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LETTERS TO THE EDITOR

HARMONIC WAVE PROPAGATION IN AN INFINITE ROTATING COMPOSITE TIMOSHENKO SHAFT

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1. INTRODUCTION

In Kim *et al.* [1], a theory was presented for the dynamics of a rotating, tapered, filament-wound composite, Timoshenko shaft. The work was motivated by the development of a light weight, stiff, extended length endmill (i.e. a part between the tool holder and the cutter) or boring bar for metal cutting operations in an effort to attain high-speed operation free of various types of instabilities (flutter, parametric resonance, regenerative effects). The underlying theory is quite complicated and involves several sets of modelling assumptions. This naturally leads to the question of the accuracy of the theory, particularly in the absence of experimental validation. A useful probe of accuracy is to investigate whether the theory supports wave propagation (i.e., is hyperbolic in nature) and if so, are the wave features "reasonable". These issues are investigated in this work for a non-tapered circular shaft.

2. EQUATIONS OF MOTION

Figure 1 shows a steadily rotating, filament-wound composite shaft, the equations of motion for which were given in Kim *et al.* [1]. In the non-tapered case and in the absence of cutting forces, these equations are

$$m\ddot{u}_x - \kappa K_V^s(u_x' - \psi_x)' + K_{VM}^o\psi_y'' = 0, \qquad m\ddot{u}_y - \kappa K_V^s(u_y' + \psi_y)' + K_{VM}^o\psi_x'' = 0, \quad (1,2)$$

$$I_y\ddot{\psi}_x - I_z\Omega\dot{\psi}_y - K_M\psi_x'' + \kappa K_{MV}^o(u_y' + \psi_y)' - \kappa K_V^s(u_x' - \psi_x) + K_{VM}^o\psi_y' = 0, \qquad (3)$$

$$I_x\ddot{\psi}_y + I_z\Omega\dot{\psi}_x - K_M\psi_y'' + \kappa K_{MV}^o(u_x' - \psi_x)' + \kappa K_V^s(u_y' + \psi_y) - K_{VM}^o\psi_x' = 0, \qquad (4)$$

$$m\ddot{u}_z - K_P u_z'' - K_{PT}\phi'' = 0, \qquad I_z\ddot{\phi} - K_T\phi'' - K_{TP}u_z'' = 0. \qquad (5,6)$$

Here xyz denote an inertial set of reference axes (z is along the shaft axis), the shaft has been taken to be rotating at a uniform rate Ω about the inertial z-axis, u_x, u_y, u_z are displacements of the neutral axis in the x, y, z directions, respectively, ψ_x and ψ_y are bending rotation angles about the y- and x-axis, respectively, and ϕ is an angle of twist. m is the mass per unit length, the I's are mass moments of inertia per unit length, and κ is a Timoshenko shear coefficient. The prime and overdot denote differentiation w.r.t. z and time (t), respectively.

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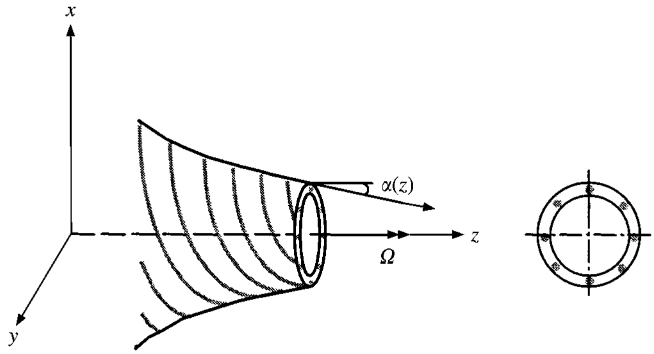

Figure 1. Single lamina of a rotating, tapered, filament-wound composite shaft.

Expressions for K_V^s, K_{VM}^o, K_M, K_{MV}^o, K_P, K_{PT}, K_T, and K_{TP} can be found in Kim *et al.* [1]. Of importance in this work are K_{VM}^o and K_{MV}^o which couple shear and bending (in the other direction).

Note that the bending motions are coupled through gyroscopic and material effects, but are not coupled to extensional and torsional motions.

Consider first the bending motions. Seeking plane wave solutions one takes

$$u_x = B_1 e^{i(kz-\omega t)}, \qquad u_y = B_2 e^{i(kz-\omega t)}, \tag{7, 8}$$

$$\psi_x = B_3 e^{i(kz-\omega t)}, \qquad \psi_y = B_4 e^{i(kz-\omega t)}. \tag{9, 10}$$

Substitution of equations (7)–(10) into equations (1)–(4) leads to the determinant of the coefficients of B_1, B_2, B_3, B_4 being equal to zero and an equation of the form

$$det = c_8(K)S^8 + c_7(K)S^7 + c_6(K)S^6 + c_5(K)S^5 + c_4(K)S^4 + c_3(K)S^3 + c_2(K)S^2 + c_1(K)S + c_0(K) = 0. \tag{11}$$

Here S and K are dimensionless frequencies and wave numbers, respectively, given by

$$S = a\sqrt{\frac{K_V^s}{m}}\,\omega, \qquad K = ak, \tag{12, 13}$$

where a is the shaft radius. The coefficients c_0–c_8 are readily found using Maple [2]. Once they are known, equation (11) can be solved to obtain frequency–wave number plots. The group velocity $c_g = d\omega/dk$, or in dimensionless form $c_g\Big/\sqrt{\frac{m}{K_V^s}} = dS/dK$ can be obtained by differentiating equation (11), which yields

$$\frac{dS}{dK} = (c_8'S^8 + c_7'S^7 + c_6'S^6 + c_5'S^5 + c_4'S^4 + c_3'S^3 + c_2'S^2 + c_1'S + c_0')/(8c_8S^7 + 7c_7S^6 + 6c_6S^5 + 5c_5S^4 + 4c_4S^3 + 3c_3S^2 + 2c_2S + c_1), \tag{14}$$

where now a prime denotes a derivative w.r.t. K. The derivatives in equation (14) can be found using Maple and then the group velocity as a function of K can be obtained.

Results for the extensional–torsional modes (stemming from equations (5) and (6)) can be obtained in a similar manner.

3. NUMERICAL RESULTS

Consider composite shafts with a hollow steel core. The outer and inner diameters are 40 and 18·4 mm, respectively, and the thickness of the steel core is 4·8 mm. The composite material used in this study is a high modulus graphite/epoxy (density = 1610 kg/m^3, $E_1 = 192$ GPa, $E_2 = 7{\cdot}24$ GPa, $G_{12} = 4{\cdot}07$ GPa, $G_{23} = 3{\cdot}0$ GPa, $\nu_{12} = 0{\cdot}24$) and the

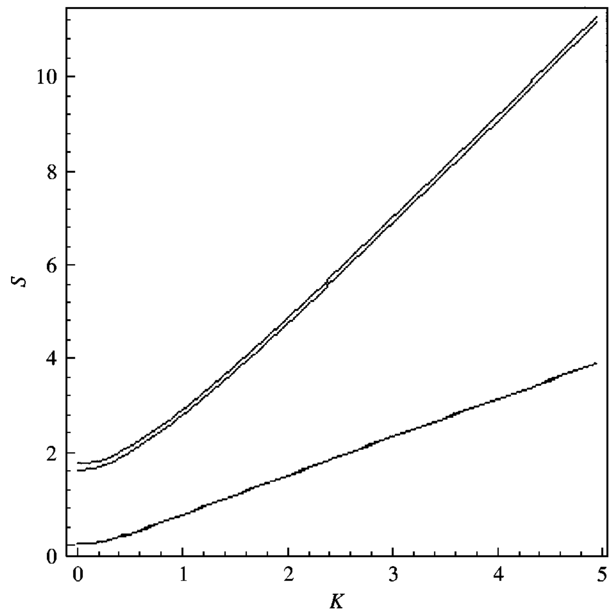

Figure 2. Dimensionless frequency (S) versus wave number (K) for $\theta = \pm 20°$ and $\Omega = 10\,000$ rad/s.

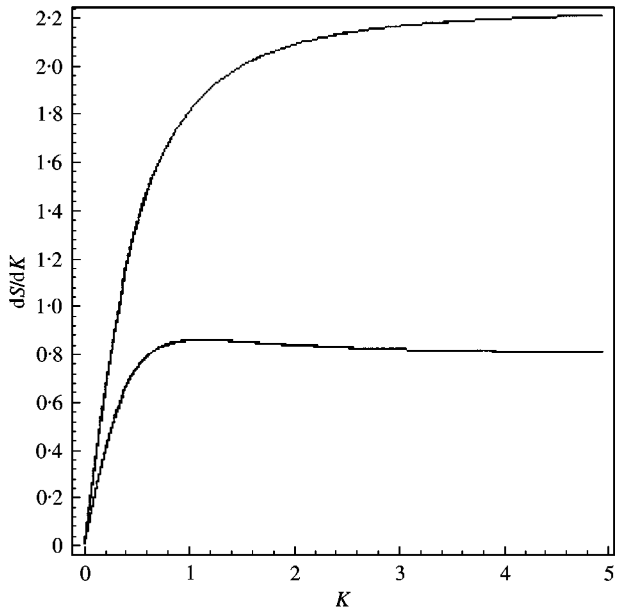

Figure 3. Dimensionless group velocity (dS/dK) versus wave number (K) for $\theta = \pm 20°$ and $\Omega = 10\,000$ rad/s.

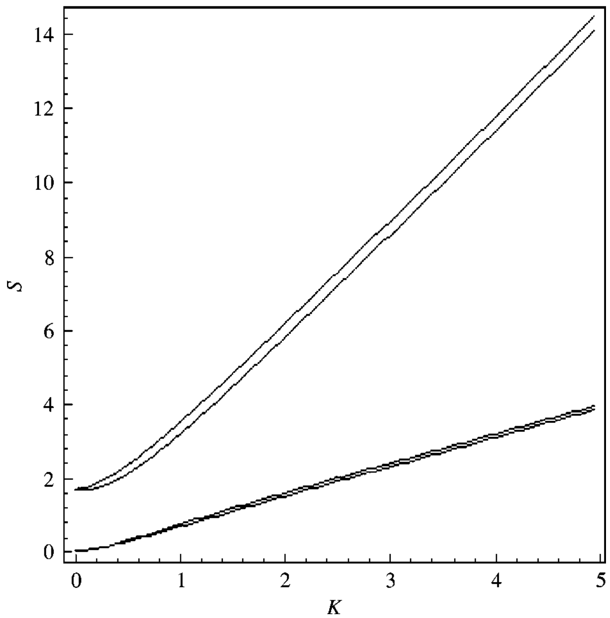

Figure 4. Dimensionless frequency (S) versus wave number (K) for $\theta = 5°$ and $\Omega = 1000$ rad/s.

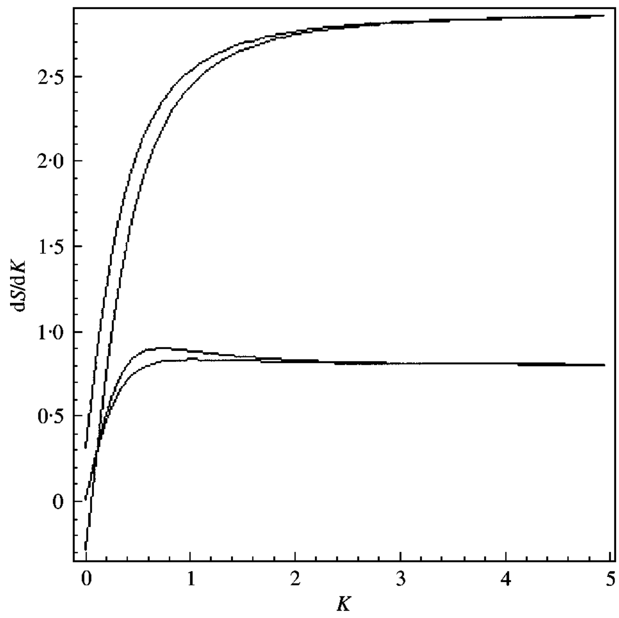

Figure 5. Dimensionless group velocity (dS/dK) versus wave number (K) for $\theta = 5°$ and $\Omega = 1000$ rad/s.

stacking sequence is steel/$(\theta)_n$ where θ is a fiber angle and n is the number of composite layers.

Shown in Figures 2 and 3 are plots of frequency and group velocity versus wave number for $\theta = \pm 20°$, $n = 20$ and $\Omega = 10\,000$ rad/s (quite high). As with the isotropic case (see reference [3]) there are four branches (the upper two stemming from the Timoshenko effects) and the effect of rotation is quite small. For real wave numbers the frequencies and

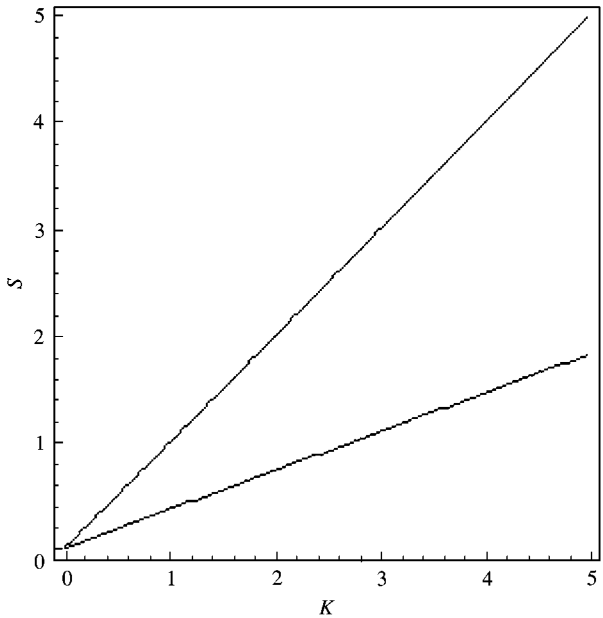

Figure 6. Dimensionless frequency (S) versus wave number (K) for $\theta = 5^\circ$ in the extensional–torsional motions.

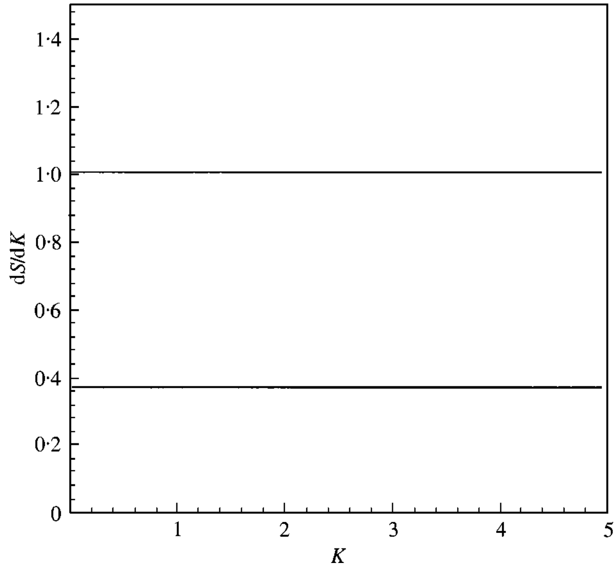

Figure 7. Dimensionless group velocity (dS/dK) versus wave number (K) for $\theta = 5^\circ$ in the extensional–torsional motions.

group velocities are real and behave in a manner similar to the isotropic case (see reference [3]). Similar results have been found for $\theta = \pm 5^\circ$. One can conclude that the theory is "healthy" for shafts of $\pm$ construction. Note that for such configurations the coupling coefficient $K^o_{VM} = K^o_{MV} = -1775$, which is quite small.

Shown in Figures 4 and 5 are plots of frequency and group velocity versus wave number for $\theta = 5^\circ$, $n = 40$ and $\Omega = 1000$ rad/s. The effect of rotation is now quite strong. Again the frequencies and group velocities are real—a good sign—but the Timoshenko branches of the group velocity curves exhibit some questionable features, namely the negative values and the intersections with the lower branches seen for a small value of K. For this shaft $K^o_{VM} = K^o_{MV} = 85\,770$, so that there is strong coupling between shear and bending in the other direction. In such cases, the lesson to be learnt is that the wave features are somewhat suspect and caution must be exercised in using the theory.

Consider now the extensional–torsional motions. Shown in Figures 6 and 7 are plots of frequency and group velocity versus wave number for $\theta = 5^\circ$. In this theory rotation has no effect on these motions and only two branches are seen, and both look reasonable. Note that since c_g is a constant, no dispersion occurs.

In overall summary, it can be concluded that the model given by equations (1)–(6) in most cases predicts reasonable physical results. However, for the $\theta = 5^\circ$ construction some anomalous features are seen for the upper (Timoshenko) branches in the bending motion.

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