(O) \tag{12}$$

and the equation, following from the relation between angular accelerations of the co-ordinate systems $O_{n-1}x_{n-1}y_{n-1}z_{n-1}$ and $O_nx_ny_nz_n$, may be written down in the way described for the rods joint as

$$\boldsymbol{\varepsilon}_n(O) = D[\boldsymbol{\varepsilon}_{n-1}(O),\ \dot{\gamma}_{n-1}(l_{n-1})\,\mathbf{i}_{n-1},\ \dot{w}'_{n-1}(l_{n-1})\,\mathbf{j}_{n-1},\\ \dot{v}'_{n-1}(l_{n-1})\ \mathbf{k}_{n-1},\ \ddot{\vartheta}_n\,\mathbf{k}_n], \tag{13}$$

where D is some linear operator; the dots denote derivation with respect to the time t.

For the representation of equation (10) in the scalar form both its members are projected on the axes of the $O_nx_ny_nz_n$ reference frame:

$$\left\|\begin{matrix} -N_{x,n}(O) \\ Q_{y,n}(O) \\ Q_{z,n}(O) \end{matrix}\right\| - B^{\mathrm{T}}_{n-1,n} \left\|\begin{matrix} -N_{x,n-1}(l_{n-1}) \\ Q_{y,n-1}(l_{n-1}) \\ Q_{z,n-1}(l_{n-1}) \end{matrix}\right\| = 0. \tag{14}$$

Here $B_{n-1,n} = A_n^{\mathrm{T}}A_{n-1}$ is the matrix of transition from the basis $\mathbf{i}_{n-1}, \mathbf{j}_{n-1}, \mathbf{k}_{n-1}$ to the basis $\mathbf{i}_n,\ \mathbf{j}_n,\ \mathbf{k}_n$; $A_n = \|\alpha^n_{ij}\|\ (i,j = 1, \ldots, 3)$ is the matrix of direction cosines $\alpha^n_{11} = \cos(X, x_n)$, $\alpha^n_{12} = \cos(X, y_n), \ldots, \alpha^n_{33} = \cos(Z, z_n)$.

Equality (9) is analogously transformed to the form

$$\left\|\begin{matrix} -M_{x,n}(O) \\ M_{y,n}(O) \\ -M_{z,n}(O) \end{matrix}\right\| - B^{\mathrm{T}}_{n-1,n} \left\|\begin{matrix} -M_{x,n-1}(l_{n-1}) \\ M_{y,n-1}(l_{n-1}) \\ -M_{z,n-1}(l_{n-1}) \end{matrix}\right\| = 0. \tag{15}$$

The matrix $B_{n-1,n}$ structure is so designed that at the chosen orientation of the cylindrical hinge axis the third equality of system (15) results in two equalities,

$$M_{z,n-1}(l_{n-1}) = -M^e_n, \qquad M_{z,n}(O) = -M^e_n, \tag{16}$$

where M^e_n is the external controlling moment in the nth joint.

For obtaining the final form of boundary equations (14)–(16) it is necessary to use the following differential relationships:

$$N_{x,n} = E_nF_n\frac{\partial u_n}{\partial x_n}, \qquad Q_{y,n} = E_nJ_{zn}\frac{\partial^3 v_n}{\partial x_n^3}, \qquad Q_{z,n} = E_nJ_{yn}\frac{\partial^3 w_n}{\partial x_n^3},$$

$$M_{x,\,n} = G_nJ_{pn}\frac{\partial \gamma_n}{\partial x_n}, \qquad M_{y,n} = E_nJ_{yn}\frac{\partial^2 w_n}{\partial x_n^2}, \qquad M_{z,n} = E_nJ_{zn}\frac{\partial^2 v_n}{\partial x_n^2}.$$

After corresponding projection equality (12) gives

$$\left\|\begin{matrix} a_{x,n}(O) \\ \\ a_{y,n}(O) \\ \\ a_{z,n}(O) \end{matrix}\right\| - B^{\mathrm{T}}_{n-1,n} \left\|\begin{matrix} a_{x,n-1}(l_{n-1}) \\ \\ a_{y,n-1}(l_{n-1}) \\ \\ a_{z,n-1}(l_{n-1}) \end{matrix}\right\| = 0. \tag{17}$$

By using relations (2)–(5), the above system is reduced to the form

$$\left\|\begin{matrix} a_{x,n}(O) \\ a_{y,n}(O) \\ \\ a_{z,n}(O) \end{matrix}\right\| - B^{\mathrm{T}}_{n-1,n} \left\|\begin{matrix} a_{x,n-1}(O) - (\omega^2_{y,n-1}(O) + \omega^2_{z,n-1}(O))l_{n-1} \\ a_{y,n-1}(O) + \varepsilon_{z,n-1}(O)l_{n-1} + \\ + \omega_{y,n-1}(O)\omega_{x,n-1}(O)l_{n-1} + \ddot{v}_{n-1}(l_{n-1}) \\ a_{z,n-1}(O) - \varepsilon_{y,n-1}(O)l_{n-1} + \\ + \omega_{z,n-1}(O)\omega_{x,n-1}(O)l_{n-1} + \ddot{w}_{n-1}(l_{n-1}) \end{matrix}\right\| = 0. \tag{18}$$

For representing equations (13) in an explicit form the equality

$$\boldsymbol{\omega}_n(O) = \boldsymbol{\omega}_{n-1}(l_{n-1}) + \dot{\vartheta}_n \mathbf{k}_n, \tag{19}$$

resulting from the theorem of addition of angular velocities at compound motion [16] is constructed. Here $\boldsymbol{\omega}_n(O)$ is the vector of angular velocity of the basis $\mathbf{i}_n, \mathbf{j}_n, \mathbf{k}_n$; $\boldsymbol{\omega}_{n-1}(l_{n-1})$ is the angular velocity vector of a trihedron, connected with the end $x_{n-1} = l_{n-1}$ of the $(n-1)$th elastic rod.

After substitution of the expression

$$\boldsymbol{\omega}_{n-1}(l_{n-1}) = \boldsymbol{\omega}_{n-1}(O) + \dot{\gamma}_{n-1}(l_{n-1})\,\mathbf{i}_{n-1} - \dot{w}'_{n-1}(l_{n-1})\,\mathbf{j}_{n-1} + \dot{v}'_{n-1}(l_{n-1})\,\mathbf{k}_{n-1},$$

which follows from ratio (6), into formula (19) and projecting on the basis $\mathbf{i}_n, \mathbf{j}_n, \mathbf{k}_n$ one obtains

$$\begin{Vmatrix} \omega_{x,n}(O) \\ \omega_{y,n}(O) \\ \omega_{z,n}(O) - \dot{\vartheta}_n \end{Vmatrix} - B^{\mathrm{T}}_{n-1,n} \begin{Vmatrix} \omega_{x,n-1}(O) + \dot{\gamma}_{n-1}(l_{n-1}) \\ \omega_{y,n-1}(O) - \dot{w}'_{n-1}(l_{n-1}) \\ \omega_{z,n-1}(O) + \dot{v}'_{n-1}(l_{n-1}) \end{Vmatrix} = 0. \tag{20}$$

The system of three equations

$$\begin{Vmatrix} \varepsilon_{x,\,n}(O) \\ \varepsilon_{y,\,n}(O) \\ \varepsilon_{z,\,n}(O) - \ddot{\vartheta}_n \end{Vmatrix} - \dot{B}^{\mathrm{T}}_{n-1,n} \begin{Vmatrix} \omega_{x,n-1}(O) + \dot{\gamma}_{n-1}(l_{n-1}) \\ \omega_{y,n-1}(O) - \dot{w}'_{n-1}(l_{n-1}) \\ \omega_{z,n-1}(O) + \dot{v}'_{n-1}(l_{n-1}) \end{Vmatrix}$$

$$- B^{\mathrm{T}}_{n-1,n} \begin{Vmatrix} \varepsilon_{x,n-1}(O) + \ddot{\gamma}_{n-1}(l_{n-1}) \\ \varepsilon_{y,n-1}(O) - \ddot{w}'_{n-1}(l_{n-1}) \\ \varepsilon_{z,n-1}(O) + \ddot{v}'_{n-1}(l_{n-1}) \end{Vmatrix} = 0. \tag{21}$$

is obtained by differentiating both parts of this equality with respect to t.

The equations of motion of the platform and tip load considered as solid bodies are used as boundary conditions for the first and the latter links. By way of example, these relations for the case of rigid connection of the load to the end $x_N = l_N$ of the Nth link are written in the form of differential equations of motion of a free solid body [16]:

$$M_L \cdot a_{x,N+1} = R_{x,N+1}, \qquad M_L \cdot a_{y,N+1} = R_{y,N+1}, \qquad M_L \cdot a_{z,N+1} = R_{z,N+1},$$

$$I_{xL} \cdot \dot{\omega}_{x,N+1} + (I_{zL} - I_{yL}) \cdot \omega_{y,N+1} \cdot \omega_{z,N+1} = M_{x,N+1},$$

$$I_{yL} \cdot \dot{\omega}_{y,N+1} + (I_{xL} - I_{zL}) \cdot \omega_{z,N+1} \cdot \omega_{x,N+1} = M_{y,N+1},$$

$$I_{zL} \cdot \dot{\omega}_{z,N+1} + (I_{yL} - I_{xL}) \cdot \omega_{x,N+1} \cdot \omega_{y,N+1} = M_{z,N+1}.$$

Here M_L is the mass of the solid body; I_{xL}, I_{yL}, I_{zL} are its principal moments of inertia; $a_{x,N+1}$, $a_{y,N+1}$, $a_{z,N+1}$ are the projections of the acceleration vector of the body centroid O_{N+1} on the axes of the local coordinate system $O_{N+1}x_{N+1}y_{N+1}z_{N+1}$, rigidly connected with the body; $\omega_{x,N+1}, \omega_{y,N+1}, \omega_{z,N+1}$ are the angular velocities of the body concerning the axes of the system $O_{N+1}x_{N+1}y_{N+1}z_{N+1}$; $R_{x,N+1}$, $R_{y,N+1}$, $R_{z,N+1}$ are the components of the vector of reactions of the end $x_N = l_N$ of the Nth link on the co-ordinate system $O_{N+1}x_{N+1}y_{N+1}z_{N+1}$; $M_{x,N+1}$, $M_{y,N+1}$, $M_{z,N+1}$ are the reactional moments at the end $x_N = l_N$ of the Nth link concerning the co-ordinate system $O_{N+1}x_{N+1}y_{N+1}z_{N+1}$.

The initial equations at $t = 0$ stem from the condition of the mechanical system's initial configuration,

$$\begin{aligned} &\varphi_k(O) = \varphi_{k,0}, \psi_l(O) = \psi_{l,0}, \vartheta_m(O) = \vartheta_{m,0}, u_n(x_n, O) = u_{n,0}(x_n), \\ &v_n(x_n, O) = v_{n,0}(x_n), w_n(x_n, O) = w_{n,0}(x_n), \gamma_n(x_n, O) = \gamma_{n,0}(x_n), \end{aligned} \tag{22}$$

and its initial state of motion,

$$\begin{aligned} &\dot{\varphi}_k(O) = \dot{\varphi}_{k,0}, \dot{\psi}_l(O) = \dot{\psi}_{l,0}, \dot{\vartheta}_m(O) = \dot{\vartheta}_{m,0}, \dot{u}_n(x_n, O) = \dot{u}_{n,0}(x_n), \\ &\dot{v}_n(x_n, O) = \dot{v}_{n,0}(x_n), \dot{w}_n(x_n, O) = \dot{w}_{n,0}(x_n), \dot{\gamma}_n\ (x_n, O) = \dot{\gamma}_{n,0}(x_n). \end{aligned} \tag{23}$$

Equations (22) and (23) provide closure of system (8), (9), (14)–(16), (18), and (21).

4. STATEMENT OF PROBLEMS OF DYNAMIC AND KINEMATIC CONTROL

Firstly, the problem of dynamic control of system (8), (9), (14)–(16), (18), and (21)–(23) will be set up. Let the program law of variation of controlling moments $M_n^e(t)$ $(n = 1, \ldots, N)$ applied to the connecting pin joints be given. It is required to determine the law of the system motion (N angles of rotations $\varphi_k = \varphi_k(t)$, $\psi_l = \psi_l(t)$, $\vartheta_m = \vartheta_m(t)$ $(k \neq l, l \neq m, m \neq k)$) and the functions of elastic displacements of the links $u_n(x_n, t)$, $v_n(x_n, t)$, $w_n(x_n, t)$, $\gamma_n(x_n, t)$ $(n = 1, \ldots, N)$.

The problem of the system kinematic control is set up as follows. Let the program law of N angles $\varphi_k = \varphi_k(t)$, $\psi_l = \psi_l(t)$, $\vartheta_m = \vartheta_m(t)$ $(k \neq l, l \neq m, m \neq k)$ variation be given. It is necessary to determine the system motion (the functions of the links elastic displacements $u_n(x_n, t)$, $v_n(x_n, t)$, $w_n(x_n, t)$, $\gamma_n(x_n, t)$ $(n = 1, \ldots, N)$ and also to find the controlling moments $M_n^e(t)$ $(n = 1, \ldots, N)$ in hinges, providing realization of the given relations $\varphi_k = \varphi_k(t)$, $\psi_l = \psi_l(t)$, $\vartheta_m = \vartheta_m(t)$ $(k \neq l, l \neq m, m \neq k)$.

Thus, in the case of dynamic control, system (8) for the nth rod is closed by 19 boundary equations (9), (14)–(16), (18) and (21), which, however, contain the additional unknown function $\ddot{\vartheta}_n$. At the kinematic control, the function $\ddot{\vartheta}_n$ is considered to be given and equation (16) is used for determination of the moments $M_n^e(t)$ $(n = 1, \ldots, N)$ in the connecting hinges.

5. TECHNIQUES FOR CONSTRUCTION OF THE MOTION EQUATIONS SOLUTIONS

The essential non-linearity of the hybrid-type equations obtained and their implicitness relative to the higher derivatives with respect to time cause significant difficulties of a theoretical and computing nature in the construction of the solutions of the problems set up of dynamic and kinematic control. In the present work a step-by-step algorithm is proposed for the solution construction. At each step of discretization in time it permits one to reduce problem (8), (9), (14)–(16), (18), and (21)–(23) to a sequence of linear multipoint-like boundary value problems for the ordinary differential equations with respect to the spatial co-ordinate, containing, however, the additional unknown algebraic values representing the magnitudes of the linear $(a_{x,n}(O), a_{y,n}(O), a_{z,n}(O))$ and angular $(\varepsilon_{x,n}(O), \varepsilon_{y,n}(O), \varepsilon_{z,n}(O))$ accelerations required at the time step considered. In doing so the implicit finite-difference scheme of the Houbolt method is used for the system (8) integration in time, according to which the derivatives $\mathrm{d}X/\mathrm{d}t$, $\mathrm{d}^2X/\mathrm{d}t^2$ of any function X with respect to time t at the time moment $t + \Delta t$ are replaced by their finite-difference analogues:

$$\begin{aligned} \dot{X}(t + \Delta t) &= \dot{X}_{t+1} = (11X_{t+1} - 18X_t + 9\,X_{t-1} - 2X_{t-2})/6\Delta t, \\ \ddot{X}(t + \Delta t) &= \ddot{X}_{t+1} = (2X_{t+1} - 5X_t + 4\,X_{t-1} - X_{t-2})/(\Delta t)^2. \end{aligned} \tag{24}$$

Here Δt is the step of integration in time, $X_{t+1} = X(t + \Delta t)$, $X_t = X(t)$, $X_{t-1} = X(t - \Delta t)$, $X_{t-2} = X(t - 2\Delta t)$. As usual the time step Δt value is chosen on the basis of the calculation accuracy practical convergence.

With allowance made for equations (24) at the moment of time $t + \Delta t$, system (8) of partial differential equations for the nth link is replaced by the sequence of the ordinary differential equations written for every step of the time discretization in the form

$$E_nF_n\frac{\mathrm{d}^2u_n}{\mathrm{d}x_n^2}\bigg|_{t+1} = \rho_nF_na_{x,n}(O)\,|_{t+1} - \rho_nF_n(\omega_{y,n}^2(O) + \omega_{z,n}^2(O))x_n\,|_{t+1},$$

$$E_nJ_{zn}\,\frac{\mathrm{d}^4v_n}{\mathrm{d}x_n^4}\bigg|_{t+1} - N_{x,n}\bigg|_t\,\frac{\mathrm{d}^2v_n}{\mathrm{d}x_n^2}\bigg|_{t+1} - \frac{\mathrm{d}N_{x,n}}{\mathrm{d}x_n}\bigg|_t\,\frac{\mathrm{d}v_n}{\mathrm{d}x_n}\bigg|_{t+1} + \frac{2\rho_nF_n}{\Delta t^2}\,v_n\bigg|_{t+1}$$

$$= -\rho_nF_na_{y,n}(O)\,|_{t+1} - \rho_nF_nx_n\varepsilon_{z,n}(O)\,|_{t+1} - \rho_nF_nx_n\omega_{y,n}(O)\omega_{x,n}(O)\,|_{t+1}$$

$$-\frac{\rho_nF_n}{\Delta t^2}\,[-5v_n\,|_t + 4v_n\,|_{t-1} - v_n\,|_{t-2}],$$

$$E_nJ_{yn}\,\frac{\mathrm{d}^4w_n}{\mathrm{d}x_n^4}\bigg|_{t+1} - N_{x,n}\bigg|_t\,\frac{\mathrm{d}^2w_n}{\mathrm{d}x_n^2}\bigg|_{t+1} - \frac{\mathrm{d}N_{x,n}}{\mathrm{d}x_n}\bigg|_t\,\frac{\mathrm{d}w_n}{\mathrm{d}x_n}\bigg|_{t+1} + \frac{2\rho_nF_n}{\Delta t^2}\,w_n\bigg|_{t+1}$$

$$= -\rho_nF_na_{z,n}(O)\,|_{t+1} + \rho_nF_nx_n\varepsilon_{y,n}(O)\,|_{t+1} - \rho_nF_nx_n\omega_{z,n}(O)\,\omega_{x,n}(O)\,|_{t+1}$$

$$-\frac{\rho_nF_n}{\Delta t^2}\,[-5\omega_n|_t + 4\omega_n|_{t-1} - \omega_n|_{t-2}],$$

$$G_nJ_{pn}\frac{\partial^2\gamma_n}{\partial x_n^2}\bigg|_{t+1} = \rho_nJ_{pn}\varepsilon_{x,n}(O)\,|_{t+1}. \tag{25}$$

Similar transformations are executed also on the basis of formulas (24) for the boundary conditions, containing partial derivatives with respect to time t.

The peculiarity of system (25) consists in the fact that the equations composing it are not coupled explicitly and the mutual dependence of the variables included in them comes only through the boundary conditions.

It is considered that the states of the manipulator at the moments of time t, $t - \Delta t$, $t - 2\Delta t$ are known; then in system (25) the functions $u_n(x_n)$, $v_n(x_n)$, $w_n(x_n)$, $\gamma_n(x_n)$ and the values of variables $a_{x,n}(O)$, $a_{y,n}(O)$, $a_{z,n}(O)$, $\varepsilon_{x,n}(O)$, $\varepsilon_{y,n}(O)$, $\varepsilon_{z,n}(O)$ at the moment $t + \Delta t$ turn out to be unknown. The presence of the last ones is associated with the hybrid type of equations (8). For their determination a special modification of the transfer matrix method is used. As the boundary value problem for equations (25) contains not only unknown boundary conditions but also the mentioned unknown linear and angular accelerations, the special designations for the missing initial values of the derivatives of desired functions and for the unknown parameters of accelerations in the right members of equations (25) for the types of control considered are introduced. In the case of the kinematic control of the system they are as follows:

$$C_1 = \mathrm{d}u_n(O)/\mathrm{d}x_n, \qquad C_2 = a_{x,n}(O), \qquad C_3 = \mathrm{d}^2v_n(O)/\mathrm{d}x_n^2, \qquad C_4 = \mathrm{d}^3v_n(O)/\mathrm{d}x_n^3,$$

$$C_5 = a_{y,n}(O), \qquad C_6 = \varepsilon_{z,n}(O), \qquad C_7 = \mathrm{d}^2w_n(O)/\mathrm{d}x_n^2, \qquad C_8 = \mathrm{d}^3w_n(O)/\mathrm{d}x_n^3,$$

$$C_9 = a_{z,n}(O), \qquad C_{10} = \varepsilon_{y,n}(O), \qquad C_{11} = \mathrm{d}\gamma_n(O)/\mathrm{d}x_n, \qquad C_{12} = \varepsilon_{x,n}(O). \tag{26}$$

At dynamic control all the indicated notations are conserved, but the constant C_3 becomes a known quantity, which acquires the value $C_3 = M_n^e/E_nJ_{zn}$. Here and further, the subindexes $t+1$ at the desired variables are omitted.

According to the transfer matrix technique the system (25) solution is represented in the form of a combination of particular solutions multiplied by the unknown constants C_i:

$$u_n(x_n) = u_n^1(x_n)C_1 + u_n^2(x_n)C_2 + u_n^q(x_n),$$

$$v_n(x_n) = v_n^3(x_n)C_3 + v_n^4(x_n)C_4 + v_n^5(x_n)C_5 + v_n^6(x_n)C_6 + v_n^q(x_n),$$

$$w_n(x_n) = w_n^7(x_n)C_7 + w_n^8(x_n)C_8 + w_n^9(x_n)C_9 + w_n^{10}(x_n)C_{10} + w_n^q(x_n),$$

$$\gamma_n(x_n) = \gamma_n^{11}(x_n)C_{11} + \gamma_n^{12}(x_n)C_{12} + \gamma_n^q(x_n). \tag{27}$$

Here $u_n^q(x_n)$, $v_n^q(x_n)$, $w_n^q(x_n)$, $\gamma_n^q(x_n)$ are the particular solutions of the Cauchy problems for equations (25) under homogeneous initial conditions and $a_{x,n}(O) = a_{y,n}(O) = a_{z,n}(O) = 0$, $\varepsilon_{x,n}(O) = \varepsilon_{y,n}(O) = \varepsilon_{z,n}(O) = 0$; $u_n^1(x_n), \dots, \gamma_n^{12}(x_n)$ the particular solutions of the Cauchy problems for equations (25), reduced to such form and under such initial conditions, that for each ith solution only one of the desired variables of group (26), appropriate to the constant C_i, obtains unit value and all the remaining parameters and right members of equations (25) obtain zero values.

Possessing the matrix of the transfer functions $u_n^1(x_n),_{,}\ \gamma_n^q(x_n)$ for each of the N elastic links, it is possible, using the 13 conditions of conjugation (14)–(16), (18), and (21) for each pair of the adjacent links and also the conditions at the origin $x_1 = 0$ of the first link and at the end $x_N = l_N$ of the last one, to count the $13N$ parameters, embracing the $12N$ constants C_i and the N angular accelerations $\ddot{\varphi}_k$, $\ddot{\psi}_l$, $\ddot{\vartheta}_m$ of relative slewing of the contiguous links in the case of dynamic control or the N external moments M_n^e $(n = 1, \dots, N)$ in the case of kinematic control. Note that as at small steps Δt of the integration in time the coefficients of the terms of equations (25), containing the small values Δt^2 in their denominators, become very large, system (25) can turn out to be rigid and contain rapidly increasing functions among its particular solutions. Therefore, the Runge–Kutta fourth-order method together with an orthogonalization procedure is used for this system numerical integration and the mentioned particular solutions construction [13, 15].

On determining the elastic manipulator state at the moment of time $t + \Delta t$, it is possible to pass to the next step of the computing process and to determine the system state at the moment $t + 2\Delta t$. For this purpose firstly on the basis of the found values φ_k, ψ_l, ϑ_m, $\omega_{x,n}(O)$, $\omega_{y,n}(O)$, $\omega_{z,n}(O)$, $\varepsilon_{x,n}(O)$, $\varepsilon_{y,n}(O)$, $\varepsilon_{z,n}(O)$, calculated at every node for the time moments $t + \Delta t$, t, $t - \Delta t$, $t - 2\Delta t$, their values for the moment $t + 2\Delta t$ are computed with the help of the Adams–Bashforth predictor–corrector method [15]. Then at the same step the matrices $B_{n-1,n}$, $\dot{B}_{n-1,n}$ are calculated. Depending on the orientation of a cylindrical hinge axial line it is possible to separate the three cases of these matrices calculation.

Case 1. Let $\varphi_n = \varphi_n(t + 2\Delta t)$, $\dot{\varphi}_n = \dot{\varphi}_n(t + 2\Delta t)$, $\psi_n = \vartheta_n = 0$, $\dot{\psi}_n = \dot{\vartheta}_n = 0$; then

$$B_{n-1,n} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{Vmatrix}, \qquad \dot{B}_{n-1,n} = \begin{Vmatrix} 0 & 0 & 0 \\ 0 & -\sin\varphi\cdot\dot{\varphi} & -\cos\varphi\cdot\dot{\varphi} \\ 0 & \cos\varphi\cdot\dot{\varphi} & -\sin\varphi\cdot\dot{\varphi} \end{Vmatrix}. \tag{28}$$

Case 2. Let $\psi_n = \psi_n(t + 2\Delta t)$, $\dot{\psi}_n = \dot{\psi}_n(t + 2\Delta t)$, $\varphi_n = \vartheta_n = 0$, $\dot{\psi}_n = \dot{\vartheta}_n = 0$; then

$$B_{n-1,n} = \begin{Vmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{Vmatrix}, \qquad \dot{B}_{n-1,n} = \begin{Vmatrix} -\sin\psi\cdot\dot{\psi} & 0 & \cos\psi\cdot\dot{\psi} \\ 0 & 0 & 0 \\ -\cos\psi\cdot\dot{\psi} & 0 & -\sin\psi\cdot\dot{\psi} \end{Vmatrix}. \tag{29}$$

Case 3. Let $\vartheta_n = \vartheta_n(t + 2\Delta t)$, $\dot{\vartheta}_n = \dot{\vartheta}_n(t + 2\Delta t)$, $\varphi_n = \psi_n = 0$, $\dot{\varphi}_n = \dot{\psi}_n = 0$; then

$$B_{n-1,n} = \begin{Vmatrix} \cos\vartheta & -\sin\vartheta & 0 \\ \sin\vartheta & \cos\vartheta & 0 \\ 0 & 0 & 1 \end{Vmatrix}, \qquad \dot{B}_{n-1,n} = \begin{Vmatrix} -\sin\vartheta\cdot\dot{\vartheta} & -\cos\vartheta\cdot\dot{\vartheta} & 0 \\ \cos\vartheta\cdot\dot{\vartheta} & -\sin\vartheta\cdot\dot{\vartheta} & 0 \\ 0 & 0 & 0 \end{Vmatrix}. \tag{30}$$

By employing the matrices $B_{n-1,n}$, the matrices

$$A_n = B_{0,1}\cdot B_{1,2}\cdots B_{n-1,n} \tag{31}$$

are computed.

Thereafter, possessing the system states at the time moments $t - \Delta t$, t, $t + \Delta t$, it is possible by using the scheme described above to determine its state at the moment $t + 2\Delta t$ and so on.

Note that this approach permits one to avoid the discretization of the constitutive (8) and reduced (25) equations in the spatial co-ordinates. Its advantage over the formulations employing mode shapes *ab initio* lies in the fact that the chosen motion modes are not imposed on the elastic system. It is likely that the advantage is more evident for spatial multilink robots with intricate regimes of motions.

6. NUMERICAL SIMULATON AND DISCUSSION OF RESULTS

The approach described was used for the numerical simulation of the dynamics of coplanar controlled motion of an elastic two-link manipulator, established on a freely driven platform of mass $M_p = 5\times10^3$ kg and principal moment of inertia $I_{zP} = 1.083\times10^4$ kg m^2. The manipulator links represent tubular rods of length $l_1 = 6$ m, $l_2 = 3.5$ m accordingly with identical outside diameters $D_1 = D_2 = 1\times10^{-1}$ m and internal ones $d_1 = d_2 = 5\times10^{-2}$ m. The material densities $\rho_1 = \rho_2 = 2{\cdot}7\times10^3$ kg/m^3, the moduli of elasticity $E_1 = E_2 = 6{\cdot}8\times10^{10}$ Pa. The load transferred by the manipulator represents a spherical body of radius $r_L = 0{\cdot}4$ m, mass $M_L = 2103$ kg and principal moment of inertia $I_{zL} = 134{\cdot}592$ kg m^2 for the case of dynamic control and $r_L = 0{\cdot}25$ m, $M_L = 513{\cdot}8$ kg and $I_{zL} = 12{\cdot}845$ kg m^2 for the case of kinematic control.

In the calculations the time step value Δt was varied and based upon the results for practical convergence it was chosen to be $\Delta t = 5\times10^{-2}$ s for dynamic control and $\Delta t = 2{\cdot}5\times10^{-2}$ s for the kinematic control.

It is assumed that the manipulator motion takes place in the plane XOY of the inertial system of reference and at the initial immovable state axes of the co-ordinate system $O_0x_0y_0$

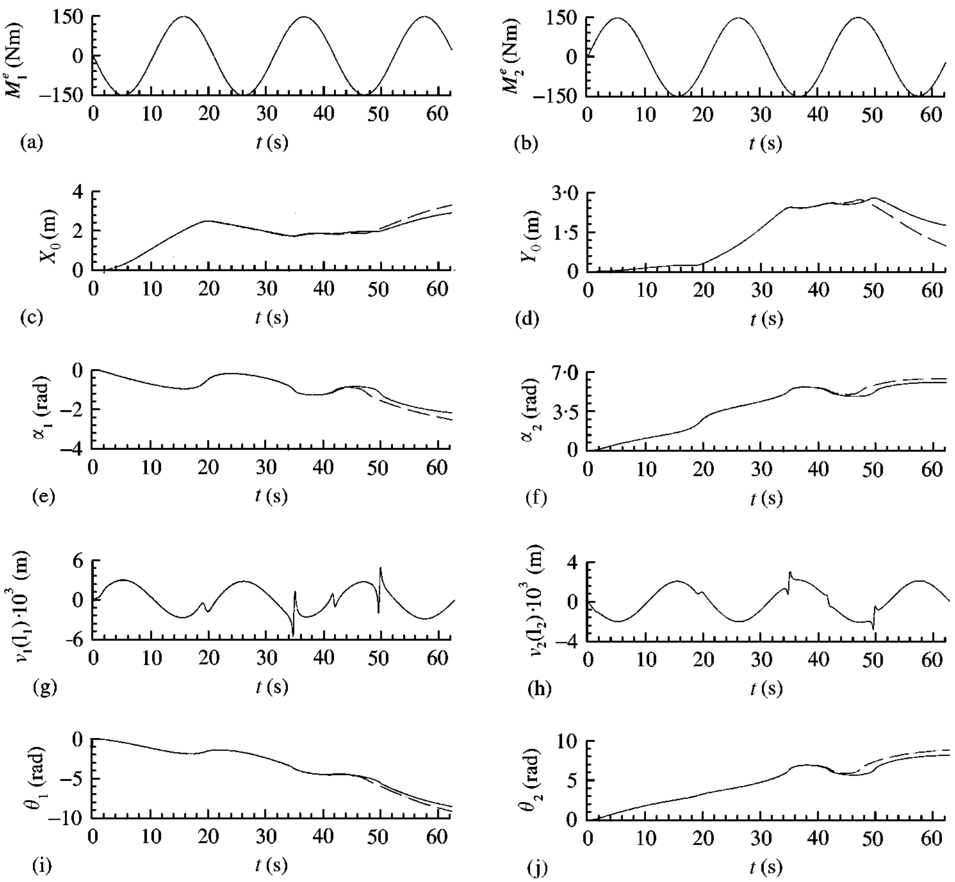

Figure 2. Functions of dynamic behaviour of the space robot manipulator at dynamic control (——, elastic model; – – –, rigid model).

bound with the platform are aligned with the corresponding axes of the system OXY. The platform mass centre coincides with the point O_0, the point O_1 of the hinged connection of the first link beginning at the platform is offset along the axis O_0x_0 by the distance $O_0O_1 = 0{\cdot}5$ m from the point O_0. At the initial state the axes OX, O_0x_0 are collinear and the axes O_1x_1, O_2x_2 are turned in relation to the axis OX through the angle $\pi/2$.

The dynamically controlled motion of the manipulator came about as a consequence of variation of the external moments $M_1^e(t)$, $M_2^e(t)$, changing under the set harmonic laws $M_1^e(t) = -150\sin(0{\cdot}3t)$ N m, $M_2^e(t) = 150\sin(0{\cdot}3t)$ N m [see Figures 2(a) and (b)]. The system motion was studied for three periods $0 \leqslant t \leqslant 63$ s of the controlling action.

The kinematically controlled motion was carried out as a consequence of variation of the angles $\vartheta_1(t)$, $\vartheta_2(t)$ under the set law for 24 s. The diagrams of their second derivatives $\ddot{\vartheta}_1(t)$, $\ddot{\vartheta}_2(t)$ are shown in Figures 3(a) and (b). The maxima of their absolute values equal $|\ddot{\vartheta}_1(t)|_{max} = 0{\cdot}01091\,\mathrm{s}^{-2}$, $|\ddot{\vartheta}_2(t)|_{max} = 0{\cdot}00818\,\mathrm{s}^{-2}$; thus, the total angles of rotations were $\vartheta_1(24) = 60°$, $\vartheta_2(24) = 45°$. The system motion was investigated in the time interval $0 \leqslant t \leqslant 28$ s.

For comparison and verification of the calculations' precision, the statements of the problems of dynamic and kinematic control of the manipulators with geometrically and

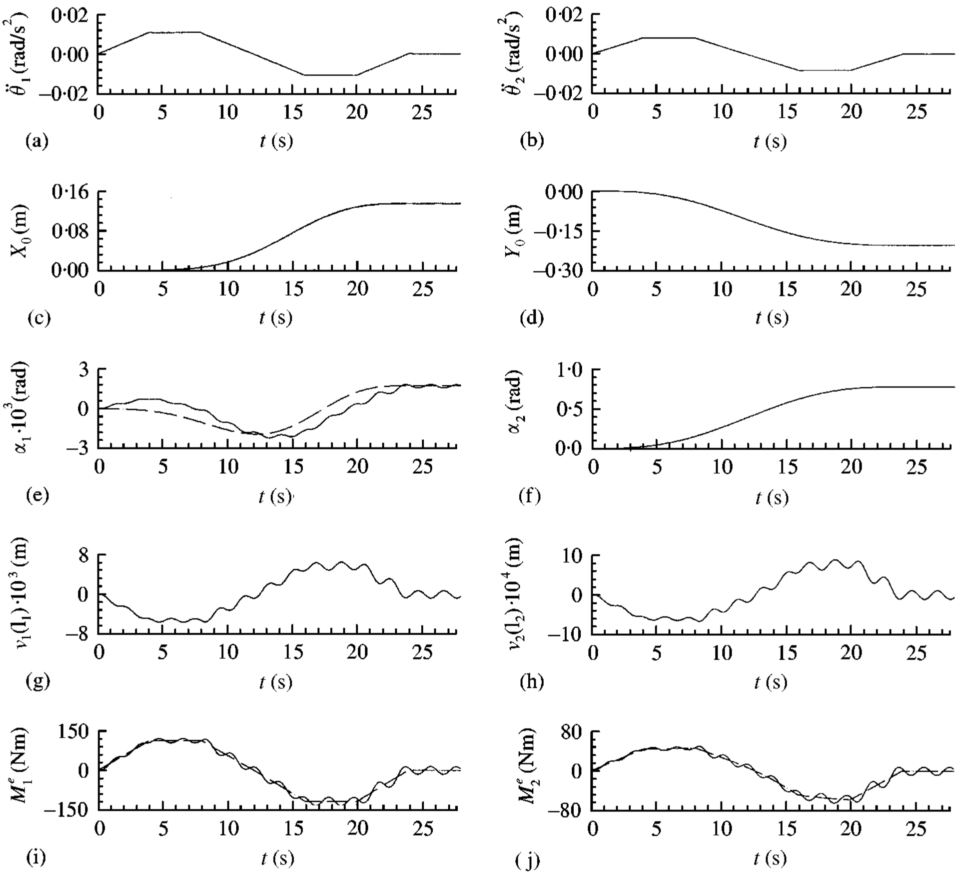

Figure 3. Functions of dynamic behaviour of the space robot manipulator at kinematic control (——, elastic model; – – –, rigid model).

inertially equivalent elastic and absolutely rigid links were considered. In the case of the rigid links the system had five degrees of freedom and was described by the Lagrange 10th-order ordinary differential equations possessing first integrals. In the diagrams of Figures 2 and 3 the appropriate functions for the rigid manipulator motion are shown by the point-like curves, and the corresponding functions for the elastic manipulator are represented by the solid lines. In the segments where the corresponding functions practically coincide they are presented by the thinner continuous lines.

The functions $X_0(t)$, $Y_0(t)$ of the carrying platform centroid C displacements relative to the inertial reference frame OXY are demonstrated in Figures 2(c) and (d) respectively for the case of the dynamic control. As the calculations testify, in the time interval $0 \leqslant t \leqslant 32$ s the functions $X_0(t)$, $Y_0(t)$ values, found for the elastic and rigid statements of the problem, coincide with a precision of three significant digits. The links elastic pliability has rendered an appreciable influence on the motion of the carrying platform mass centre from the time moment $t = 32$ s.

The specified functions are presented in Figures 3(c) and (d) for the case of kinematic control. It is possible to notice that in this situation the effect of the links elastic pliability on the platform motion is insignificant and the corresponding functions, obtained in the study of the elastic and rigid models of the manipulator, practically coincide.

The angles of rotations $\alpha_1(t)$, $\alpha_2(t)$ of the reference frames $O_1x_1y_1z_1$, $O_2x_2y_2z_2$ connected to the links relative to the inertial co-ordinate system OXY at the dynamic control are displayed in Figures 2(e) and (f). The values found of the angles $\alpha_1(t)$, $\alpha_2(t)$ for the elastic and rigid statements coincide with a precision of three significant digits only in the interval $0 \leqslant t \leqslant 32$ s. At subsequent moments of time the distinction between the angles values for the specified statements becomes easily discernible visually [see Figures 2(e) and (f)]. At kinematic control the angles of rotations $\alpha_1(t)$, $\alpha_2(t)$ of the local co-ordinate systems are presented in Figures 3(e) and (f). Contrary to the angles $\vartheta_1(t)$, $\vartheta_2(t)$, which vary under the set laws and do not depend on the elastic pliability of the links, the former ones include also the angles of elastic bendings. Presented in Figures 3(e) and (f) are the dependences of these angles on time; these point to appreciable influence of the rods elastic deformations on their values in the regime of controlled motion ($0 \leqslant t \leqslant 24$ s). It becomes more distinct in the stages of the manipulator acceleration and retardation. Due to this, the graph of the function $\alpha_1(t)$ possesses the most complicated character, where the first link high-frequency elastic oscillations are preserved even after removal of the controlling action, taking place $t > 24$ s.

The above noted tendencies in the behaviour of the system considered are largely conditioned by the process of the manipulator links elastic deformation in time. Consider now Figures 2(g) and (h) and Figures 3(g) and (h), where the functions of variation in time of the elastic displacements $v_1(l_1)$, $v_2(l_2)$ of the links ends are represented accordingly for the cases of dynamic and kinematic controls. Firstly, analyze the functions $v_1(l_1, t)$, $v_2(l_2, t)$, obtained at the dynamic control. It is easy to see that in the interval of time $0 \leqslant t \leqslant 32$ s these functions are smooth and the process of the manipulator links deformation proceeds quasi-statically. However, despite the smoothness of the controlling action, in the subsequent moments of time some separate sections of nearly discontinuous variation of the functions $v_1(l_1, t)$, $v_2(l_2, t)$ in the compressed time scale used take place. These effects appear to be induced by the rapid change of the system mass geometry accompanying folding of its links. In response to the decrease of inertia moments of the system's separate parts they acquire large accelerations. Similar behaviour occurs at a sharp change in the carrying platform trajectory curvature [see Figures 2(c) and (d)]. In the neighbourhood of the specified states the functions maximal values are attained. For example, at the moment of time $t = 35$ s the elastic displacements of the links reach the values $v_1(l_1) = -5{\cdot}24 \times 10^{-3}$ m, $v_2(l_2) = 3{\cdot}12 \times 10^{-3}$ m. At kinematic control the curves $v_1(l_1, t)$, $v_2(l_2, t)$ outlines [see Figures 3(g) and (h)] are constituted by superposition of elastic high-frequency oscillations on the profiles of the curves, which are close to the profiles of the set functions of angular accelerations $\ddot{\vartheta}_1(t)$, $\ddot{\vartheta}_2(t)$, taken with the opposite signs. On completion of the motion control ($t > 24$ s) the elastic displacements $v_1(l_1)$, $v_2(l_2)$ of the links ends become rather small and their influence on the system motion as a whole appears rather inappreciable.

The values of the angles $\vartheta_1(t)$, $\vartheta_2(t)$ calculated during analysis of the manipulator dynamic control are indicated in Figures 2(i) and (j). The graphs of these functions, constructed for the elastic and rigid statements, have essential differences only at $t > 32$ s, when the process of deformation of elastic links of the manipulator has a dynamic nature and exerts an influence on the dynamics of the whole system. In this case the calculations based on the use of a rigid model result in essential overestimation of the angles $\vartheta_1(t)$, $\vartheta_2(t)$ values.

In Figures 3(i) and (j) the diagrams of the functions of the controlling moments $M_1^e(t)$, $M_2^e(t)$ determined by solution of the problem of kinematic control are presented. The profiles of these curves for the rigid manipulator are close to the profiles of the functions of angular accelerations $\ddot{\vartheta}_1(t)$, $\ddot{\vartheta}_2(t)$. The allowance made for the manipulator links flexibility

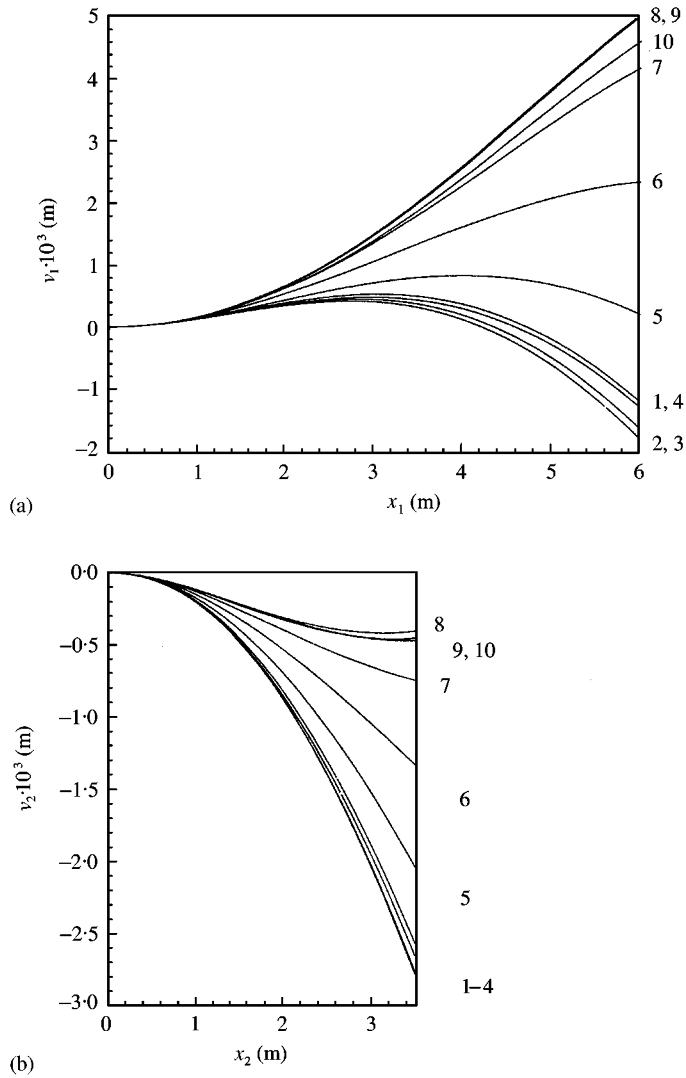

Figure 4. Modes of dynamic bending of the first (a) and second (b) links of the manipulator at dynamic control.

results in appreciable alteration of the functions $M_1^e(t)$, $M_2^e(t)$, which differ from zero at $t > 24$ s as well.

During the motion considered of the manipulational system the incessant evolution of modes of the links axial lines $v_1(x_1)$, $v_2(x_2)$ takes place.

In Figure 4 the modes 1–10 of dynamic bending, the first (a) and second (b) links at the moments of time $t_1 = 49{\cdot}5$ s, $t_2 = 49{\cdot}55$ s, ..., $t_{10} = 49.95$ s, obtained in the case of the system dynamic control, are given. For the case of kinematic control, the modes 1–10 are shown in Figures 5(a) and (b) accordingly at the moments of time $t_1 = 12{\cdot}425$ s, $t_2 = 12{\cdot}45$ s, ..., $t_{10} = 12{\cdot}525$ s. Their analysis testifies that they represent a superposition of at least the first two partial modes of free vibrations.

Thus, the comparison of the results of the cosmic manipulational system study obtained on the basis of its rigid and elastic models, allows one to make the conclusion that the elastic pliability of links has an essential influence on the calculated dynamic characteristics of the

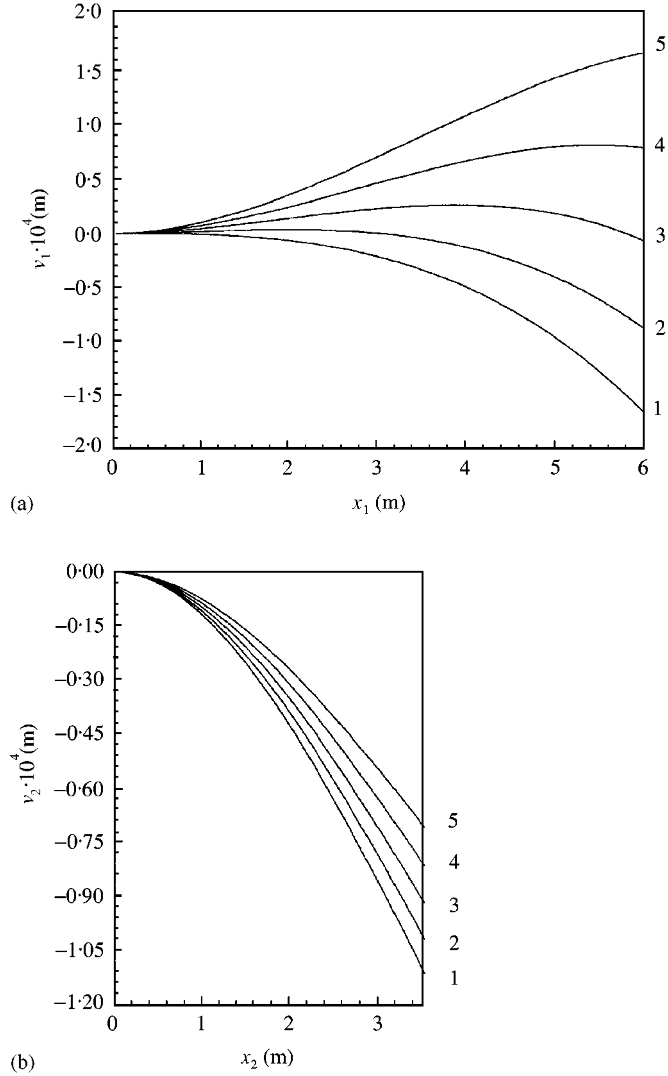

Figure 5. Modes of dynamic bending of the first (a) and second (b) links of the manipulator at kinematic control.

system as a whole and on the motion of the useful load displaced by it. The allowance made for the distributed properties of elasticity and inertia of the links permits one not only to describe more precisely the dynamics of real-space robots (in particular, the angles of their links slewing), but also to give a quantitative estimation of amplitudes of elastic oscillations of their actuators. Hence, even at smooth controlling actions it is expedient to investigate the dynamics of such systems on the basis of the mathematical model of an elastic continuum system.

In conclusion it should be emphasized, that for the selected ways of system control the equations of the manipulator motion, both with flexible and with absolutely rigid links, possess cyclic integrals. Therefore, the cyclic variables determining the co-ordinates of the manipulator centroid, should conserve their values, and the angular positions of all the links and terminal bodies after termination of the control process and the angular motion slowdown should coincide for both types of the manipulators with the precision of small displacements induced by the relative elastic oscillations.

In the numerical calculations these integrals can be employed for exclusion of some desired variables and reduction of the d.o.f. number, though in this case the constituent equations become much more cumbersome. Therefore, in our calculations the cyclic co-ordinates were not excluded and were used for verification of the computations accuracy. In all the cases the first integrals were satisfied with high precision.

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