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The cracked beam theory presented in the paper entitled "Free Vibrations of Beams with a Single Edge Crack" was a part of a previous research effort to develop a predictive model for the vibrational response of damaged structures. The ultimate aim was to use this approach for the on-line monitoring of structural integrity and the non-destructive identification of damage (e.g., location and severity), for structures such as turbines, aircraft structural components, and power systems.

In the paper, an analytical modelling technique was developed for predicting the natural frequencies and mode shapes of beams with cracks as a function of damage, namely crack location and size. Transverse bending vibrations were considered, described by a fourth order partial differential equation. The main idea was to introduce kinematic assumptions into the Bernoulli–Euler beam model in order to mimic the distribution of stress and strain near the crack tip and the change in neutral axis caused by the crack.

The kinematic assumption $u(x, z, t) = (-z + \varphi(x, z))w'(x, t)$, or $u'(x, z, t) = (-z + \varphi(x, z))w''(x, t) + \varphi'(x, z)w'(x, t)$, was indeed what was proposed to account for the shift of the neutral axis near the crack (here a prime denotes a derivative with respect to x). However, it was shown that the term $\varphi'(x, z)w'(x, t)$ contributes in a negligible way to the beam's bending energy, and therefore only the first term of u' [i.e., $u' = (-z + \varphi(x, z))w''(x, t)$], along with $u(x, z, t) = (-z + \varphi(x, z))w''(x, t)$, was substituted into the extended Hu–Washizu variational principle. This led to a much simpler equation of motion [equation (36) in our paper or equation (8) in the Letter of Comments] than that proposed in the Letter of Comments [equation (16)]. In our paper, the equation of motion of the cracked beam was then solved using a Galerkin procedure with 60–100 terms. The results of this study showed that the complex equation of motion derived with the complete expression, $u'(x, z, t) = (-z + \varphi(x, z))w''(x, t) + \varphi'(x, z)w'(x, t)$, provided negligible improvement in the accuracy for the natural frequencies and mode shapes of the cracked beam, compared to the simpler solution obtained with the simplified expression of u'. Finally, the solution procedure presented in our paper produced a self-adjoint (i.e., symmetric) stiffness matrix. This was accomplished by distributing the bending energy symmetrically during the Galerkin procedure.