*Journal of Sound and Vibration* (2001) **243**(5), 929–944
doi:10.1006/jsvi.2000.3476, available online at http://www.idealibrary.com on IDEAL®

# RESPONSE OF SLIDING STRUCTURES TO BI-DIRECTIONAL EXCITATION

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(*Received* 10 *August* 1998, *and in final form* 21 *November* 2000)

Dynamic response of structures supported on the sliding systems to bi-directional (i.e., two horizontal components) earthquake and harmonic ground motion is investigated. The superstructure is assumed to be rigid and the frictional forces mobilized at the interface of sliding system are assumed to have the ideal Coulomb-friction characteristics. Coupled differential equations of motion of the structure with sliding system in two orthogonal horizontal directions are solved in the incremental form using Newmark's method with iterations. The iterations are required due to dependence of the frictional forces on the response of the system. The response of the system with bi-directional interaction is compared with those without interaction (i.e., two-dimensional idealization in two directions) in order to investigate the effects of bi-directional interaction of frictional forces. These effects are investigated under important parametric variations. The important parameters considered include the isolator properties (i.e., period, damping and friction coefficient) and the characteristics of the harmonic excitation (namely excitation frequency, amplitude ratio and phase difference). It is shown that if the effects of bi-directional interaction of frictional forces are neglected then the sliding base displacements will be underestimated which can be crucial from the design point of view. Further, the bi-directional interaction effects are found to be more severe for the sliding systems without restoring force in comparison with the systems with restoring force.



## 1. INTRODUCTION

Base isolation is an aseismic design approach in which the building is protected from the hazards of earthquake forces by a mechanism which reduces the transmission of horizontal acceleration into the structure. The main concept in base isolation is to reduce the fundamental frequency of structural vibration to a value lower than the predominant energy-containing frequencies of earthquake ground motions. The other purpose of an isolation system is to provide an additional means of energy dissipation and thereby, reduce the transmitted acceleration into the superstructure. Accordingly, by using a base isolation device in the foundation, the structure is essentially uncoupled from the ground motion during earthquakes. Buckle and Mayes [1] and Jangid and Datta [2] have provided excellent reviews of earlier and recent works on base isolation system.

Several base isolation systems including laminated rubber bearing, frictional bearing and roller bearing have been developed to study the effectiveness of base isolation. A significant amount of the recent research in base isolation has focused on the use of frictional elements to concentrate flexibility of the structural system and to add damping to the isolated structure. The most attractive feature of the frictional base isolation system is its effectiveness over a wide range of frequency input. The other advantage of a frictional type system is that it ensures maximum acceleration transmissibility equal to maximum limiting frictional force. The simplest friction-type device is the pure-friction referred to as the P-F system [3, 4]. More advanced devices involve pure-friction elements in combination with a restoring force. The restoring force in the system reduces the base displacements and brings back the system to its original position after an earthquake. Some of the commonly proposed sliding isolation systems with restoring force include the resilient-friction base isolator (R-FBI) system [5], Alexisismon isolation system [6], the friction pendulum system (FPS) [7] and the rolling rods [8, 9]. The sliding systems perform very well under a variety of severe earthquake loading and are quite effective in reducing the large levels of the superstructure's acceleration without inducing large base displacements [5]. Jangid and Datta [10] investigated that the sliding systems are less sensitive to the effects of torsional coupling in asymmetric base-isolated structures. A comparative study of base isolation systems has shown that the response of a sliding system does not vary with the frequency content of earthquake ground motion [11–13]. Recently, Jangid [14] has shown that there exists an optimum value of friction coefficient of sliding system for minimum superstructure acceleration under earthquake motion.

Most of the above studies on the sliding isolation systems are based on the two-dimensional (2-D) planar model of the isolated structure subjected to uni-directional excitation. Such a model of the isolated structures ignores the bi-directional interaction effects of the frictional forces mobilized in the isolation system in two horizontal directions. An experimental study by Mokha *et al.* [15] has shown that there exists interaction between the orthogonal components of the frictional forces mobilized at the sliding interface. These effects were further confirmed by Jangid [3] for the pure-friction sliding system. Analysis of the sliding structures without considering the interaction of the frictional forces generally underestimates the sliding base displacements which can be very crucial for the effective design of the sliding systems.

Herein, the response of structures isolated by the sliding system to two horizontal components of harmonic and real earthquake ground motion is investigated. The specific objectives of the present study are summarized as follows: (1) to present a method for dynamic analysis of sliding structures to bi-directional ground motion which incorporates the interaction effects of the frictional forces, (2) to study the effects of bi-directional interaction on the response of the sliding structures (by comparing the response of the system with and without interaction), and (3) to investigate the influence of important parameters on bi-directional interaction effects of frictional forces. The important parameters considered include the isolator properties (i.e., period, damping and friction coefficient) and the characteristics of the harmonic motion such as the excitation frequency, amplitude ratio and phase difference.

## 2. MODEL OF SUPERSTRUCTURE AND THE SLIDING SYSTEM

Figure 1 shows the model of a structure supported on the sliding system. The various assumptions made for the system under consideration are as follows:

1. Superstructure is considered to be symmetric and modelled as a rigid mass. This is based on the fact that when base isolation is used the superstructure behaves roughly as a rigid body [1,2].
2. Frictional forces mobilized in the sliding isolation system have the ideal Coulomb-friction characteristics (i.e., the coefficient of friction remains constant and

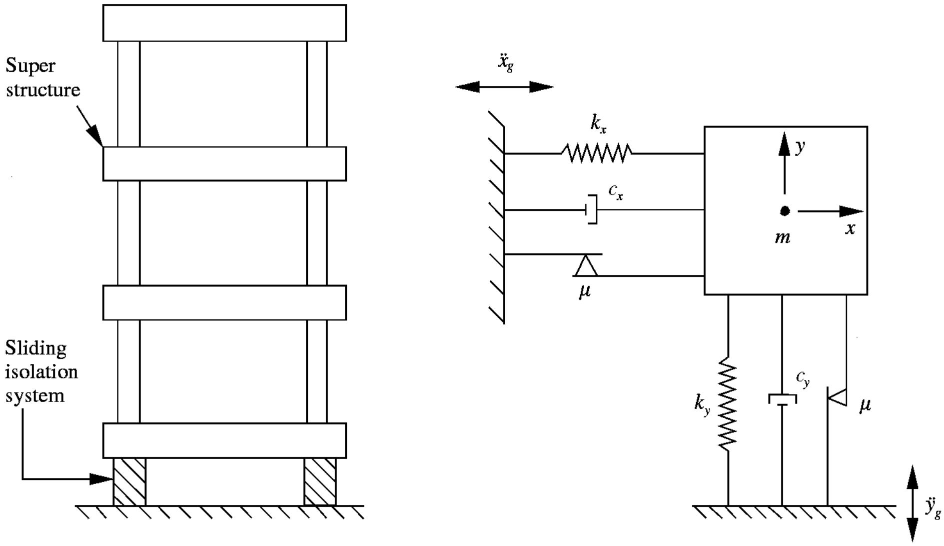


Figure 1. Model of the superstructure and the sliding system.

independent of the pressure and the velocity), although the friction coefficient of various proposed sliding systems is typically dependent on the relative velocity and the interface deformations. However, Fan and Ahmadi [16] has shown that this dependence of the friction coefficient does not have noticeable effects on the peak response of the isolated systems.

3. Restoring force provided by the sliding systems is linear (i.e., proportional to relative displacement). In addition, the isolation systems also provide viscous damping.
4. The sliding system is isotropic i.e., the coefficient of friction in two orthogonal directions of the motion in the horizontal plane is the same.
5. No overturning or tilting takes place in the superstructure during sliding over the isolation system.
6. The ground accelerations act along both horizontal and orthogonal directions (referred to as $x$ and $y$ directions respectively) of the structure.

### 2.1. GOVERNING EQUATIONS OF MOTION

The system considered has essentially two degrees of freedoms (d.o.f.) which are the displacements of the rigid mass ($x$ and $y$) relative to the ground in the $x$ and $y$ directions respectively. The governing equations of motion corresponding to these DOF are expressed in the matrix form as

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix}\begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}\begin{Bmatrix} x \\ y \end{Bmatrix} + \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = -\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{Bmatrix} \ddot{x}_g \\ \ddot{y}_g \end{Bmatrix}, \tag{1}$$

where $m$ is the mass of the rigid superstructure; $c_x$ and $c_y$ are the damping and $k_x$ and $k_y$ are the stiffness of the sliding system in the $x$ and $y$ directions, respectively; $x$ and $y$ are the displacement of the rigid mass relative to the ground in the $x$ and $y$ directions, respectively; $\ddot{x}_g$ and $\ddot{y}_g$ and $F_x$ and $F_y$ are the ground accelerations and the frictional forces in the $x$ and $y$ directions of the system respectively.

The limiting value of the frictional force, $F_s$ to which the sliding system can be subjected in a particular direction is expressed as

$$F_s = \mu m g, \tag{2}$$

where $\mu$ is the friction coefficient of the sliding system, and $g$ is the acceleration due to gravity.

## 2.2. CRITERIA FOR SLIDING AND NON-SLIDING PHASES

In a non-sliding phase ($\ddot{x} = \ddot{y} = 0$ and $\dot{x} = \dot{y} = 0$) the resultant of the frictional forces mobilized at the sliding system interface is less than the limiting frictional force (i.e., $\sqrt{F_x^2 + F_y^2} < F_s$). The system starts sliding ($\ddot{x} \neq \ddot{y} \neq 0$ and $\dot{x} \neq \dot{y} \neq 0$) as soon as the resultant of the frictional forces attains the limiting frictional force. Thus, the sliding of the system takes place if

$$\left(\frac{F_x}{F_s}\right)^2 + \left(\frac{F_y}{F_s}\right)^2 = 1. \tag{3}$$

Note that equation (3) indicates a circular interaction between the frictional forces mobilized at the interface of the sliding system as shown in Figure 2(a). The system remains in the non-sliding phase inside the interaction curve. Further, the governing equations of motion in two orthogonal directions of the structures supported on the sliding type of isolators are coupled during the sliding phases due to interaction between the frictional forces. However, this interaction effect is ignored if the structural system is modelled as a 2-D system. In such cases the corresponding curve which separates the sliding and non-sliding phases is a square as shown in Figure 2(a) by dashed lines. Further, the system changes to non-sliding phase from the sliding phase whenever the resultant velocity of the rigid mass (i.e., $\sqrt{\dot{x}^2 + \dot{y}^2}$) approaches zero.

Since the frictional forces oppose the motion of the system, the direction of the sliding of the system with respect to the $x$ direction is expressed as

$$\theta = \tan^{-1}\left(\frac{\dot{y}}{\dot{x}}\right), \tag{4}$$

where $\dot{x}$ and $\dot{y}$ are the velocities of the rigid mass relative to the ground in $x$ and $y$ directions, respectively.

## 2.3. SOLUTION OF EQUATIONS OF MOTION

The frictional forces mobilized in the sliding system are non-linear functions of the displacement and velocity of the system in two orthogonal directions. Also, during the sliding phase of motion, the mobilized frictional forces are coupled with each other by the circular interaction curve [refer equation (3)]. As a result, the equations of motion are solved in the incremental form. Newmark's method has been chosen for the solution of governing differential equations, assuming linear variation of acceleration over the small time interval, $\Delta t$. The incremental equations in terms of unknown incremental displacements are expressed as

$$[K_{eff}]\{\Delta z\} = \{P_{eff}\} + \{\Delta F\}, \tag{5}$$

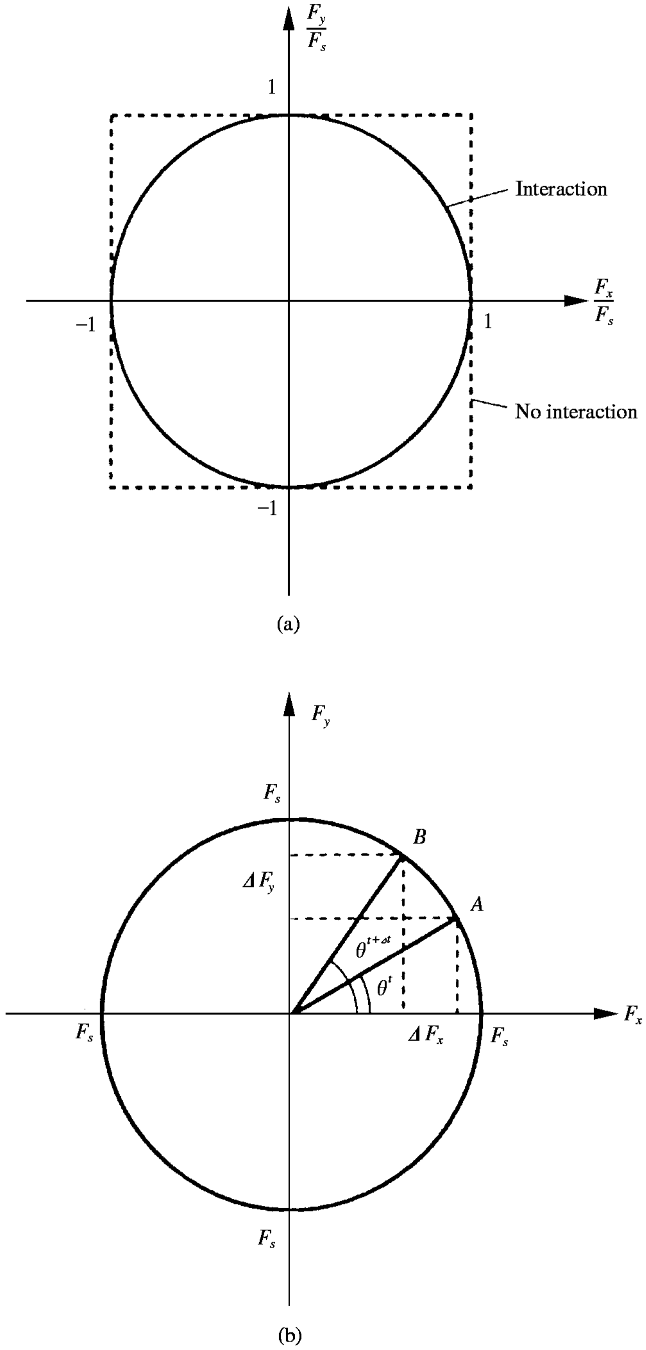


Figure 2. Interaction curve between frictional force and the incremental frictional forces during the sliding phase.

where $[K_{eff}]$ is the effective stiffness matrix; $\{\Delta z\} = \{\Delta x,\ \Delta y\}^{\mathrm{T}}$ is the incremental displacement vector; $\{P_{eff}\}$ is the effective excitation vector; $\{\Delta F\} = \{-\Delta F_x,\ -\Delta F_y\}^{\mathrm{T}}$ is the incremental frictional force vector; $\Delta F_x$ and $\Delta F_y$ are the incremental frictional forces in the $x$ and $y$ directions, respectively; and $T$ denotes the transpose.

In order to determine the incremental frictional forces, consider Figure 2(b). Assume that at time $t$ the frictional forces are at point $A$ on the interaction curve and move to point $B$ at time $t + \Delta t$. Therefore, the incremental frictional forces are expressed as

$$\Delta F_x = F_s \cos(\theta^{t+\Delta t}) - F_x^t, \tag{6a}$$

$$\Delta F_y = F_s \sin(\theta^{t+\Delta t}) - F_y^t. \tag{6b}$$

The superscript denotes the time. Since the frictional forces are opposite to the motion of the system, the angle $\theta^{t+\Delta t}$ is expressed in terms of the relative velocities of the system at time $t + \Delta t$ by

$$\theta^{t+\Delta t} = \tan^{-1}\left(\frac{\dot{y}^{t+\Delta t}}{\dot{x}^{t+\Delta t}}\right). \tag{7}$$

Substituting for $\theta^{t+\Delta t}$ in equation (6), the incremental frictional forces are expressed as

$$\Delta F_x = F_s \frac{\dot{x}^{t+\Delta t}}{\sqrt{(\dot{x}^{t+\Delta t})^2 + (\dot{y}^{t+\Delta t})^2}} - F_x^t, \tag{8a}$$

$$\Delta F_y = F_s \frac{\dot{y}^{t+\Delta t}}{\sqrt{(\dot{x}^{t+\Delta t})^2 + (\dot{y}^{t+\Delta t})^2}} - F_y^t, \tag{8b}$$

In order to solve the incremental matrix equation (5), the incremental frictional forces ($\Delta F_x$ and $\Delta F_y$) should be known at any time interval. The incremental frictional forces involve the system velocities at time $t + \Delta t$ [refer equation (8)] which in turn depend on the incremental displacements ($\Delta x$ and $\Delta y$) at the current time step. As a result, an iterative procedure is required to obtain the required incremental solution. The steps of the procedure considered are as follows:

1. Assume $\Delta F_x = \Delta F_y = 0$ for iteration, $j = 1$ in equation (5) and solve for $\Delta x$ and $\Delta y$.
2. Calculate the incremental velocity $\Delta \dot{x}$ and $\Delta \dot{y}$ using $\Delta x$ and $\Delta y$.
3. Calculate the velocities at time $t + \Delta t$ using incremental velocities (i.e., $\dot{x}^{t+\Delta t} = \dot{x}^t + \Delta \dot{x}$ and $\dot{y}^{t+\Delta t} = \dot{y}^t + \Delta \dot{y}$) and compute the revised incremental frictional forces $\Delta F_x$ and $\Delta F_y$ from equation (8).
4. Iterate further, until the following convergence criteria are satisfied for both incremental frictional forces i.e.

$$\frac{|(\Delta F_x)^{j+1}| - |(\Delta F_x)^j|}{|(\Delta F_x)^j|} \leqslant \varepsilon, \tag{9a}$$

$$\frac{|(\Delta F_y)^{j+1}| - |(\Delta F_y)^j|}{|(\Delta F_y)^j|} \leqslant \varepsilon, \tag{9b}$$

where $\varepsilon$ is a small threshold parameter. The superscript to the incremental forces denotes the iteration number.

When the convergence criteria are satisfied, the velocity of the sliding structure at time $t + \Delta t$ is calculated using incremental velocity. In order to avoid the unbalance forces, the acceleration of the system at time $t + \Delta t$ is evaluated directly from the equilibrium of system equation (1). At the end of each time step the phase of the motion of the system should be checked. The response of the sliding structures is quite sensitive to the time interval, $\Delta t$ and initial conditions at the beginning of sliding and non-sliding phases. For the present study, the results are obtained with time interval, $\Delta t = 0{\cdot}0001$ s. The number of iterations in each time step is taken as 10 to determine the incremental frictional forces at the sliding support. Further, the sliding velocity less than $1 \times 10^{-8}$ m/s is assumed to be zero for checking the transition from sliding to non-sliding phase.

## 3. NUMERICAL STUDY

Response of the rigid mass with sliding system to bi-directional harmonic and earthquake ground motion is investigated. The response quantities of interest for a base-isolated structure are the absolute acceleration and the relative displacement ($x$ and $y$). The former is directly proportional to the forces that are exerted in the system due to ground motion. The latter is a measure of displacement between the isolated structure and the ground which is crucial from the design point of view. The above two response quantities can be controlled at the expense each other. However, it is found that the bi-directional interaction effects of the frictional forces decrease the absolute acceleration and increase the sliding base displacement [3, 15]. In the present study, similar effects are observed for the absolute acceleration of the rigid mass. Hence, the effects of bi-directional interaction are mainly investigated on the relative displacement of the sliding system. Further, in order to study the interaction effects, the displacements of the system are expressed in the normalized form defined as

$$\text{Normalized sliding displacement} = \frac{\text{Resultant peak displacement of the system with considering the interaction of frictional forces}}{\text{Corresponding resultant peak displacement of the system without interaction of frictional forces}}. \qquad (10)$$

The normalized sliding displacement is an index of the bi-directional interaction effects of frictional forces and values significantly different from unity imply significant interaction effects. On the other hand, values close to unity justify the 2-D idealization of the system and the interaction of the frictional forces may be ignored.

In the present study, properties of the sliding system (such as stiffness, damping and friction) are kept the same in both $x$ and $y$ directions. Thus, the sliding isolation system can be completely defined by the three parameters namely the period of isolation $T = 2\pi/\omega$; and $\omega = \sqrt{m/k_x} = \sqrt{m/k_y}$), the damping ratio ($\xi = c_x/2m\omega = c_y/2m\omega$) and the coefficient of friction ($\mu$). The effects of bi-directional interaction are investigated for the three types of commonly used sliding base isolation systems. These systems are the pure-friction (P-F) system, the friction pendulum system (FPS) and the resilient-friction base-isolator (R-FBI). The recommended dynamic properties of these isolation systems are expressed in Table 1. The P-F system isolates the structure by dissipating the seismic energy by simple friction. On the other hand, both R-FBI and FPS systems provide the restoring force (which is modelled by the period $T$). The restoring force in FPS and R-FBI systems is provided by the gravity action and inside rubber core respectively. The rubber core of R-FBI system also adds viscous damping which is modelled by $\xi$.

### 3.1. RESPONSE TO HARMONIC EXCITATION

Response of a non-linear system to different harmonic frequencies gives considerable insight into the dynamic characteristics of the system which may be useful in interpreting

TABLE 1

*Dynamic properties of the three types of isolation systems*

| S.N. | Sliding system | $T$ (s) | $\xi$ | $\mu$ |
|---|---|---|---|---|
| 1 | P-F | — | — | 0·1 |
| 2 | FPS | 2 | — | 0·1 |
| 3 | R-FBI | 4 | 0·1 | 0·04 |

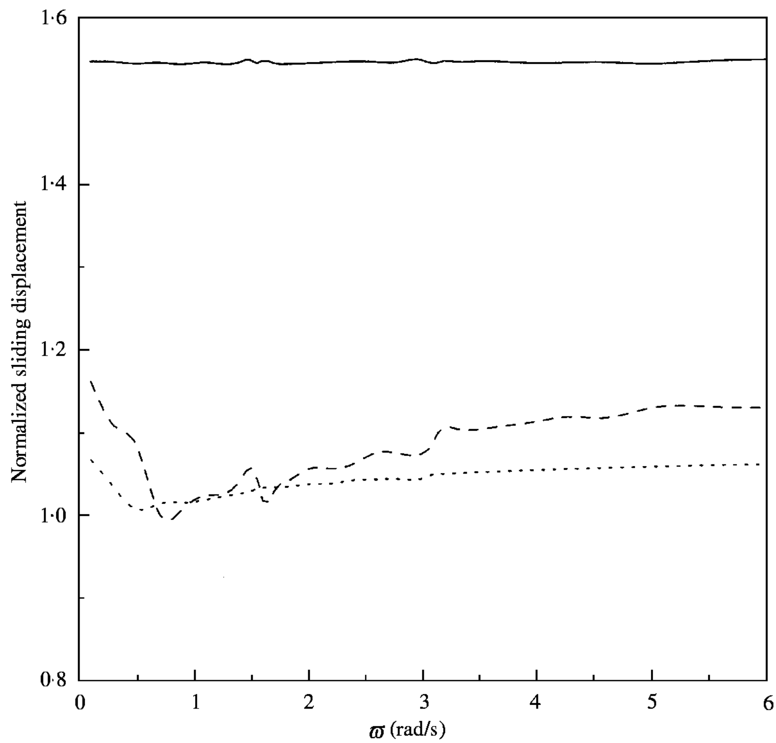


Figure 3. Plot of the normalized resultant sliding displacement of three isolation system against the harmonic excitation frequency ($A_y/A_x$ = 0·85 and $\phi$ = 30°): ——, P-F; – – – –, FPS; ·······, R-FBI.

the response to other types of excitation. The harmonic ground accelerations considered in two orthogonal directions are expressed as

$$\ddot{x}_g = A_x \sin(\varpi t), \tag{11a}$$

$$\ddot{y}_g = A_y \sin(\varpi t + \phi), \tag{11b}$$

where $\varpi$ is the frequency of harmonic excitation; $\phi$ is the phase angle; and $A_x$ and $A_y$ are the amplitudes of the harmonic ground acceleration, in the $x$ and $y$ directions respectively.

The amplitude of harmonic excitation in the $x$ direction is kept constant and taken equal to 0·5g. The duration of the harmonic ground motion in both directions is taken as 20 cycles (i.e., $40\pi/\varpi$). The effects of bi-directional interaction of frictional forces are studied for the important parameters of the harmonic excitation such as the frequency of excitation ($\varpi$), the ratio of the amplitudes in two directions ($A_y/A_x$) and the angle of phase difference ($\phi$).

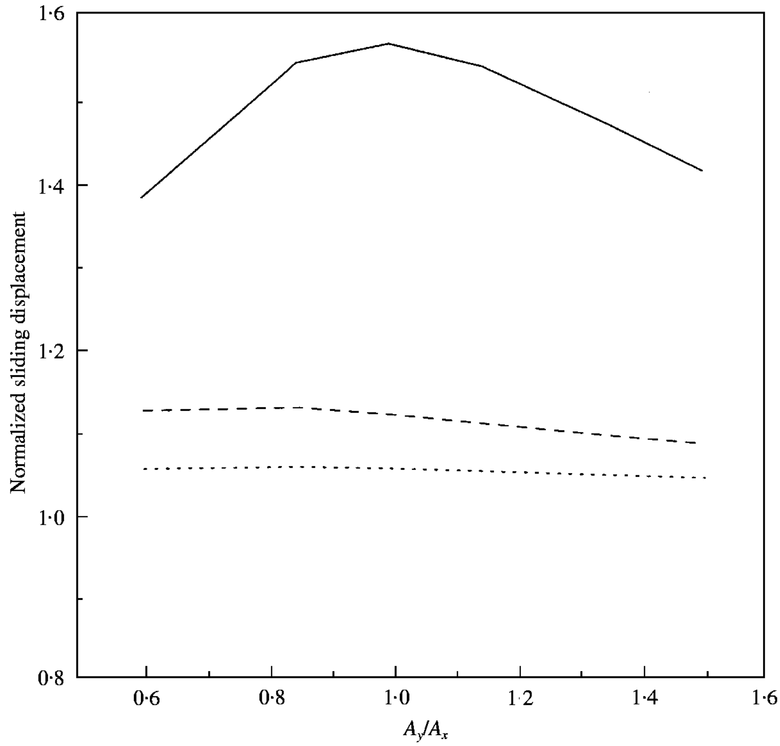


Figure 4. Plot of the normalized resultant sliding displacement of three isolation system against the amplitude ratio, $A_y/A_x$ ($\varpi = 5$ rad/s and $\phi = 30°$): ——, P-F; – – – –, FPS; ·······, R-FBI.

Figure 3 shows the variation of the normalized sliding displacement of three isolation systems against the frequency of harmonic excitation, $\varpi$. The other parameters taken are $A_y/A_x = 0{\cdot}85$ and $\phi = 30°$. The figure indicates that the normalized displacement for all isolation systems is greater than unity for all frequencies of excitation. This indicates that the sliding displacement under bi-directional ground motion considering the effects of interaction of frictional forces is higher in comparison with the same, but without interaction. This is due to fact that when the interaction is taken into consideration the structure starts sliding at a relatively lower value of the frictional forces mobilized in the sliding system [refer to the sliding equation (3)]; as a result, there is more sliding displacement in the isolated system. It is to be noted that for the P-F system the effects of bi-directional interaction are not sensitive to excitation frequency. This is due to the fact that this type of isolation system does not have any unique frequency; as a result, the interaction effects are the same for all excitation frequencies. On the other hand, for the FPS and R-FBI systems the normalized sliding displacement is influenced by the excitation frequency. The normalized displacement is higher for the lower frequencies and it decreases with the increase of excitation frequency attaining the minimum value occurring in the vicinity of resonating frequency ($\omega$). Further, the normalized displacement increases mildly with further increase of excitation frequency. It is interesting to note that the effects of bi-directional interaction are more pronounced for the P-F system in comparison with the FPS and R-FBI systems. This happens due to the presence of restoring force in the latter isolation systems. The restoring force prevents the drift (sudden increase) of sliding displacement which takes place in the P-F system. Thus, if the interaction of the frictional forces of the sliding system is ignored then the sliding displacements will be underestimated

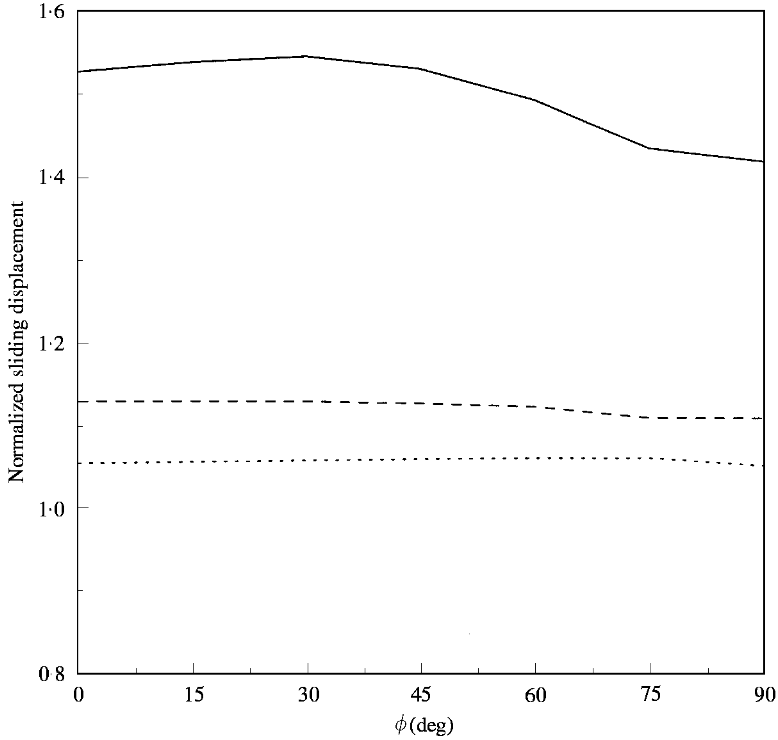


Figure 5. Plot of the normalized resultant sliding displacement of three isolation systems against the phase difference of harmonic ground motion, $\phi(\varpi = 5$ rad/s, $A_y/A_x = 0{\cdot}85)$: ——, P-F; – – – –, FPS; ·······, R-FBI.

which are crucial from the point of view of design of the isolation system. Further, the effects of bi-directional interaction are found to be less significant for the sliding systems having restoring force in comparison with those without restoring force.

In Figure 4, variation of the normalized sliding displacement is plotted against the amplitude ratio of the harmonic excitation, $A_y/A_x$. The response is shown for $\varpi = 5$ rad/s and $\phi = 30°$. The normalized displacement remains insensitive to the ratio $A_y/A_x$ for the FPS and R-FBI systems. However, for the P-F system there are significant effects of the $A_y/A_x$ ratio. The effects are maximum for the ratio $A_y/A_x$ equal to unity. Thus, if the intensity of excitation in two orthogonal directions is the same, the effects of bi-directional interaction are maximum.

The effects of phase difference of harmonic excitation, $\phi$ on the bi-directional interaction of frictional forces are shown in Figure 5. The figure indicates that the normalized response decreases with the increase of phase difference of the harmonic ground motion, being more pronounced for the P-F system in comparison with the FPS and R-FBI systems. Thus, the effects of bi-directional interaction are relatively severe if the excitation in two orthogonal directions is acting in the same phase.

### 3.2. RESPONSE TO EARTHQUAKE EXCITATION

In this section, the normalized sliding displacement and absolute acceleration of the isolated system is investigated for two real earthquake (i.e. El-Centro 1940, and Taft 1952)

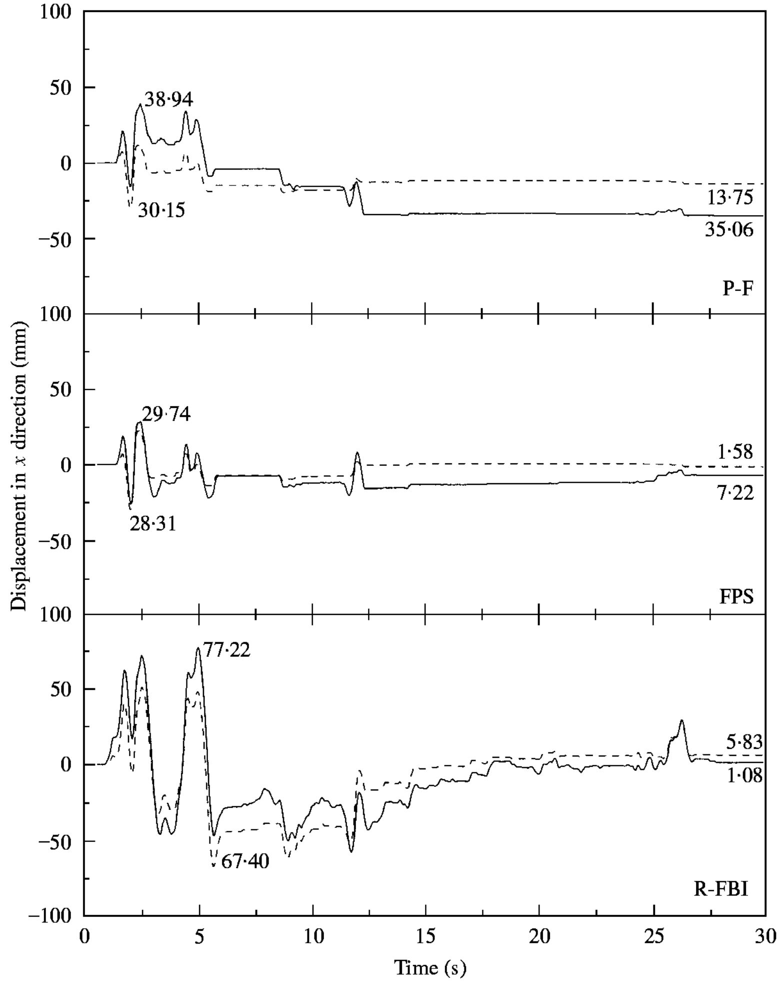


Figure 6. Time variation of relative sliding displacement in the $x$ direction of three isolation systems under El Centro, 1940 earthquake motion: ——, with interaction; – – – – –, no interaction.

ground motions. The duration of these ground motions is taken as 30 s. The components N00E and S69E of El-Centro and Taft earthquake, respectively, are applied in the $x$ direction of the system (with the orthogonal component applied in the $y$ direction).

In Figure 6, time variation of relative sliding displacement of rigid mass in the $x$ direction is plotted for El-Centro, 1940 earthquake excitation. The response of the three types of isolation systems is shown for both considering and ignoring the interaction of friction forces. It is observed that the sliding displacements are relatively more for considering the interaction effects in comparison with that without interaction effects. For the P-F system, the peak sliding displacements are 38·94 and 30·15 mm for with and without interaction effects respectively. This implies that there is significant underestimation of the sliding displacement if the bi-directional interaction effects are ignored and the system is idealized as a 2-D system. In addition, there is also significant difference in the residual base displacement of the system for two cases. Further, the FPS and R-FBI systems also depict the similar effects of the bi-directional interaction of frictional forces under earthquake excitation. However, as observed for sinusoidal excitation, the effects for the FPS and

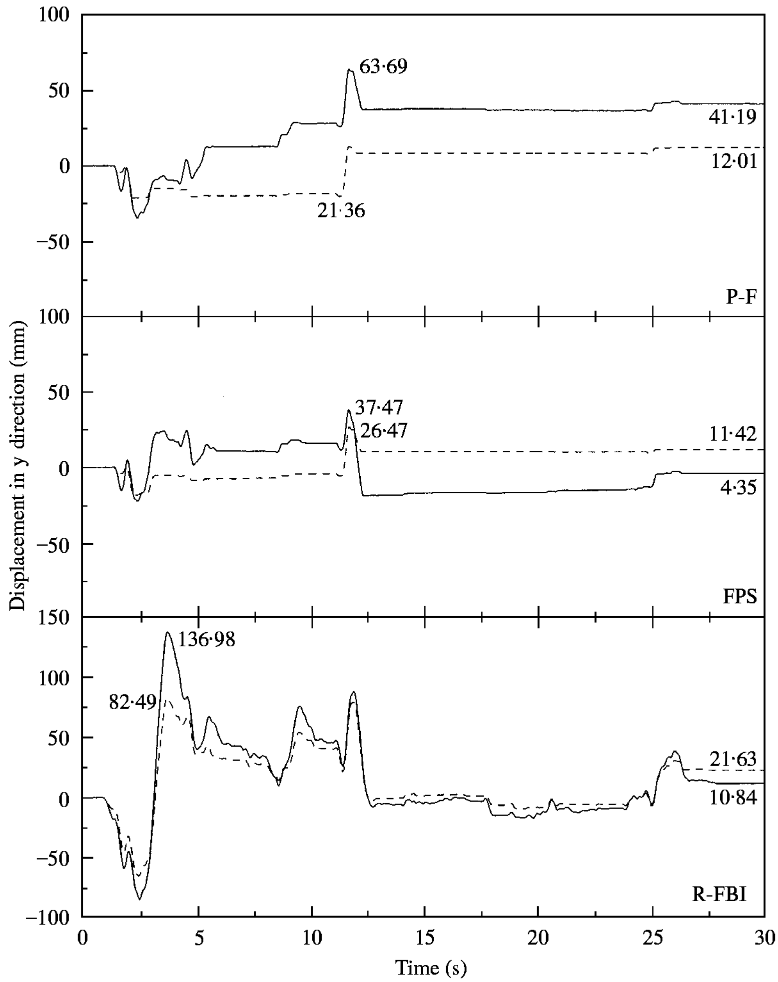


Figure 7. Time variation of relative sliding displacement in the $y$ direction of three-isolation systems under El Centro, 1940 earthquake motion: ——, with interaction; – – – – –, no interaction.

R-FBI systems are less pronounced in comparison with the P-F system. Similar effects of bi-directional interaction on the corresponding sliding displacement in the $y$ direction of the system are depicted in Figure 7.

In Table 2, peak resultant absolute acceleration of the superstructure under different isolation systems is shown. The response is shown for two earthquake motions and compared with the corresponding response of an unprotected system (referred to as non-isolated). It is observed from the table that the peak response of the isolated system is less in comparison with the non-isolated system indicating that the sliding system is effective in reducing the earthquake response of the system. Further, the acceleration of isolated system with considering the interaction effects of frictional forces is less in comparison with the corresponding response without interaction. These effects are more pronounced for the P-F and FPS systems as compared to R-FBI system. This is due to the higher friction coefficient of P-F and FPS systems. Thus, acceleration response of the superstructure decreases due to interaction effects of frictional forces of the isolation system.

So far, the effects of bi-directional interaction of frictional forces have been investigated for the fixed parameters of the isolation system. However, it will be interesting to study the

TABLE 2

*Peak superstructure acceleration response under earthquake excitation*

| | | Peak resultant acceleration ($g$) | | |
|---|---|---|---|---|
| | | | Isolated | |
| Excitation | Sliding system | Non-isolated | Interaction | No interacton |
| El-Centro, 1940 | P-F | 0·363 | 0·1 | 0·141 |
| | FPS | 0·363 | 0·137 | 0·166 |
| | R-FBI | 0·363 | 0·079 | 0·081 |
| Taft, 1952 | P-F | 0·236 | 0·1 | 0·141 |
| | FPS | 0·236 | 0·111 | 0·146 |
| | R-FBI | 0·236 | 0·050 | 0·064 |

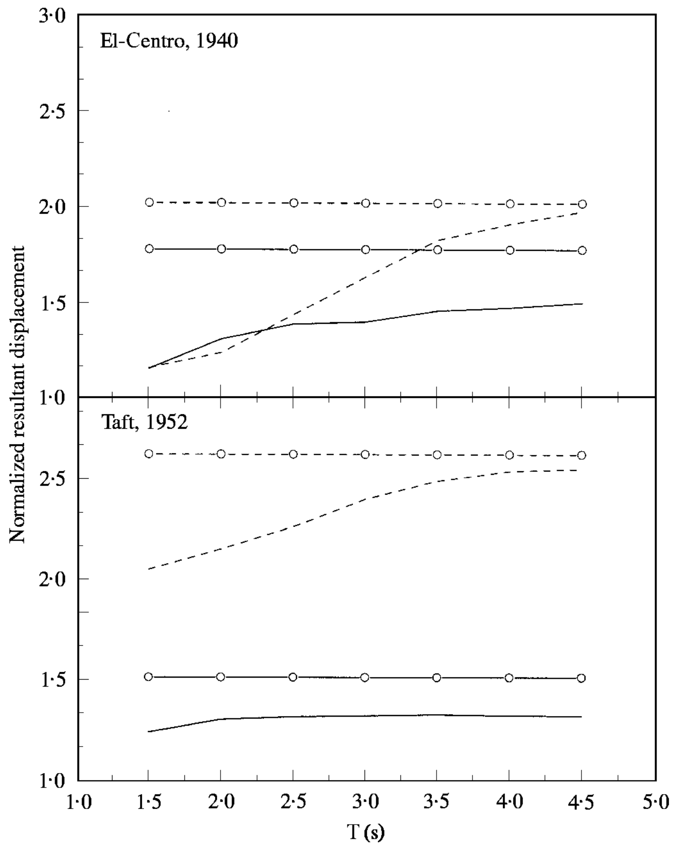


Figure 8. Effects of isolation period on normalized resultant sliding displacement of the system ($\xi = 0{\cdot}1$): ——, $\mu = 0{\cdot}04$; – – – – -, $\mu = 0{\cdot}1$; —○—, $\mu = 0{\cdot}04$ (P-F); – -○- –, $\mu = 0{\cdot}1$ (P-F).

influence of isolator parameters (such as period, damping and friction coefficient) on the bi-directional interaction effects. In Figure 8, variation of the normalized sliding displacement is plotted against the isolator time period, $T$ for $\mu = 0{\cdot}04$ and 0·1. The

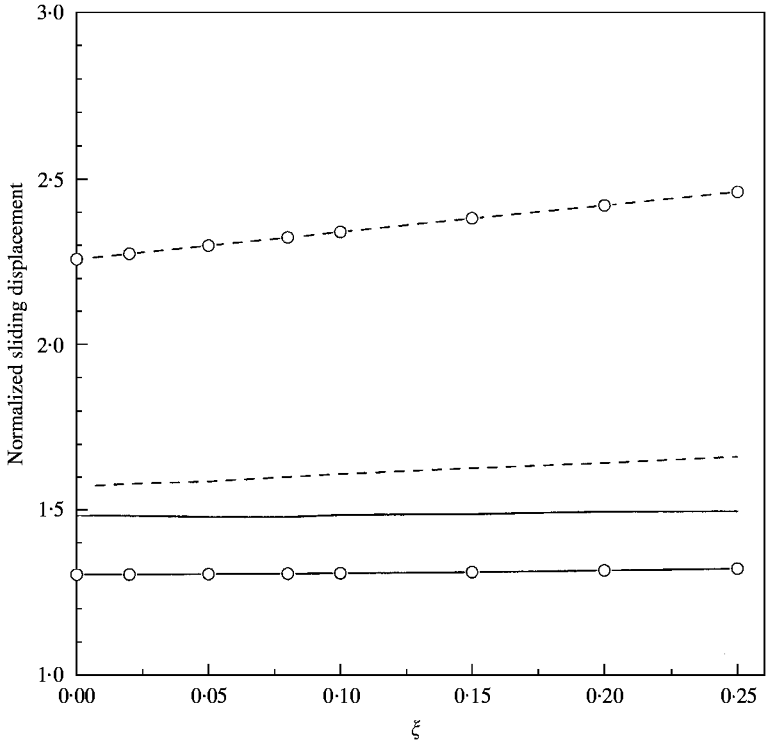


Figure 9. Effects of isolation damping on normalized resultant sliding displacement of the system ($T = 3$ s): ——, $\mu = 0{\cdot}04$ (El-Centro, 1940); ; - - - - -, $\mu = 0{\cdot}1$ (El-Centro, 1940); —○—, $\mu = 0{\cdot}04$ (Taft, 1952); - -○- -, $\mu = 0{\cdot}1$ (Taft, 1952).

damping ratio of the sliding system, $\xi$ is taken as 0.1. The normalized responses are compared with the corresponding responses of the P-F system having the same value of the friction coefficient. The normalized displacement increases with the increase of period of isolation system and is maximum for the corresponding P-F system (for which the $T = \infty$). Thus, the effects of bi-directional interaction are less pronounced for the system with strong restoring force in comparison with that with weak restoring force. Further, the normalized sliding displacements are more for $\mu = 0.1$ as compared to that of $\mu = 0{\cdot}04$. This indicates that the effects of bi-directional interaction are relatively less for the low values of friction coefficient of the sliding system. This is due to the fact that for lower values of friction coefficient, the isolation system remains most of the time in the sliding phase for both cases of excitation (i.e., with and without interaction). As a result, the difference in the sliding displacement is relatively less and hence the normalized displacement is less. Thus, the effects of bi-directional interaction are relatively severe for the isolation systems having higher friction coefficient and weak restoring force as compared to the corresponding isolation system with lower value of friction coefficient and strong restoring force.

Figure 9 shows the variation of normalized sliding displacement against the damping ratio of the isolation system for $\mu = 0{\cdot}04$ and $0{\cdot}1$. The period of the sliding system, $T$ is taken as 3 sec. The figure indicates that the normalized displacement is not much influenced by the damping of the isolation system (except for Taft excitation with $\mu = 0{\cdot}1$). Thus, the variation in the isolator damping does not significantly influence the bi-directional interaction effects of the frictional forces.

## 4. CONCLUSIONS

The response of the rigid superstructure supported on the sliding isolation system to bi-directional harmonic and real earthquake ground motion is investigated. The interaction between the frictional forces of the sliding system in two orthogonal directions is duly considered. In order to study the effects of the bi-directional interaction of frictional forces, the response of the sliding structures with interaction are compared with those without interaction. In addition, the effects of the bi-directional interaction are investigated under parametric variations. From the trends of the results of the present study, the following conclusions may be drawn:

1. If the interaction of the frictional forces of the sliding system is ignored then the sliding displacements which are crucial from the design point of view will be underestimated.
2. The effects of bi-directional interaction are not sensitive to excitation frequency for the P-F system. However, the excitation frequency has influence on the FPS and R-FBI systems.
3. The effects of bi-directional interaction are maximum if the harmonic excitation in two orthogonal directions is identical.
4. The effects of bi-directional interaction are found to be quite significant for the P-F system as compared to the FPS and R-FBI systems. In general, the effects of bi-directional interaction are found to be relatively severe for the isolation systems having higher friction coefficient and weak restoring force as compared to the corresponding isolation system with lower value of friction coefficient and strong restoring force.
5. Variation in the damping of the sliding system does not much influence the bi-directional interaction effects of the frictional forces.

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## APPENDIX A: NOMENCLATURE

| | |
|---|---|
| $A_x$ | amplitude of the harmonic motion in $x$ direction |
| $A_y$ | amplitude of the harmonic motion in $y$ direction |
| $c_x$ | damping of sliding system in $x$ direction |
| $c_y$ | damping of sliding system in $y$ direction |
| $F_x$ | frictional force in $x$ direction |
| $F_y$ | frictional force in $y$ direction |
| $F_s$ | limiting frictional force |
| $g$ | acceleration due to gravity |
| $j$ | iteration number |
| $k_x$ | stiffness of the sliding system in $x$ direction |
| $k_y$ | stiffness of the sliding system in $y$ direction |
| $[K_{eff}]$ | effective stiffness matrix |
| m | mass of the superstructure |
| $\{P_{eff}\}$ | effective excitation vector |
| $t$ | time |
| $T$ | period of isolation |
| $x$ | displacement of rigid mass in $x$ direction |
| $\dot{x}$ | relative velocity of rigid mass in $x$ direction |
| $\ddot{x}$ | relative acceleration of rigid mass in $x$ direction |
| $\ddot{x}_g$ | ground acceleration in $x$ direction |
| $y$ | displacement of the rigid mass in $y$ direction |
| $\dot{y}$ | relative velocity of rigid mass in $y$ direction |
| $\ddot{y}$ | relative acceleration of rigid mass in $y$ direction |
| $\ddot{y}_g$ | ground acceleration in $y$ direction |
| $\mu$ | friction coefficient of the sliding system |
| $\theta$ | direction of the sliding of the rigid mass |
| $\varepsilon$ | small threshold parameter |
| $\xi$ | damping ratio of sliding isolation system |
| $\bar{\omega}$ | frequency of harmonic excitation |
| $\omega$ | isolation frequency |
| $\phi$ | phase angle |
| $\Delta t$ | time interval |
| $\Delta F_x$ | incremental frictional force in $x$ direction |
| $\Delta F_y$ | incremental frictional force in $y$ direction |
| $\Delta x$ | incremental displacement of rigid mass in $x$ direction |
| $\Delta y$ | incremental displacement of rigid mass in $y$ direction |
| $\{\Delta z\}$ | incremental displacement vector |
| $\{\Delta F\}$ | incremental frictional force vector |