Journal of Sound and Vibration (2001) **244**(3), 395–405
doi:10.1006/jsvi.2000.3483, available online at http://www.idealibrary.com on IDEAL®

PIEZOELECTRIC ARRAY SENSING OF VOLUME DISPLACEMENT: A HARDWARE DEMONSTRATION

A. FRANÇOIS, P. DE MAN AND A. PREUMONT

*Active Structures Laboratory, Université Libre de Bruxelles, C.P.*165/42, 50 *Av. Roosevelt*, 1050 *Bruxelles, Belgium. E-mail: apreumon@ulb.ac.be*

(*Received* 14 *February* 2000, *and in final form* 2 *August* 2000)

This paper presents the results from a hardware demonstration of a noise radiation sensor for a baffled rectangular plate. The sensor consists of an array of independent piezoelectric patches connected to a linear combiner. The coefficients of the linear combiner are computed in order to reconstruct (in real time) the volume displacement (or velocity). The robustness of the volume displacement sensor with respect to the location of the disturbance source is investigated.

1. INTRODUCTION

The purpose of this research is to develop a robust feedback control of the sound power radiated by a baffled plate; it is based on the premise that structure-borne sensors are preferable to microphones, to eliminate the time delay due to sound propagation, and that the performance and the robustness of the control system can be enhanced by using a single output signal which is a close representation of the performance metric. The volume velocity is known to be closely related to the sound power radiated by a baffled plate at low frequency. Indeed, by decomposing the surface velocity into a number of velocity distributions which radiate independently (radiation modes), it has been shown [1] that the net volume velocity of the surface is a good estimate of the first radiation mode. Furthermore, at low frequency, the first radiation mode accounts for the majority of the sound power radiation.

An array of accelerometers placed on a regular mesh on the radiating plate could, in principle, be used to reconstruct the volume velocity [2]. Indeed, the time integration of the output is proportional to the volume velocity, including the rigid-body mode. However, acceleration sensors, especially small ones, are less sensitive at low frequency, where the volume velocity is the most relevant. For a plate without rigid-body modes, various sensors have been proposed based on a shaped piezopolymer film bonded to the plate, for sensing the volume displacement or velocity [3, 4], for reconstructing the sound pressure at a given point in the far field [5], or for evaluating the radiated sound power [6]; these sensors can have complex shapes which may make them costly, difficult to manufacture, and sensitive to minor changes in their geometry, bonding conditions or material properties. Alternative strategies for evaluating the total radiated acoustic power from a set of discrete strain information at a regular mesh were developed in reference [7], but these approaches were computationally intensive and were not suited to real-time applications with the existing microprocessor technology.

0022-460X/01/280395 + 11 $35.00/0

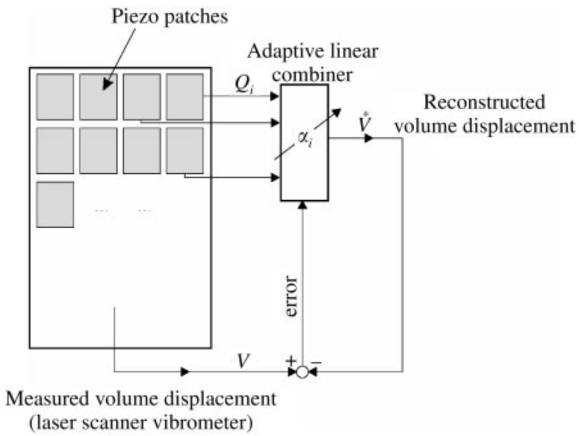

Figure 1. Principle of the volume displacement sensor for a rectangular plate.

A previous study [8] attempted to eliminate the drawbacks associated with the aforementioned methods by reconstructing the volume displacement through a set of discrete strain sensors connected to an adaptive linear combiner (see Figure 1). This paper describes an experimental demonstration of the volume displacement sensor and reexamines the way that the coefficients of the linear combiner can be obtained experimentally.

2. PRINCIPLE OF THE ARRAY SENSOR

The volume displacement sensor uses an array of piezoelectric strain sensors; such sensors cannot be used in the presence of rigid-body modes. The array sensor is based on the following facts.

The electric charges Q_i generated on every piezoelectric strain sensor by the plate deformation is a linear combination of the modal amplitudes z_j,

$$Q_i = \sum_j q_{ij} z_j, \tag{1}$$

where q_{ij} is the electric charge generated on sensor i by a unit amplitude of mode j.

The volume displacement V is a linear combination of the modal amplitudes,

$$V = \sum_j V_j z_j, \tag{2}$$

where V_j is the modal volume displacement of mode j.

At low frequency, equation (2) is dominated by the contribution of the first few modes and therefore only these modal amplitudes need to be reconstructed. This leads to the idea of reconstructing the modal amplitudes of the dominant modes, z_j, $j = 1, \ldots, m$ from the electric charges Q_i, $i = 1, \ldots, n$ produced by a redundant set of piezoelectric strain sensors ($n > m$), leading to

$$z_j = \sum_i a_{ji} Q_i, \tag{3}$$

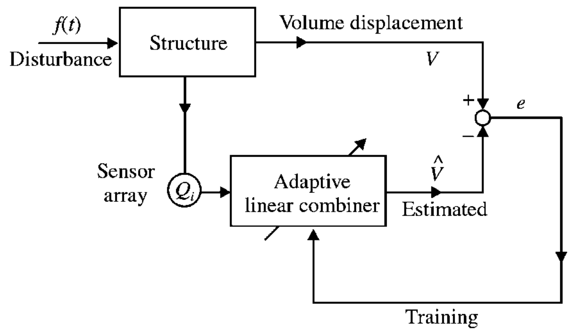

Figure 2. Principle of the adaptive linear combiner sensor.

where the coefficients a_{ji} are unknown at this stage. Combining equation (3) with equation (2) yields

$$V = \sum_j \sum_i V_j a_{ji} Q_i = \sum_i \alpha_i Q_i, \tag{4}$$

where

$$\alpha_i = \sum_j V_j a_{ji}. \tag{5}$$

Equation (3) can be regarded as the pseudo-inverse of equation (1); however, as will be seen later, the sensitivity of the coefficients α_i to the measurement noise depends strongly on the way this pseudo-inverse is defined. The final equation (4) relating the volume displacement to the electric charges has the form of a linear combiner with constant coefficients α_i.

In reference [8], the coefficients α_i were determined analytically for a beam, starting from the orthogonality relations. It was also shown numerically that they can be reconstructed from an adaptive linear combiner (see Figure 2) which minimizes the mean-square error between the estimated volume displacement $\hat{V}$, the output of the linear combiner, and the actual volume displacement V. The latter can be obtained either analytically or numerically, if a model is available, or experimentally with a scanner vibrometer. The α_i coefficients were obtained with a classical LMS algorithm [9]. The numerical results reported in reference [8] showed a good convergence; the effect of the size of the array on the bandwidth of the reconstructed volume velocity was also investigated.

3. EXPERIMENTAL SET-UP

Figure 3 shows a view of the experimental configuration that consists of a 4 mm thick glass plate (0·58 m × 1·28 m) mounted in a standard window chassis that is fixed on a concrete box enclosing a noise disturbance source (loudspeaker). The sensor array consists of 4 × 8 piezoceramic (PZT) patches (13·75 mm × 25 mm) glued on the plate according to a regular mesh. The experimental set-up includes a scanner vibrometer (Polytec). The linear combiner is materialized by programmable multiplying digital to analog converters (MDAC) with 12-bits resolution (see Figure 4). The experimental natural frequencies are given in Table 1.

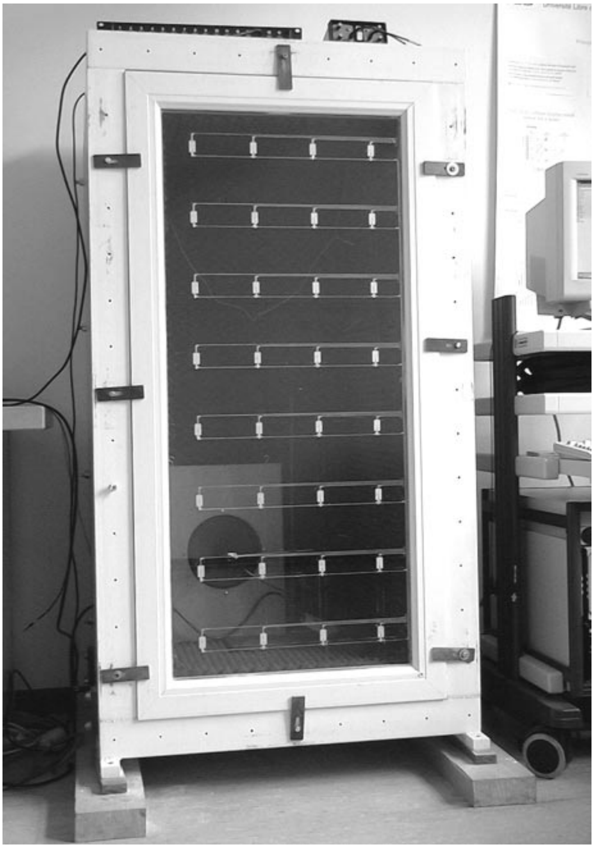

Figure 3. Experimental set-up: glass plate covered by an array of 4×8 piezoelectric patches.

TABLE 1

Natural frequencies of the glass plate

Mode	Frequency (Hz)
(1, 1)	42·5
(1, 2)	55·5
(1, 3)	86·9
(2, 1)	118·0
(1, 4)	128·0
(2, 2)	137·5
(2, 3)	171·3
(1, 5)	176·3
(2, 4)	210·0
(1, 6)	242·0
(5, 2)	263·1

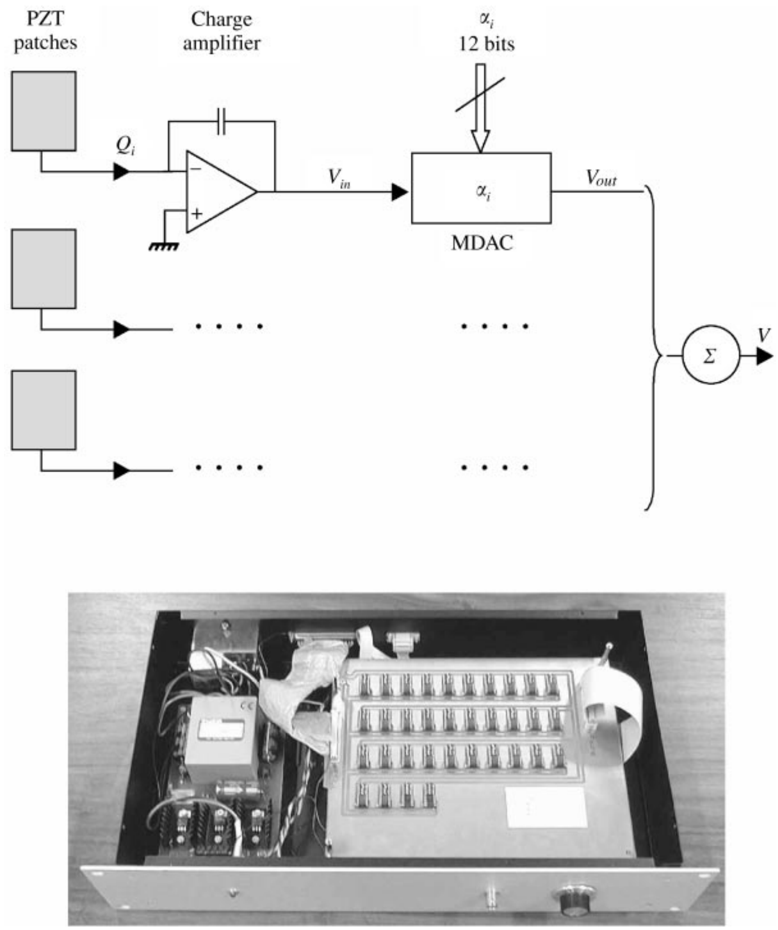

Figure 4. The linear combiner. The α_i coefficients are materialized with multiplying digital to analog converters (MDAC).

4. DETERMINATION OF α_i

The experimental set-up uses a scanner vibrometer which measures the surface velocity (or displacement) at the nodes of a regular mesh on the plate. The scanner does not measure the surface velocity simultaneously at all nodes, but rather one node after the other. As a result, the numerical determination of the coefficients of the linear combiner from experimental data must be performed in the frequency domain. This contrasts with the time domain approach used in reference [8].

For a wideband excitation applied to the disturbance source, the frequency response functions (FRF) $Q_i(\omega)$ between the voltage applied to the disturbance loudspeaker and the electric charges produced by the piezoelectric patches are readily obtained. On the other hand, the scanner vibrometer supplies the FRF $D_k(\omega)$ between the disturbance source and the nodal displacements on the plate. If Δ is the area associated with each node, the FRF $V(\omega)$ between the disturbance source and the volume displacement is given by

$$V(\omega) = \Delta \sum_{k=1}^{N} D_k(\omega), \tag{6}$$

where N is the total number of nodes involved in the scanning process.

From equation (4), one has

$$V(\omega) = \sum_{i=1}^{n} \alpha_i Q_i(\omega), \tag{7}$$

where the coefficients α_i are the unknowns. If this equation is written at a set of l discrete frequencies ω_i ($l > n$) regularly distributed over the frequency band of interest, function equality (7) can be transformed into a redundant system of linear equations,

$$\begin{pmatrix} Q_1(\omega_1) & \dots & Q_n(\omega_1) \\ Q_1(\omega_2) & \dots & Q_n(\omega_2) \\ & \dots & \\ Q_1(\omega_l) & \dots & Q_n(\omega_l) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} V(\omega_1) \\ V(\omega_2) \\ \vdots \\ V(\omega_l) \end{pmatrix} \tag{8}$$

or in matrix form

$$Q\boldsymbol{\alpha} = \mathbf{V}, \tag{9}$$

where the rectangular matrix Q, of dimension (l, n), and the vector $\mathbf{V}$ are complex quantities and the vector $\boldsymbol{\alpha}$ of the linear combiner coefficients is real.

The solution of this redundant system of equations requires some care to eliminate the effect of noise.

To begin with, since Q and $\mathbf{V}$ are complex quantities, the solution of equation (9) is in general complex. However, since the coefficients α_i are supposed to be real, only the real part of the solution is meaningful; the imaginary part is usually much smaller than the real part and its magnitude gives a measure of the quality of the solution. Alternatively, real coefficients α_i can be secured by restricting equation (9) to its real part; the two approaches have been found equivalent.

The coefficients resulting from the use of the pseudo-inverse in the mean square sense,

$$\boldsymbol{\alpha} = Q^{+}\mathbf{V} \tag{10}$$

with $Q^{+} = (Q^{\mathrm{T}}Q)^{-1}Q^{\mathrm{T}}$ are highly irregular and highly dependent on the disturbance source. This difficulty can be overcome by using a singular-value decomposition of Q [10],

$$Q = U_1 \Sigma U_2^{\mathrm{H}}, \tag{11}$$

where U_1 and U_2 are unitary matrices containing the eigenvectors of QQ^{H} and $Q^{\mathrm{H}}Q$, respectively, and Σ is the rectangular matrice of dimension (l, n) with the singular values σ_i on the diagonal (equal to the square root of the eigenvalues of QQ^{H} and $Q^{\mathrm{H}}Q$). If $\mathbf{u}_i$ are the column vectors of U_1 and $\mathbf{v}_i$ are the column vectors of U_2, equation (11) can be written as

$$Q = \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^{\mathrm{H}} \tag{12}$$

and the pseudo-inverse reads

$$Q^{+} = \sum_{i=1}^{n} \frac{1}{\sigma_i} \mathbf{v}_i \mathbf{u}_i^{\mathrm{H}}. \tag{13}$$

This equation shows clearly that, because of the presence of $1/\sigma_i$, the lowest singular values tend to dominate the pseudo-inverse; this is indeed responsible for the high variability of the coefficients resulting from equation (10). The problem can be eliminated by truncating the singular-value expansion, equation (13), and deleting the contribution relative to smaller singular values which are dominated by the noise. Numerical simulations have shown that, in a system without noise, the number of singular values which are significant, that is the rank of the system, is equal to the number of modes which respond significantly in the

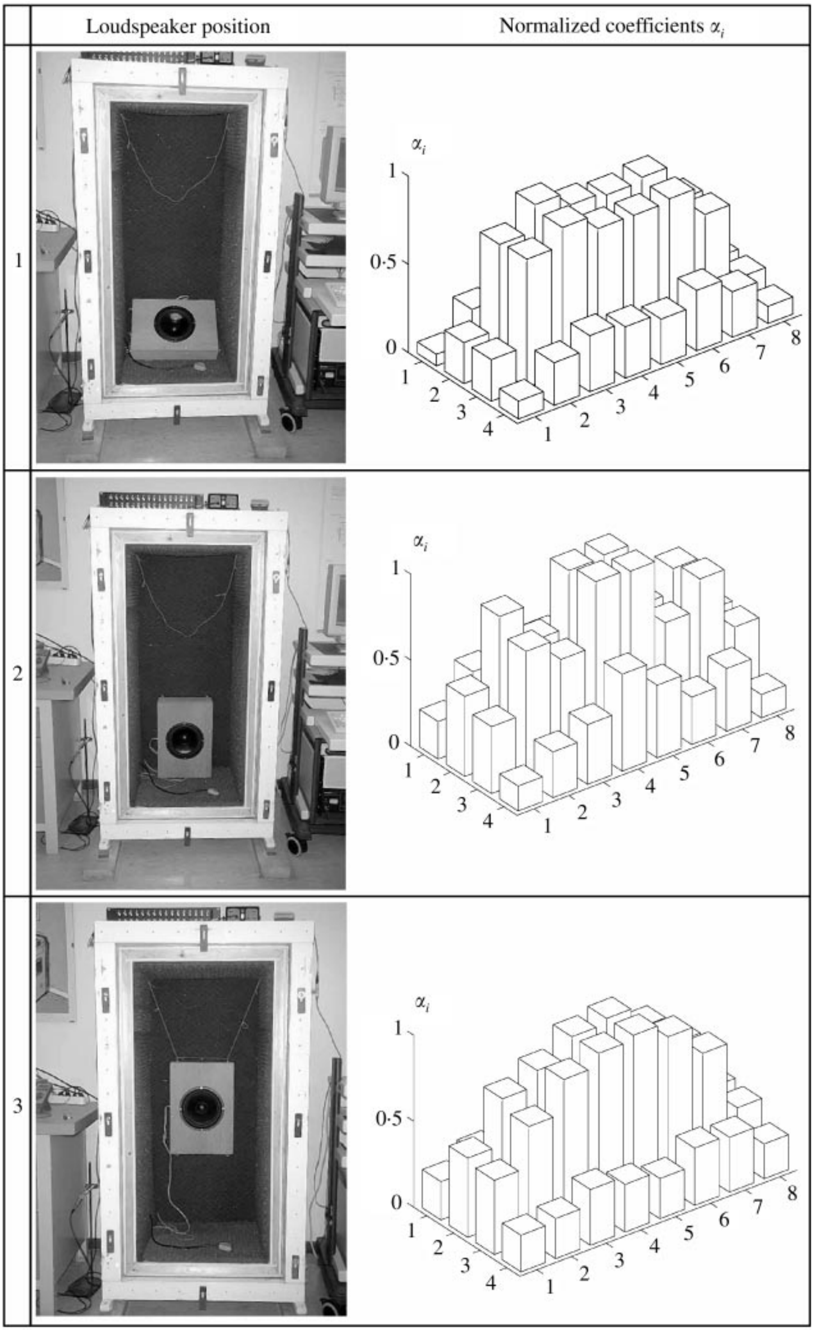

Figure 5. Influence of the location of the disturbance source on the coefficients α_i of the 4×8 sensor array.

frequency band of interest (upon assuming this number is smaller than the number of sensors in the array). In the presence of noise, the selection is slightly more complicated, because the gap in magnitude between significant and insignificant singular values disappears, and some trial and error is needed to identify the optimum number of singular values in the truncated expansion. Similar observations have been made in modal identification (see, e.g., references [11, 12]).

Note that the singular values of Q depend on the excitation, and it is important that all the modes contributing to the volume displacement be properly excited. From our experience, the coefficients α_i and the reconstructed volumetric displacement are not very sensitive to the number l of frequency points used in equation (8) provided they span the whole frequency range where the volume displacement must be reconstructed.

The coefficients α_i vary moderately with the location of the disturbance source; the robustness of the sensor can be improved by including several disturbances in the experiment; this is illustrated below.

5. EXPERIMENTAL RESULTS

The disturbance source consists of a loudspeaker excited by a band-limited white-noise voltage in the frequency band (25–400 Hz). Figure 5 shows the coefficients α_i of the 4×8

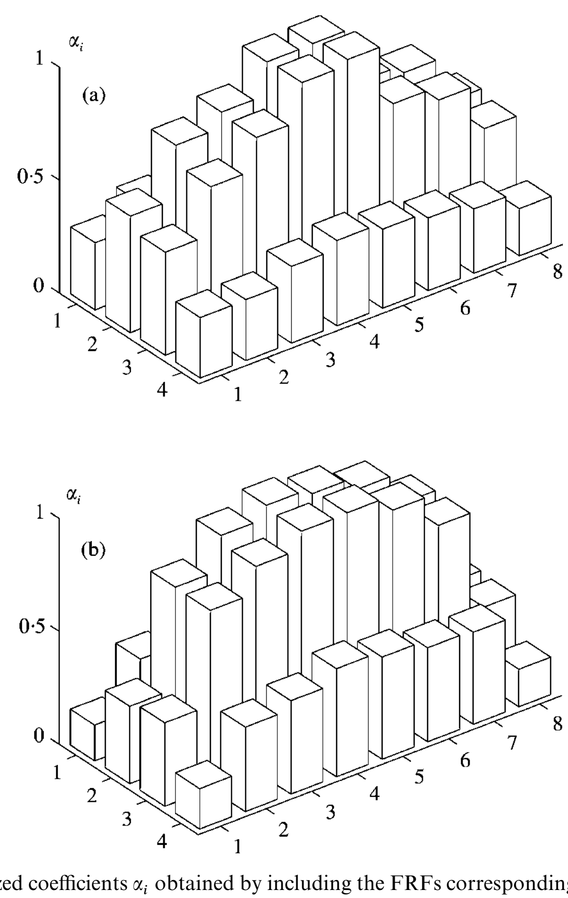

Figure 6. (a) Normalized coefficients α_i obtained by including the FRFs corresponding to the three locations of the disturbance source. (b) Analytical predictions based on a simply supported plate.

array sensor used in this study, obtained by solving equation (9) for three different locations of the disturbance source. The coefficients exhibit some dependence on the source location. Also, the distribution is not fully symmetric because it takes into account the variability of the piezoelectric constant and bonding conditions of the piezo patches. Figure 6(a) shows the coefficients obtained by solving equation (9) after including FRF corresponding to the

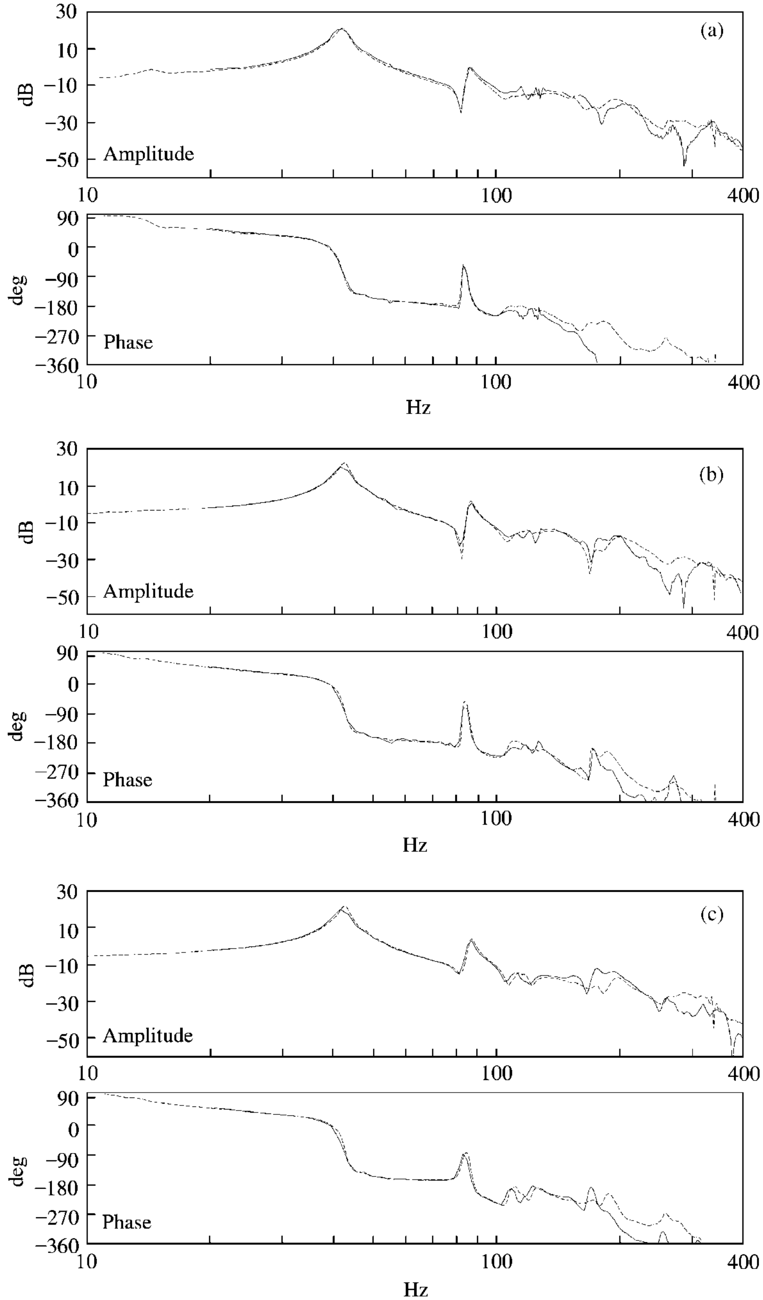

Figure 7. Volume displacement FRF (——) measured and (- - -) reconstructed according to equation (7) with the average coefficients of Figure 6(a) for the three locations of the disturbance source.

various source locations. The distribution looks very similar to the theoretical predictions based on the analytical model of a simply supported plate and shown in Figure 6(b).

Figure 7 compares the volume displacement FRF $V(\omega)$ reconstructed according to equation (7) with the coefficients α_i of Figure 6(a) (dotted line) with the FRF $V(\omega)$ calculated according to equation (6) from FRF $D_k(\omega)$ measured by the scanner vibrometer (full line). The comparison is shown for the three locations of the disturbance source; Figure 7(a)–7(c) correspond, respectively, to the loudspeaker in positions 1–3 in Figure 5. The agreement is good for all source locations.

6. CONCLUSIONS

A piezoelectric array sensor connected to a linear combiner has been developed to reconstruct the volume displacement in real time. The linear combiner is materialized by MDAC components. The linear combiner coefficients have been obtained in the frequency domain from the displacement FRF measured with a laser scanner vibrometer for various locations of the disturbance source; the solution of the redundant system of linear equations is based on a singular-value decomposition. The FRF reconstructed with the volume displacement sensor are in good agreement with those reconstructed from displacement measurements, for various locations of the disturbance source.

Further work is under way to include this sensor in a physically based strategy for robust feedback control of noise radiation from a baffled plate [13].

ACKNOWLEDGMENT

This work was supported by the Inter University Attraction Pole IUAP-IV-24 on Intelligent Mechatronics Systems and by the Ministry of *Region Wallonne* (DGTRE) under grants no. 3512 and no. 3363 (FIRST).

REFERENCES

1. M. E. JOHNSON and S. J. ELLIOTT 1995 *Journal of the Acoustical Society of America* **98**, 2174–2186. Active control of sound radiation using volume velocity cancellation.
2. J. P. MAILLARD and C. R. FULLER 1998 *Journal of the Acoustical Society of America* **103**, 396–400. Comparison of two structural sensing approaches for active structural acoustic control.
3. J. REX and S. J. ELLIOTT 1992 *Proceeding MOVIC-1, Yokohama*, September 339–343. The QWSIS — a new sensor for structural radiation control.
4. F. CHARETTE, A. BERRY and C. GUIGOU 1997 *Journal of the Acoustical Society of America* **103**, 1493–1503. Active control of sound radiation from a plate using a polyvinylidene fluoride volume displacement sensor.
5. R. CLARK, R. BURDISSO and C. FULLER 1993 *Journal of Intelligent Material Systems and Structures* **4**, 354–365. Design approaches for shaping polyvinylidene fluoride sensors in active structural acoustic control (ASAC).
6. S. D. SNYDER, N. TANAKA and Y. KIKUSHIMA 1996 *Journal of Vibration and Acoustics* **118**, 112–121. The use of optimally shaped piezo-electric film sensors in the active control of free field structural radiation. Part 2: feedback control.
7. P. MASSON, A. BERRY and J. NICOLAS 1998 *Journal of the Acoustical Society of America* **102**, 1588–1599. Active structural acoustic control using strain sensing.
8. A. PREUMONT, A. FRANÇOIS and S. DUBRU 1999 *Journal of Vibration and Acoustics* **121**, 446–452. Piezoelectric array sensing for real-time, broad-band sound radiation measurement.
9. B. WIDROW and S. D. STEARNS 1985 *Adaptive Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall.

10. G. STRANG 1988 *Linear Algebra and its Applications*. New York: Harcourt Brace Jovanovich, third edition.
11. B. PEETERS and G. DE ROECK 1999 *Mechanical Systems and Signal Processing* **13**, 855–878. Reference-based stochastic subspace identification for output-only modal analysis.
12. M. SMAIL, M. THOMAS and A. LAKIS 1999 *Mechanical Systems and Signal Processing* **13**, 803–819. Assessment of optimal arma model orders for modal analysis.
13. P. DE MAN, A. FRANÇOIS and A. PREUMONT 2000 *Internoise*-2000, *Nice*, August. A physically based strategy for robust feedback control of noise radiation from a baffle plate.